

Analysis and Mitigation of Cascading Failure Spatial Propagation in Real Utility Outage Data

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Abstract—In this paper, the spatial propagation of cascading failures is studied for real utility outage data from Bonneville Power Administration (BPA). The spatial propagation features based on geographical distances are revealed by the proposed analysis method. Furthermore, a critical component identification method is proposed based on a new metric that combines the information of the expected number of outages and that of the spatial distance. A cascading failure mitigation strategy is further proposed based on the upgrading of the identified critical components. The effectiveness of the proposed mitigation strategy in terms of suppressing spatial propagation is validated on the 14-year real utility outage data from BPA.

Index Terms—Blackout, cascading failure, interaction matrix, mitigation, real data, spatial propagation, utility outage data.

I. INTRODUCTION

Large-scale cascading blackouts, such as the 2003 U.S.-Canadian blackout [1] and the 2012 Indian blackout [2], have serious economic and social impacts. Investigating and analyzing the mechanisms of cascading blackouts can help identify critical components and further provide effective mitigation.

Traditional cascading failure study is based on various simulation models, such as hidden failure model [3] and OPA model [4]. The simulated data has been studied for extracting failure propagation properties, such as by the branching process (BP) model [5], [6], multi-type BP model [7], and the component interaction models including influence graph [8], interaction network [9], and coupled interaction network [10]. However, simulation models and the simulated data are very difficult to benchmark or validate.

Therefore, in recent years real outage data has been directly analyzed. A 14-year real outage dataset from Bonneville Power Administration (BPA) [11], [12] has been studied based on an influence graph in [13]. The same data is investigated by a generation-dependent interaction network estimated from the expectation maximization (EM) algorithm in [14], and a critical component based mitigation strategy is proposed to suppress cascading failures. After upgrading the critical components that lead to the highest expected number of outages, the mitigated failure propagation is validated by the decreasing probability of large cascades.

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However, only focusing on the number of outages cannot provide a detailed description of the cascading failure impacts. How long time a cascading failure lasts and how wide an area a cascading failure can spread to are also critical factors that need to be considered. Therefore, a cascading failure analysis utilizing the system topology information is useful. The failure propagation paths and spatial distances between outages need to be explicitly considered in the mitigation strategy, helping prevent large-scale blackouts in a wide area.

In [15], the propagation path is described by electrical distance, which is the equivalent impedance between two components. The component interaction, the number of outages, and the amount of load shedding are combined in the mitigation strategy design. However, using electrical distance to measure spatial distance could lose the topology information such as the actual geographical lengths of the components, leading to ineffective spatial propagation mitigation. The topology information of the BPA outage data is studied in [16] based on cascade spreading statistics, providing a new direction for mitigation. However, critical components are not identified by directly considering spatial propagation.

In this paper, analysis and mitigation of cascading failure spatial propagation based on the spatial topology are proposed for the 14-year real utility outage data from BPA. The major contributions of this paper are listed as follows.

- 1) We propose a cascading failure spatial propagation analysis method by defining the spatial distance between two generations of outages, the total spatial distance between the outages in a cascade, and also the average spatial propagation velocity for two consecutive generations.
- 2) We propose a critical component identification method based on a new metric called the total spatial distance which combines the information of the expected number of outages and that of the spatial distance. A cascading failure mitigation strategy is further proposed based on the upgrading of the identified critical components. Both large cascading blackouts and the spatial propagation can be effectively suppressed.

The remainder of this paper is organized as follows. Section II describes the real utility outage data and the failure interaction matrices that are estimated from the data. Section III

explains the spatial propagation analysis method for studying spatial features in cascading failures. Section IV proposes a critical component identification method considering spatial propagation and a cascading failure mitigation strategy based on the identified critical components. Section V presents the results of spatial propagation analysis for the BPA data and validates the effectiveness of the proposed mitigation strategy. Finally, conclusions are drawn in Section VI.

