

# Reference Governor for Constrained Data-Driven Control of Aerospace Systems with Unknown Input-Output Dynamics

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**Abstract**— This work considers the design of a reference governor to satisfy pointwise-in-time output and control constraints in the setting of data-driven control of aerospace systems with unknown input-output dynamics. This unknown dynamics lumps together the combined effects of unknown internal (state space) dynamics, disturbance forces and torques, and unknown internal (mass/inertia) parameters. The unknown dynamics are modeled by a control-affine ultra-local model (ULM) in discrete time. The reference governor is an add-on scheme that enforces the output and control constraints by modifying, when required, a reference command to the system with unknown input-output dynamics. The reference command is determined on the basis of constraint admissible sets constructed in a data-driven setting and exploiting our ULM. A Lyapunov analysis is carried out to ensure that the output of the reference governor-based control system converges to a desired output trajectory that meets the constraints. Numerical simulation results for aircraft longitudinal flight control are reported with this reference governor-based data-driven control scheme, which demonstrate the performance of the controller and the enforcement of the constraints.

## I. INTRODUCTION

Feedback control of autonomous systems with unknown or uncertain dynamics as well as with state and control constraints is a challenging research problem of current interest. In recent years, there has been significant research focused on handling both state and control constraints, which are primarily model-based [1]–[3], [6], [11]–[16]. Examples of state constraints in real-world applications include collision avoidance and pointing direction constraints, while magnitude and rate limitations on actuators are examples of control constraints. The framework proposed here provides an approach to satisfy pointwise-in-time state and control constraints for a system with unknown dynamics.

Many methodologies have been proposed to deal with constraint handling. One approach is to design the controller based on the model predictive control (MPC) framework, which is a well-established industry standard. The applications of MPC, which leverages the idea of receding horizon optimal control, span several fields including but not limited to, unmanned aerial vehicles [1], [2], spacecraft control [3], [4] and automotive applications [5], [7]. Another existing method to handle constraints involves the use of artificial potential functions, which have minima at the desired target and maxima in the set of constraints [6]. This approach is computationally inexpensive, thus making it suitable for

hardware applications. However, the feedback system may settle in unwanted equilibria due to the existence of saddle points or multiple critical points. A variety of ideas have been proposed to resolve this issue [8], [9]. A related method is that of using barrier functions. A barrier function of states of a dynamical system is a function that increases indefinitely in value when the state approaches the boundary of a desired or feasible region from inside this region which is a sub-level set of this function. A control barrier function is a barrier function for a controlled dynamical system, such that the system with control inputs satisfies the barrier function condition [10]. These functions can be used to satisfy output, state, or control input constraints [11], [12]. Control barrier functions in the context of barrier certificates and control Lyapunov functions are studied in [13], while their relations to control Lyapunov functions are studied in [14]. The framework of integral control barrier functions can be used to simultaneously satisfy control input and state exclusion zone constraints [15]. A modification of this approach, where the barrier function was selected based on known initial states and initial control inputs and did not increase to infinite values for finite values of states and control inputs, was proposed in [16].

An alternate and elegant way to handle constraints is the use of reference governors [17]. A reference governor is an add-on scheme that enforces pointwise-in-time state and control constraints by modifying the reference command to a closed-loop system. This is the approach used in this work.

The above-mentioned approaches have primarily been used for systems with well-known dynamics models with little or no uncertainties. However, there has been recent growth and interest in applying data-driven control methods to systems with uncertain or unknown dynamics. Model-free control approach based on intelligent PID is proposed in [18]. The idea is to replace the complex mathematical model for a SISO system with an ultra-local model that describes the unknown input-output dynamics of the system. A generalization of the ultra-local model to MIMO nonlinear systems was formulated in [19]. Recently, a data-enabled predictive control design for an unknown system was proposed in [20] which computes a data-driven optimal control using a receding horizon approach. This approach is analogous to MPC for a linear time-invariant case. The application of model-free control to a quadrotor is considered in [21]. The integration of MPC with an ultra-local model is proposed in [22]. An example of a learning-based reference governor for constraint satisfaction is proposed in [23] and [24]. This scheme modifies the governor parameters over time to ensure

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constraint satisfaction. In the former work, the constraints are satisfied only after the learning phase of the model, while in the latter work, constraints are satisfied during the learning phase of the model as well.

