A Two-stage Peer-to-peer Energy Trading Model for Distribution Systems with the Participation of the Utility

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***Abstract*—With the growing penetration of distributed energy resources (DER) in distribution systems, the traditional utility dominated tariff-based business model may no longer meet the need for further development. As a result, the transformation from the traditional tariff-based business model to the emerging peer-to-peer energy trading model has been acknowledged by researchers and policy makers. In this paper, a two-stage peer-to-peer energy trading model is proposed while considering the role of the utility. Specifically, energy transactions between buyers and sellers are optimized in the first stage; the cleared transactions are submitted to the utility for approval in the second stage, which solves a transaction approval model to verify the transactions from the perspective of secure system operations. Indeed, certain transactions may be disapproved to ensure that all network constraints, such as voltage and line flow limitations, are satisfied. In addition, a comprehensive trading tariff is designed to recover the hidden costs of the utility, such as those associated with network usage, system losses, and ancillary service provision. A modified 33-bus distribution system is adopted to verify the proposed model.**

***Index Terms*—Peer-to-peer energy trading, distributed energy resources, distribution system, energy market.**

NOMENCLATURE

Indices and Sets

b	Index of buyers.
$b-s$	Index of the energy transaction from buyer b to seller s.
c	Index of capacitor banks.
i, j	Indices of buses.
$i-j$	Index of the line connecting buses i and j.
sub	Index of the substation bus.
s	Index of sellers.
$s-b$	Index of the energy transaction from seller s to buyer b.
$\boldsymbol{\mathcal{B}}$	Set of buyers.
$\boldsymbol{\mathcal{B}}_i$	Set of buyers on bus i.
$\boldsymbol{\mathcal{B}}_s$	Set of buyers that can trade with seller s.
$\boldsymbol{\mathcal{C}}$	Set of capacitor banks.
$\boldsymbol{\mathcal{C}}_i$	Set of capacitor banks on bus i.
$\boldsymbol{\mathcal{I}}$	Set of buses.
$\boldsymbol{\mathcal{K}}$	Set of linearization segments, $\boldsymbol{\mathcal{K}} \triangleq \{1,2 \dots K\}$, where K is the number of segments.
$\boldsymbol{\mathcal{L}}$	Set of lines.
$\boldsymbol{\mathcal{S}}$	Set of sellers.
$\boldsymbol{\mathcal{S}}_i$	Set of sellers on bus i.
$\boldsymbol{\mathcal{S}}^{CN}$	Set of sellers that allow partial curtailment of transactions.
$\boldsymbol{\mathcal{S}}^{IN}$	Set of sellers that only allow full curtailment of transactions.

Parameters

B_c	Susceptance of capacitor bank c.
B_{i-j}	Susceptance of line $i-j$.
C_{b-s}^T	Comprehensive trading tariff on the transaction between buyer b and seller s.
C_b^U	Selling tariff of the utility to buyer b.
C_s^U	Purchasing tariff of the utility to seller s.
G_{i-j}	Admittance of line $i-j$.
M	A large positive number.
P_b^{UC}	Submitted transaction from the utility to buyer b.
P_s^{UC}	Submitted transaction from seller s to the utility.
P_b^{UB}	Upper bound of power demand of buyer b.
P_b^{LB}	Lower bound of power demand of buyer b.
P_s^{UB}	Upper bound of power demand of seller s.
P_s^{LB}	Lower bound of power demand of seller s.
P_{s-b}^{TC}	Submitted transaction from seller s to buyer b.
P^{UB}	Upper bound of active power injection through the substation bus.
P_{i-j}^{UB}	Active power limit of line $i-j$.
Q^{UB}	Upper bound of reactive power injection through the substation bus.
S_k	Slope of linearization segment k.
V_i^{LB}	Lower bound of voltage magnitude of bus i.
V_i^{UB}	Upper bound of voltage magnitude of bus i.
V^{SUB}	Given voltage magnitude of the substation bus.
$\alpha_{k,i-j}$	Value on linearization segment k of line $i-j$.
β_{b-s}^{TC}	Weight for curtailment of transaction from seller s to buyer b.
β_s^{UC}	Weight for curtailment of transaction from seller s to the utility.
β_b^{UC}	Weight for curtailment of transaction from the utility to buyer b.
δ_s	Power factor of seller s.
δ_b	Power factor of buyer b.
φ^{UB}	Maximum voltage phase angle difference.

Continuous Variables

c_b^U	Cost of buyer b for purchasing power from the utility.
c_s^U	Payment to seller s for selling power to the utility.
c_s^T	Comprehensive trading cost of seller s to the

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	utility.
p_b	Active power scheduled to buyer b.
p_s	Active power scheduled from seller s.
p^S	Active power injection through the substation bus.
p_{b-s}^T	Power transaction from buyer b to seller s.
p_{s-b}^T	Power transaction from seller s to buyer b.
p_{s-b}^{TC}	Curtailment on energy transaction from seller s to buyer b.
p_b^{UC}	Curtailment on energy transaction from the utility to buyer b.
p_s^{UC}	Curtailment on energy transaction from seller s to the utility.
p_b^U	Energy transaction from the utility to buyer b.
p_s^U	Energy transaction from seller s to the utility.
p_{i-j}	Active power on line $i-j$.
q_b	Reactive power scheduled to buyer b.
q_s	Reactive power scheduled from seller s.
q_c	Reactive power provided by capacitor bank c.
q^S	Reactive power injection through the substation bus.
q_{i-j}	Reactive power on line $i-j$.
v_i	Voltage magnitude of bus i.
θ_i	Voltage phase angel of bus i.
φ_{i-j}^{PS}	Auxiliary variable representing positive voltage phase angle difference between buses i and j.
φ_{i-j}^{NG}	Auxiliary variable representing negative voltage phase angle difference between buses i and j.

