

1 Domain textures in the fractional quantum Hall effect

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14 **Impacts of domain textures on low-lying neutral excitations in the bulk of fractional quantum Hall
15 effect (FQHE) systems are probed by resonant inelastic light scattering. We demonstrate that large
16 domains of quantum fluids support long-wavelength neutral collective excitations with well-
17 defined wave vector (momentum) dispersion that could be interpreted by theories for uniform
18 phases. Access to dispersive low-lying neutral collective modes in large domains of FQHE fluids
19 such as long wavelength magnetorotons at filling factor $\nu = 1/3$ offer significant experimental access
20 to strong electron correlation physics in the FQHE.**

22
23 Current geometrical theories of strongly correlated phases in fractional quantum Hall effect
24 (FQHE) fluids of two-dimensional electron systems (2DES) identify low-lying neutral collective
25 excitations (known as magnetorotons [1]) as chiral gravitons [2–7]. The bulk of FQHE fluids is non-
26 uniform and formation of domain textures [8–11] have significant impact on low-lying collective
27 excitations [12–14]. The impact of bulk domain textures in strongly correlated phases is the subject
28 of increasing attention, with prominent examples in FQHE fluids [15–20] and in high temperature
29 superconductors [21–24]. To investigate the insulating bulk of quantum Hall phases, studies on
30 thermal transport and electronic interference by edge modes are interpreted under bulk-edge
31 correspondence [25–31]. Experimental methods that directly probe low-lying neutral collective
32 excitations in the bulk of FQHE fluids are crucial tools in the quest to understand strong electron
33 correlation physics.

34
35 In the second Landau level (SLL) at $\nu = 7/3$ a seemingly conventional FQHE is actually very
36 distinct from its counterpart at $\nu = 1/3$ in the lowest Landau level (LLL) [32–38]. Even under a small
37 in-plane magnetic field, appearance of anisotropic longitudinal resistance indicates emergence of
38 domains of nematic phases [39,40] that coexist with domains of the bulk FQHE fluid [41]. Newly
39 discovered collective modes under a small in-plane magnetic field are interpreted as long-
40 wavelength plasmons of nematic domains in the filling factor range $2 < \nu < 3$ [42]. Studies of the
41 plasmons under a small in-plane magnetic field serve as a probe of the impact of domain textures
42 on low-lying collective excitations.

43 Access to magnetoroton modes of FQHE fluids and to plasmons of nematic liquids is provided
 44 by resonant inelastic light scattering (RILS) methods [13,42]. Here we report that RILS by nematic
 45 plasmons at $\nu = 7/3$ under a small in-plane magnetic field reveals a wide range of nematic domain
 46 sizes from around $1 \mu\text{m}$ to characteristic sizes larger than several microns, and establishes FQHE-
 47 nematic fluids at $\nu = 7/3$ as an ideal platform to study impacts of domain textures on low-lying
 48 neutral excitations. The domains with dimensions much larger than the inelastic scattering
 49 wavelength (about $1 \mu\text{m}$) support low-lying collective excitations with well-defined wave vector
 50 (momentum) dispersions. Very sharp dispersive long-wavelength magnetoroton modes have been
 51 reported in the LLL at $\nu = 1/3$ [13]. We surmise that the observed modes, with well-defined wave
 52 vectors, are excitations of the FQHE fluid in large domains. Results at $\nu = 1/3$ reported in previous
 53 works are examined to highlight possible experimental insights on long-wavelength magnetoroton
 54 excitations that are identified as chiral gravitons under strong electron correlation in recent theory
 55 for uniform FQHE phases [4–7].

56
 57 Characteristic domain sizes are obtained by modelling the RILS intensity of nematic plasmons.
 58 While the matrix element depends on plasmon-electron interactions [12], we focus here on the
 59 photon frequency dependence and on breakdown of wave vector conservation due to formation of
 60 domain textures. We find that intensity is [43]:
 61

$$62 \quad I \propto \left| \frac{1}{(\omega_I - \omega_1 - i\gamma_1)} \frac{1}{(\omega_o - \omega_2 - i\gamma_2)} \right|^2 \frac{1}{(\omega(q) - \omega(k))^2 + (\Delta\omega/2)^2} \quad (1)$$