II. INTERACTION MATRIX OF UTILITY OUTAGE DATA

In this paper, the 14-year real outage data since January 1999 from BPA in the Transmission Availability Data System (TADS) is used for cascading failure analysis [11], [12]. Since cascading is defined as the uncontrolled successive loss of system elements by North American Electric Reliability Corporation (NERC), only 10,779 automatic outages are used for analysis. The outage data is grouped into different cascades and generations according to the gaps in start time between successive outages [13], [14]. One cascade corresponds to one cascading failure sample while one generation corresponds to one stage in a cascade. Each cascade starts with initial outages in generation 0 followed by outages grouped into further generations until the cascade stops. Let $\mathcal{F}_g^{(m)}$ be the set of the failed components in generation g of cascade m . We have a total of $M = 6,687$ cascades listed below.

	generation 0	generation 1	generation 2	...
cascade 1	$\mathcal{F}_0^{(1)}$	$\mathcal{F}_1^{(1)}$	$\mathcal{F}_2^{(1)}$...
cascade 2	$\mathcal{F}_0^{(2)}$	$\mathcal{F}_1^{(2)}$	$\mathcal{F}_2^{(2)}$...
\vdots	\vdots	\vdots	\vdots	\vdots
cascade M	$\mathcal{F}_0^{(M)}$	$\mathcal{F}_1^{(M)}$	$\mathcal{F}_2^{(M)}$...

There are 346 buses and $n = 582$ transmission lines involved in the outages. The transmission lines are considered components. Based on the data, the component interactions in cascading failures are organized into an interaction matrix. Due to obvious evolution among generations and high heterogeneity among cascades, an interaction matrix \mathbf{B}_g is estimated for any two consecutive generations based on the EM algorithm developed in [14]. The algorithm estimates the interaction matrix by updating $p_{ij}^{m,g}$ for each cascade m , which is the probability of component j outage in generation $g+1$ following the component i outage in generation g . Since the largest generation number $G = 109$, there are 108 interaction matrices ($\mathbf{B}_g = [b_{ij}^g] \in \mathbb{R}^{n \times n}$, $g = 0, \dots, 107$). The element b_{ij}^g is the empirical probability that component j will fail in generation $g+1$ after component i fails in generation g .

III. SPATIAL DISTANCE IN CASCADING FAILURE

The power system topology of the transmission lines (components) involved in the BPA outage data in Section II is shown in Fig. 1. The green dots are buses and the lines are components. Since the actual geographical layout of the buses is not available, we use line thickness to represent the length of the components. In power systems, the topological distance $d_{i_1 i_2}^t$ between buses i_1 and i_2 can be defined as the number

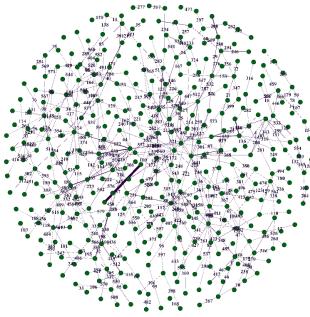


Fig. 1. System topology based on the BPA outage data.

of components (transmission lines) along the shortest path between these two buses. The geographical distance $d_{i_1 i_2}^g$ can be defined as the length of the components on the shortest path. Since we want to keep the information of component length for practical considerations, the distance between component $i : i_1 \rightarrow i_2$ and component $j : j_1 \rightarrow j_2$ is defined as:

$$d_{ij} = \min\{d_{i_1 j_1}^g, d_{i_1 j_2}^g, d_{i_2 j_1}^g, d_{i_2 j_2}^g\}, \quad (1)$$

which is the shortest geographical distance among all possible geographical distances between one bus of component i and one bus of component j . Note that $d_{ij} = d_{ji}$ and $d_{ij} = 0$ when components i and j share at least one common bus.

Then the spatial distance between the outage components in two successive generations g and $g+1$ of cascade m can be calculated based on the final $p_{ij}^{m,g}$ obtained in interaction matrix estimation as:

$$d_{g \rightarrow g+1}^{(m)} = \sum_{j \in \mathcal{F}_{g+1}^{(m)}} \sum_{i \in \mathcal{F}_g^{(m)}} \frac{p_{ij}^{m,g}}{\sum_{i \in \mathcal{F}_g^{(m)}} p_{ij}^{m,g}} d_{ij}. \quad (2)$$

Note that a weighted averaging is performed for the component pairs in successive generations based on the final $p_{ij}^{m,g}$ that indicates the dependencies between components i and j .