Binary variables

I_c	Switched ON/OFF indicator of capacitor bank c (1: switched ON; 0, switched OFF).
I_{s-b}	Curtailment indicator of transaction from seller s to buyer b (1: curtailment; 0, otherwise).
$Y_{k,i-j}$	Marginal segment indicator for segment k of line $i-j$ (1: active; 0, otherwise).
Z_{i-j}	Auxiliary variable to represent voltage phase angle difference.

Symbols

$C_s(\cdot)$	A step cost function of seller s.
$U_b(\cdot)$	A step benefit function of buyer b.

I. INTRODUCTION

In recent years, we have witnessed a rapid growth of electric energy generation from distributed energy resources (DERs) in distribution systems, presenting a steep growth curve that is far from an inflection point. By 2040, electric energy generation from DERs is projected to be 317,323 GWh, from 185,334 GWh in 2015 [1]. Among all the driving forces, the proliferation of solar photovoltaic panels, small wind turbines, and energy storage systems is extremely prominent. Indeed, electricity generation by local DERs brings multiple benefits to both the distribution and transmission sectors. On the one hand, DERs can reduce system losses, relieve the overload of upstream distribution lines and transformers, and enhance the reliability of distribution systems. On the other hand, DERs also benefit transmission systems by alleviating congestions and mitigating electricity peak demands/prices [2].

Under the current utility dominated tariff-based business model, it is a common practice that excessive electricity from DERs is purchased through power purchase agreements (PPAs) based on a long-term fixed tariff [3] set by the utility; meanwhile, the utility sells the purchased electricity to residential and small merchant consumers at a much higher tariff. Although the long-term purchasing tariff can reduce financial risks of DER investors to some extent, the resulting squeezed profit space may not provide sufficient economic incentives to support a further growth of DERs. To this end, peer-to-peer energy trading has been recently explored, which allows electricity producers and users in the distribution system to trade directly, respectively seeking high-price buyers and low-price sellers. Peer-to-peer energy trading could incentivize more active investments in DERs with higher profits, associated with higher financial risks, while the reduced electricity price provokes higher energy consumptions and further promotes the DER deployment. Indeed, this new business model is not a departure from the utility's benefits. On the one hand, meeting energy demand of customers locally could reduce the utility's financial risks of bidding into the bulk power market with highly volatile market prices; On the other hand, with an increase in the power consumption, the utility's profits from network usage and ancillary services would also increase.

Many peer-to-peer energy trading mechanisms for distribution systems have been proposed in literature. References [4] and [5] extensively reviewed existing studies, classifying the typical designs into three categories: (i) game theory-based mechanisms [6], [7]; (ii) auction-based mechanisms [8], [9]; and (iii) consensus-based mechanisms [10], [11]. Among the three categories, game theory based trading mechanisms are usually the most abstract and complex, and generally require certain assumptions and specific rules to ensure the existence of an equilibrium and the reachability of the equilibrium via proper solution algorithms. The auction-based mechanisms collect the bids from buyers and sellers, and conduct a bid matching process to determine the transactions and the clearing prices. The matching process is iterative, allowing buyers and sellers to adjust bids gradually and add new transactions. The consensus based mechanisms are the closest to the current bulk power market practice, which can be shown as a centralized optimization problem with the objective of maximizing the social welfare or minimizing the total cost. Considering the potential privacy concerns, this model can be decomposed into local optimization models of traders and solved iteratively in a decentralized manner [12]–[14]. In each iteration, individual traders exchange necessary information with others that are coupled via global constraints, and update their own local optimization models. This iterative process continues until certain stopping criteria are met, indicating that a consensus among all traders has been reached.

However, physical limitations of the distribution networks are usually not explicitly considered in existing studies on the peer-to-peer energy trading model, because of two main challenges. First, involving the network constraints would significantly complicate the trading model. It could invalid certain assumptions that are critical to guarantee the existence and reachability of the equilibrium, causing difficulties in the matching process, and changing the characteristics of the underlying optimization problem with significantly extra computational complexities. Secondly, the network information is a sensitive public safety topic, and generally not publicly available to the traders. Because satisfying network constraints is critical to ensuring secure system operations, a potential solution is to further verify the transactions out of the trading market, requiring the intervention of an administrator.

When transforming from the traditional tariff-based business model to the emerging peer-to-peer energy trading model, a common assumption is that the utility is augmented to a distribution system operator (DSO) [14], who on the one hand independently administrates the distribution system like independent system operators (ISO) in the wholesale electricity markets, and on the other hand conducts transactions with traders in its control area. Indeed, the later feature would cause a dilemma to the DSO on its independence, and also impose economic risks to the DSO when participating in the wholesale electricity market. However, the DSO's economic risks and revenue profiles are often not well justified. Indeed, the existing utility shall not be excluded from, but can play a critical role in the peer-to-peer energy trading. Specifically, relying on its rich experiences and sophisticated tools in operating the distribution system and interacting with the bulk energy market, the utility can assume the responsibility of transaction verification and secure system operations. In addition, the utility can regulate transaction price caps to both buyers and sellers. Moreover, if the peer-to-peer energy trading cannot be economically settled, buyers and sellers can still directly trade with the utility to satisfy their basic demands with the utility tariffs.

In this paper, a two-stage model, including a peer-to-peer energy trading model and a utility transaction approval model, is proposed. Specifically, the peer-to-peer energy trading among buyers and sellers is cleared at the first stage, and the cleared transactions are submitted to the utility for approval in the second stage. If the full realization of these cleared transactions results in potential violations on network physical limitations, curtailments on certain transactions will be applied until all network constraints are respected. This is achieved by solving the transaction approval model. In addition, the comprehensive trading tariff is designed to recover the hidden costs of the utility, such as those associated with network usage, system losses, and ancillary service. With the proposed model, this paper focuses on revealing the role that the current utilities could play in peer-to-peer energy trading markets of distribution systems, which is not fully explored in existing studies.

