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Analytical Construction of Koopman EDMD Candidate Functions for Optimal Control of Ackermann-Steered Vehicles

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Abstract: The path-tracking control performance of an autonomous vehicle (AV) is crucially dependent upon modeling choices and subsequent system-identification updates. Traditionally, automotive engineering has built upon increasing fidelity of white- and gray-box models coupled with system identification. While these models offer explainability, they suffer from modeling inaccuracies, non-linearities, and parameter variation. On the other end, end-to-end black-box methods like behavior cloning and reinforcement learning provide increased adaptability but at the expense of explainability, generalizability, and the sim2real gap. In this regard, hybrid data-driven techniques like Koopman Extended Dynamic Mode Decomposition (KEDMD) can achieve linear embedding of non-linear dynamics through a selection of "lifting functions". However, the success of this method is primarily predicated on the choice of lifting function(s) and optimization parameters. In this study, we present an analytical approach to construct these lifting functions using the iterative Lie bracket vector fields considering holonomic and non-holonomic constraints on the configuration manifold of our Ackermann-steered autonomous mobile robot. The prediction and control capabilities of the obtained linear KEDMD model are showcased using trajectory tracking of standard vehicle dynamics maneuvers and along a closed-loop racetrack.

Keywords: Koopman Operator, Lie Brackets, Data-Driven Control, Autonomous Vehicles

1. INTRODUCTION

Wheeled mobile robots, henceforth referred to as WMRs find applications in increasingly diverse domains ranging from manufacturing shop floors, warehouses, remote surveillance etc. all the way up to surveying, rescue missions and planetary exploration. Success in these application domains is predicated on the path-tracking control capabilities of the system, crucially dependent upon modeling choices and subsequent system-identification updates. In this context, we focus on car-like WMRs, a popular architectural choice in the mentioned application spaces. The Ackermann-steered vehicles (car-like robots) exhibit high controllability and smooth cornering capabilities without undergoing energy losses and wear from tire skidding, making them an ideal choice for medium- or high-speed use cases. However, the motion of these systems is characterized by non-holonomic velocity constraints making path-tracking a challenging control problem.

Traditionally, modeling of robotic systems was based on differential geometric theory were championed by Barraquand and Latombe (1989), Lafferriere and Sussmann (1991), Laumond et al. (1994) etc., derived upon on the configuration space of the robot and motion constraints. Similary, model linearization through techniques like input-output feedback linearization and non-linear passivity-based control design are captured in Morin and Samson (2008).

While these approaches were revolutionary in providing a fundamental framework for modeling of robotic systems and design of feedback controllers for path tracking problems, they came at the expense of complicated mathematical formulation and non-linearities that made controller design a challenging task. Thus, simplified geometric and linear models were developed, enabling mathematically

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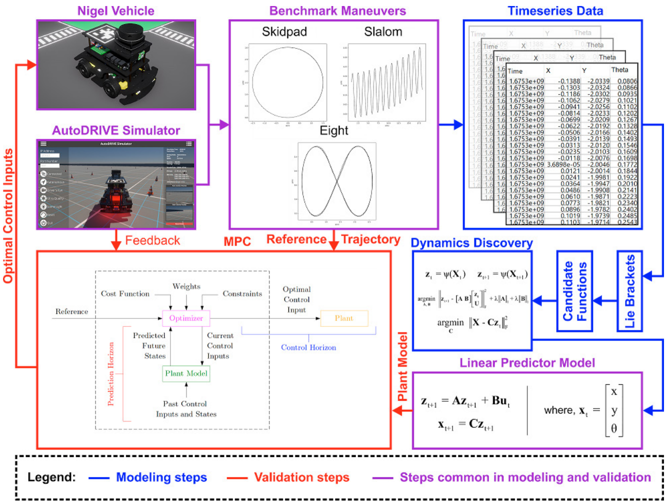


Fig. 1. Proposed approach involving timeseries data collection of benchmark maneuvers, analytical construction of candidate functions using Lie bracket formulation, discovery of dynamics for identification of linear predictor model using Koopman EDMD, formulation of linear MPC and experimental validation.

and computationally feasible path-tracking capabilities. Controllers like the Pure Pursuit Coulter (1992), Stanley Hoffmann et al. (2007) and kinematic model-based feedback control techniques have proven effective for low-speed, planar path tracking problems but fail to capture aspects of operation like wheel terrain interactions, parameter variation, etc., limiting their capabilities.

