

# Variable Stiffness Floating Spring Leg: Performing Net-Zero Energy Cost Tasks Not Achievable Using Fixed Stiffness Springs

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**Abstract**—Sitting down and standing up from a chair and, similarly, moving heavy objects up and down between factory lines are examples of cyclic tasks that require large forces but little to no net mechanical energy. Motor-driven artificial limbs and industrial robots can help humans do these tasks, but motors require energy to provide force even if they supply no net mechanical energy. Springs are energetically conservative mechanical elements useful for building robots that require no energy when performing cyclic tasks. However, conventional springs can be limited by their non-customizable force-deflection behavior—for example, when they cannot meet the force demand despite storing enough energy to perform a cyclic task. Variable stiffness springs are a special type of spring with customizable force-deflection behavior, but most typical variable stiffness springs require energy to amplify force similar to motors. In this paper, we introduce a new type of variable stiffness spring design which is energetically conservative despite having a customizable force-deflection behavior. We present the theory of these springs and demonstrate their utility in performing a net-zero mechanical energy cost lifting task that requires force amplification and as such is not realizable using conventional springs. Energetically conservative springs with customizable force-deflection behavior may find their place in assistive devices, exoskeletons, and industrial robots that can perform a larger class of tasks than conventional springs using little to no external energy.

**Index Terms**—Mechanism Design, Compliant Joints and Mechanisms, Physically Assistive Devices, Variable Stiffness Springs, Floating Spring Mechanisms

## I. INTRODUCTION

**S**PRINGS are mechanical elements that can store and release energy. Springs can be used in place of motors in robots, assistive devices, and exoskeletons to help humans perform static tasks by providing force at zero energy cost, or

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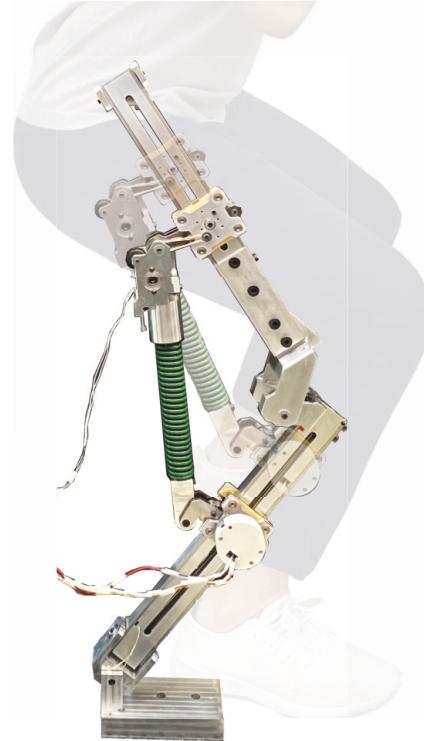


Fig. 1. The floating spring leg.

cyclic tasks by recuperating negative work to perform net-zero mechanical energy cost tasks.

Springs placed in series or parallel with human limbs help reduce the energy supplied by a human during cyclic tasks [1]—for example during lifting [2], [3], walking [4], [5], [6], and running [7]. Similarly, springs can be used in parallel or series with a motor in robot actuators to reduce the energy supplied by the motor when holding a heavy weight or performing a repetitive task [8], [9], [10], [11], [12]. However, conventional springs can be limited by their fixed force-deflection behavior, i.e. fixed stiffness. For example, when a conventional spring supports a mass at static equilibrium, the energy stored by the spring cannot be released to move the mass because the spring cannot increase the force to accelerate the mass resting at static equilibrium. This example shows that springs may be limited in performing a task for which they do not meet the force demand even if they meet the energy demand of the task.

Variable stiffness springs are a special type of springs that have customizable force-deflection behavior. Variable stiffness

springs are often used in robot joints and typically consist of a conventional spring with non-customizable force-deflection behavior and a transmission mechanism that changes the stiffness of the spring perceived by the environment. There are many different types of variable stiffness springs [13], but in most designs, the apparent stiffness of the spring is changed by pre-tensioning the springs [14], [15], [16], changing the active length of the spring [17], [18], [19], [20], or adding a transmission mechanism between the spring and the load [21], [22], [23], [24], [25], [26], [27]. The latter two types of variable stiffness spring designs have the advantage of theoretically zero-energy cost stiffness modulation while the spring is un-deflected and stores no energy [28]. However, when the spring is deflected and stores energy, increasing the apparent stiffness requires external energy proportional to the energy stored by the spring [29]. This feature lessens the benefit of variable stiffness springs in tasks that require increasing stiffness to meet the force demand when the spring stores energy, even if the task requires no net mechanical energy.