63
 64 ω_1 (ω_2) and γ_1 (γ_2) are the energies and broadenings of the incoming (outgoing) resonant channels
 65 of optical excitations of the GaAs quantum well that hosts 2DES. When both incoming (ω_I) and
 66 outgoing (ω_o) photon energies are in the vicinity of these optical excitations, a large intensity
 67 enhancement occurs through double resonant inelastic light scattering (DRILS). $\omega(q)$ is the
 68 plasmon energy at wave vector q [43]. Energy conservation in inelastic scattering requires $\omega(q) =$
 69 $\omega_I - \omega_o$. $\Delta\omega$ is the range of the plasmon energies due to breakdown of wave vector conservation.
 70

71 The inelastic scattering wave vector \mathbf{k} is along the in-plane magnetic field [42]. Wave vector
 72 conservation ($\mathbf{k} = \mathbf{q}$) occurs for domain sizes $D \gg \lambda = 2\pi/k$, where λ is the inelastic scattering
 73 wavelength, $k = |\mathbf{k}|$ and D is a characteristic length of nematic domains along the direction of \mathbf{q} .
 74 Wave vector conservation is broken under a finite domain size, so mode wave vectors have an
 75 uncertainty $\Delta q = 2\pi/D$ [49]. We have
 76

$$77 \quad \Delta q = \left(\frac{dq}{d\omega} \right) |_{q=k} \Delta\omega = 2k\Delta\omega/\omega(k) \quad (2)$$

78
 79 for nematic plasmons with dispersion $\omega(q)$ [43]. Estimates of values of D can be obtained with Eq.
 80 (2) and determinations of $\Delta\omega$ from inelastic light scattering spectra.
 81

82 At $\nu = 7/3$ there is a set of DRILS spectra for which breakdown of wave vector conservation
 83 has very minor impact. Here estimated nematic domain sizes are $D \approx 5.3 \mu\text{m} \gg \lambda = 1.2 \mu\text{m}$. In the
 84 same device, strong impact of breakdown of wave vector conservation in another set of spectra is

85 found for domains that have D comparable to λ . The optical resonances in large nematic domains
86 involve quasiparticle states close to the Fermi level. This could be evidence showing that large
87 nematic domains are responsible for anisotropic electrical conduction in the FQHE at $v = 7/3$ under
88 tilted magnetic fields [41]. Domains of FQHE fluid with $D > \lambda$ and those with $D < \lambda$ should coexist
89 in the non-uniform bulk of FQHE states. Albeit the non-uniformity, low-lying neutral collective
90 excitations in large domains with $D \gg \lambda$ accessed by RILS and DRILS would manifest the
91 fundamental correlation physics in the uniform bulk of FQHE phases in the LLL and the SLL.

92
93 The ultraclean 2DES is confined in a 30-nm-wide, symmetric, modulation-doped single
94 GaAs/AlGaAs quantum well. Carrier mobility of the wafer measured by transport is $\mu = 23.9 \times 10^6$
95 cm^2/Vs at 300 mK. The sample is mounted on the cold finger of a $^3\text{He}/^4\text{He}$ -dilution refrigerator with
96 windows for direct optical access and inserted in the bore of a 16T superconducting magnet. All
97 measurements were performed at $T \leq 45$ mK. The electron density under illumination is directly
98 determined in each cool down by RILS measurements of spin waves at $v = 3$ (Fig. S2) and yields n
99 $= 2.8 \times 10^{15} \text{ m}^{-2}$ [43]. The stability of electron density and sample quality against illumination are
100 confirmed by photoluminescence (PL) and RILS measurements under zero magnetic field [43].
101 Figure 1(a) describes the back-scattering geometry at a small tilt angle $\theta = 20^\circ$. The finite wave
102 vector transfer in back-scattering is $k = |\mathbf{k}_I - \mathbf{k}_o| \sin\theta$, where \mathbf{k}_I and \mathbf{k}_o are wave vectors of the
103 incoming and outgoing photons. DRILS spectra are excited with the linearly polarized tunable
104 emission from a Ti:sapphire laser that is finely tuned to match the ω_1 excitation. The incident power
105 density was kept well below 10^{-4} W/cm^2 . The outgoing photons are dispersed by a triple grating
106 spectrometer and recorded by a CCD camera.