The rest of the paper is organized as follows. The mechanism of the two-stage model is presented in Section II. Section III proposes the peer-to-peer energy trading model, and the utility transaction approval model is introduced in Section IV. Numerical case studies are conducted in Section V, and the conclusions are presented in Section VI.

II. Mechanism of the Two-stage Model and the Critical Role of the Utility

The mechanism of the two-stage model and the comprehensive trading tariff are introduced in this section. The structure of the two-stage model is shown in Fig. 1 and described as follows.

A. Peer-to-Peer Energy Trading Model

With the proposed trading mechanism, each trader registers as a buyer or a seller within a trading window. After the peer-to-peer energy trading market is open, the utility will release its selling and purchasing tariff, which could vary for different traders. In addition, comprehensive trading tariffs with respect to individual pairs of buyers and sellers will be released to related sellers as well. The selling tariff, the purchasing tariff, and the comprehensive trading tariff would be dynamically updated over various trading windows, reflecting changes of the wholesale market prices and system operational conditions. On this basis, combined with information on available trading between buyers and sellers as well as their benefit functions (for buyers) and cost functions (for sellers), a peer-to-peer energy trading model can be built. The model can be solved either in a centralized or decentralized manner [15], depending on the consensus of traders on privacy. Both manners have their advantages and disadvantages.

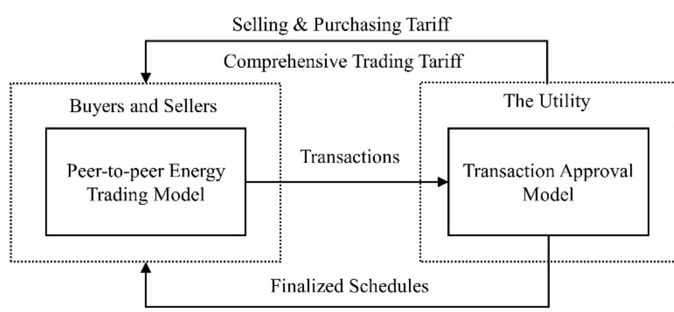

Fig. 1. The structure of the two-stage model.

If a centralized framework is adopted for the peer-to-peer energy trading market, the utility cannot act as the market operator, because it is also a market participant and exposing the benefit and cost information of other traders to the utility will raise fairness concerns. Therefore, an independent third-party agent is needed, responsible for collecting information from the traders and the utility and solving the peer-to-peer trading model during each trading window. The agent can provide web-based services for the traders and the utility to submit their bidding information and verify their compliance. The third-party agent is also responsible for ensuring confidentiality of the collected data. Traders and the utility may pay a registration fee or annual fee to receive this service, which is then used by the third-party agent to offset its operating cost.

If a decentralized framework is adopted, such as the Relaxed Consensus plus Innovation method proposed in reference [10], the information exposure can be avoided even without a third-party agent. However, this requires the traders' capabilities of instant communication and solving local optimization problems, which undoubtedly puts forward a high requirement on the infrastructure and raises the bar for participating in the market. Another challenge of the decentralized framework is its dramatically high computational burden [10], especially when the market scales up in terms of the number of traders. It is also emphasized that, under the decentralized framework, releasing utility tariffs to individual traders is critical for addressing the local benefit maximization or cost minimization problems.

B. Utility Transaction Approval and Curtailment

Due to network and operating information confidentiality to both the traders and the third-party agent, the peer-to-peer trading model is unable to include physical system operation limits. However, because the physical limitations are of critical importance to ensure secure system operations, fully implementing all the cleared transactions may not be physically feasible. Therefore, after solving the peer-to-peer trading model, the cleared transactions are submitted to the utility for verification with respect to physical limits, and the utility will realize the cleared transactions as much as possible. When a full realization cannot be achieved, curtailments of certain transactions will be applied until secure operations are guaranteed. This is done by solving the transaction approval model. Thereafter, the approved quantities of transactions will be finalized and released to traders, which are used to settle the financial payment with all entities.

C. Comprehensive Trading Tariff

The comprehensive trading tariff is designed to recover hidden costs of the utility for realizing transactions of traders. It consists of three components:

- *Compensation for system loss*: The power transmission between buyers and sellers will undoubtedly incur power losses. However, in the transaction settlement, the measurement of the buyer's power withdraw and the seller's power injection are at their interconnection buses, which means the losses are not explicitly considered. To this end, the losses are covered by the utility, and have to be compensated. It is worthwhile to mention that losses are related to the interconnection locations of buyers and sellers, as well as the current system operating status.
- *Compensation for ancillary service*: Considering the fact that the sellers in the peer-to-peer energy trading are usually of small scale and primarily consisting of renewable DERs with limited controllability and predictability, they could be faced with a relatively high risk of being unable to fully follow the cleared transactions. To this end, the utility acts as the regulating reserve provider to balance the resulting energy shortage or surplus. In addition, the utility also provides other important grid services, such as the voltage support by operating its capacitor banks and other reactive power resources.
- *Compensation for network usage* [16]: Needless to say, the transmission of electricity energy between traders must use the physical network, which is owned by the utility. The tariff of network usage will be charged to related traders.

Unlike selling and purchasing tariffs of the utility that have a certain degree of pricing freedom, the freedom of changing the comprehensive trading tariff would be rather limited, and the pricing method shall be agreed among traders, regulated by supervisory committees, and compliant to polices. In fact, among the three components, system losses are compensated by the energy purchase of the utility from the wholesale market, and ancillary service provided by the utility is also purchased from the wholesale market, which means these tariffs shall be linked to the corresponding wholesale market prices. In addition, the network usage tariff is usually a long-term fixed price. Therefore, pricing of these three parts will set the basis of the comprehensive trading tariff, limiting its variability.

It is noteworthy that the three components are not uniform for all traders, instead, they would vary according to the system operational status, resource types and power factors of sellers, and the interconnection buses of the buyers and sellers. In this paper, the corresponding costs associated with the comprehensive trading tariff are all included in the total cost of the sellers.