The *total spatial distance* between the outages in cascade m can be further calculated as:

$$d_{\text{total}}^{(m)} = \sum_{g=0}^{G-1} d_{g \rightarrow g+1}^{(m)}. \quad (3)$$

Investigating the relationship between the total number of outages of the cascades, generations, and their $d_{\text{total}}^{(m)}$ could help reveal the detailed features in spatial propagation.

Then the *average spatial propagation velocity* from generation g to generation $g+1$ can be calculated as:

$$\bar{v}_{g \rightarrow g+1} = \frac{1}{M_{g \rightarrow g+1}} \sum_{m=1}^{M_{g \rightarrow g+1}} d_{g \rightarrow g+1}^{(m)}, \quad (4)$$

where $M_{g \rightarrow g+1}$ is the number of cascades that contains generations g and $g+1$. Here one generation is considered as one discrete time step.

In [17], based on very simple cascading overload simulations, it is found that the propagation velocity of a Euclidean distance of the failures from the center of the initial failure is approximately constant and is similar for different networks. However, in many real cascading blackouts such as the infamous 2003 U.S.-Canadian blackout [1] the spatial

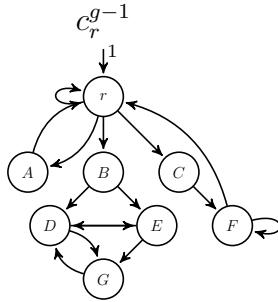


Fig. 2. Subgraph starting with r .

propagation actually accelerated significantly as cascading progressed. Here we will reveal the average spatial propagation velocity for real historical outage data.

IV. CRITICAL COMPONENT IDENTIFICATION AND CASCADING FAILURE MITIGATION

In [14], critical components are identified by calculating the expected number of outages starting with each component, and the larger the expected number of outages is, the more critical the corresponding component is in cascading failure propagation. A mitigation strategy is also developed based on the identified critical components. However, this mitigation does not consider component spatial distances and thus cannot guarantee to suppress cascading failure spatial propagation.

To develop a more comprehensive mitigation strategy, the expected number of outages and the spatial distance between components are combined to create a new metric for the identification of critical components.

A. Expected Number of Outages Starting with a Component

The expected number of outages that one root component could cause is obtained based on the subgraph in the failure interaction network starting with the root component. A major challenge is that the subgraphs may have loops (directed circles or self-loops) due to complicated interactions among outages, as illustrated in Fig. 2. This problem could be overcome by solving a set of linear equations [14].

For \mathbf{B}_g , the subgraph starting with the root node r is denoted by $\mathcal{G}_r^g(\mathcal{C}_r^g, \mathcal{L}_r^g)$, where the cardinality of the subgraph component set \mathcal{C}_r^g is N_r^g . The node numbers in the subgraph are re-ranked from 1 which corresponds to component r . For the new node number j we denote j^0 as its old number before the re-ranking. An adjacency matrix $\mathbf{C}^g = [c_{uv}^g] \in \mathbb{R}^{N_r^g \times N_r^g}$ is built, for which $c_{uv}^g = b_{u^0v^0}^g$ if there is a link from node v^0 to node u^0 in \mathbf{B}_g . Let the vector of the expected number of outages for the nodes be $\mathbf{e}^g \in \mathbb{R}^{N_r^g \times 1} = [e_1^g \ e_2^g \ \dots \ e_{N_r^g}^g]^\top$. Then \mathbf{e}^g is obtained by solving:

$$\mathbf{e}^g - \mathbf{C}^g \mathbf{e}^g = \mathbf{c}, \quad (5)$$

where $\mathbf{c} = [c_r^{g-1} \ 0 \ \dots \ 0]^\top$ and c_r^{g-1} is the number of times the root node is assumed to fail in generation $g-1$. This provides a simple and efficient way to consider the complicated interactions among various components.