In this paper, we present the floating spring leg – a quasi-passive variable stiffness robotic spring leg, shown in Fig. 1, that can change its apparent stiffness while the spring stores a significant amount of mechanical energy. The leg is able to store energy at one stiffness and then change to another stiffness without changing the amount of energy stored in the spring, thereby maintaining the “energetically conservative” behavior that characterizes conventional fixed stiffness springs. By changing the stiffness of the leg while storing energy, the floating spring provides customizable forces not only between, but also within a typical compression-expansion work cycle of the leg. Consequently, our novel design improves upon state-of-the-art variable stiffness mechanisms that can maintain energetically conservative behavior when the spring is not deflected (and stores no energy) [18], [19], [30], [20] or when the spring is deflected by a prescribed amount (and stores a prescribed amount of energy) [21], [31], [32].

The novel feature of the floating spring leg can be exemplified by considering the human first compressing the leg at a low stiffness, subsequently locking and reorienting the spring to achieve high leg stiffness, and finally releasing the energy stored by the spring. Increasing leg stiffness and force is required to increase dynamic performance in tasks such as weight lifting [33], jumping [34], and running [35], [36], [37]. Increasing the leg force can also be beneficial in assisting humans with weak lower limb muscles in a sit-to-stand task [38], [39], [40]. Our findings show promise in these applications, in addition to other applications, that include but are not limited to, spring-driven robots that could move heavy objects up and down between factory lines [41], [42].

This paper extends the author’s recent work [29] that introduced the concept of the floating variable stiffness spring in two important ways: (i) First, we present the design of the first fully functional floating spring leg (Section II). (ii) Second, we experimentally validate the theoretically predicted benefits of the floating spring leg in a weight-lifting task (Section III). We conclude with the limitations and potential future applications of the energetically conservative variable

stiffness spring concept introduced in this paper (Section IV).

## II. THE FLOATING SPRING LEG

In this section, we introduce the working principle and design of the floating spring leg. The concept of the floating spring leg has been previously presented in [29] and will be briefly covered in the following. The prototype is shown in Fig. 1. The CAD model is shown in Fig. 3. The leg consists of three main components – the skeleton, the stiffness modulation mechanism, and the lockable compression spring.

### A. Working principle

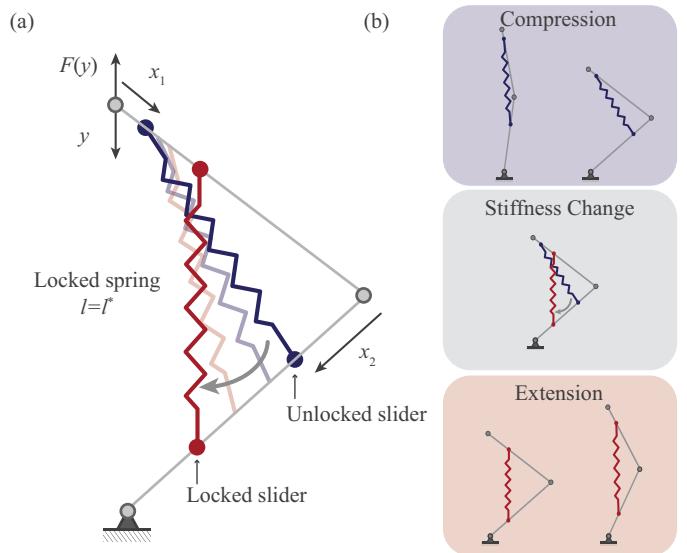


Fig. 2. Floating variable stiffness spring mechanism [29]. (a) The deflection of the mechanism is  $y$  while the force provided by the mechanism is  $F(y)$ . (b) Work cycle: The mechanism is compressed in a low-stiffness configuration (blue). After compression, the stiffness of the mechanism is increased by moving the endpoints of the spring to a high-stiffness configuration (red). Finally, the mechanism releases the energy stored by the spring in the high-stiffness configuration.

Figure 2(a) shows a conceptual design of the floating spring. The mechanism is comprised of two rigid links (gray) connected with a cylindrical joint and a conventional fixed stiffness spring (blue/red), i.e. a leg skeleton. We assume that the length of the spring can be locked at any desirable length  $l = l^*$ , i.e. a lockable spring. We further assume that the endpoints of the spring,  $x_1$  and  $x_2$ , can be moved along the rigid links and locked on the rigid links at positions  $x_1 = x_1^*$  and  $x_2 = x_2^*$ , i.e. a stiffness modulation mechanism. By moving the endpoints of the locked spring, the force felt at the output,  $F(y)$ , can be adjusted, thereby changing the output stiffness of the mechanism during the deflection cycle, Fig. 2(b).