III. Peer-to-peer Energy Trading Model

The peer-to-peer energy trading model is formulated as in (1)-(13), with the objective of maximizing the total social welfare. In the objective function (1), the first term represents the net benefit of buyers, equal to its benefit function minus the payment to the utility; the second term represents the total cost of sellers, including the payment from the utility and the comprehensive trading cost. In this paper, both the benefit function of a buyer $U_b(\cdot)$ and the cost function of a seller $C_s(\cdot)$ are formulated as step functions, consistent with the current bulk energy market practice [17].

The comprehensive trading cost of seller b is calculated as in (2). The power purchase cost of buyer b from the utility is calculated as in (3), and the power selling payment of seller s to the utility is calculated as in (4). Equality constraint (5) calculates the total power schedule of buyer b, where the first term is power from sellers and the second term represents energy from the utility. Constraint (6) calculates the total power schedule of seller s, slimier to constraint (5). Constraint (7) forces that the pair of variables representing the traded power in opposite directions are of equal quantity. It is referred to as the reciprocity constraint in reference [10]. The power lower and upper bounds of buyer b and seller s are represented in constraints (8) and (9). Variables of power from buyer b to seller s and to the utility are non-negative as in constraints (10) and (11), while variables of power from seller s to buyer b and to the utility are non-positive as in constraints

(12) and (13).

$$\max \textstyle\sum_{b\in\mathcal{B}}\{U_b(p_b)-c_b^U\}-\sum_{s\in\mathcal{S}}\{C_s(p_s)-c_s^U+c_s^T\} \tag{1}$$
$$c_s^T=-\textstyle\sum_{b\in\mathcal{B}_s} C_{b-s}^T\cdot p_{b-s}^T; \qquad \forall s\in\mathcal{S} \tag{2}$$
$$c_b^U=C_b^U\cdot p_b^U; \qquad \forall b\in\mathcal{B} \tag{3}$$
$$c_s^U=-C_s^U\cdot p_s^U; \qquad \forall s\in\mathcal{S} \tag{4}$$
$$p_b=\textstyle\sum_{s\in\mathcal{S}_b} p_{s-b}^T+p_b^U; \qquad \forall b\in\mathcal{B} \tag{5}$$
$$p_s=-\textstyle\sum_{b\in\mathcal{B}_s} p_{b-s}^T-p_s^U; \qquad \forall s\in\mathcal{S} \tag{6}$$
$$p_{b-s}^T+p_{s-b}^T=0; \qquad \forall b\in\mathcal{B}_s, \forall s\in\mathcal{S} \tag{7}$$
$$P_b^{LB}\le p_b\le P_b^{UB}; \qquad \forall b\in\mathcal{B} \tag{8}$$
$$P_s^{LB}\le p_s\le P_s^{UB}; \qquad \forall s\in\mathcal{S} \tag{9}$$
$$p_{s-b}^T\ge 0; \qquad \forall b\in\mathcal{B}_s, \forall s\in\mathcal{S} \tag{10}$$
$$p_b^U\ge 0; \qquad \forall b\in\mathcal{B} \tag{11}$$
$$p_{b-s}^T\le 0; \qquad \forall b\in\mathcal{B}_s, \forall s\in\mathcal{S} \tag{12}$$
$$p_s^U\le 0; \qquad \forall s\in\mathcal{S} \tag{13}$$

The peer-to-peer energy trading model (1)–(13) is a linear programing (LP) problem that has a well-defined dual problem and holds strong duality. Referring to [10], the dual variable λ_{b-s} to the equality constraint (7) is the clearing price of the transaction between buyer b and seller s. The buyer will pay at this price, while the seller will be paid at part of this price, because λ_{b-s} also includes the comprehensive trading tariff to the utility. That is, λ_{b-s} will be split into two parts of $\lambda_{b-s}-C_{b-s}^T$ and C_{b-s}^T, and the seller will be paid at the former, while the utility will be paid at the latter. For transactions with the utility, the buyers will pay at the selling tariff and the sellers will be paid at the purchasing tariff. The money flows and energy flows of the peer-to-peer energy trading model are shown in Fig. 2. It is noteworthy that only energy flow directions are presented in Fig. 2, while the final energy flow quantities will be determined by the transaction approval model.

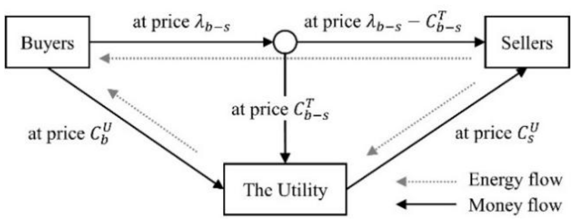

Fig. 2. Money and energy flows of entities.

IV. Utility Transaction Approval Model

The utility transaction approval model is formulated as in (14)–(46). The objective is to minimize the weighted transaction curtailment with respect to a linearized AC power flow model [18]. In the objective function (14), curtailment weights of individual transactions may vary. For example, transactions of clean energy from renewable resources may enjoy larger weights, so that a higher priority to follow the transactions can be maintained. In practice, the utility can define multiple categories of transactions and assign weights accordingly, and each transaction can be classified into a category and follow the weight of that category.

Constraints (15) and (16) represent the nodal active and reactive power balance for individual non-substation buses, while that of the substation bus are represented by constraints (17) and (18). Active and reactive power injections from the main grid through the substation bus are limited by constraints (19) and (20).

Transaction curtailments are modeled via constraints (21)–(26). Constraints (21) and (22) recalculate the power schedules of buyers and sellers after curtailment. It is worthwhile to mention that because the derivation of λ_{b-s} is no longer needed and constraint (7) is not included, only the curtailment variable p_{s-b}^{TC} of the corresponding power trade variable p_{s-b}^T is present. Constraints (23) and (24) limit the curtailment on transactions with the utility to be within the cleared values. For transactions between traders, two optional modes are defined, namely partial curtailment and full curtailment, as in constraints (25) and (26). Specifically, the partial curtailment allows the transaction being realized at any value between zero and the cleared value, while the full curtailment cuts off the entire transaction once applied.