The main contribution of this work is the design of reference governors to enforce pointwise-in-time output and control constraints for data-driven control systems with unknown dynamics. This is carried out in the setting of an ultra-local model (ULM), which models the unknown dynamics as described in [19] and as depicted in Fig. 1. The robust observer that estimates the unknown dynamics guarantees finite-time stable convergence of observer errors. These estimates compensate for the unknowns in the nonlinearly stable tracking control law. A reference governor is then used to modify a given reference output trajectory in order to satisfy pointwise-in-time output and control constraints. The tracking control scheme then ensures stable tracking of this modified reference trajectory. We note that there is synergy between the application of the reference governor and the use of ULM, in that the reference governor slows down reference changes and naturally makes ULM more accurate.

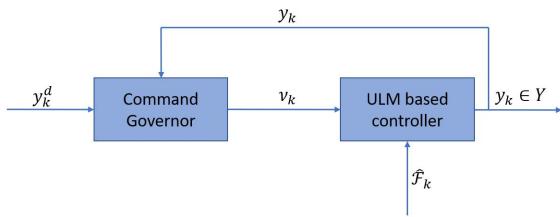


Fig. 1. ULM-based reference governor schematic

The remainder of this paper is organized as follows. Section II outlines preliminary concepts and assumptions on the system model for the data-driven control approach. The design of constraint admissible sets and reference governor for data-driven control is developed in Section III. Section IV reports numerical simulation results that demonstrate satisfaction with the output and control constraints for this data-driven approach. Finally, section V presents a summary of the proposed work and planned future directions.

## II. PROBLEM FORMULATION

In this section, we describe the concept of a “black box” ultra-local model (ULM) that models the unknown input-output dynamics. This ULM is used to design a finite-time stable observer for unknown dynamics.

### A. Ultra-local model for unknown systems

The ultra-local model representing a system with  $n$  inputs and  $m$  outputs with unknown internal dynamics in discrete time is given by:

$$y_{k+v} = \mathcal{F}_k + \mathcal{G}_k u_k, \quad (1)$$

where  $y_k \in \mathbb{R}^n$  is the output vector which denotes the measured output variables,  $\mathcal{F}_k \in \mathbb{R}^{n \times n}$  represents the unknown dynamics which lumps together effects of internal states and

parameters, external disturbance and torque,  $u_k \in \mathbb{R}^m$  is the control input vector and  $\mathcal{G}_k \in \mathbb{R}^{n \times m}$  is a control gain matrix that is part of the controller design. Additionally, the system is sampled in an increasing sequence of time instants  $t_k$ , where  $k \in \mathbb{W} = \{0, 1, 2, \dots\}$  and  $\mathbb{W}$  denotes the index set of whole numbers. The unknown dynamics modeled by the ultra-local model (1) satisfies the following assumptions. These assumptions ensure that a given output trajectory can be tracked by the control inputs.

*Assumption 1:* The control gain matrix  $\mathcal{G}_k$  is a full-rank matrix with the same number of inputs and outputs ( $m = n$ ).

*Assumption 2:* The unknown dynamics  $\mathcal{F}_k$  and the control gain matrix  $\mathcal{G}_k$  are Lipschitz continuous such that,

$$\begin{aligned} \|\mathcal{F}_{k+1} - \mathcal{F}_k\| &\leq L_F \|\xi_{k+1} - \xi_k\|, \\ \|\mathcal{G}_{k+1} - \mathcal{G}_k\| &\leq L_G \|\xi_{k+1} - \xi_k\|, \end{aligned} \quad (2)$$

where  $L_F$ ,  $L_G$  are Lipschitz constants and  $\xi_k = (y_k, y_{k+1}, \dots, y_{k+v-1}, z_k, u_k, t_k)$  where  $z_k$  is a vector of unknown internal states and parameters. Here and in the sequel,  $\|\cdot\|$  denotes the 2-norm unless specified otherwise.