### B. Leg skeleton

The skeleton of the leg, shown in Fig. 3(a), has five degrees of freedom (DOFs) and has been designed to represent the structure of a two-link leg with the thigh, shank, and associated joints. There are two rotational joints at the top of the thigh (flexion-extension and abduction-adduction), one

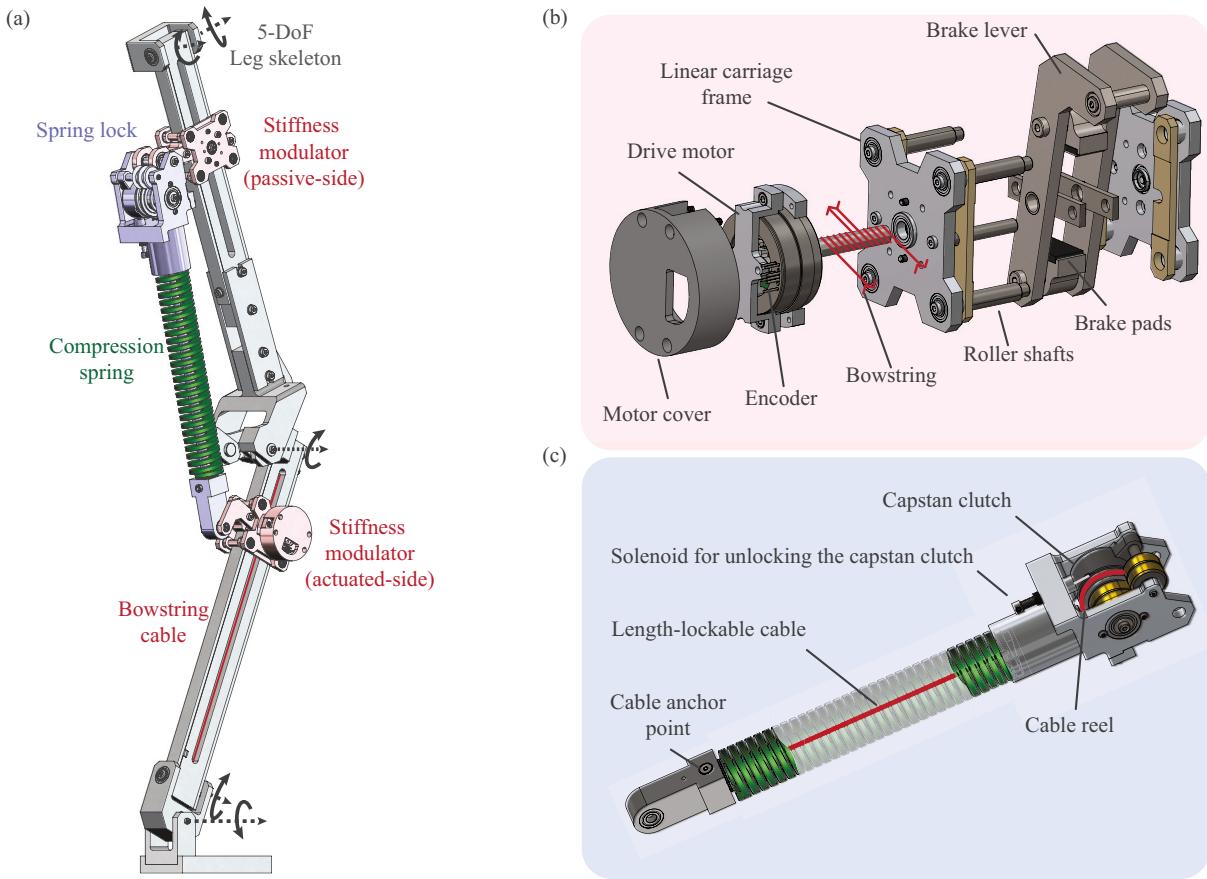


Fig. 3. Floating spring leg. (a) CAD model of the leg with the main components. (b) Design of the stiffness modulation mechanism; the motorized self-locking slider. (c) CAD model of the lockable spring.

connecting the thigh and shank (flexion-extension), and two at the bottom of the shank resembling an ankle (flexion-extension and pronation-supination). Connected to the shank through the ankle is the foot that serves to maintain ground contact.