B. Critical Component Identification Considering Cascading Failure Spatial Propagation

Previous research only considers the expected number of outages to identify critical components [14]. Here, the expected number of outages will be combined with spatial distance information to help better identify critical components, reducing both the number of outages and spatial propagation.

For every node $u^0 \in \mathcal{C}_r^g \setminus \{r\}$ in the failure interaction network from \mathbf{B}_g , if it is in the subgraph \mathcal{G}_r^g starting with node r and its re-ranked node number is u , the outage measure from component r to component u^0 will be $s_{r,u^0}^g = e_u^g$. If node u^0 is node r , then $s_{r,r}^g = e_1^g - c_r^{g-1}$. After completing all calculations for each node in each failure interaction matrix, we combine the expected number of outages and the spatial distances to calculate a new metric for each node in \mathcal{C}_r^g , the *expected spatial propagation* $I_d^g(r)$:

$$I_d^g(r) = \sum_{i=1}^{N_r^g} s_{r,i}^g d_{ri}. \quad (6)$$

Further, the *total spatial propagation*, $I_d(r)$, over all generations is obtained as:

$$I_d(r) = \sum_{g=0}^{G-1} I_d^g(r). \quad (7)$$

The proposed metric considers not only the number of outages following a component failure but also the extent of spatial propagation after that component failure. A component is considered critical if its failure leads to many outages with extensive spatial propagation.

C. Mitigation Strategy Based on Critical Components

To design the mitigation strategy for reducing the spatial propagation, we identify 20 critical components with the highest I_d values, and the set of these critical components is denoted by \mathcal{C}_d . Mitigation based on \mathcal{C}_d could be implemented by reducing the failure probabilities of the critical components in \mathbf{B}_g . Specifically, for the j th component in \mathcal{C}_d in descending order of I_d , the corresponding column in \mathbf{B}_g is multiplied by $\alpha_j = j/40$. The greater I_d is, the heavier the suppression is on the corresponding component. Then the mitigated interaction matrices \mathbf{B}_g^d are used by the generation-dependent interaction model in [14] to generate mitigated cascades. By recalculating the spatial propagation velocities based on the generated cascades, the mitigation effect will be validated.

V. SPATIAL PROPAGATION ANALYSIS AND CASCADING FAILURE MITIGATION

Cascading failure spatial propagation before and after mitigation is discussed in this section. To validate the effectiveness of the proposed spatial propagation mitigation, the mitigation based on the critical components that only focus on the expected number of outages in [14], denoted by \mathcal{C}_e , is compared.

A. Spatial Propagation Analysis Before Mitigation

Most cascades have one generation, and only 752 cascades have positive $d_{\text{total}}^{(m)}$. Fig. 3 shows the complementary cumulative distribution (CCD) of the relatively large $d_{\text{total}}^{(m)}$'s (i.e.

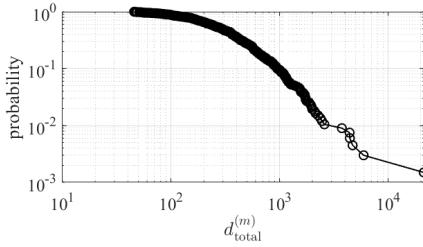


Fig. 3. CCD of large $d_{\text{total}}^{(m)}$'s.

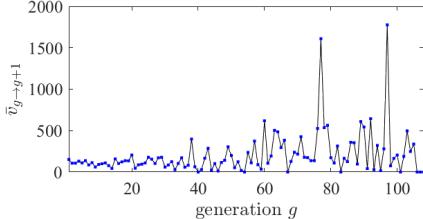


Fig. 4. $\bar{v}_{g \rightarrow g+1}$ without mitigation.

$d_{\text{total}}^{(m)} > 10^{-2}$ which is around 90th percentile of $d_{\text{total}}^{(m)}$ for all cascades), indicating that there is a dramatic difference between different cascades. The maximum $d_{\text{total}}^{(m)}$ is 2.13×10^4 for $m = 4, 005$ that has the largest number of generations.

