2023 IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm) | 978-1-6654-5556-5/23/$31.00 ©2023 IEEE | DOI: 10.1109/SmartGridComm57358.2023.10333939

Inexactness of Second Order Cone Relaxations for Calculating Operating Envelopes

Hannah Moring
Electrical and Computer Engineering
University of Michigan
Ann Arbor, MI, USA
hmoring@umich.edu

Johanna L. Mathieu
Electrical and Computer Engineering
University of Michigan
Ann Arbor, MI, USA
jlmath@umich.edu

***Abstract*—As the number of distributed energy resources participating in power networks increases, it becomes increasingly more important to actively manage network constraints to ensure safe operation. One proposed method that has gained significant attention and implementation, particularly in Australia, is the use of dynamic operating envelopes. Operating envelopes represent net export limits set by the system operator on every node in the distribution network that change as system conditions change. They are calculated using an optimal power flow problem, frequently using a linearization or relaxation of the nonlinear power flow equations. This paper presents two case studies and some numerical analysis to explain why a second order cone relaxation of the power flow equations will lead to ineffective operating envelopes. A modification to the objective function which allows the second order cone relaxation to nearly recover the solution to the nonlinear formulation is also presented.**

***Index Terms*—Distributed Energy Resources, Operating Envelopes, Safe Operation**

I. Introduction

The number of small-scale distributed energy resources (DERs), like roof-top solar photovoltaics (PV), batteries, and electric vehicles, connected to distribution networks has been growing rapidly in recent years [1]. DERs will play an important role in the decarbonization of the electricity sector, as they provide needed flexibility [2]. This flexibility refers to the capacity to adjust power production/demand to maintain safe network operations. Individually, DERs are too small to provide significant flexibility to the system, but aggregated together they can have a much bigger impact. Active participation by DERs and DER aggregations in the distribution network can lead to network violations like over- and under-voltages [3], [4]. Numerous solutions have been proposed to maintain safe operations in distribution networks under the presence of DERs and DER aggregations, including coordination strategies between aggregators and distribution network operators (DNOs) [5], constructing a constraint set on aggregator controls [6], constructing a convex inner approximation of the optimal power flow (OPF) to quantify feeder capacity [7], and constructing dynamic operating envelopes [8]–[11].

Operating envelopes represent the import and/or export limits at each active node in the distribution network that would prevent unsafe operations [11]. An active node is defined as any node within the distribution network that has a controllable DER capable of exporting or importing power connected to it. An efficient way to do this is to solve a modified version of the OPF problem. The OPF problem is an optimization problem which was originally formulated to find the lowest cost generator dispatch that lies within network constraints [12]. Since its introduction, it has been generalized so that OPF can refer to any optimization problem subject to the power flow equations and other operational constraints [13]. The OPF problem is nonlinear and non-convex, making it difficult to solve and computationally intractable for large networks. The use of convex relaxations of the power flow equations can alleviate this issue. However, relaxations do not always give exact or feasible solutions [14]. Here we investigate how they work for operating envelope problems, which have been proposed to ensure safe operations of active distribution networks.

The objective of this paper is to highlight issues that can arise when using a SOC relaxation of the power flow equations to calculate operating envelopes. A review of existing literature suggests that either a SOC relaxation or a linearized version of the power flow equations is typically utilized in the calculation of operating envelopes. For example, the current injection-based three-phase power flow equations are linearized about a predefined voltage point using a first-order Taylor series in [9], [15]. In [11], the three-phase branch flow power flow equations are linearized about an estimate of the real and reactive power flows. Both of these linearization techniques require an estimate or measurement of either voltage or power and may run into problems if the estimates are inaccurate or if there is a significant change in the system state. Alternatively, a second-order cone (SOC) relaxation of the power flow equations was used in [10]. A SOC relaxation provides computational tractability, but does not require estimates or measurements like linearization-based approaches. However, SOC relaxations do not always produce feasible solutions to the original nonlinear problem.

There are two main contributions of this paper. The first is a discussion of the use of the SOC relaxation of the power flow equations to calculate operating envelopes. Specifically, the paper outlines why the SOC relaxation of the power flow equations will not produce feasible solutions to the original nonlinear operating envelope problem and should not be used

This work was supported by NSF Grant 1845093.

to calculate operating envelopes. Two case studies illustrate this and a discussion of the numerical results is presented. The second contribution of the paper is a modification to the objective function of the SOC formulation that results in a solution very close to the solution of the full nonlinear problem.