Constraints (27) and (28) repeat the lower bounds of constraints (8) and (9). Reactive power from buyer b and seller s are calculated by (29) and (30) with given power factors. Capacitor banks are modeled as in (31) and (32), which can be switched ON or OFF per system needs. In (31), the squared voltage magnitude v_i^2 is linearized as $2\cdot v_i-1$. Equality constraints (33) and (34) calculate active and reactive power flow on line $i-j$. It is emphasized that due to system losses, power flow from bus i to bus j is not equal to that from bus j to bus i. The third terms of (33) and (34) respectively represent line active and reactive power losses, linearized from $(G_{i-j}/2)\cdot(\theta_i-\theta_j)^2$ and $-(B_{i-j}/2)\cdot(\theta_i-\theta_j)^2$. The linearization constraints are formulated as in (35)–(42) and referring to [19]. Line active power flow limits are enforced by constraint (43), and voltage magnitude limits are enforced by constraint (44). Constraint (45) sets the substation bus as the reference bus with a voltage phase angle of 0, and constraint (46) sets its voltage magnitude as V^{SUB}.

$$\min \textstyle\sum_{s\in\mathcal{S}}\sum_{b\in\mathcal{B}_s}\beta_{b-s}^{TC}\cdot p_{b-s}^{TC} + \sum_{b\in\mathcal{B}}\beta_b^{UC}\cdot p_b^{UC}-\sum_{s\in\mathcal{S}}\beta_s^{UC}\cdot p_s^{UC}; \tag{14}$$
$$\textstyle\sum_{i-j\in\mathcal{L}} p_{i-j}+\sum_{b\in\mathcal{B}_i} p_b=\sum_{j-i\in\mathcal{L}} p_{j-i}+\sum_{s\in\mathcal{S}_i} p_s; \qquad \forall i\in\mathcal{J}/\{sub\} \tag{15}$$
$$\textstyle\sum_{i-j\in\mathcal{L}} q_{i-j}+\sum_{b\in\mathcal{B}_i} q_b=\sum_{j-i\in\mathcal{L}} q_{j-i}+\sum_{s\in\mathcal{S}_i} q_s+\sum_{c\in\mathcal{C}_i} q_c; \qquad \forall i\in\mathcal{J}/\{sub\} \tag{16}$$
$$\textstyle\sum_{i-j\in\mathcal{L}|i=sub} p_{i-j}=p^S; \tag{17}$$
$$\textstyle\sum_{i-j\in\mathcal{L}|i=sub} q_{i-j}=q^S; \tag{18}$$
$$-P^{UB}\le p^S\le P^{UB}; \tag{19}$$
$$-Q^{UB}\le q^S\le Q^{UB}; \tag{20}$$
$$p_b=\textstyle\sum_{s\in\mathcal{S}_b}(P_{s-b}^T-p_{s-b}^{TC})+(P_b^U-p_b^{UC}); \quad \forall b\in\mathcal{B} \tag{21}$$
$$p_s=-\textstyle\sum_{b\in\mathcal{B}_s}(P_{b-s}^T+p_{s-b}^{TC})-(P_s^U-p_s^{UC}); \ \forall s\in\mathcal{S} \tag{22}$$
$$0\le p_b^{UC}\le P_b^U; \qquad \forall b\in\mathcal{B} \tag{23}$$
$$P_s^U\le p_s^{UC}\le 0; \qquad \forall s\in\mathcal{S} \tag{24}$$
$$0\le p_{s-b}^{TC}\le P_{s-b}^T; \qquad \forall b\in\mathcal{B}_s, \forall s\in\mathcal{S}^{CN} \tag{25}$$
$$p_{s-b}^{TC}=P_{s-b}^T\cdot(1-I_{s-b}); \qquad \forall b\in\mathcal{B}_s, \forall s\in\mathcal{S}^{IN} \tag{26}$$
$$P_b^{LB}\le p_b; \qquad \forall b\in\mathcal{B} \tag{27}$$
$$P_s^{LB}\le p_s; \qquad \forall s\in\mathcal{S} \tag{28}$$

$$q_b = \left(\sqrt{1-\delta_b^2}/\delta_b\right) \cdot p_b; \qquad \forall b \in \mathcal{B} \qquad (29)$$

$$q_s = \left(\sqrt{1-\delta_s^2}/\delta_s\right) \cdot p_s; \qquad \forall s \in \mathcal{S} \qquad (30)$$

$$B_c \cdot (2 \cdot v_i - 1) - M \cdot (1 - I_c) \leq q_c \leq B_c \cdot (2 \cdot v_i - 1) + M \cdot (1 - I_c); \quad \forall c \in \mathcal{C}_i, \forall i \in \mathcal{I} \qquad (31)$$

$$0 \leq q_c \leq M \cdot I_c; \qquad \forall c \in \mathcal{C} \qquad (32)$$

$$p_{i-j} = G_{i-j} \cdot (v_i - v_j) - B_{i-j} \cdot (\theta_i - \theta_j) + (G_{i-j}/2) \cdot \textstyle\sum_{k \in \mathcal{K}} S_k \cdot \alpha_{k,i-j}; \qquad \forall i-j \in \mathcal{L} \qquad (33)$$

$$q_{i-j} = B_{i-j} \cdot (v_i - v_j) - G_{i-j} \cdot (\theta_i - \theta_j) - (B_{i-j}/2) \cdot \textstyle\sum_{k \in \mathcal{K}} S_k \cdot \alpha_{k,i-j}; \qquad \forall i-j \in \mathcal{L} \qquad (34)$$

$$\theta_i - \theta_j = \varphi_{i-j}^{PS} - \varphi_{i-j}^{NG}; \qquad \forall i-j \in \mathcal{L} \qquad (35)$$