### B. First order model-free finite time stable observer

A discrete-time first-order observer for the unknown dynamics  $\mathcal{F}_k$  in (1) is given in [19] and used in this work. Let  $\hat{\mathcal{F}}_k$  be the estimate of the unknown  $\mathcal{F}_k$ , which is based on previously estimated values of  $\mathcal{F}_i$  obtained from the measured outputs  $y_i$  for  $i \in \{0, \dots, k-1\}$ . Define the estimation error:

$$e_k^{\mathcal{F}} := \hat{\mathcal{F}}_k - \mathcal{F}_k. \quad (3)$$

*Assumption 3:* The unknown dynamics  $\mathcal{F}_k$  and its estimate  $\hat{\mathcal{F}}_k$  are bounded.

Now, we state the following results which are proved in [19].

*Proposition 1:* Consider the estimation error  $e_k^{\mathcal{F}}$  as defined in (3) and let  $r \in ]1, 2[$  and  $\lambda > 0$  be the constants. Let the first order finite difference of the unknown dynamics  $\mathcal{F}_k$ , given by

$$\Delta \mathcal{F}_k := \mathcal{F}_{k+1} - \mathcal{F}_k \quad (4)$$

be bounded as defined in (2). Let the control influence matrix  $\mathcal{G}_k$  be bounded as defined in (2). Let the nonlinear observer be given by

$$\begin{aligned} \hat{\mathcal{F}}_{k+1} &= \mathcal{D}(e_k^{\mathcal{F}}) e_k^{\mathcal{F}} + \mathcal{F}_k, \\ \text{where } \mathcal{D}(e_k^{\mathcal{F}}) &= \frac{((e_k^{\mathcal{F}})^T e_k^{\mathcal{F}})^{1-1/r} - \lambda}{((e_k^{\mathcal{F}})^T e_k^{\mathcal{F}})^{1-1/r} + \lambda}, \end{aligned} \quad (5)$$

and  $\mathcal{F}_k = y_{k+v} - \mathcal{G}_k u_k$  from (1). Then the estimation error  $e_k^{\mathcal{F}}$  converges to a bounded neighborhood of  $0 \in \mathbb{R}^n$  in finite time, where bounds on this neighborhood can be determined from bounds on  $\Delta \mathcal{F}_k$ .

The above result proves the stability of the observer for the unknown dynamics  $\mathcal{F}_k$ . The following result proved in [19], gives the robustness of this observer.

*Proposition 2:* Consider the observer design in (5) for the unknown  $\mathcal{F}_k$  in the ultra-local model (1). Let the first order difference  $\Delta \mathcal{F}_k$  from (4) be bounded according to

$$\|\Delta \mathcal{F}_k\| \leq B^{\mathcal{F}}, \quad (6)$$

where  $B^{\mathcal{F}} \in \mathbb{R}^+$  is a known constant. Then the observer error  $e_k^{\mathcal{F}}$  converges to the neighbourhood given by

$$\mathcal{E} := \{e_k^{\mathcal{F}} \in \mathbb{R}^n : \sigma(e_k^{\mathcal{F}}) \|e_k^{\mathcal{F}}\| \leq B^{\mathcal{F}}\}, \quad (7)$$

for some finite  $k > N$ ,  $N \in \mathbb{W}$ , where

$$\sigma(e_k^{\mathcal{F}}) := 1 + \mathcal{D}(e_k^{\mathcal{F}}). \quad (8)$$

Note that the above result gives ultimate bounds on the estimation error of the unknown (or disturbance) dynamics  $\mathcal{F}_k$ , where the bounds depend on its rate of change  $\Delta\mathcal{F}_k$ .