Section II presents the paper's notation and OPF formulations for calculating operating envelopes. The modified objective function and discussion of its merits are given in Section II-C. Section III presents two case studies to illustrate issues with using the SOC relaxation within the operating envelope problem. Conclusions are given in Section IV.

II. Problem Formulations

Consider a radial distribution network with a set of nodes $\mathcal{N}$ and a set of lines $\mathcal{L}$. For simplicity, we assume the network is balanced and therefore can be represented by its single-phase equivalent circuit. We will use the branch flow method for representing power flow [14], where the voltage and current angles of the system can be omitted by writing the power flow and voltage difference equations in terms of the voltage and current magnitudes squared. Let $z_{ij} = r_{ij} + jx_{ij}$ represent the impedance on the line connecting nodes i and j. The reactive power demand at node i is q_i^{d}. The apparent power flow limit on the line connecting nodes i and j is $\overline{s}_{ij}$. The per unit squared voltage limits at each bus are $\underline{v}, \overline{v}$.

Equity or fairness is an important consideration when formulating an optimization problem to find operating envelopes [9]–[11], [15]. When a linear objective function is used to maximize the aggregated operating envelopes, nodes further down the feeder receive significantly smaller allocations [11]. This means that a customer with PV who is far from the feeder head will have less ability to sell their excess power production than would a customer who is close to the feeder head. Therefore, the operating envelope formulation in this paper utilizes the objective function proposed in [10], which maximizes the smallest operating envelope to allocate operating envelopes fairly across active nodes. We also analyzed a version of the formulation presented in this paper with the objective of maximizing the sum of the operating envelopes across the feeder. The results were similar in terms of inexactness and infeasibility of the solution with respect to the original nonlinear problem. Those results are not included for brevity.

A. AC-OPF Branch Flow Formulation

We first define the full nonlinear operating envelope problem, which leverages the AC power flow equations. This formulation is based on the formulation given in [10] but differs in that we do not explicitly define the operating envelopes as the difference between PV generation and load at every node. Rather, we define the operating envelopes as the net export at every node. Additionally, we do not model an upper limit on the operating envelopes based on DER capacity.

Our decision variables are $\mathbf{p}^{\exp}$, a vector with elements $p_i^{\exp}$ representing the operating envelope at each node i, i.e., the maximum net real power export at each node i; $\underline{p}^{\exp}$, the smallest magnitude operating envelope in the network; $\mathbf{P}$, a matrix of active branch power flows with elements p_{ij} representing the active power flowing from node i to node j; $\mathbf{Q}$, a matrix of reactive branch power flows with elements q_{ij} representing the reactive power flowing from node i to node j; $\mathbf{v}$, a vector with elements representing the squared voltage magnitude $v_i = |V_i|^2$ at each node i; and $\mathbf{L}$, a matrix of squared current magnitudes with elements $l_{ij} = |I_{ij}|^2$ representing the squared current magnitude on the line connecting nodes i and j. Let $\mathbf{x}$ be a vector of stacked decision variables. We compute operating envelopes $\mathbf{p}^{\exp}$ by solving,

$$\max_{\mathbf{x}} \ \underline{p}^{\exp} \tag{1a}$$
$$\text{s.t. } \underline{p}^{\exp} \leq p_i^{\exp}, \quad \forall i \in \mathcal{N} \tag{1b}$$
$$\sum_{i:i\to j} (p_{ij} - r_{ij} l_{ij}) + p_j^{\exp} = \sum_{k:j\to k} p_{jk}, \forall j \in \mathcal{N} \tag{1c}$$
$$\sum_{i:i\to j} (q_{ij} - x_{ij} l_{ij}) - q_j^{\mathrm{d}} = \sum_{k:j\to k} q_{jk}, \forall j \in \mathcal{N} \tag{1d}$$
$$v_i = v_j + 2(r_{ij}p_{ij} + x_{ij}q_{ij}) - (r_{ij}^2 + x_{ij}^2) l_{ij}, \forall i \in \mathcal{N} \tag{1e}$$
$$p_{ij}^2 + q_{ij}^2 = l_{ij} v_i, \quad \forall ij \in \mathcal{L} \tag{1f}$$
$$p_{ij}^2 + q_{ij}^2 \leq \overline{s}_{ij}^2, \quad \forall ij \in \mathcal{L} \tag{1g}$$
$$\underline{v} \leq v_i \leq \overline{v}, \quad \forall i \in \mathcal{N} \tag{1h}$$