The spatial propagation velocity $\bar{v}_{g \rightarrow g+1}$ is shown in the Fig. 4. When $g = 97$, $\bar{v}_{g \rightarrow g+1}$ has the highest value which is 1,700. When $g = 39, 53, 66, 82, 100, 105, 106, 107$, $\bar{v}_{g \rightarrow g+1} = 0$, which is due to the components in these generations sharing the same buses. Based on (2), if two components i and j connect to at least one common bus, the responding d_{ij} is zero which further leads to zero $\bar{v}_{g \rightarrow g+1}$.

The $\bar{v}_{g \rightarrow g+1}$ for different generations is significantly different. Meanwhile, the number of outages is 2,658 for $g = 0$ while it decreases sharply in further generations. To better reveal the cascading failure spatial propagation properties, the outages from successive generations are grouped. Specifically, the generation grouping rules are listed below.

- 1) Starting with generation 0, a generation group combines several consecutive generations until the total number of outages in this group is larger than a predetermined threshold. This is repeated for the remaining generations. In a special case, if the number of outages of a single generation meets the condition, it is a group.
- 2) Before generation $g_e = 41$ each generation has hundreds or even thousands of outages while after generation g_e each generation only has less than ten outages. Therefore, we choose two different thresholds for the number of outages: for a group whose first generation is before generation g_e , the threshold is chosen as $O_1 = 129$ (the 95th percentile of the number of outages in all generations); for the remaining generation groups, the threshold is set as $O_2 = 65$, which is about half of O_1 .
- 3) If the remaining un-grouped generations do not have greater than or equal to O_2 number of outages, a few generations in the previous generation group will be

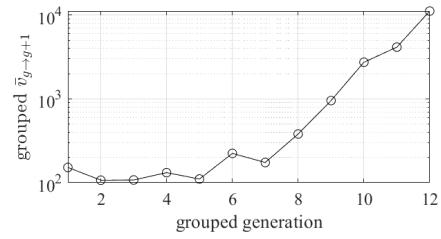


Fig. 5. $\bar{v}_{g \rightarrow g+1}$ for grouped generations.

TABLE I
COMPARISON OF CRITICAL COMPONENTS

Rank	\mathcal{C}_e	$\mathcal{C}_d (I_d^g)$	Rank	\mathcal{C}_e	$\mathcal{C}_d (I_d^g)$
1	83	24 (1.54×10^4)	11	201	41 (6.86×10^3)
2	17	126 (1.47×10^4)	12	61	61 (6.48×10^3)
3	234	83 (1.15×10^4)	13	56	179 (6.48×10^3)
4	24	92 (9.60×10^3)	14	59	4 (5.99×10^3)
5	76	116 (9.52×10^3)	15	126	76 (5.98×10^3)
6	2	17 (8.46×10^3)	16	26	42 (5.91×10^3)
7	85	2 (8.20×10^3)	17	92	101 (5.87×10^3)
8	8	234 (8.02×10^3)	18	73	59 (5.70×10^3)
9	101	446 (7.86×10^3)	19	126	75 (5.64×10^3)
10	187	23 (7.02×10^3)	20	42	13 (5.20×10^3)

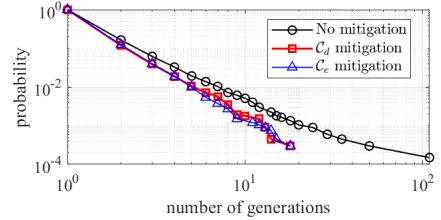


Fig. 6. CCDs of the number of generations of each cascade with and without mitigation.

combined with the remaining un-grouped generations until the number of outages in the last generation group is greater than or equal to O_2 .