### III. REFERENCE GOVERNOR

In this section, we enhance the model-free control approach based on ULM with a reference governor to enforce constraints without delay ( $v = 1$ ). The modified reference command, which is subjected to a constraint admissible set, is designed by this reference governor.

#### A. Nominal controller

Consider the ultra-local model defined in (1), with the following nominal controller:

$$\mathcal{G}_k u_k = -\hat{\mathcal{F}}_k + \Phi y_k + \Lambda \nu_k, \quad (9)$$

where  $\nu_k := \nu(t_k)$  is the modified reference command,  $\Phi, \Lambda \in \mathbb{R}^{m \times m}$  are tuning matrices,  $\Phi$  is Schur (all eigenvalues are located in the interior of the unit disk) and the estimate  $\hat{\mathcal{F}}_k$  acts as a feed-forward cancellation term. Substituting the nominal controller in the ultra-local model (1) leads to the following closed-loop model:

$$y_{k+1} = \Phi y_k + \Lambda \nu_k - e_k^{\mathcal{F}} \quad (10)$$

Output constraints, which may arise from state constraints, are described by:

$$y_k \in Y \quad (11)$$

where  $Y \subset \mathbb{R}^n$  is a known constraint set. We make the following assumption about this constraint set.

*Assumption 4:* The set  $Y$  is compact, convex, and symmetric about the origin, which is in its interior.

#### B. Constraint admissible set

The constraint admissible set  $O_\infty$  is defined as a set of all initial conditions such that the predicted response of (10) corresponding to the initial state and constant command  $\nu$  satisfies constraints defined in (11). More formally,

$$O_\infty = \{(y_0, \nu) : y_k(y_0, \nu, \{e_k^{\mathcal{F}}\}) \in Y, \forall \{e_k^{\mathcal{F}}\} \in \mathcal{E}\}, \quad (12)$$

where  $y_k(y_0, \nu, e_k^{\mathcal{F}})$  is the solution of system (1) given by

$$y_{k+1} = \Phi^{k+1} y_0 + \sum_{i=0}^k (\Phi^{k-i} \Lambda \nu + \Phi^{k-i} e_i^{\mathcal{F}}), \quad (13)$$

and  $\mathcal{E}$  and  $Y$  are defined in (7) and (11), respectively. Note that our reference governor design will assume that the observer error has converged to the set given in (7). To obtain the constraint admissible set, the concept of the Pontryagin

difference is used here. For  $U$  and  $V \subset \mathbb{R}^n$  the Pontryagin difference  $U \sim V$  is given by the set,

$$U \sim V := \{x \in \mathbb{R}^n : x + v \in U, \forall v \in V\}. \quad (14)$$

Consider,

$$\begin{aligned} Y_0 &= Y, \\ Y_1 &= Y \sim \mathcal{E}, \\ &\vdots \\ Y_{k+1} &= Y \sim \mathcal{E} \sim \dots \sim \Phi^k \mathcal{E}. \end{aligned} \quad (15)$$

Then, from [25] we get,

$$\begin{aligned} O_k = \Big\{ (y_0, \nu) \in \mathbb{R}^n \times \mathbb{R}^n : \Phi^i y_0 \\ + \sum_{i=0}^k \Phi^{k-i} \Lambda \nu \in Y_i, i = 0, \dots, k \Big\}. \end{aligned} \quad (16)$$

Using (15) and (16), we can obtain the following recursions:

$$Y_{k+1} = Y_k \sim \Phi^k \mathcal{E} \text{ where } Y_0 = Y, \quad (17)$$

$$\begin{aligned} O_0 &= \{(y_0, \nu) \in \mathbb{R}^n \times \mathbb{R}^n : y_0 \in Y\}, \\ O_{k+1} &= O_k \cap \{(y_0, \nu) \in \mathbb{R}^n \times \mathbb{R}^n : \\ &\quad \Phi^{k+1} y_0 + \Phi^k \Lambda \nu \in Y_{k+1}\}. \end{aligned} \quad (18)$$

where  $k \in \mathbb{Z}^+$ . Therefore, from (12), we obtain the constraint admissible set as

$$O_\infty = \bigcap_{k \in \mathbb{Z}^+} O_k. \quad (19)$$

To ensure finite-termination of the set sequence, typically a strict feasibility constraint on  $\nu$  is added; see [26] for details. This constraint has not been used in the present work; instead  $O_k$  was computed up to a sufficiently large index  $k$  and used as an approximation to  $O_\infty$ . The design of the reference governor is given in the following subsection.