### C. Stiffness modulation mechanism

Changing the mechanical advantage between the spring and the leg is achieved by moving the endpoints of the spring along the links of the skeleton. The spring is connected to the links with self-locking sliders, as shown in Fig. 3(a). The slider at the bottom of the spring is motorized while the slider at the top of the spring is not motorized. The motor at the bottom of the spring is used to move both sliders simultaneously only when the spring is locked. The exploded view of the bottom motorized slider is shown in Fig. 3(b). The top slider only differs from the bottom slider by the absence of the motor, and as such, the exploded view of the top slider is not shown.

The self-locking capability of the sliders is accomplished using braking levers that clamp the sliders onto the leg skeleton when the spring leg is loaded, see Fig. 4(a)-(b). The spring end is connected to the lever in such a way that the force of the spring has a mechanical advantage to generate a sufficient normal force on two high-friction brake pads and prevent translation, shown in Fig. 4(b). The carriage is free to move only when there is no spring force applied to the lever,

which occurs when the spring is fully extended or locked in extension.

Once the carriages are free to move, a bowstring actuator drives the carriages along the limbs of the skeleton. The bowstring actuator is shown in Fig. 3(b). It comprises a 48 W brushless DC motor (Allied Motion HT02000) with an 8 mm motor shaft, 0.65 mm kevlar cable, and a rotary encoder (AMS AS5145). The motor shaft runs through the width of the carriage, and the kevlar cable is wound around the shaft between the steel reinforcements inside the carriage. The ends of the cable are terminated on either end of the shank, Fig. 3(a) red. As a result, when the motor turns, the carriage translates along the shank proportionally to the output shaft diameter and the rotation of the motor.

Figure 4(c) shows the position tracking of the bowstring actuator and the power required by the motor to change the mechanical advantage of the spring for a low, medium, and high leg stiffness  $x_2 \in [25.1, 75.4, 125.7]$  mm with low, medium, and high speeds  $\dot{x}_2 \in [25.1, 75.4, 125.7]$  mm/s. When the spring is locked, the motor requires minimal effort to move the carriages housing the ends of the spring, and the required power only scales with the speed at which the spring ends are moved, independent of the energy stored in the spring [29]. In the absence of a spring force, the motor requires a 55 mNm holding torque with an average power of 0.86 W to

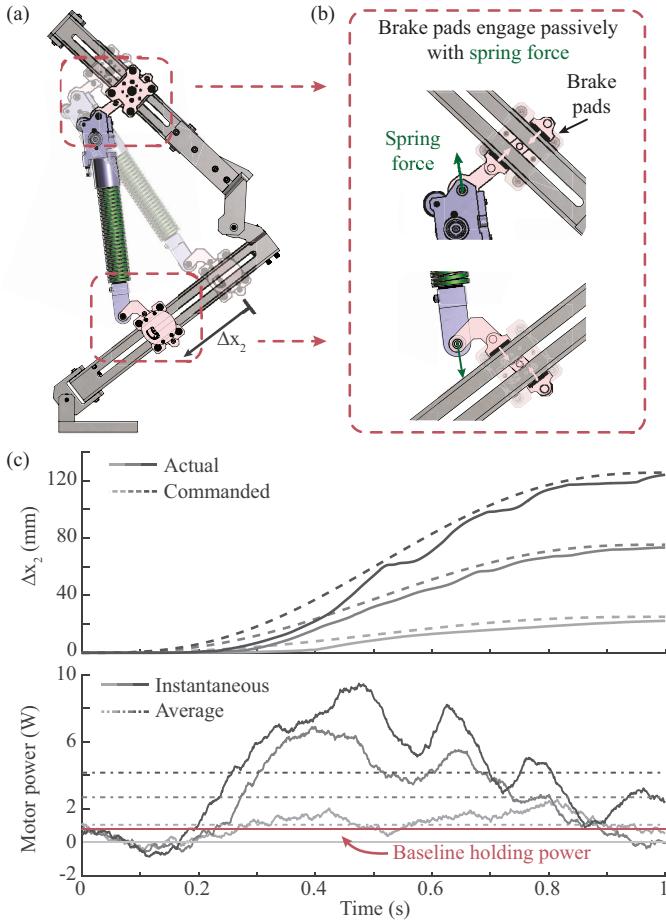


Fig. 4. Changing the stiffness of the mechanism. (a) The endpoints of the spring are moved using the motorized slider (bottom red). (b) Detailed view of the passive braking mechanism inside the sliders. (c) The motion of the motorized slider and the electrical power required by the bowstring actuator.

stop the carriages, see Fig. 4(c) red line. No power is needed to maintain the positions of the carriages once the spring is loaded.