Then $\bar{v}_{g \rightarrow g+1}$ is calculated for the grouped generations and is shown in Fig. 5, where a clear increasing tendency of the spatial propagation velocity is revealed, indicating that the extent of spatial propagation is increased in later generations. This is consistent with the previous blackouts such as the 2003 U.S.-Canadian blackout [1] in which the spatial propagation accelerated significantly as the cascading progressed.

B. Mitigation Result of Critical Spatial Component

Mitigation based on critical components is implemented and the spatial propagation features after mitigation are analyzed. The two sets of critical components, \mathcal{C}_e based on the expected number of outages [14] and \mathcal{C}_d based on the proposed total spatial propagation, are listed in Table I. It is seen that the critical components in \mathcal{C}_d are different from those in \mathcal{C}_e . Compared with \mathcal{C}_e , eight new components (highlighted by bold font) are identified as critical components in \mathcal{C}_d .

M cascades are generated under the mitigation based on the two critical component sets. Fig. 6 shows the CCDs of

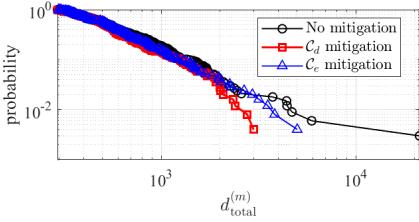


Fig. 7. CCDs of of large $d_{\text{total}}^{(m)}$'s with and without mitigation.

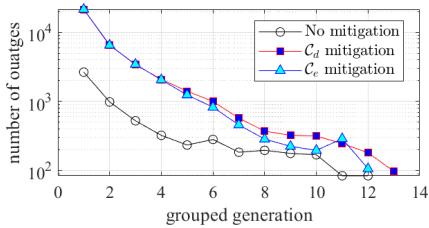


Fig. 8. Number of outages in grouped generations with and without mitigation.

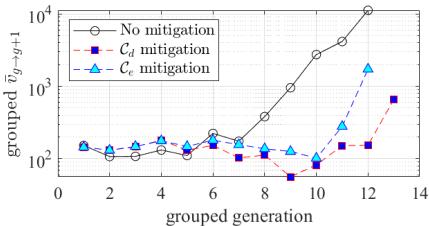


Fig. 9. $\bar{v}_{g \rightarrow g+1}$ for grouped generations with and without mitigation.

the number of generations of each cascade with and without mitigation. Compared with the case without mitigation, two mitigation strategies can restrict failure propagation by decreasing the number of generations. In Fig. 7, the CCDs of $d_{\text{total}}^{(m)}$ with and without mitigation are shown. Note that only $d_{\text{total}}^{(m)} > 129$ (95th percentile of $d_{\text{total}}^{(m)}$) is displayed. Compared with \mathcal{C}_e mitigation, the proposed \mathcal{C}_d mitigation can more significantly suppress cascading failure spatial propagation.

To illustrate the mitigation effect on $\bar{v}_{g \rightarrow g+1}$, we generate $10M$ cascades and group the generations following the same rules in Section V-A. Fig. 8 shows the number of outages in the grouped generations. The proposed \mathcal{C}_d mitigation strategy has a similar number of outages in the grouped generations as that under the \mathcal{C}_e mitigation that only focuses on the expected number of outages. In Fig. 9, the $\bar{v}_{g \rightarrow g+1}$ for the grouped generations with and without mitigation is shown. Compared to the case without mitigation, both \mathcal{C}_d and \mathcal{C}_e mitigation can suppress the spatial propagation of cascading failures. The proposed \mathcal{C}_d mitigation has the smallest average spatial propagation velocity, thus validating the effectiveness of the proposed mitigation strategy based on the identified critical components from the defined total spatial propagation.

VI. CONCLUSION

In this paper, a cascading failure spatial propagation analysis method and a critical component based mitigation strategy are proposed. The cascading failure mitigation is based on the identified critical components using a metric that considers both the number of outages and the spatial distance. The results of the 14-year BPA data show that there is an increasing tendency of the spatial propagation velocity. Also, the proposed mitigation strategy based on total spatial propagation significantly outperforms the traditional mitigation strategy that only focuses on the number of outages in terms of suppressing cascading failure spatial propagation.

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