In recent decades, WMRs have seen significant hardware improvements due to advances in mobile computing, sensor availability, and modular architectural designs. Concurrently, software stacks have adopted the "Sense-Think-Act" paradigm, enabling autonomous navigation. Utilizing neural network-based architectures, motion commands are generated without explicitly modeling the system, making black-box control methods widely adopted in Imitation Learning and Reinforcement Learning based approaches, as demonstrated by Joglekar et al. (2022), Salvi et al. (2022). While end-to-end learning can adapt to different operating conditions and improve path-tracking control capabilities, they often suffer from explainability, generalizability, and sim2real transition gaps, which are critical for safety-critical applications.

The design of explainable, high-performance, and safe control is paramount for any robotic application Huang et al. (2019). As a result, data-driven methods enabling explicit model identification through the temporal snapshots of the system have gained prominence in the community Vaidya (2022). In this milieu, data-driven Koopman operator-based algorithms such as the Extended Dynamic Mode Decomposition (EDMD) as highlighted by Korda and Mezić (2018) have gained popularity due to their potential to extract a linear system model utilizing only the temporal data or snapshots of the system. While adoption of the Koopman EDMD techniques has been showcased in many non-linear and chaotic systems, this manuscript explores its application within a vehicle dynamics modeling context. The model obtained from the Koopman EDMD technique depends on the choice of candidate lifting functions $\mathbf{\Psi}$ used as highlighted in 2.2.1. While there are generalized candidate function libraries like the polynomial-, radial basis function- library, they do not consider the application-specific requirements posed by the plant model/ dynamical system. Often the choice of these candidate functions is decided heuristically and is subject to parameter tuning.

This study presents an approach to construct a candidate function library for our non-holonomic WMR analytically. We leverage the long-standing knowledge of differential geometry-based analysis of non-holonomic systems to obtain control vector fields that serve as coefficients for the coordinate basis transform. This approach improves over the Dubins car model-based lifting functions Joglekar et al. (2023). The prediction capabilities of the linear model, along with the efficacy of the feedback control law (MPC), are highlighted using a path-tracking problem involving vehicle dynamics maneuvers and eight-loop racetrack traversal.

2. PRELIMINARY

This section presents mathematical notation and concepts concerning the kinematics constraints on a robot's configuration space. The idea behind Lie brackets for controllable vector fields is also introduced. Finally, a mathematical formulation for the Koopman EDMD algorithm enabling data-driven linear embedding of non-linear systems based on lifted temporal snapshots of states/control is presented.

2.1 Kinematic Constraints on Robot Configuration

Consider a Wheeled Mobile Robot (WMR) with n dimensional configuration vector q and configuration velocities $\dot{q}$ subject to: (i) k holonomic constraints (configuration space constraints) and (ii) $m-k$ non-holonomic constraints (velocity-level constraints). Combinedly, the constraints can be treated at the velocity-level and expressed as:

$$A(q)\dot{q} = 0 \tag{1}$$

Here, $A(q)$ is a full rank matrix spanning the $\mathbb{R}^{m\times n}$ manifold. Let $G(q)$ be a full-rank matrix spanning the smooth and linearly independent vector fields in null space of $\mathcal{N}(A)$ given by $A(q)g_i(q) = 0$ for $i = 1, \cdots, n-m$. $G(q)$ can now be written as:

$$G(q) = [g_1(q) \cdots g_{n-m}(q)] \tag{2}$$

where Δ is the manifold spanned by these linearly independent vector fields in $G(q)$, which may or may not be involute.

$$\Delta = span\{g1(q) \cdots g_{n-m}(q)\} \tag{3}$$

Let Δ^* be the smallest involute distribution encompassing Δ. The following conditions dictate the holonomic and non-holonomic constraints on the system as highlighted by Campion et al. (1991).