107
108 Optical excitons contributing to the DRILS matrix element are built from transitions between
109 quasiparticles in conduction states and holes in valence states shown in steps 1 and 3 in Figs. 1(b)
110 and 1(c). These resonant channels are identified from PL and resonant Rayleigh scattering spectra
111 at $v = 7/3$ as shown in Fig. S1 [43]. At $v = 7/3$ excitons participating in DRILS are extremely sharp,
112 indicating that Landau levels in the bulk of nematic-FQHE fluid system support well-defined states
113 even under non-uniform conditions that must prevail due to coexistence of two distinct phases (the
114 FQHE fluid and the nematic liquid). FQHE fluid at $v = 7/3$ is characterized by the determination of
115 low energy magnetoroton excitations, which is suppressed by increasing temperatures or deviation
116 of filling factors (Fig. S3) [43,46].

117
118 Two types of DRILS spectra of plasmons are reported. They are defined by the outgoing
119 resonances in steps 3 shown in Figs. 1(b) and 1(c). In Low Outgoing Resonance (LOR) mode the
120 outgoing photon resonates with L transitions that involve states of correlated quantum fluids in the
121 SLL [50]. In High Outgoing Resonance (HOR) mode the outgoing photon resonates with the
122 excitonic X transition involving empty states in the partially populated SLL [50]. Figures 2 and 3
123 report DRILS results by nematic plasmons at $v = 7/3$.

124
125 In results for DRILS-LOR shown in Fig. 2(a) there is a sharp nematic plasmon peak that
126 slightly blueshifts with higher incoming photon energies within the small energy range $\Delta\omega_S$. The
127 small blueshift is due to minor impact of breakdown of wave vector conservation. Figure 2(b) plots
128 the intensities of plasmons as a function of outgoing (red dots) or incoming (blue dots) photon

129 energies. The blue dots indicate a clear incoming resonance with the X exciton, and the red dots
130 indicate an outgoing resonance associated with the Fermi level of correlated phases [near the high
131 energy edge of L emission in Fig. 2(b)]. We expect the DRILS matrix element to be prominent when
132 symmetry of the final state in L is connected to X exciton through plasmon-electron interactions.
133 The sharp exciton in the outgoing resonance in LOR provides critical spectroscopic insight into
134 links of large nematic domains with the Fermi level of 2DES. The results reveal interplays of
135 topological order in the FQHE with nematic order that could impact anisotropic transport by edge
136 modes [41].

137

138 In LOR, an additional broad continuum with energy range $\Delta\omega_B$ shown in Fig. 2(a) is weakly
139 excited without double resonance. We find that in HOR this continuum of modes is DRILS active
140 and gives rise to a series of plasmon modes within $\Delta\omega_B$ shown in Fig. 3(a). Different energy ranges
141 of double resonant plasmons in LOR and HOR spectra reveal a rich set of domain sizes. As domain
142 sizes increase ($D \gg \lambda$), the wave vector dispersions of elementary excitations would tend to be
143 similar to those of a uniform electron fluid.

144

145 Modeling DRILS spectra with Eqs. (1) and (2) yields nematic domain sizes $D_S \approx 5.3 \mu\text{m}$ in
146 LOR and $D_B < 2.2 \mu\text{m}$ in HOR [43]. Figure 2(c) displays nematic plasmons generated by DRILS
147 model in LOR. Figures 2(d) and 3(b) show DRILS model fits of plasmon energies and intensities
148 as a function of incoming photon energies in LOR and HOR, which are the two factors directly
149 capturing the resonance evolution of plasmons. The fits successfully reproduce DRILS-LOR results.
150 It supports the interpretation that LOR spectra are from large nematic domains with small impact of
151 breakdown of wave vector conservation and thus represent well-defined bulk-like collective modes.
152 In contrast, the HOR incoming resonance [Fig. 3(b)] has a flatter top with a few outliers. When
153 domain sizes are comparable to the inelastic scattering wavelength $\lambda = 1.2 \mu\text{m}$, wave vectors are no
154 longer good quantum numbers and modeling of DRILS spectra may have to be modified.

155

156 It is highly significant that DRILS-LOR involves larger domains where quantum fluids with a
157 sharp optical transition at the Fermi level are better defined. In contrast, DRILS-HOR involves
158 intermediate states of electron-hole pairs forming X and X^+ excitons which are associated with
159 empty states in the partially populated SLL. As a result, LOR and HOR effectively probe plasmons
160 in two different ranges of nematic domain sizes: larger domains close to the Fermi level in LOR,
161 and smaller domains producing stronger impact of breakdown of wave vector conservation in HOR
162 [Fig. 1(d)]. FQHE-nematic fluids at $\nu = 7/3$ with DRILS-active plasmons thus clearly reveal distinct
163 impacts of domain textures depending on their sizes.