#### C. Reference governor design

The reference governor considers the modified reference input  $\nu_k$  as an optimization variable which is obtained at each sampling instant  $k$  by solving an optimization problem. It is obtained as the solution to the following minimization problem:

$$\begin{aligned} \nu_k^* = \arg \min_{\nu} & \| \nu - r_k \|_R^2 \\ \text{s.t. } & (y_k, \nu) \in O_\infty, \end{aligned} \quad (20)$$

where  $R$  is a positive definite matrix,  $\|x\|_R^2 = x^T R x$  and  $O_\infty$  is given in (12).

#### D. Control constraints

We define the control constraints in the following form:

$$\|u_k\| \leq \mu, \quad (21)$$

where  $\mu$  is the upper bound on control inputs. Multiplying both sides of the above equation by the control gain matrix  $\mathcal{G}_k$  results in

$$\|\mathcal{G}_k u_k\| \leq \|\mathcal{G}_k\| \|u_k\| \leq \|\mathcal{G}_k\| \mu. \quad (22)$$

The control constraint can be satisfied through an appropriate design of the control gain matrix  $\mathcal{G}_k$ . From (9), we define

$$\omega_k := -\hat{\mathcal{F}}_k + \Phi y_k + \Lambda \nu_k. \quad (23)$$

Then, using (9), (22), and (23) the following inequality is obtained:

$$\|\omega_k\| = \|\Phi y_k + \Lambda \nu_k - \hat{\mathcal{F}}_k\| \leq \|\mathcal{G}_k\| \mu, \quad (24)$$

which can be written as

$$\|\mathcal{G}_k\| \geq \frac{1}{\mu} \|\omega_k\|. \quad (25)$$

This leads to the following condition for the control gain  $\mathcal{G}_k$ :

$$\|\mathcal{G}_k\| \geq \frac{1}{\mu} \|\Phi y_k + \Lambda \nu_k - \hat{\mathcal{F}}_k\|. \quad (26)$$

The control gain matrix  $\mathcal{G}_k$  is then designed as

$$\mathcal{G}_k := \alpha_k \check{\mathcal{G}}, \quad (27)$$

where  $\alpha_k$  is a tuning parameter and  $\check{\mathcal{G}}$  is a positive diagonal matrix. This tuning parameter can be designed as

$$\alpha_k = \frac{1}{\mu \check{g}^i} \|\Phi y_k + \Lambda \nu_k - \hat{\mathcal{F}}_k\|, \quad (28)$$

where  $\check{g}^i$  denotes as the smallest diagonal element in  $\check{\mathcal{G}}$ .

### E. Stability Analysis

The stability and robustness analysis of the disturbance (unknown dynamics) observer and control law is presented in [19]. Here we characterize the ultimate bound on the tracking error for a constant  $\nu$ . Such a bound is useful in verifying conditions for the convergence of the reference governor with constrained inputs [26].

With the control law given in §III-A, we define the output trajectory tracking error as

$$e_k^y = \nu - y_k. \quad (29)$$

Subtracting  $\nu$  from both sides of (10), we get

$$\nu - y_{k+1} = \nu - \Phi y_k - \Lambda \nu + e_k^{\mathcal{F}}. \quad (30)$$

The gain matrices  $\Phi$ , and  $\Lambda$  are defined in §III-A and can be designed such that

$$\Lambda = I - \Phi, \quad (31)$$

where  $I$  is the  $n \times n$  identity matrix. Using this design of gain matrices we obtain