Briefly summarized, there are 3 possible cases:

- If $k = m$, then the system is holonomically constrained, and Δ is involute and spans the entire manifold.
- If $k = 0$, then the system has only non-holonomic constraints, thus, Δ^* spans the entire manifold.
- If $0 < k < m$, there are k holonomic constraints reducing the configuration space where $dim(\Delta^*) = n-k$.

Controllability using Lie Brackets: Assuming that our system has non-holonomic constraints imposed on it, there are constraints on the tangent space of the configuration. Physically, it means that a direct path to a certain configuration may not yield a feasible trajectory for our systems. This is where the concept of controllability dictates a locally accessible set of motions given the non-holonomic constraints. Consider the following systems with n dimensional configuration space $q \in R^n$ and m control inputs $u \in R^m$. The control vector fields can be given as $\dot{q} = G(q)u$ where $G(q) \in R^{n\times m}$. Assuming, r non-holonomic constraint equations restrict the span of the distribution to $(n-r) \rightarrow \Delta$. These constraints are written as control vector fields in $G(q)$ allowing for a locally accessible set of motions in our configuration space. Assume there are currently 2 vector control fields in $G(q) = [g_1(q), g_2(q)]$, let $X = [g_1]$ and $Y = [g_2]$. Lie brackets of the vector fields X and Y can be written as $[X,Y] = \partial X.Y - \partial Y.X$. The Lie algebra of our distribution $LA(\Delta)$ is the smallest distribution containing Δ and is closed under the Lie brackets. The Lie Alegbra Rank Condition (LARC) is said to be full rank at configuration q_t, if there exists a neighbourhood in this configuration where all points are reachable by the system. We apply Lie brackets to obtain linearly independent n vectors enabling control of the robot across the neighbourhood of q_t through a series of motions thus satisfying the Small Time Local Controllability (STLC) condition. The Lie brackets-based formulation, while allowing for linearly independent control vectors spanning the configuration space, contains non-linear terms. Detailed formulation highlighting application for our Ackermann-steered robot can be found in 4.2.2.

2.2 Finite-Dimension Koopman Operator Theory

This section discusses the Koopman operator in the form of Koopman EDMD, allowing for the linear embedding of our non-linear system. The fundamental idea behind the Koopman operator is a mathematical framework enabling linearization of non-linear dynamics by transforming the basis vectors to a new high-dimensional coordinate space, also called "lifting" of the system.

Consider a non-linear system,

$$\mathbf{x}_{t+1} = \mathbf{f}(\mathbf{x}_t, \mathbf{u}_t) \tag{4}$$

where $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^n$ and $\mathbf{u} \in \mathcal{U} \subseteq \mathbb{R}^m$; $\mathbf{f} : (\mathcal{X},\mathcal{U}) \rightarrow \mathcal{X}$ governs the evolution of dynamics in time. Let $(g(x))$ be a set of new basis vectors transforming our vector space to a functional space. While $f(x)$ propagates the dynamics in the vector space, the Koopman operator $\mathcal{K}$ is a linear operator in the functional space.

$$[\mathcal{K}g](x) = g \circ \mathbf{f}(x, u) \tag{5}$$

This basis transformation lifts the system to an infinite dimensional space in the seminal work by Koopman et al. Koopman (1931). However, as highlighted in the next section, we need a finite-dimensional approximation of the Koopman operator for practical purposes.

Koopman EDMD for Finite-Dimensional Linear Predictor: Finite dimension approximation of the Koopman operator allows us to leverage the mathematical framework discovery of linear dynamics while retaining computational feasibility. This extends to applying traditional mathematical tools for system analysis and designing linear optimal feedback control. The Extended Dynamic Mode Decomposition (EDMD) method allows for approximating finite dimensional Koopman eigenfunctions and modes. As we demonstrate, the EDMD technique allows for constructing these basis functions through standard libraries or analytically.