#### D. Lockable axial compression spring

Figure 3(c) shows the lockable spring. The spring comprises a light-duty compression die spring and a friction-based capstan mechanism. The capstan mechanism is housed in series with one end of the spring and the connection to the brake hinge at the other end. The spring (green) is locked in extension using a cable (red) connected to the capstan clutch at one end of the spring and the anchor point at the other, Fig. 3(c). The spring is unlocked using a solenoid that releases the capstan clutch and allows the cable to extend. The design of the capstan clutch and the lockable spring is detailed in [43]. The mass of the spring assembly is 1.9 kg. The linear compression spring has a stiffness of 14.8 N/mm and can generate over 1200 N of force.

Figure 5(a) shows the spring upon compression and locking. Figure 5(b) shows the force of the spring upon compression and locking. The path by which the force decreases when the spring is being locked represents the loss incurred from the lock not being instantaneous or rigid. Due to this loss,

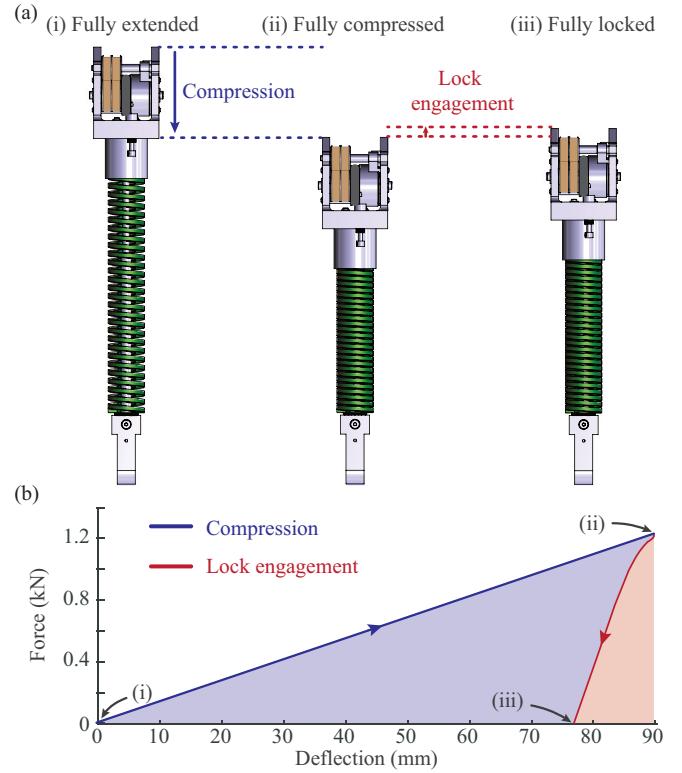


Fig. 5. Lockable axial compression spring. (a) Configurations of the lockable spring during compression: (i) the lockable spring is at free length and fully extended, (ii) the spring is compressed to a minimum length, and (iii) the lock engages and the lockable spring does not extend. (b) Force of the spring.

the spring is 80% efficient in returning stored spring energy. When the lock is released, the force of the spring transfers to the thigh and shank through the end points of the spring. In this way, the spring generates a torque about the knee of the exoskeleton to extend the leg.

### III. LIFTING WITH A VARIABLE STIFFNESS LEG

The floating variable stiffness spring could be used to increase the peak vertical force by storing energy at an initially lower stiffness and releasing the stored energy at a higher stiffness [29]. This may enable the leg to perform a weight-lifting task otherwise impossible with a fixed stiffness spring. To validate the behavior in the built prototype, the floating spring leg was used to perform the energy storage and release experiment shown in Fig. 6.

#### A. Experimental setup

Figure 6 exemplifies the task and shows the experimental setup. During the experiment, the device was vertically constrained, see Fig. 6(b). In particular, the hip flexion joint of the leg was placed in a block that houses two linear bearings while the bearings slide along rails in the vertical direction. The ankle flexion joint was vertically aligned with the hip flexion joint, and the foot was fixed to a mechanical breadboard.