$$\nu - y_{k+1} = \nu - \Phi(y_k - \nu) - \nu + e_k^{\mathcal{F}}. \quad (32)$$

Finally, substituting (29) in the above equation leads to the following error dynamics:

$$e_{k+1}^y = \Phi e_k^y + e_k^{\mathcal{F}}. \quad (33)$$

*Theorem 1: Consider the closed-loop model for the unknown system (10), the control law (9) and the observer law (5). Let the estimation error in (3) be bounded according to*

$$\|e_k^{\mathcal{F}}\| \leq B \text{ for } k > N, \quad (34)$$

where bound  $B \in \mathbb{R}^+$  and  $N \in \mathbb{W}$  are known. Then the output trajectory tracking error  $e_k^y$  will converge to the neighborhood given by

$$N^y := \{e_k^y \in \mathbb{R}^n : \rho(\|\Phi\|) \|e_k^y\| \leq B\}, \quad (35)$$

for  $k > N' > N$  where  $N, N' \in \mathbb{W}$ , and

$$\rho(\|\Phi\|) := 1 - \|\Phi\|. \quad (36)$$

*Proof:* Consider the following discrete Lyapunov function for output tracking error,

$$V_k^y = \frac{1}{2} (e_k^y)^T e_k^y. \quad (37)$$

The first difference of the Lyapunov function is given by,

$$V_{k+1}^y - V_k^y = (e_{k+1}^y + e_k^y)^T (e_{k+1}^y - e_k^y). \quad (38)$$

Substituting the error dynamics (33) in above expression and after some algebraic manipulations, we obtain

$$\begin{aligned} V_{k+1}^y - V_k^y &= (\Phi e_k^y)^T (\Phi e_k^y) - (e_k^y)^T e_k^y \\ &\quad + 2(e_k^{\mathcal{F}})^T (\Phi e_k^y) + (e_k^{\mathcal{F}})^T e_k^{\mathcal{F}}. \end{aligned} \quad (39)$$

Using the bounds defined on  $e_k^{\mathcal{F}}$  given in (34), the first difference of the Lyapunov function can be upper bounded as follows:

$$\begin{aligned} V_{k+1}^y - V_k^y &\leq \|\Phi e_k^y\|^2 - \|e_k^y\|^2 + 2B\|\Phi\| \|e_k^y\| + B^2 \\ &= \|\Phi\|^2 \|e_k^y\|^2 - \|e_k^y\|^2 + 2B\|\Phi\| \|e_k^y\| + B^2, \end{aligned}$$

and will be simplified as

$$V_{k+1}^y - V_k^y \leq -(1 - \|\Phi\|^2) \|e_k^y\|^2 + 2B\|\Phi\| \|e_k^y\| + B^2. \quad (40)$$

The right-hand side of the inequality (40) will be negative for large initial transients in  $e_k^y$ , and can be expressed as a quadratic inequality expression in  $\|e_k^y\|$  as follows,

$$(1 - \|\Phi\|^2) \|e_k^y\|^2 - 2B\|\Phi\| \|e_k^y\| - B^2 > 0. \quad (41)$$

Then, the condition

$$\|e_k^y\| > \frac{B}{1 - \|\Phi\|}, \quad (42)$$

for positive real roots of  $\|e_k^y\|$  ensures that  $V_{k+1}^y - V_k^y < 0$ . This implies that the discrete Lyapunov function  $V_k^y$  is monotonically decreasing when the tracking error  $\|e_k^y\|$  is large enough to satisfy the inequality condition (42) for some finite  $k > N'$ , where  $N' > N$ . Therefore, the output trajectory tracking error  $e_k^y$  will converge to the neighbourhood  $N^y$  of  $0 \in \mathbb{R}^n$  given by (35). ■

It is worth noting that, in absence of a reference governor we have  $\Phi = 0$ . From [19], this leads to convergence of  $e_k^y$  to the bounded neighborhood of  $0 \in \mathbb{R}^n$  in finite time, where the bounds are the same as that of estimation error  $e_k^{\mathcal{F}}$ .