Numerical Optimization for Linear Predictor: Consider temporal snapshots of the states and control inputs as $\mathbf{X}_t = [\mathbf{x}_0, \ldots, \mathbf{x}_{t-1}] \in \mathbb{R}^{n\times t}$, $\mathbf{U} = [\mathbf{u}_0, \cdots, \mathbf{u}_{t-1}] \in \mathbb{R}^{m\times t}$, with $\mathbf{X}_{t+1} = [\mathbf{x}_1, \ldots, \mathbf{x}_t] \in \mathbb{R}^{n\times t}$ being a single time step progression of $\mathbf{X}_t$. The relationship between elements of $\mathbf{X}, \mathbf{X}_{t+1}$ and $\mathbf{U}$ matrix can be written as a non-linear discrete system in the vector space as:

$$\mathbf{x}_{t+1} = \tilde{A}\mathbf{x}_t + \tilde{B}\mathbf{u}_t \tag{6}$$

Let $\mathbf{\Psi}$ be a functional basis vector in lifted space such that $\mathbf{\Psi} = [\mathbf{\Psi}_1(x), \ldots, \mathbf{\Psi}_N(x)] \in \mathbb{R}^N$. The temporal snaphots in $\mathbf{X}_t$ and $\mathbf{X}_{t+1}$ are lifted along the directions of $\mathbf{\Psi}$. The lifted linear system can be represented as

$$\mathbf{\Psi}(x_{t+1}) = \mathbf{A}\mathbf{\Psi}(x_t) + \mathbf{B}u_t \tag{7}$$

Let $\mathbf{z}_t = \mathbf{\Psi}(\mathbf{X}_t)$ and $\mathbf{z}_{t+1} = \mathbf{\Psi}(\mathbf{X}_{t+1})$ denote the lifted states where $\mathbf{z}_t, \mathbf{z}_{t+1} \in \mathbb{R}^{N\times t}$ where $N >> n$. From Eq.(7), we have:

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{u}_t \tag{8}$$

The state matrix corresponding and control matrices in the lifted space are $\mathbf{A}$ and $\mathbf{B}$, respectively. $\mathbf{C}$ is the inverse transform allowing estimation of the state vector from the lifted state. Numerically, the analytical solution for obtaining $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ as highlighted by Korda and Mezić (2018) is:

$$\underset{\mathbf{A},\mathbf{B}}{\arg\min} \left\| \mathbf{z}_{t+1} - [\mathbf{A}\ \mathbf{B}] \begin{bmatrix} \mathbf{z}_t \\ \mathbf{U} \end{bmatrix} \right\|_F^2 \tag{9}$$

$$\underset{\mathbf{C}}{\arg\min} \|\mathbf{X} - \mathbf{C}\mathbf{z}_t\|_F^2 \tag{10}$$

3. MOTIVATION AND PROBLEM STATEMENT

The primary contribution of this study is to assess the performance of a data-driven Koopman EDMD model for path-tracking capabilities if components of vector fields are derived from Lie brackets of a non-holonomic Ackermann-steered vehicle. The mathematical framework for analytical construction of lifting function $\mathbf{\Psi}$ along with model identification with EDMD are presented, enabling dynamics prediction. A feedback law in the form of linear MPC is implemented to enable path tracking on critical dynamic maneuvers.

To support our research, we utilized the AutoDRIVE Ecosystem Samak et al. (2023) to collect dynamics data of Nigel, a 1:14 scale robotic vehicle platform suitable for in-lab AV research experiments. The AutoDRIVE Simulator recorded time-synchronized percepto-dynamic data of the ego vehicle across various trajectory profiles and maneuvers. For this study, we selected a subset of this data, focusing on vehicle dynamics. The choice of this ecosystem was driven by its seamless sim2real transferability, ensuring the applicability of our approach in real-world scenarios going forward.