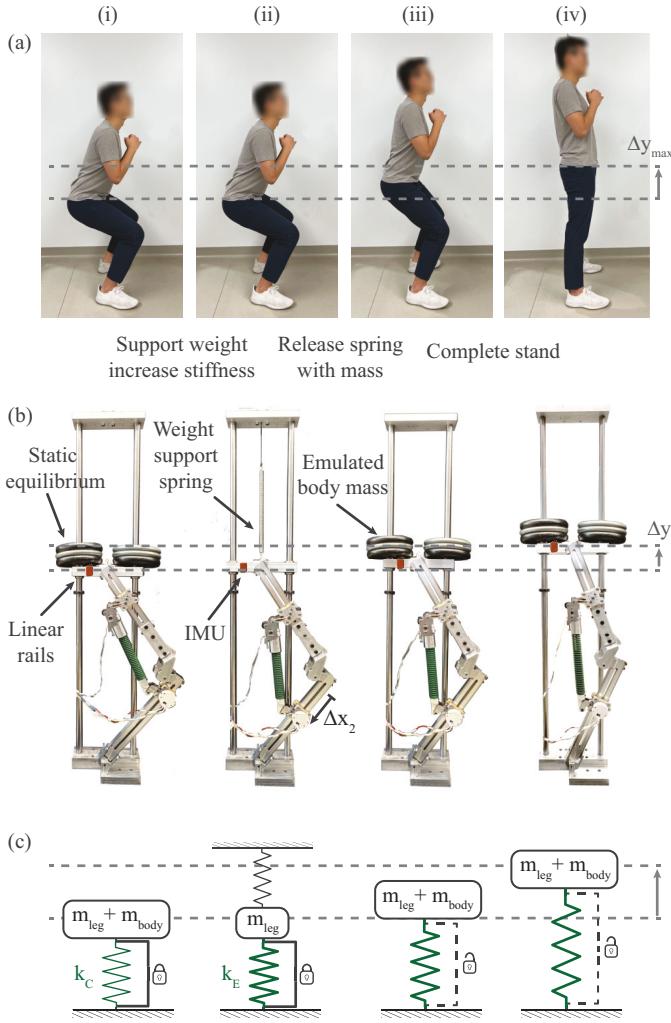


Fig. 6. Experiment of the squat-to-stand task. (a) A squat-to-stand task exemplified by a human. (b) A squat-to-stand task performed by the floating spring leg. (c) A squat-to-stand task illustrated by a spring-mass model. (i) During the squat, the leg has low stiffness while the mass lowers until the force of the leg fully supports the mass. (ii) Subsequently, the leg stiffness is increased while maintaining the same height and supporting the mass with the human or the extension spring. (iii) Next, the spring leg releases the energy it stored at the bottom of the squat to move the mass. (iv) Finally, maximum height is achieved and the squat-to-stand task is completed. The video of the experiment is provided in the supplementary material.

### B. Experimental procedure

Using the experimental setup, a weighted squat-to-stand task was performed with the floating spring leg. The video of the experiment is provided in the supplementary material.

Figure 6(a) shows a human performing a squat-to-stand motion to contextualize the task. Figure 6(a-i) shows the human lowering the body into the squat. When squatting, the legs use energy to generate the force to support the body. Next, the human initiates an upward motion, shown in Fig. 6(a-ii). Here energy is required to increase the leg force and accelerate the weight, instead of merely supporting the weight. Finally, the human moves upwards back to the standing position, Figs. 6(a-iii and iv). In this case, energy is required to compensate for the work done by gravity.

Figure 6(b) shows the task performed with the floating

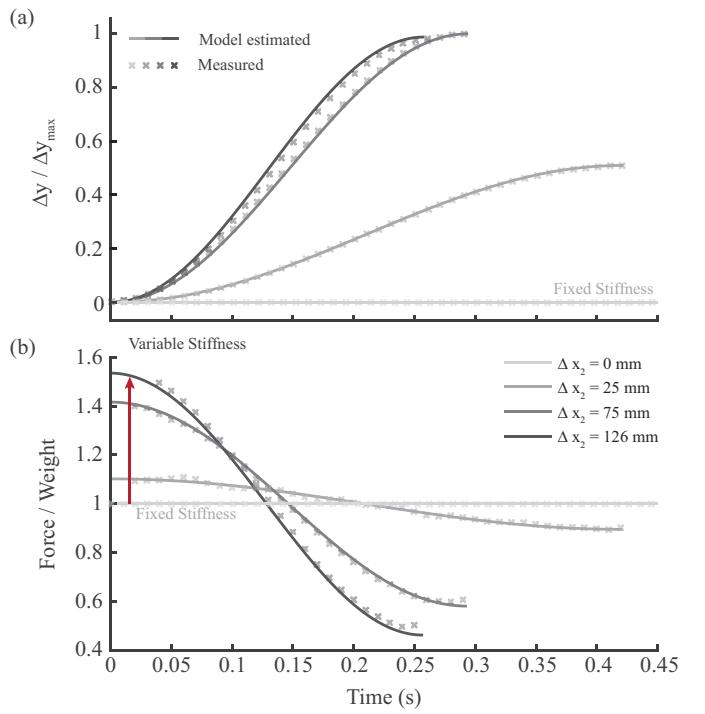


Fig. 7. Experimental results for the squat-to-stand task. (a) Measured vertical displacement of the hip joint of the floating spring leg normalized by the maximum displacement achieved with the highest stiffness increase. (b) Estimated spring force (solid) and the associated measured vertical force using the IMU at the hip. The transient IMU data is not shown.

spring leg. Here, we wish to show that the benefit of the energetically conservative floating spring leg is independent of the force provided by the human. Consequently, we used the spring leg alone to perform the task.