$$\textstyle\sum_{k \in \mathcal{K}} \alpha_{k,i-j} = \varphi_{i-j}^{PS} + \varphi_{i-j}^{NG}; \qquad \forall i-j \in \mathcal{L} \qquad (36)$$

$$0 \leq \varphi_{i-j}^{PS} \leq \varphi^{UB} \cdot Z_{i-j}; \qquad \forall i-j \in \mathcal{L} \qquad (37)$$

$$0 \leq \varphi_{i-j}^{NG} \leq \varphi^{UB} \cdot (1 - Z_{i-j}); \qquad \forall i-j \in \mathcal{L} \qquad (38)$$

$$0 \leq \alpha_{k,i-j} \leq \frac{\varphi^{UB}}{K}; \qquad \forall k \in \mathcal{K}, \forall i-j \in \mathcal{L} \qquad (39)$$

$$\alpha_{k,i-j} \leq \alpha_{k-1,i-j}; \qquad \forall k \in \mathcal{K}/\{1\}, \forall i-j \in \mathcal{L} \qquad (40)$$

$$\frac{\varphi^{UB}}{K} - \alpha_{k,i-j} \leq \frac{\varphi^{UB}}{K} \cdot Y_{k,i-j}; \qquad \forall k \in \mathcal{K}/\{K\}, \forall i-j \in \mathcal{L} \qquad (41)$$

$$\alpha_{k,i-j} \leq \frac{\varphi^{UB}}{K} \cdot (1 - Y_{k-1,i-j}); \qquad \forall k \in \mathcal{K}/\{1\}, \forall i-j \in \mathcal{L} \qquad (42)$$

$$-P_{i-j}^{UB} \leq p_{i-j} \leq P_{i-j}^{UB}; \qquad \forall i-j \in \mathcal{L} \qquad (43)$$

$$V_i^{LB} \leq v_i \leq V_i^{UB}; \qquad \forall i \in \mathcal{I} \qquad (44)$$

$$\theta_{i|i=sub} = 0; \qquad (45)$$

$$v_{i|i=sub} = V^{SUB}; \qquad (46)$$

It is emphasized that the curtailment weights in (14) and the curtailment mode options as described in (25) and (26) are pre-registered by traders with the utility. Because of the presence of binary variables, such as I_{s-b} and $Y_{k,i-j}$, models (14)–(46) are a mixed-integer linear programing (MILP) problem. After solving it, the quantity of curtailment and the final power schedules of traders can be obtained.

It is worthwhile to mention that although transaction curtailments are allowed, problems (14)–(46) may still encounter infeasibility, if the system cannot supply the basic demands of the traders (i.e., P_b^{LB} of buyers or P_s^{LB} of sellers) under submitted transactions. If this occurs, slack variables can be introduced into (27)–(28) and penalized in the objective to guarantee feasibility. In addition, extra transactions with the utility can be introduced to meet the basic demands of buyers and sellers.

V. Case Studies

A. Test System Setup

The 33-bus distribution system is used to validate the proposed two-stage model. Two lines are added to the original radial system to build a looped network. All original 32 fixed loads are converted into buyers. The power lower and upper bounds of each buyer are respectively set as 25% and 150% of its original demand level. In total, 8 sellers are added into the system at different buses. Any buyer is allowed to trade with any seller, leading to 32 × 8=256 available transactions in total, and the corresponding comprehensive trading tariffs are set according to their interconnect locations. Benefit functions of buyers and cost functions of sellers all have 5 segments and are carefully turned. The selling and purchasing tariffs to traders are set as more expensive and less profitable compared with participating in peer-to-peer trading.

In the transaction approval model, weights of all transactions, including those between traders and with the utility, are set as 1, which means all transactions are considered to have equal priority. The power factor of a buyer is calculated based on its basic demand values, while sellers are considered to provide active power only with a unity power factor. Voltage magnitude of the substation bus V^{SUB} is set as 1.05 p.u. V_i^{LB} and V_i^{UB} are respectively set as 0.95 p.u. and 1.05 p.u. The curtailment mode of 10 transactions related with 1 particular seller is set as full curtailment, and all other transactions are set as partial curtailment. In addition, 2 capacitor banks are connected to the system. In the linearization model, we set φ^{UB} as 0.0349 rad (2°) and K as 30. The detailed system data can be found in reference [20].

The time window of 15 minutes is studied for the two-stage model. We solve the peer-to-peer energy trading model in a centralized manner. The LP based peer-to-peer energy trading model and the MILP based transactive approval model are implemented in MATLAB through YALMIP [21], and solved by Gurobi 9.0.0. The MILP problem is solved to be a zero MIP gap for fair comparison.

B. Analysis on Peer-to-peer Energy Trading Result

After solving the peer-to-peer energy trading model, the cleared transactions can be obtained. The results are compared with those from the traditional utility dominated tariff-based business model, in which all buyers and sellers can only trade with the utility at the selling and purchasing tariff. The comparison is summarized in Table I.

TABLE I

SUMMARIZED RESULT OF THE 33-BUS SYSTEM (KWH)

Business Model	Total energy **to** (15 minutes)		Total energy **from** (15 minutes)	
	Buyers	Utility	Sellers	Utility
Peer-to-peer	590	0	590	0
Traditional	-	250	-	232

It can be seen that, in the peer-to-peer energy trading model, all cleared transactions are between traders, and the total amount of traded energy is much higher than that with the traditional business model, showing a more active market. The total 590 kWh of traded energy from the peer-to-peer business model is made up of 39 transactions. Although the difference of traded energy between the two business models will be affected by the specified tariff settings, there is no doubt that the peer-to-peer trading business model can provide higher incentives to encourage more energy consumption.

