Gallery of Fluid Motion

Sculpting the Sphinx

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Some geological evidence suggests that the Great Sphinx was a natural landform before its surface was modified by the ancient Egyptians [1,2]. Is this controversial theory at all plausible? Some support comes from a class of landforms called yardangs that resemble seated lions, but how and why they take on such shapes is mysterious [3,4]. Fluid mechanical investigations may provide insights by showing what kinds of formations can be carved by fluidic erosion, and visualization studies can reveal the mechanisms linking the observed shapes to the flows.

We explored this problem by conducting laboratory experiments on the erosion of bodies made of clay that are washed by fast flowing water. Based on accounts of the nonuniform composition of the rock making up the Sphinx [5], we tested the effect of hard, nonerodible inclusions within mounds of softer clay. As an idealization of a prevailing wind pattern, we subjected the bodies to the unidirectional flow of a water tunnel. Under suitable conditions, we find that Sphinx-like "sculptures" are carved by the flow, as shown in Figs. 1 and 2.

Supposing the initial form should be featureless, we built up a mound of bentonite clay into a half ellipsoid whose long axis is aligned with the flow from a water tunnel (Engineering Laboratory Design). The clay is a powder to which water is added in the proportion 2:1 to form a stiff paste that is smeared layer by layer onto a platform that serves as the "bedrock." Material inhomogeneity takes the simple form of a short plastic cylinder that is resistant to erosion and is initially fully included within the clay. The photograph of Fig. 1 shows a later stage that resembles a lion in repose. The cylindrical inclusion, now largely excavated, has become the "head" facing into the wind. It is undercut to form a "neck" that connects to the body while "paws" are left at the base. What causes this anatomy?

The laboratory setting compresses time and space as compared to what happens in nature. Our structures develop over hours, and their size allows us to record the 3D morphology at regular intervals with an optical scanner (Shinning 3D EinScan-SE). Removed from the flow, the body is shone with light patterns and stereoscopically imaged from many angles, these data feeding into a digital reconstruction with a resolution of about 0.1 mm. The clay sculpture, being ephemeral, is a poor subject of flow visualization. Instead, we convert the surface scan at the Sphinx-like stage to a

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FIG. 1. A laboratory Sphinx in the current of a water tunnel. The object is coated with clay and fluorescein dye, and the photograph captures the "streak volume" or 3D region of flow that has at some time entered the boundary layer and eroded the surface.

solid geometry file that is 3D printed from plastic resin using a stereolithographic printer (Formlabs Form 3L).

The photo of Fig. 1 is obtained by painting a thin layer of clay mixed with fluorescein dye over the printed form [6], which is then returned to the flow tunnel for imaging with a digital camera (Nikon D610). The neon green hues of the original image are converted to sandy and ochre tones. High contrast is obtained with matte black background panels and illumination from bright white lamps aimed so as to enhance the dye without casting shadows. Using a scaled-up body proved useful to attain the same Reynolds number *Re* at lower flow speed, which leads to less dispersion of the dye. Lower speed is also helpful for extending the working time to gather photos, which is limited in the recirculating tunnel by the return of the polluted fluid to the test section.

Figure 1 can be interpreted as the "streak volume" or the 3D region swept out by all fluid that has passed within the boundary layer and eroded the surface. Evidently, much of the body is enveloped by separated flows and the turbulent wake. Vortices shed from the head form the wavy and billowing "mane" of the lion, and these flows seem responsible for the enhanced erosion just downstream of the head that give rise to the arched back.

The windward features of the lion are better explained by streak-line imaging with filaments of dye released upstream of the structure, as shown in the side- and top-view photos of Fig. 2. Here, we employ arrays of hypodermic tubing through which fluorescein dye is fed, and the images are again recolored. Achieving straight rather than sinuous filaments requires slender tubes whose ends are carefully deburred and chamfered, and which are gently bent to smoothly come into alignment