First, Fig. 6(b-i) shows the spring leg starting from 1 m height and stopping at 0.885 m at the bottom of the squat. At the bottom of the squat, the spring leg produces the force to support a 27 kg mass placed on the top of the slider block, emulating a human applying part of their body weight as an input force during the eccentric squat phase. Figure 6(b-ii) shows the removal of the 27 kg mass and activation of the spring lock. The locked spring was subsequently reoriented from the initial low stiffness setting shown in Fig. 6(b-i) to the final high stiffness setting shown in Fig. 6(b-ii). The removal of the mass and the addition of the auxiliary extension spring in Fig. 6(b-ii) emulate the human supporting their mass for a short time duration while the stiffness of the robot leg is changed, without adding energy to the spring. Finally, the extension spring was removed, the mass was reapplied, and the spring was unlocked, Fig. 6(b-iii). Consequently, the spring leg moved the mass back to a standing height, see Fig. 6(b-iv). The spring-mass model of the experiment is shown in Fig. 6(c).

The experiment was performed for one fixed stiffness lifting task ( $\Delta x_2 = 0$  mm), where the stiffness was not increased before the spring was released, and three variable stiffness lifting tasks ( $\Delta x_2 \in [25.1, 75.4, 125.7]$  mm) where the stiffness was increased before the spring was released. In both cases, the energy stored by the spring was the same. The motor was

only used to reorient the spring while the length of the spring was locked and the sliders could freely move along the leg. Therefore, the motor did not provide any force to help lift the load. The acceleration of the mass was recorded using an inertial measurement unit (IMU), Fig. 6(b).

### C. Results

Figure 7 shows the measured displacement and the estimated spring force during the squat-to-stand experiment. We observe that the fixed stiffness spring leg could not provide enough force to move the mass resting at the bottom of the squat, see Fig. 7(a), whereas the variable stiffness spring leg could accelerate the mass by increasing the force at the bottom of the squat, see Fig. 7(b). The variable stiffness cases resulted in vertical displacements and speeds that were dependent on the amount of force amplification. In particular, a small force amplification did not allow the mass to move up to the standing position, while a large force amplification did not allow the mass to move beyond the standing position, but allowed the mass to reach that position faster.

Figure 7 solid lines show the prediction using the spring-mass model, represented in Fig. 6(c) and defined by

$$\ddot{y} + \frac{k_E}{m}y = g, \quad (1)$$

where we estimated the effective linear spring stiffness values normalized by the load mass –  $k_E/m \in [55.6, 114.8, 148.1] \text{ N/m/kg}$  ( $R^2 > 0.99$ ) – using the measured displacement, Fig. 7(a), and the measured acceleration, Fig. 7(b). A comparison of the solid lines and the experimental data in Fig. 7 shows that the behavior of the spring leg exoskeleton closely resembles that of the theoretical model: it amplifies the force and power output of the leg without altering the energy stored by the spring to perform a weight lifting task, which is impossible to perform with a fixed stiffness spring.

Differences in the experimental data and the analytical prediction can be attributed to losses in the system and model parameter uncertainty. For instance, the model assumes that the release of the spring happens instantaneously, whereas, in the physical design, there is both an electrical and mechanical response time to power the solenoid and release the spring lock. Additionally, damping not included in the model affects the achievable height, especially at high stiffness settings where the motion is faster due to higher spring forces. Nevertheless, the experimental results have shown that there exists a threshold for the force amplification to complete the task of reaching the standing height. If the force amplification is too low, then the task cannot be completed. If the force amplification is high enough, then the task can be completed, and the time to complete the task will be shorter as the force amplification becomes higher.

## IV. DISCUSSION AND CONCLUSION

In this work, we present the design of the first functional floating spring leg (Section II), and experimentally demonstrated the theoretical benefit of customizable force-deflection behavior using the energetically conservative variable stiffness

spring leg in a typical weight-lifting task (Section III). We found that energetically conservative variable stiffness legs that can change stiffness and force can be used to leverage the energy stored by the spring; whereas, the same energy could not be released by a conventional fixed stiffness spring.