4. METHODOLOGY

4.1 Data Collection

Consider the 1:14 scale Nigel vehicle performing standard vehicle dynamics maneuvers (skidpad, fishhook, slalom and eight) within the "Testing Track" scenario of AutoDRIVE Simulator. The said maneuvers are described below:

- **Skidpad:** Skidpad maneuvers were realized using an open-loop controller, which set the throttle and steering inputs of the vehicle to a set of constant values sampled uniformly from within the actuation limits of the vehicle (refer Table 1). This made the vehicle drive in circular trajectories of varying radii.
- **Slalom:** Slalom maneuvers were performed using an open-loop controller, setting the vehicle's throttle to a constant value within its actuation limits (refer Table 1). The steering actuator was subjected to a time-shifted and limit-scaled sinusoidal signal input ($\delta = \delta_{lim} * \texttt{sin}(\pi/2 + t)$), leading the vehicle to follow sinusoidal trajectories with varying amplitudes.
- **Eight:** Eight maneuvers were realized using a human-in-the-loop teleoperation approach, wherein the throttle of the vehicle was set to a constant value sampled uniformly from within the actuation limits of the vehicle (refer Table 1) and the human operator teleoperated the steering actuator using an analog input modality (dragging a standard computer mouse across the screen to proportionately set the steering angle). This made the vehicle drive in slightly variable trajectories around the eight track.

This work utilized data collected at a target sampling rate of 30 Hz, which included the 2D positional coordinates x, y and yaw angle θ of the vehicle w.r.t. the static world frame, linear speed v and yaw rate ω. Additionally, the low-level throttle and steering control inputs (τ, δ) provided to the vehicle were recorded and used to deduce the standard torque-velocity (τ, v) mapping for the vehicle to be used in conjunction with the steering input δ.

4.2 Analytical Construction of Candidate Functions

This section highlights two main approaches to constructing candidate basis functions for obtaining the lifted dynamics. The first method is the Dubins car model, highlighted in Joglekar et al. (2023)

Dubins Car based Candidate Function Selection: The Dubins car model provides a framework for generating a smooth feasible path between two points in a configuration space for car-like robots as highlighted by Dubins (1957) Lamiraux and Laumond (2001). Utilizing the robot's configuration space and Dubin's car model, an analytical library of lifting functions can be obtained for Koopman EDMD. We have successfully demonstrated this formulation in Joglekar et al. (2023).

Lie Brackets based STLC Candidate Functions: This section builds on the construct of Lie brackets, as highlighted in 2.1.1, for a car-like robot having non-holonomic velocity constraints on the motion. Iterative Lie brackets result in vector fields allowing smooth motion across the configuration space through a series of locally accessible maneuvers. Consider a car-like robot with configuration space $q = [x, y, \theta, \delta]^T$ with control inputs linear velocity v and steering input δ. The feasible motion under this configuration space is the translation and rotational motion of the vehicle chassis corresponding to velocities in $\dot{x}, \dot{y}$, and $\dot{\theta}$ direction along with the angular motion of the steering wheel $\dot{\delta}$ bounded by the steering limits. The non-holonomic constraints to this configuration space arise from lateral velocity constraints on the front and rear axle.

Considering discrete-time snapshots of the system for our Koopman EDMD model, the state vector consists of x, y, θ, δ coordinates. The instantaneous center of rotation for any steered maneuver aligns with the vehicle's rear axle. Assuming there is no wheel slip, there are two constraints on the motion imposed by the rear and front wheels. The movement of the rear axle w.r.t the reference point can be given as:

$$-\sin(\theta)\dot{x} + \cos(\theta)\dot{y} = 0 \tag{11}$$

Similarly, the motion of the front axle is:

$$-\sin(\theta+\delta)\dot{x} + \cos(\theta+\delta)\dot{y} + l\cos\delta\dot{\theta} = 0 \tag{12}$$

These constraints can be written in the form:

$$A(q)\dot{q} = 0 \tag{13}$$

where,

$$A(q) = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 & 0 \\ -\sin(\theta+\delta) & \cos(\theta+\delta) & l\cos\delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