C. Analysis on the Clearing Price

The clearing prices of transactions to individual buyers are shown in Fig. 3. It can be seen that the clearing prices of transactions to individual sellers are different, but are all lower than the selling tariff. It indicates that the buyers encounter cost savings compared to directly trading with the utility. This observation of lower clearing prices may not be general to all cases. In fact, the clearing price of a transaction could be higher than the selling tariff; however, the corresponding transaction quantity would be zero, i.e., no deal will be made between the buyer and the seller of this transaction. Fig. 4 shows the prices ($\lambda_{b-s} - C_{b-s}^{T}$) at which the sellers will be paid. Prices of a seller with different buyers are the same, because the benefit and cost functions are step functions. Similarly, we can see that these prices are higher than the purchasing tariff from the utility, which means the sellers can gain more profits through peer-to-peer energy trading.

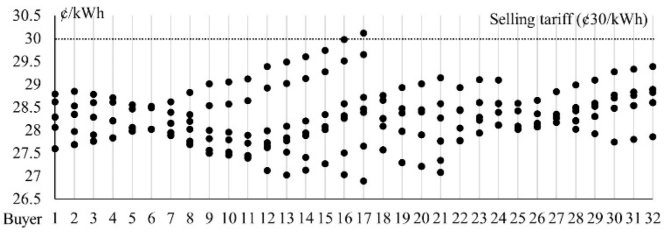

Fig. 3. Clearing prices of transactions to individual buyers.

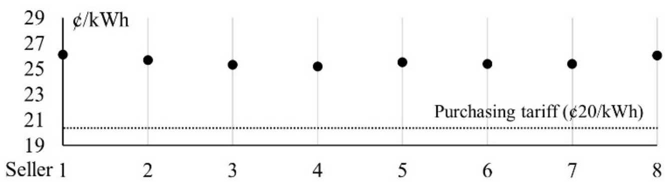

Fig. 4. Prices of transactions to individual sellers.

D. *Impacts of a Comprehensive Trading Tariff*

Intuitively, the comprehensive trading tariff would impact the cleared transaction quantities as well as the clearing prices. Considering the setup in the above sections B and C as the base case, we gradually increase the comprehensive trading tariff from +0% (i.e., base case) to +100% with the step-up being 10%. Fig. 5 and Fig. 6 show the total cleared transaction quantities and the average clearing price over all transactions between traders for those cases.

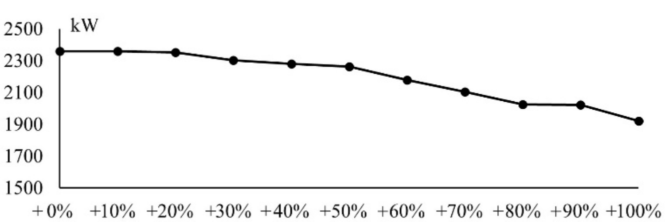

Fig. 5. Total cleared transaction quantities between traders.

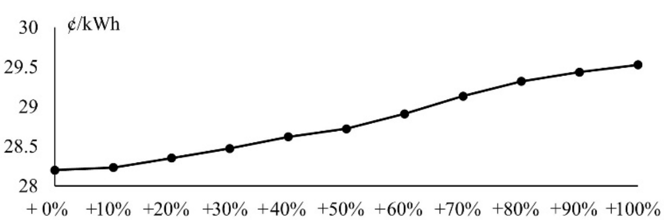

Fig. 6. Average clearing price over all transactions.

Unsurprisingly, it can be seen that as the comprehensive trading tariff gradually increases, the total cleared transaction quantity reduces, and the clearing price increases. When the comprehensive trading tariff increases to a certain level, the average clearing price could be even higher than the selling tariff. Quantities of most transactions are zero, because clearing prices are higher than the selling tariff.

It should be emphasized that the setting of the comprehensive trading tariff shall reflect the true hidden costs of the utility on each transaction, i.e., shall be sufficient to cover the related financial losses. Reference [22] provided an idea that the sensitivities of the financial losses against active power injections can be used to estimate the incremental losses and determine the corresponding tariff. A systematic way to reasonably set the trading tariff will be explored in a future study.

E. *Transaction Approval*

The cleared transactions from the peer-to-peer energy trading model will be submitted to the utility for verification via the transaction approval model. The transaction approval model is solved in 23.27 seconds. The result shows that 6 out of the 39 transactions are curtailed, and the total amount of curtailment is 239.35 kW, triggered by potential violations on the voltage upper bounds (44). Fig. 7 shows the voltage magnitude profile if all cleared transactions are fully realized. It can be seen that the voltage magnitudes of buses 20–22 are above the upper bound of 1.05 p.u. This is because 3 sellers are connected at these buses, and their power injections raise the voltage magnitudes. After applying the curtailment, the voltage magnitudes of these three buses are contained within the limitations, as shown in Fig. 8.

After solving the transaction approval model, the system losses can be obtained. The active power loss is compensated via a 10.51 kW power injection through the substation bus. Because the sellers do not provide reactive power, 1123.23 kVar reactive power is injected through the substation bus to supply buyers. System loss is extremely low because power is supplied locally by DERs. In addition, capacitor banks are all switched OFF to avoid potential violations on the voltage upper bound.

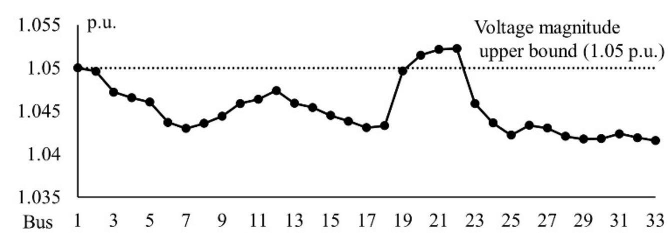

Fig. 7. The voltage profile if no transactions are curtailed.

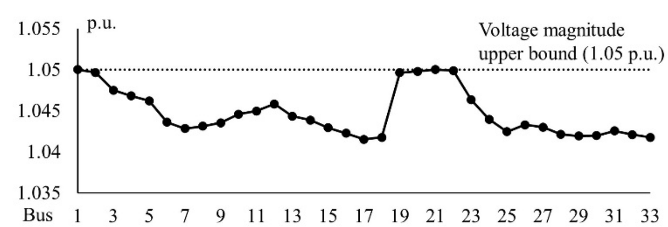

Fig. 8. The voltage profile after applying curtailment.