Our leg prototype provides one practical implementation of the energetically conservative variable stiffness spring concept, while our experiment provides one example that illustrates the theoretically predicted benefit of energetically conservative variable stiffness springs in tasks that require high force but little to no net mechanical energy. Examples of these tasks are sitting down and standing up from a chair by a human assisted with a robotic spring leg or moving objects up and down between factory lines by a spring-assisted industrial robot.

### A. Limitations

Although the floating spring leg introduces customizability in force-deflection behavior at a low energy cost, this new feature adds design complexity compared to conventional fixed stiffness springs. In particular, the floating spring leg requires the integration of three key mechanisms – an actuator to reorient the spring, a clutch to lock the orientation of the spring, and a clutch to lock the energy stored by the spring (Section II). These three mechanisms reduce the mass-energy density of the spring, add frictional losses, and require careful consideration for seamless integration. While this paper demonstrates the proper functionality of all these components with reasonably high energy storage-and-release efficiency, the weight of the spring leg was not optimized – for example, the mass of the skeleton (6.3 kg) is approximately three times the mass of the floating spring (1.9 kg). Consequently, the mass of the leg can be significantly reduced by optimizing the shape and material of the skeleton, with an aim to reduce the weight of the skeleton to the weight of the floating spring.

Another difference between conventional spring legs and floating spring legs is that a small amount of energy is still needed to reorient the spring, and therefore, change the stiffness of a floating spring leg. However, the energy to reorient the spring is independent of the energy already stored in the spring, because reorienting the locked spring does not compress or extend the spring. Therefore, the energy to reorient the spring can be made negligible compared to the energy stored by the spring, see Fig. 4(c) and [29]. Nevertheless, the energy cost to reorient the spring at the same speed could be reduced by minimizing the weight of the spring and using an optimal control strategy to move the spring using the least amount of energy [44]. Alternatively, the motor could be replaced by an interface that allows the human hands to reorient the spring, and thereby increase or decrease the apparent stiffness of the leg.

### B. Applications in assistive devices

The experiment in Section III showed the potential benefit of using a floating spring leg exoskeleton over a more conventional spring leg exoskeleton in assisting a human with weakened legs – for example, an elderly person having difficulty producing sufficient vertical force with their legs

to stand up from a chair [45], [46], [47]. The spring leg could increase stiffness to provide high forces to assist difficult portions of tasks, such as the onset of standing from a chair. However, releasing the same spring energy at a higher stiffness would reduce the range of motion over which the spring leg can amplify force. To provide assistive force over the entire range of motion, the spring leg may be complemented with a motor – for example, at the knee. Similarly, increasing the leg force may also benefit athletic performance in other physically demanding tasks such as jumping and running [34]. Finally, customizing the leg force may not only benefit performance but also improve user comfort. In our experiment, the same standing height could be achieved by a smaller force exerted over a longer time or a larger force over a shorter time, as shown in Fig. 7. Therefore, by minimizing the assistive spring force, within all possible forces that can achieve the task, user comfort may be also improved [48], [49] without compromising the task.

However, future work is required before the floating spring leg can be used to effectively assist or augment humans. In particular, reducing the effect of added device weight and interfacing the robot leg with the human body are the most outstanding challenges. Extra mass will reduce the effect of assistive force and energy transferred from the robot leg to the human body [50], [51], [52], as well as require more user effort to support while the apparent leg stiffness is changed. The robot leg should be made more lightweight, Sect. IV-A, and adding a clutch at the knee may help alleviate the burden of the user to support the extra mass as the stiffness is changed. Also, the interface between the robot leg and the human should enable effective force and efficient energy transfer [53], [54], especially since in applications, the device may not be vertically constrained.

### C. Applications in spring-assisted robots

The design presented in this paper shows one practical realization of the energetically conservative variable stiffness spring in the form of a robotic leg. However, the concept of energetically conservative variable stiffness springs may find its place in other areas of robotics. For example, gravity compensators are devices that use springs designed and optimized for particular, application-dependent trajectories and payloads based on the fixed force-deflection behavior of a conventional spring [55], [56], [57]. Energetically non-conservative variable stiffness springs enable these devices to adapt to different trajectories and payloads using external energy proportional to the payload, or more precisely, the energy stored by the spring [58], [59], [60], [61]. Energetically conservative variable stiffness springs could enable similar adaptability at an energy cost that is independent of the payload and the energy stored by the spring, potentially allowing variable payloads along a desired trajectory with small actuation effort.

In summary, energetically conservative variable stiffness springs can enable the design of spring-driven robots that combine the energetic benefit of conventional springs with the customizability of variable stiffness springs.

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