Let $G(q) = [g_1(q), g_2(q)]$ be our two control vector fields derived from Eq.(14) satisfying $A(q)G(q) = 0$.

$$G(q) = \begin{bmatrix} 0 & \cos(\theta) \\ 0 & \sin(\theta) \\ 0 & \frac{\tan\delta}{l} \\ 1 & 0 \end{bmatrix} \tag{15}$$

The 2-dimensional control vector field does not span the distribution of our entire configuration space. We compute successive Lie brackets to span the four-dimensional space of the configuration space. As a result, we compute successive Lie brackets that do not lie in the distribution spanned by g_1 and g_2. The resultant are $g_3 = [g_1(q), g_2(q)]$ and $g_4 = [g_1(q), g_3(q)]$.

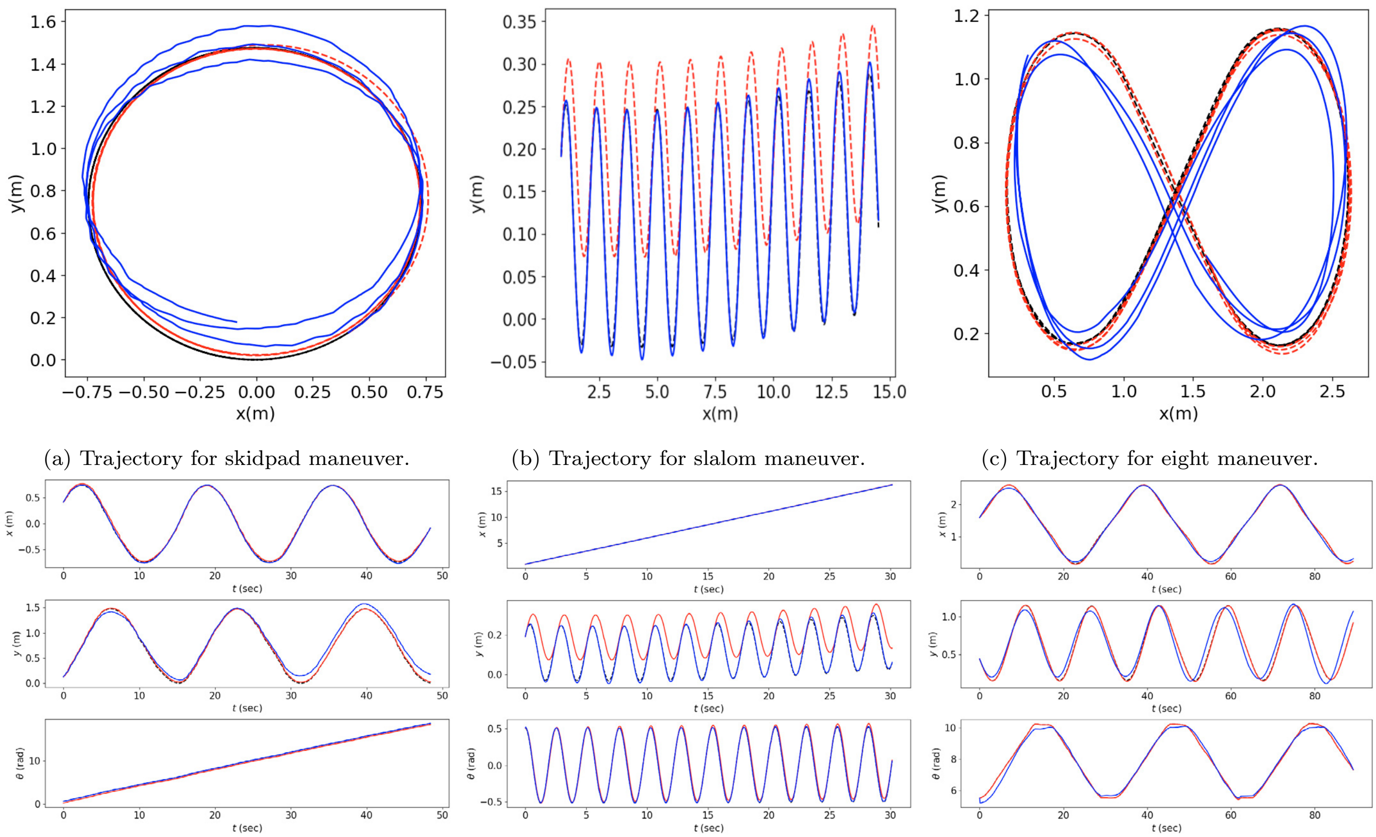
$$G(q) = [g_1, g_2, g_3, g_4] = \begin{bmatrix} 0 & \cos(\theta) & 0 & \frac{\sin\theta}{l\cos^2\theta} \\ 0 & \sin(\theta) & 0 & \frac{-\cos\theta}{l\cos^2\theta} \\ 0 & \frac{\tan\delta}{l} & \frac{1}{l\cos^2\theta} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