164

165 Domain textures at other filling factors can also be resolved by DRILS. Figure 4(a) shows
166 strong nematic plasmons of a non-FQHE state at $\nu = 2.68$. DRILS model reproduces main resonance
167 features, as shown in Fig. 4(b). Compared to $\nu = 7/3$ in LOR, the broader resonance reveals smaller
168 domain sizes $D \approx 3.5 \mu\text{m}$, which exerts a larger impact of breakdown of wave vector conservation
169 and is consistent with higher disorder level in the upper half of the SLL.

170

171 Large domains of FQHE fluids support neutral collective excitations with well-defined wave
172 vector dispersion that could be observed in RILS and DRILS spectra which access bulk states.

173 Extremely sharp long-wavelength magnetoroton excitations at $\nu = 1/3$ were observed [13,51]. Single
174 resonance is considered here due to the lack of a pair of well-understood resonant channels with
175 energy difference matching the magnetoroton energy. The narrow linewidth can be regarded as a
176 consequence of very small breakdown of wave vector conservation in RILS spectra from large
177 domains of the bulk FQHE fluid and the small curvature of the magnetoroton dispersion near zero
178 momentum. The low-lying neutral collective excitations from bulk-like FQHE fluid in large
179 domains could be interpreted in terms of theories for uniform FQHE phases. With increasing wave
180 vectors, dispersive long-wavelength modes observed in RILS at $\nu = 1/3$ split in a manner consistent
181 with a two-roton bound state [13].

182

183 Long-wavelength magnetoroton excitations play key roles in recent geometrical theories of
184 electron correlation in bulk FQHE phases [2,3]. Magnetoroton modes occur here as spin 2 chiral
185 gravitons, corresponding to the long-wavelength quantum fluctuations of an internal dynamic
186 metric [4–7]. At $\nu = 1/3$ only the mode with spin $S = -2$ is active in RILS experiments in the $k \rightarrow 0$
187 limit [4–6]. The results in Ref. 13 that reveal a magnetoroton doublet offer a key experimental
188 insight that needs to be interpreted. In the framework of geometrical theories, a doublet may occur
189 in RILS spectra at finite k because of coupling between magnetorotons with $S = -2$ and $+2$ [4,6].

190

191 In summary, RILS and DRILS methods investigate impacts of domain textures on low-lying
192 collective excitations in the non-uniform bulk of FQHE of 2DES in GaAs quantum structures. For
193 domains larger than the inelastic light scattering wavelength ($\lambda = 1.2 \mu\text{m}$) there is nearly full
194 conservation of wave vector in the light scattering events. Magnetoroton excitations have been
195 identified in RILS spectra from $\nu = 5/2$ FQHE fluids [45]. RILS measurements with higher
196 resolution in large domains of FQHE fluids at $\nu = 5/2$ could identify narrow long-wavelength neutral
197 collective excitations with well-defined wave vector dispersion. Such experiments may reveal
198 features of topological domain textures involving Pfaffian and anti-Pfaffian orders [15–20]. Suitable
199 design of GaAs quantum structures [28], enhanced growth protocols [52] and characterization by
200 RILS and DRILS may lead to creation and identification of large domains of quantum fluids likely
201 to host novel electron correlation effects.