Constraint (1b) transforms a $\max\min$ objective into a linear objective by defining the smallest operating envelope to be maximized. Constraints (1c) and (1d) enforce active and reactive power balance, where notation $i : i \to j$ specifies that we should sum over all lines ij injecting power into j, and $k : j \to k$ specifies that we should sum over all lines jk consuming power from j. Constraint (1e) defines the voltage drop between bus i and the downstream bus j. Constraint (1f) defines the squared apparent power flowing from bus i to j and (1g) limits it. Finally, constraint (1h) enforces the voltage limits at each bus. This formulation assumes monotonicity, i.e., if $p^{\exp}$ defines a valid operating envelope, then any injection less than $p^{\exp}$ is feasible. We note that this may not always be true in practice since (1f) is nonconvex.

With the exception of the objective and constraint (1b), problem (1) is a standard OPF formulation. Constraint (1f) is non-convex, making the problem difficult to solve and impractical to scale to large systems. For this reason, operating envelopes are generally calculated using a linearization or relaxation of the power flow equations [9]–[11], [15].

B. Second-Order Cone Relaxation

The SOC relaxation was first proposed for the branch flow model in [14]. The only difference from the full nonlinear formulation (1) is that constraint (1f) is replaced with the SOC constraint

$$\left\| \begin{matrix} 2p_{ij} \\ 2q_{ij} \\ l_{ij} - v_i \end{matrix} \right\|_2 \leq l_{ij} + v_i, \qquad \forall ij \in \mathcal{L}, \tag{2}$$

which is equivalent to $p_{ij}^2 + q_{ij}^2 \leq l_{ij} v_i$, a relaxation of (1f). Then, the SOC formulation is

$$\max \quad \underline{p}^{\exp} \tag{3a}$$
$$\text{s.t.} \quad (1b) - (1e), (1g), (1h), (2). \tag{3b}$$

The solution given by this formulation is exact, i.e., it is the solution to the full nonlinear problem, if it is feasible in the full nonlinear problem, i.e., $p_{ij}^2 + q_{ij}^2 = l_{ij} v_i$ for every $ij \in \mathcal{L}$. For a solution of the SOC problem that is not exact, the difference between the left-hand and right-hand sides of this equation can be used to quantify the inexactness of the solution.

While this SOC relaxation has been successfully used for a variety of OPF problems, the SOC relaxation is problematic when used in operating envelope problems, as we will show in Section III. We next give some intuition for why this is the case. In this SOC formulation, there is no direct calculation of squared current l_{ij}. Instead it is constrained by (1c)-(1e), (2), and indirectly by the remaining constraints. Some upper and lower bounds on the squared currents can be derived from these constraints. For example, (2) gives us a lower bound

$$l_{ij} \geq \frac{p_{ij}^2 + q_{ij}^2}{v_i}, \tag{4}$$

and (1e) together with (1h) give us an upper bound

$$l_{ij} \leq \frac{\underline{v} - v_j - 2(r_{ij} p_{ij} + x_{ij} q_{ij})}{(r_{ij}^2 + x_{ij}^2)}, \tag{5}$$

and a lower bound

$$l_{ij} \geq \frac{\overline{v} - v_j - 2(r_{ij} p_{ij} + x_{ij} q_{ij})}{(r_{ij}^2 + x_{ij}^2)}. \tag{6}$$

Further bounds can be derived from the other constraints. Within these bounds, the squared currents can take any value. However, only one of the possible values is a solution to the power flow equations for a given voltage, real power flow, and reactive power flow, i.e., the value that satisfies (1f). The SOC formulation will not find that solution because larger currents can result in calculated feasible voltages corresponding to larger operating envelopes. Specifically, the flexibility introduced by the SOC relaxation of the power flow equations allows for artificial increases in the squared current. A larger squared current in (1c) permits larger values of $p^{\exp}$ without significant changes to real power flow, and simultaneously, decreases the change in voltage between node j and node i in (1e). Hence, this flexibility will lead to operating envelopes that are too large to be effective.