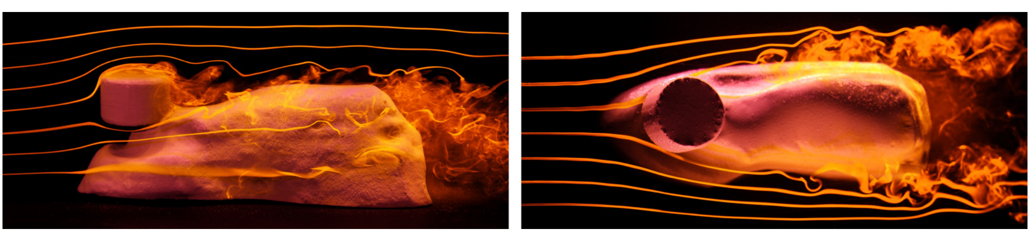

FIG. 2. Visualization of flow streak lines in vertical (left) and horizontal (right) planes. Not shown is an array of hypodermic tubing that release filaments of fluorescein dye into the upstream current.

with the flow. A larger model and slower flow speed again helps to minimize dispersion of the dye. It is unnecessary but helpful to match the density of the dye by adding alcohol and to match the flow speed, which we accomplish with a Mariotte bottle that also maintains constant feed rate.

The portions of the streak lines just upstream and to the sides of the head region are observed to be steady. They can thus be interpreted as streamlines whose spacings indicate local flow speed, with more closely spaced lines associated with faster flow by mass conservation of the incompressible fluid. The side-view image reveals faster flows in the region of the lion's neck as current is funneled in from the head above and the paws below. The top-view image also shows faster flows in this region, which, in the horizontal plane, is due to the incoming stream splitting and deflecting around the neck. These compounding effects could explain the locally high shear stress and high erosion rate just under the head and hence why strong carving digs the neck and reveals the paws.

Our laboratory system is at best inspired by and qualitatively related to landforms. We study objects of typical size 10 cm in water flows of typical speed 10 cm/s, yielding $Re = \mathcal{O}(10^4)$ that are orders of magnitude lower than those that occur for natural yardangs. Clay in running water obeys a simple law in which the erosion rate varies with the local fluidic shear stress on the solid surface [7,8]. In contrast, the relevant aoelian or wind-driven processes include abrasion by windborne grains and attrition or transport of loosened grains by turbulent flows [9,10]. Nonetheless, perhaps some general aspects of the shape-flow evolution problem are robust to system details.

Our results suggest that Sphinx-like structures can form under fairly commonplace conditions. These findings hardly resolve the mysteries behind yardangs and the Great Sphinx, but perhaps they provoke us to wonder what awe-inspiring landforms ancient peoples could have encountered in the deserts of Egypt and why they might have envisioned a fantastic creature.

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[1] F. El-Baz, Desert builders knew a good thing when they saw it, Smithsonian **12**, 116 (1981).

[2] G. Gerster and F. El-Baz, Egypt's desert of promise, National Geographic **161**, 190 (1982).

[3] A. S. Goudie, Mega-yardangs: A global analysis, Geography Compass **1**, 65 (2007).

[4] A. J. Parsons and A. D. Abrajams, *Geomorphology of Desert Environments*, 2nd ed. (Springer, Dordrecht, 2009).

[5] K. L. Gauri, Geologic study of the sphinx, Newsletter of the American Research Centre In Egypt **127**, 24 (1984).

[6] K. Amin, J. M. Huang, K. J. Hu, J. Zhang, and L. Ristroph, The role of shape-dependent flight stability in the origin of oriented meteorites, Proc. Natl. Acad. Sci. USA **116**, 16180 (2019).

[7] M. N. J. Moore, L. Ristroph, S. Childress, J. Zhang, and M. J. Shelley, Self-similar evolution of a body eroding in a fluid flow, Phys. Fluids **25**, 116602 (2013).

[8] L. Ristroph, M. N. J. Moore, S. Childress, M. J. Shelley, and J. Zhang, Sculpting of an erodible body by flowing water, Proc. Natl. Acad. Sci. USA **109**, 19606 (2012).

[9] R. S. Anderson, Erosion profiles due to particles entrained by wind: Application of an eolian sediment-transport model, Geological Society of America Bulletin **97**, 1270 (1986).

[10] M. S. Yalin, *Mechanics of Sediment Transport* (Pergamon, Oxford, 1976).