202

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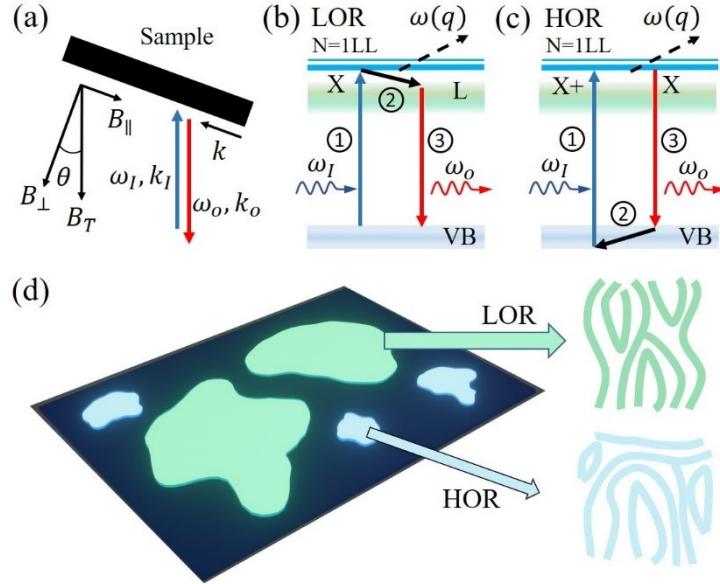
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342 FIG. 1 (color online). (a) Schematic description of the experimental back-scattering geometry at a tilt
 343 angle $\theta = 20^\circ$. Incoming and outgoing photons have energies ω_I , ω_o and wave vectors k_I , k_o ,
 344 respectively. The total magnetic field B_T produces a perpendicular component B_{\perp} and a parallel
 345 component B_{\parallel} . The inelastic scattering wave vector is parallel to B_{\parallel} . (b)(c) Different DRILS processes
 346 between valence bands and the SLL in (b) LOR and (c) HOR. A plasmon mode $\omega(q)$ is generated in the
 347 second step. (d) Schematic plot of domain textures probed by DRILS in nematic-FQHE fluids. Large
 348 (small) nematic domains are active in LOR (HOR). Smaller domains are more disordered.

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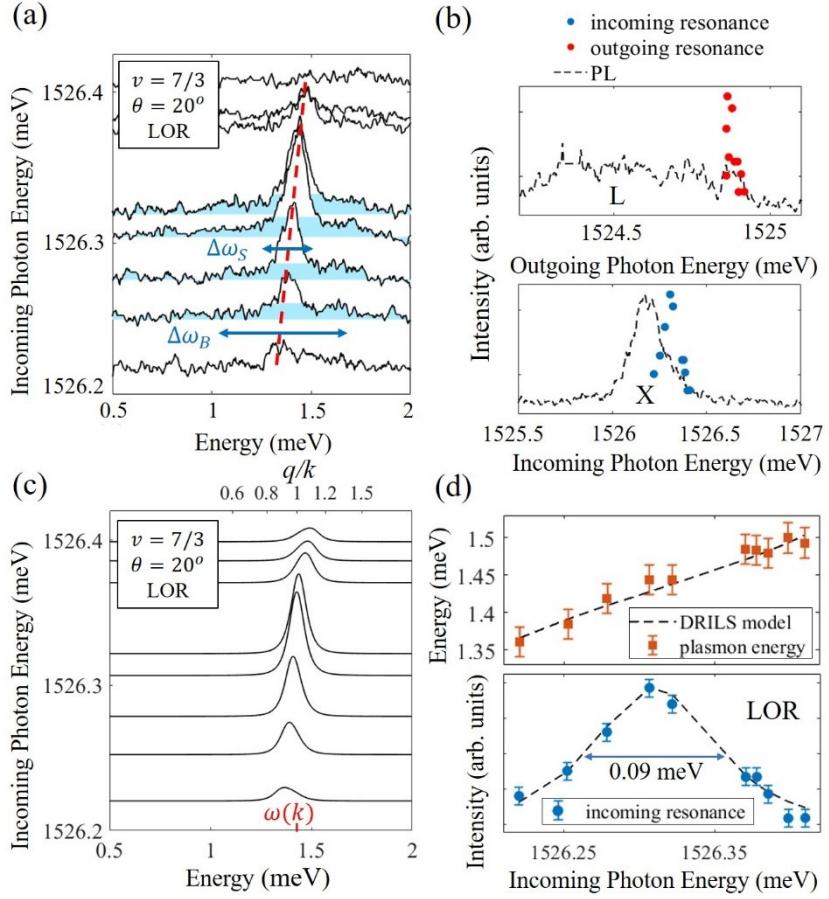
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359 FIG. 2 (color online). (a) DRILS-LOR spectra of nematic plasmon modes measured at $v = 7/3$. The
 360 dashed line marks their blueshift. Areas filled in cyan indicate the underlying continuum. Arrows mark
 361 the different energy ranges of plasmons and the continuum. (b) Outgoing (top panel) and incoming
 362 (bottom panel) resonances of plasmons. (c) LOR plasmon spectra generated by DRILS model, where
 363 $(\omega_1, \gamma_1) = (1527.31 \text{ meV}, 0.06 \text{ meV})$, $(\omega_2, \gamma_2) = (1524.88 \text{ meV}, 0.06 \text{ meV})$, $\omega(k) = 1.43 \text{ meV}$ and $\Delta\omega_S$
 364 $= 0.16 \text{ meV}$. The corresponding non-conserved wave vector q/k is labelled on the top x-axis. (d) DRILS
 365 model fits (dashed lines) of plasmon energies (top panel) and incoming resonance (bottom panel) using
 366 the parameters above. The width of incoming resonance is reduced from $2\gamma_1$ due to the impact of plasmon
 367 wave vector distribution.