Furthermore, the objective of maximizing the smallest operating envelope across the network does not meet the conditions needed to prove the exactness of a SOC relaxation of an OPF formulation utilizing the branch-flow model outlined in [14]. Namely, the cost function is not strictly decreasing in current or line losses.

C. Objective Function Modification

In this section, a possible method for improving the efficacy of operating envelopes calculated using SOC relaxations of the power flow equations is presented.

As mentioned in the previous section, the solution from the SOC formulation takes advantage of the flexibility in the squared current variables to permit larger operating envelopes, which fail to ensure safe operation of the actual (nonlinear) power network. A possible remedy for this is to penalize power losses in the objective, as they are a function of current squared. We believe that this makes the objective function strictly decreasing in both current and line losses, satisfying conditions in [14]. The simplest augmented objective function would be

$$\max_{\mathbf{x}} \; \underline{p}^{\exp} - \sum_{ij \in \mathcal{L}} l_{ij} r_{ij}. \tag{7}$$

However, differences in magnitude between the two terms can lead to undesirable outcomes. Because the losses term is a sum over the entire network, it can have a greater magnitude than $\underline{p}^{\exp}$. If so, the solution will prioritize loss minimization, and therefore power flowing through each branch, rather than maximizing operating envelopes.

In order to achieve a solution close to the full nonlinear operating envelope problem solution, we can add a weight to the losses term such that the augmented objective function becomes

$$\max_{\mathbf{x}} \; \underline{p}^{\exp} - \lambda \sum_{ij \in \mathcal{L}} l_{ij} r_{ij}, \tag{8}$$

where λ is a weighting parameter found heuristically.

The issue with this approach is that the solution to the full nonlinear operating envelope problem must be known in order to precisely tune the weighting parameter. However, it can be possible to approximately tune the parameter without prior knowledge of the full nonlinear solution using the inexactness as a guide, as we will show in Section III. We note that the best choice of weighting parameter is dependent on the system loading conditions, meaning the parameter must be re-tuned as the system conditions change. This tuning process could be challenging in real time.

III. Case Study

A. Setup

In this section, two case studies will be used to highlight the implications of using the SOC relaxation of the operating envelope problem. The first case study will be on a simple 4-bus network, based on the MATPOWER "case4_dist" test case [16], where numerical results will provide clear insights on the impacts of using a SOC relaxation. Subsequently, a 56-bus network, which was modified from the IEEE 123-bus network and presented in [17], will be used to illustrate the extent of the problem in a larger, more realistic network. The 4-bus network is shown in Fig. 1(a) and the 56-bus network is shown in Fig. 1(b). Note that the substation is considered to be the last bus, or bus 4 in the 4-bus network and bus 56 in the 56-bus network. In both cases, bus 1 is the bus

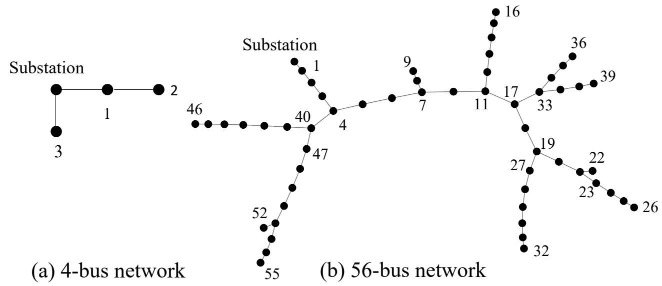

Fig. 1. One line diagrams of the (a) 4-bus network, based on a network available from [16], and (b) 56-bus network presented in [17].

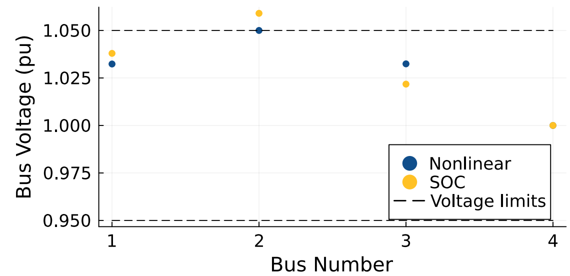

Fig. 2. The voltage at each bus in the 4-bus network resulting from solving AC power flow with power exports at each node equal to the operating envelope limits calculated using the nonlinear and SOC operating envelope formulations.

connected to the substation. In this study, we assume there are no capacitor banks in the network and we assume there are no voltage regulators except for one at the substation. The voltage at the substation is set to 1.0 p.u. For simplicity, we assume that there is a controllable DER at each bus. This means that power can be injected at every bus and therefore every bus needs an operating envelope. Voltages between 0.95 and 1.05 p.u. are considered safe. Any operation that leads to values below or above these limits will be considered unsafe.