## IV. NUMERICAL SIMULATION RESULTS

This section presents numerical simulation results of the proposed reference governor design for the constrained ULM-based control. The proposed scheme is applied to control the pitch angle of an aircraft as a SISO system, for which the dynamics is unknown to the controller.

### A. Aircraft model

The dynamics model for the aircraft flying at constant velocity and altitude is given by [27]:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} \delta \quad (43)$$

$$y = \theta \quad (44)$$

where  $\alpha$  is the angle of attack,  $q$  is the pitch rate and  $\theta$  is the pitch angle. This model, assumed unknown to the tracking control law, is used to generate an initial reference (or desired) output trajectory for tracking.

For this SISO system, the input is the elevator deflection angle  $\delta$  and the output is the pitch angle  $\theta$ . The pitch angle output is subjected to the constraint  $|\theta| \leq 5^\circ$  and the control input is subjected to the constraint  $|\delta| \leq 0.1^\circ$ .

We apply the nominal controller designed in (9), which leads to the closed-loop form of ULM as described in (10). The output trajectories are illustrated in Fig. 2. It shows that the nominal output trajectory violates the output constraint, whereas the actual output trajectory tracks the modified reference trajectory while satisfying the output constraint.

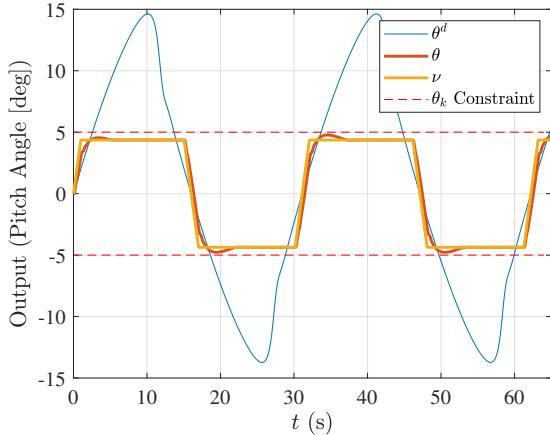


Fig. 2. Desired, modified, and actual output trajectories

Simulation results for the control input are shown in Fig. 3. Note that the control input constraints are satisfied and the inputs stay within the required limits. Fig. 4 illustrates the error in the estimation of the unknown dynamics  $\mathcal{F}$  for the ultra-local model according to the observer design from section II-B.

### B. Effect of initial selections of $\hat{\mathcal{F}}$ and $\mathcal{G}$

Here, we study the effects of initial selections for the estimate of the unknown model,  $\hat{\mathcal{F}}_0$ , and the control gain matrix,  $\mathcal{G}_0$ , on the control input and the output trajectories.

The plots in Fig. 5 illustrate the control inputs obtained for different initial selections for the ULM. This figure shows that a decrease in the value of  $\mathcal{G}_0$  leads to higher transients in the control input and an increase in the value of  $\hat{\mathcal{F}}_0$  generally causes high initial transients in the control input. It also shows that the control constraints are always satisfied.