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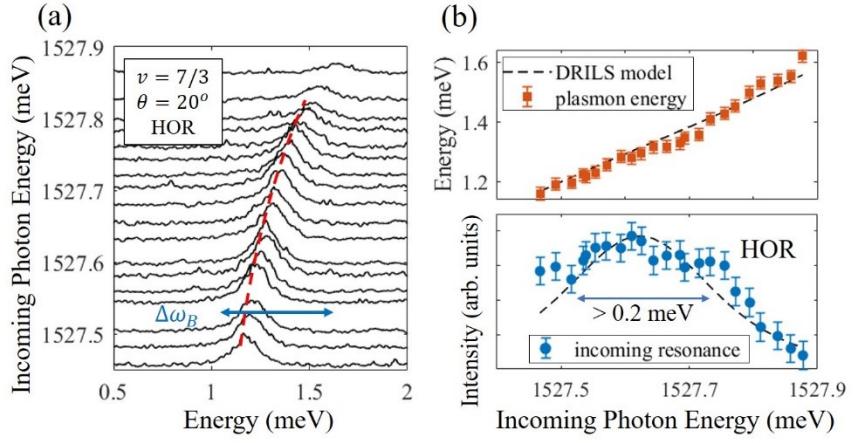
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378 FIG. 3 (color online). (a) DRILS-HOR spectra of nematic plasmon modes within the energy range $\Delta\omega_B$
 379 measured at $\nu = 7/3$. The dashed line marks their blueshift. (b) DRILS model fits (dashed lines) of
 380 plasmon energies (top panel) and incoming resonance (bottom panel) in HOR, where $(\omega_1, \gamma_1) = (1527.61$
 381 meV, 0.25 meV), $(\omega_2, \gamma_2) = (1526.31 \text{ meV}, 0.06 \text{ meV})$, $\omega(k) = 1.32 \text{ meV}$ and $\Delta\omega_B = 0.35 \text{ meV}$ (lower
 382 limit). The incoming resonance overlaps $X+$ transitions in PL, which may be a superposition of several
 383 optical excitons.

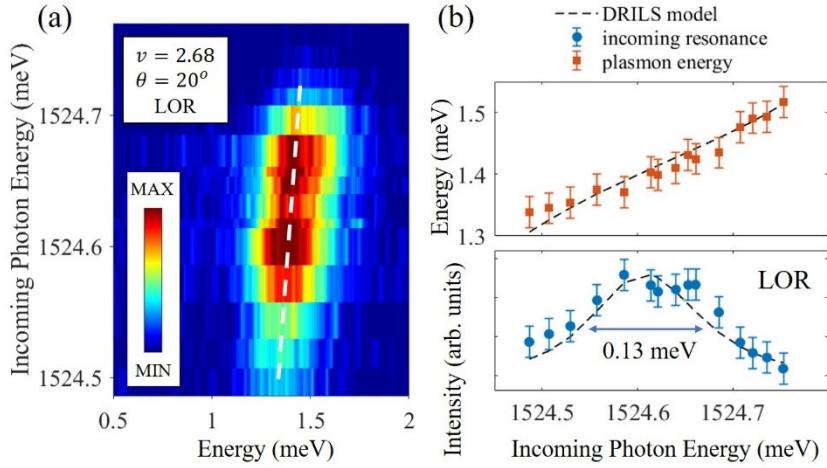
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390 FIG. 4 (color online). (a) DRILS spectra of nematic plasmon modes at $\nu = 2.68$. The dashed line marks
 391 their blueshift. (b) Reproduction of plasmon energy (top panel) and incoming resonance (bottom panel)
 392 by DRILS model, where $(\omega_1, \gamma_1) = (1524.6 \text{ meV}, 0.08 \text{ meV})$, $(\omega_2, \gamma_2) = (1523.21 \text{ meV}, 0.08 \text{ meV})$, $\omega(k)$
 393 = 1.41 meV and $\Delta\omega = 0.24 \text{ meV}$. The color code is linear with intensity.