To show how the SOC relaxation of the power flow equations impacts the calculation of operating envelopes, both the full nonlinear operating envelope formulation and the SOC formulation will be used to calculate operating envelopes for each node in the network. The solutions will be compared in terms of the magnitude of the obtained operating envelopes, and the exactness of the SOC relaxation will be discussed. To analyze the efficacy of the resulting operating envelopes, the nonlinear power flow equations will be solved for each formulation assuming the net power inject at each node is equal to the upper limit set by the operating envelopes.

Both the nonlinear and SOC problems were solved in Julia using the JuMP package [18]. The solvers used were IPOPT and CPLEX 12.7.1 for the nonlinear and SOC problems, respectively.

B. Results

This section presents the solutions to the full nonlinear operating envelope formulation and the SOC formulation of the operating envelope problem for both the 4-bus and the 56-bus networks. Also presented in this section are the results for solving the AC power flow equations given the net real power injections corresponding to the limits of the operating envelope at each bus in the network.

1) 4-Bus Network: The results in this section are for the 4-bus network. Under the full nonlinear operating envelope formulation, the operating envelopes found are 6.84 MW at buses 1 and 2, and 12.47 MW at bus 3. When the active power injection into the network is equal to these operating envelopes, the voltages at each bus correspond to the values shown in Fig. 2. As can be seen in the figure, the voltage at every bus is within safe limits. Using the SOC relaxation, the operating envelopes found are 8.28 MW at buses 1 and 2, and 10.19 MW at bus 3. The bus voltages resulting from solving the AC power flow when power equal to the corresponding operating envelope is injected at buses 1-3 in the network are also shown in Fig. 2. The operating envelopes found by the SOC relaxation do not prevent unsafe operations.

TABLE I
NUMERICAL RESULTS FOR SOC FORMULATION

Bus	1	2	3
Operating Envelope (MW)	8.28	8.28	10.19
Voltage (p.u.)	1.015	1.031	1.026
Line	4-1	1-2	4-3
Active Power (MW)	-13.414	-7.283	-9.810
Reactive Power (MW)	6.713	2.204	0.978
Current (kA)	26.797	18.278	11.386

Fig. 3 shows the difference between $l_{ij}v_i$ and $p_{ij}^2 + q_{ij}^2$ as a reflection of the inexactness of the relaxation. The values for each of the variables in the SOC formulation solution are given in Table I and the results from solving the AC power flow assuming the active power injections are equal to the operating envelopes found by the SOC relaxation are given in Table II. The values for each of the variables in the full nonlinear operating envelope problem solution are given in Table III. A comparison of the three values for each variable can be used to illustrate why the SOC relaxation is not exact, and why the resulting operating envelopes are ineffective. Compared to the SOC formulation, the AC power flow using the operating envelopes generated from the SOC formulation results in larger active power flows through the branches, smaller reactive power flows through the branches, smaller current magnitudes through the branches, and higher voltage magnitudes at the buses. Using these trends and (1c), it can be deduced that the SOC relaxation is using the flexibility in the current values to increase the operating envelopes by increasing the line losses, which are represented by $r_{ij}l_{ij}$ in (1c). The total system losses in the solution to the full nonlinear operating envelope problem are 1.085 MW. In comparison, the total system losses in the solution to the SOC problem and the AC power flow using the resulting operating envelopes are 3.545 and 1.196 MW, respectively.