After solving the proposed transaction approval model, power injections to buses can be calculated, in which an AC power flow problem is solved with the Newton-Raphson method to derive the accurate system state. To verify the linearization error of the adopted power flow model, bus voltage magnitudes calculated from the transaction approval model are compared with those from the AC power flow solution. The maximum absolute error is merely 3.44×10^{-4} p.u. and the maximum relative error is only 0.033%, showing a high accuracy of the linearized power flow model.

F. Case Study on Revenue

Revenues of buyers, sellers, and the utility from the proposed two-stage peer-to-peer energy trading model and the traditional utility dominated tariff-based business model are compared in Table II. Within the former, the cases with and without applying the comprehensive trading tariff are further studied. From the revenue volume, it is verified again that the peer-to-peer energy trading encourages a much more active market. In addition, although the price difference of selling and purchasing tariff is relatively large (¢10/kWh), the arbitrage profit of the utility is limited due to shrinking transactions.

Comparing the cases with and without applying the comprehensive trading tariff, the latter enables a more active market with a higher revenue, which is consistent the observations from Fig. 5 and Fig. 6. From Table I, since all buyers are supplied by sellers, the utility will not profit from transactions. Indeed, the arbitrage profit of the utility has been transferred to savings of buyers and profits of sellers through peer-to-peer energy trading. The comprehensive trading tariff is designed to extract a part of the deprived profit to compensate the utility's hidden costs.

TABLE II

REVENUE BREAKUP

Revenue ($)		Traditional	Comprehensive trading tariff	
			Not applied	Applied
Buyer payment		0.00	179.66	162.73
Seller paid		0.00	179.66	150.99
Utility	Payment	75.00	0.00	0.00
	Paid	50.00*	0.00	0.00
	Profit	25.00	0.00	0.00
	Compensation		0.00	11.74

* Energy price from wholesale market is set as ¢20/kWh.

G. Case Study on the Modified IEEE 123-Bus System

To further validate performance of the proposed two-stage model in terms of computational efficiency and linearization accuracy, a new case study on the modified IEEE 123-bus distribution system is conducted. This study includes 85 buyers converted from fixed loads and 10 newly added sellers, leading to 85×10=850 potential transactions between the buyers and sellers. Settings on benefit functions, cost functions, and comprehensive trading tariffs are carefully determined. In addition, settings for voltages and the linearization model are the same as the 33-bus system. The detailed system data can be found in reference [20].

The results of the peer-to-peer energy trading model and the traditional utility dominated tariff-based business model are compared in Table III. The same observation that the peer-to-peer energy trading model enables a more active market can be made. In this case, 94 transactions are cleared in the peer-to-peer energy trading model, with a total amount of 2820 kW power from sellers to supply all buyers for the 15-minute trading interval.

The clearing prices of transactions to buyers are shown in the radar chart of Fig. 9. It can be seen that clearing prices of transactions for buyers vary less significantly than in the 33-bus distribution system. The majority of them are between ¢26/kWh and ¢28/kWh, and all are lower than the selling tariff from the utility. The buyers can save costs through transactions with sellers. As observed earlier, prices of a seller to different buyers are the same and all are above the purchasing tariff.

Table III

SUMMARIZED RESULT OF 123-BUS SYSTEM

Business Model	Total energy **to** (15 minutes)		Total energy **from** (15 minutes)	
	Buyers (kWh)	Utility (kWh)	Sellers (kWh)	Utility (kWh)
Peer-to-peer	705	0	705	0
Traditional	-	218		300

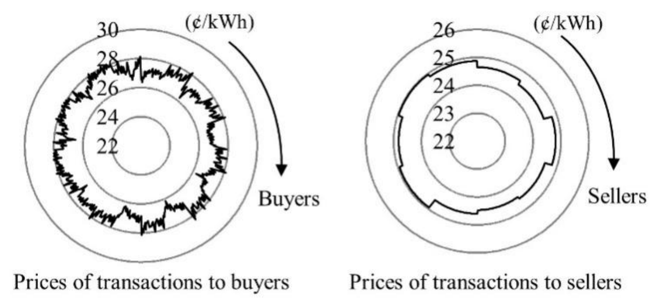

Fig. 9. Prices of transactions to buyers and sellers.

The transaction approval model approves all the 94 cleared transactions, and schedules a 7.80 kW power injection through the substation bus to balance system losses and 1475.26 kVar reactive power injection to meet the buyers' reactive power demand. The transaction approval model is solved in 2.94 seconds, which is even faster than the 33-bus system. This is because no physical constraints are binding. Compared with the power flow result, the maximum absolute error on the bus voltage magnitude is 5.61×10^{-4}p.u., and the maximum relative error is 0.056%. It can be seen that the linearization accuracy is slightly reduced compared to the 33-bus system. The overall accuracy is acceptable for practical applications.

VI. CONCLUSION

In recognizing that the emerging peer-to-peer trading model presents a better potential to promote a deeper DER penetration than the traditional tariff-based business model, a two-stage model is proposed in this paper, including a peer-to-peer trading model and a utility transaction approval model. The former optimizes transactions between traders and with the utility, and the latter verifies that the cleared transactions will not violate physical network limitations. Numerical case studies clearly show that the proposed peer-to-peer trading model can provide effective incentives to promote energy trading and energy consumption. In addition, when potential physical network violation occurs, it can effectively curtail certain transactions to secure system operations. The clearing prices of transactions for different buyers and sellers would vary, but they can bring higher financial benefits to buyers and sellers than the traditional tariff-based business model. The comprehensive trading tariff is one of the key factors affecting the peer-to-peer trading results, and a systematic way to determine it will be explored in our future study.

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