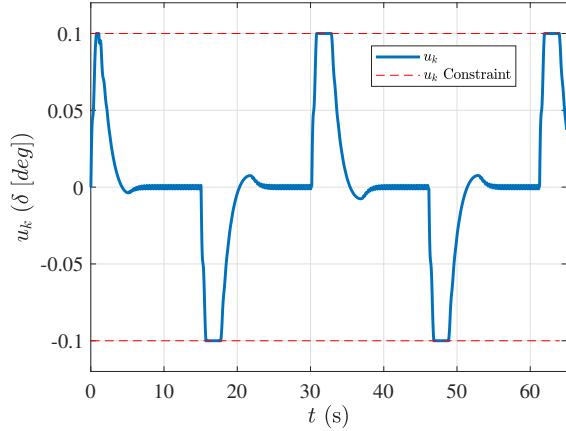


Fig. 3. Control input with constraints

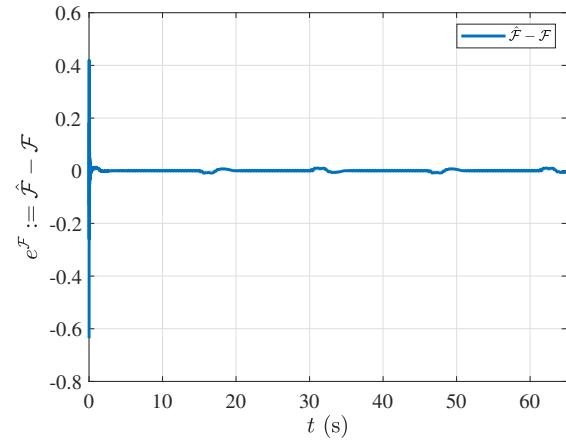


Fig. 4. Model estimation error  $e^F$

The output trajectories in Fig. 6 illustrate the effect of different initial values, primarily for  $\hat{\mathcal{F}}_0$ . As is shown in this figure, higher values of  $\hat{\mathcal{F}}_0$  tend to delay the convergence of output trajectories  $y_k$  to the modified reference  $\nu_k$ .

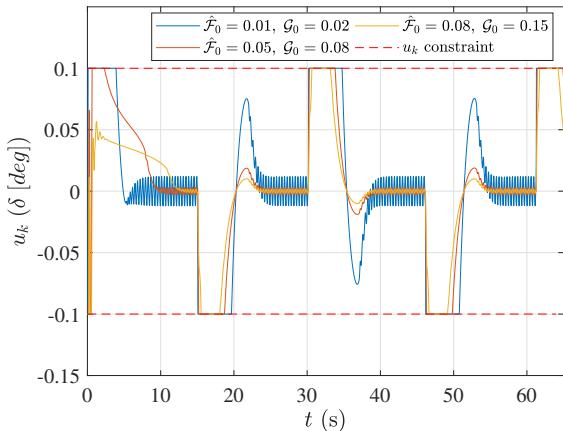


Fig. 5. Control input for different initial conditions

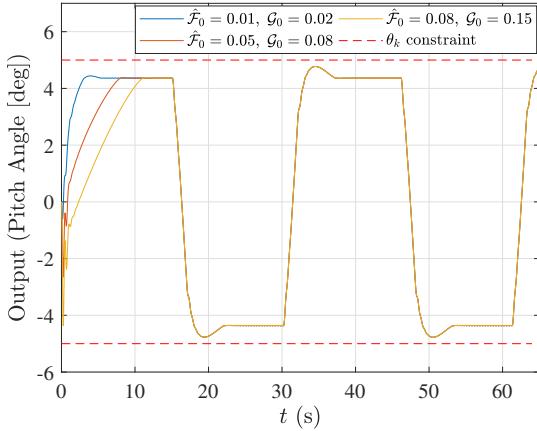


Fig. 6. Output trajectory for different initial conditions

## V. CONCLUSION

This work considers the design of a reference governor to enforce pointwise-in-time output and control constraints for a system with unknown input-output dynamics modeled by an ULM. At each time instant, a modified command is generated that minimizes the constrained cost function. This command is determined on the basis of a constraint admissible set that is designed to enforce output constraints. The control gain matrix is designed to satisfy the control constraints. A Lyapunov stability analysis shows the convergence of the tracking error and the observer error to bounded neighborhoods of zero error. A numerical simulation is performed for an aircraft pitch control system, subjected to output and control constraints. The simulation results demonstrate the tracking of a reference output trajectory while satisfying the output and control constraints. In the current framework, the output constraint satisfaction for the output trajectory is conservative. This issue will be considered in future work. In addition, the effect of initial selections of the estimate of the unknown model and the control gain matrix on the control input and output response is also illustrated through a numerical simulation study. Future research will explore the design of reference governors for systems with uncertainty (“gray box” models), monotonically non-increasing set bounds for the observer error, and systems with delay.

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