2) 56-Bus Network: The following results are from the 56-bus network. Under the full nonlinear operating envelope formulation, the operating envelope at every node is equal to

TABLE II
NUMERICAL RESULTS FOR AC POWER FLOW USING OPERATING ENVELOPES GENERATED FROM SOC FORMULATION

Bus	1	2	3
Operating Envelope (MW)	8.28	8.28	10.19
Voltage (p.u.)	1.038	1.059	1.026
Line	4-1	1-2	4-3
Active Power (MW)	-15.638	-8.102	-9.903
Reactive Power (MW)	2.265	0.567	0.792
Current (kA)	15.507	7.890	9.804

TABLE III
NUMERICAL RESULTS FOR FULL NONLINEAR OPERATING ENVELOPE FORMULATION

Bus	1	2	3
Operating Envelope (MW)	6.84	6.84	12.47
Voltage (p.u.)	1.032	1.050	1.032
Line	4-1	1-2	4-3
Active Power (MW)	-13.045	-6.718	-12.040
Reactive Power (MW)	1.693	0.455	1.076
Current (kA)	13.155	6.523	12.088

161.5 kW. When every node is injecting 161.5 kW of power into the network, the voltages at each bus correspond to the values shown in Fig. 4. As can be seen in the figure, the voltage at every bus is within safe limits. A few of the buses far away from the substation are at the upper voltage limit, but this is to be expected at the edge of safe operation.

When the SOC relaxation of the power flow equations is used with the same objective function, the solution found gives an operating envelope at every node equal to 627.9 kW. This value is significantly larger than the value given by the full nonlinear operating envelope formulation. The bus voltages resulting from solving the AC power flow when power equal to the corresponding operating envelope is injected at each bus in the network are also shown in Fig. 4. It is clear from the figure that these operating envelopes do not prevent unsafe operation. Using the limits calculated from the SOC relaxation would likely lead to extremely high over-voltages.

These unsafe voltages calculated according to the nonlinear power flow equations do not match the voltages given as part of the solution to the SOC operating envelope problem. The difference between the voltages calculated from the SOC formulation and from the AC power flow equations arises from the dependence on current. Fig. 5 shows the difference

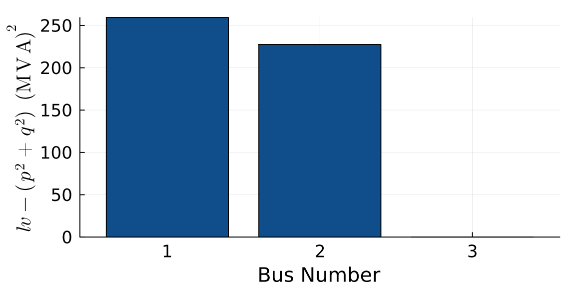

Fig. 3. The inexactness of the SOC relaxation of the power flow equations for the 4-bus network.

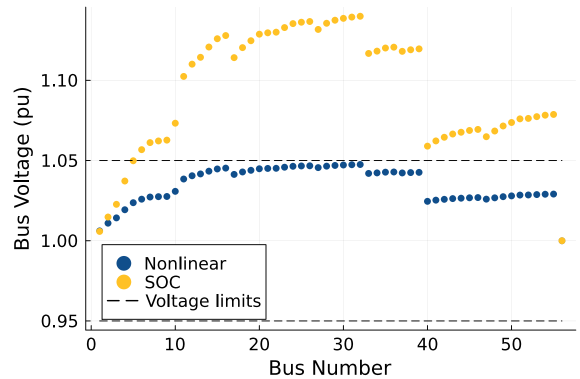

Fig. 4. The voltage at each bus in the 56-bus network resulting from solving AC power flow with power exports at each node equal to the operating envelope limits calculated using the nonlinear and SOC operating envelope formulations.

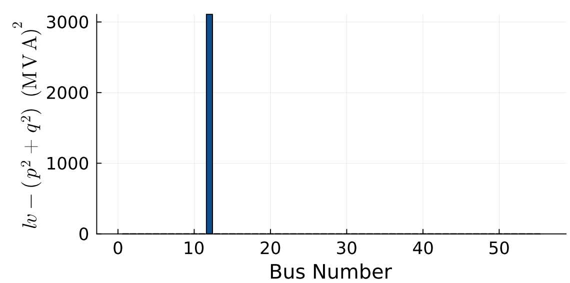

Fig. 5. The inexactness of the SOC relaxation of the power flow equations for the 56-bus network.

between $l_{ij}v_i$ and $p_{ij}^2 + q_{ij}^2$ for each bus in the network. If the SOC relaxation were exact and the power flow equations were satisfied, the difference at each bus would be zero. As can be seen in the figure, that is not the case. Given that the calculated voltages must be between 0.95 and 1.05 p.u. to be a feasible solution to (3), the very large differences must arise from large current values. These large currents would lead to large real power losses. The total system losses in the solution to the full nonlinear operating envelope problem and in the solution to the SOC problem are 0.491 and 13.756 MW, respectively.

If the objective function for the SOC formulation is modified to take the form of (8) with $\lambda = 0.15$, the resulting operating envelopes have maximum export limits of 161.5 kW at each bus, nearly matching the nonlinear solution. Fig. 6 shows the bus voltages resulting from solving the AC power flow when power equal to the corresponding operating envelope generated with the modified SOC formulation is injected at each bus in the network. Also, Fig. 7 shows the inexactness of the modified SOC formulation; it is very small compared to that of Fig. 5. Fig. 8 shows the sum of the inexactness as the weighting parameter is increased. It also shows the resulting operating envelope for each bus (i.e., all buses have the same operating envelope). In this case, to achieve low inexactness with relatively large operating envelopes, one should choose λ between 0.1 and 0.17. Choosing a value of λ higher will lead to overly conservative operating envelopes, but choosing

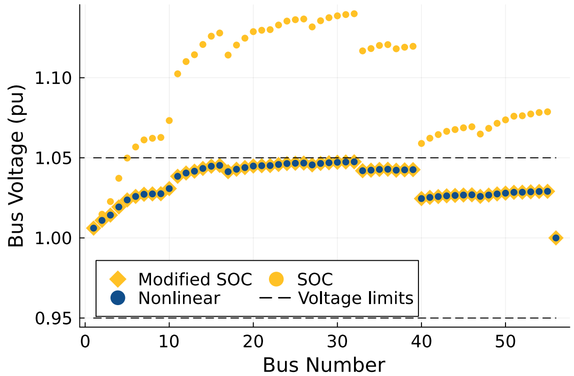

Fig. 6. The voltage at each bus in the 56-bus network resulting from solving AC power flow with power exports at each node equal to the operating envelope limits calculated using the nonlinear, original SOC, and modified SOC operating envelope formulations.

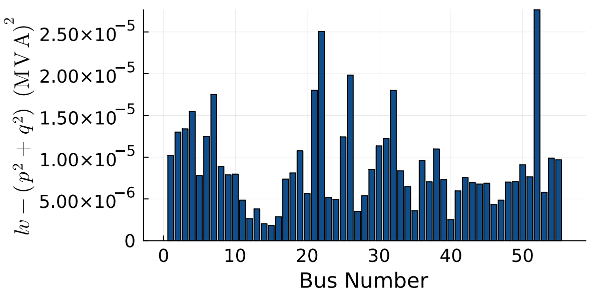

Fig. 7. The inexactness of the modified SOC relaxation of the power flow equations for the 56-bus network. Note the different y-axis scale from Fig. 5.

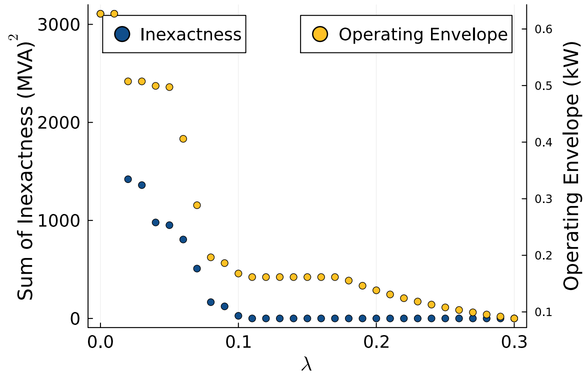

Fig. 8. The sum of the inexactness, i.e., $lv-(p^2+q^2)$, at every bus, and the resulting operating envelope for each bus as the weighting parameter λ is increased.

a smaller λ will lead to unsafe operating envelopes.

IV. Conclusions

This paper presented two case studies to illustrate the issues that can arise when a SOC relaxation of the power flow equations is used to compute operating envelopes. Accompanying the case studies is a numerical explanation as to why the SOC relaxation leads to ineffective operating envelopes. In brief, the SOC relaxation introduces flexibility in what values the branch currents can take. This flexibility allows for larger operating envelopes with calculated voltages within the limits. However, this flexibility does not exist in real power flow, which is governed by the nonlinear power flow equations, and so the large operating envelopes result in unsafe voltages in reality.

A potential fix to the issues discussed was also presented. By modifying the objective function to also minimize a weighted sum of the system losses, the true optimal solution can nearly be recovered. The flaw in this method is that knowing how to tune the weighting parameter relies on knowing the solution to the nonlinear problem.

References

[1] IEA, "Unlocking the potential of distributed energy resources," IEA, Tech. Rep., 2022.

[2] P. D. Lund, J. Lindgren, J. Mikkola, and J. Salpakari, "Review of energy system flexibility measures to enable high levels of variable renewable electricity," *Renewable and Sustainable Energy Reviews*, vol. 45, pp. 785–807, 2015.

[3] M. Ebad and W. M. Grady, "An approach for assessing high-penetration PV impact on distribution feeders," *Electric Power Systems Research*, vol. 133, pp. 347–354, 2016.

[4] X. Chen, E. Dall'Anese, C. Zhao, and N. Li, "Aggregate power flexibility in unbalanced distribution systems," *IEEE Transactions on Smart Grid*, vol. 11, no. 1, pp. 258–269, 2020.

[5] S. Ross and J. Mathieu, "Strategies for network-safe load control with a third-party aggregator and a distribution operator," *IEEE Transactions on Power Systems*, vol. 36, no. 4, pp. 3329–3339, 2021.

[6] S. Jang, N. Ozay, and J. L. Mathieu, "Data-driven estimation of probabilistic constraints for network-safe distributed energy resource control," in *58th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, 2022, pp. 1–8.

[7] N. Nazir and M. Almassalkhi, "Grid-aware aggregation and realtime disaggregation of distributed energy resources in radial networks," *IEEE Transactions on Power Systems*, vol. 37, no. 3, pp. 1706–1717, 2022.

[8] M. Z. Liu, L. F. Ochoa, P. K. C. Wong, and J. Theunissen, "Using OPF-based operating envelopes to facilitate residential DER services," *IEEE Transactions on Smart Grid*, vol. 13, no. 6, pp. 4494–4504, 2022.

[9] K. Petrou, A. T. Procopiou, L. Gutierrez-Lagos, M. Z. Liu, L. F. Ochoa, T. Langstaff, and J. M. Theunissen, "Ensuring distribution network integrity using dynamic operating limits for prosumers," *IEEE Transactions on Smart Grid*, vol. 12, no. 5, pp. 3877–3888, 2021.

[10] Y. Yi and G. Verbič, "Fair operating envelopes under uncertainty using chance constrained optimal power flow," *Electric Power Systems Research*, vol. 213, p. 108465, 2022.

[11] M. Z. Liu, L. N. Ochoa, S. Riaz, P. Mancarella, T. Ting, J. San, and J. Theunissen, "Grid and market services from the edge: Using operating envelopes to unlock network-aware bottom-up flexibility," *IEEE Power and Energy Magazine*, vol. 19, no. 4, pp. 52–62, 2021.

[12] J. Carpentier, "Optimal power flows," *International Journal of Electrical Power and Energy Systems*, vol. 1, no. 1, pp. 3–15, 1979.

[13] S. Frank, I. Steponavice, and S. Rebennack, "Optimal power flow: a bibliographic survey I," *Energy Systems*, vol. 3, p. 221–258, 2012.

[14] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification—part I," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, 2013.

[15] K. Petrou, M. Z. Liu, A. T. Procopiou, L. F. Ochoa, J. Theunissen, and J. Harding, "Operating envelopes for prosumers in LV networks: A weighted proportional fairness approach," in *2020 IEEE PES Innovative Smart Grid Technologies Europe (ISGT-Europe)*, 2020, pp. 579–583.

[16] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.

[17] S. Bolognani and S. Zampieri, "On the existence and linear approximation of the power flow solution in power distribution networks," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 163–172, 2016.

[18] M. Lubin, O. Dowson, J. D. Garcia, J. Huchette, B. Legat, and J. P. Vielma, "JuMP 1.0: Recent improvements to a modeling language for mathematical optimization," *Mathematical Programming Computation*, 2023, in press.