The distribution spanned by the $G(q)$ is involutive and spans the configuration space. The vectors in the $G(q)$ matrix are linearly independent, allowing the robot to access any configuration in terms of position and orientation with incremental motion sequences. This satisfies the STLC conditions.

The terms in the $G(q)$ matrix can be utilized along with the state measurements to construct a candidate basis function library $\mathbf{\Psi}$. The original coordinate space is transformed along the directions of coefficients of the vector fields in $G(q)$. The lifted space is represented by $\mathbf{z}_t := \mathbf{\Psi}(\mathbf{x}_t)$ and captures the kinematically feasible motion of our non-holonomic system. The basis vectors in the lifted

Table 1. Parameter variation for collection of time-synchronized vehicle state-input data.

Maneuver	Throttle τ (norm%)	Steering Angle δ (rad)	Initial x (m)	Initial y (m)	Initial θ (rad)
Skidpad	{0.2, 0.4, 0.6, 0.8, 1.0}	{0.1047, 0.2094, 0.3142, 0.4189, 0.5236}	0.0	0.0	0.0
Slalom	{0.2, 0.4, 0.6, 0.8, 1.0}	{0.1047, 0.2094, 0.3142, 0.4189, 0.5236}	0.0	0.0	0.0
Eight	{0.2, 0.4, 0.6, 0.8, 1.0}	{0.1047, 0.2094, 0.3142, 0.4189, 0.5236}	1.327	0.711	5.498



(a) Trajectory for skidpad maneuver. (b) Trajectory for slalom maneuver. (c) Trajectory for eight maneuver.

(d) State propagation for skidpad maneuver. (e) State propagation for slalom maneuver. (f) State propagation for eight maneuver.

Fig. 2. Experimental validation of vehicle trajectory and state propagation: Kinematic bicycle model with NMPC feedback law (red) and Koopman EDMD model with linear MPC feedback law (blue) against ground-truth data (black) for various maneuvers.

space $\mathbf{z}_t$ consist the follow terms:

$$\left[x \;\; y \;\; v \;\; \delta \;\; v\cos\theta \;\; v\sin\theta \;\; \frac{\tan\delta}{l} \;\; \frac{1}{l\cos^2\theta} \;\; \frac{\sin\theta}{l\cos^2\theta} \;\; \frac{-\cos\theta}{l\cos^2\theta}\right] \quad (17)$$

4.3 Linear Approximation using Koopman EDMD

In section 4.2.1 we described a set of candidate functions $\mathbf{\Psi}$ using the Lie brackets formulation for our WMR. Using the lifting function, we transform our data matrix $\mathbf{X}$ and $\mathbf{Y}$ as $(\mathbf{z}_t, \mathbf{z}_{t+1})$ with control inputs $(\mathbf{u}_t)$. The lifted system's state and control vectors ($\mathbf{A}$ and $\mathbf{B}$) can be determined from Eq.(9). This is the finite dimension approximation of the Koopman operator for our controlled dynamical system.

4.4 Linear MPC for Trajectory Tracking

A feedback control law similar to Joglekar et al. (2023) is designed for our finite dimensional Koopman operator for trajectory tracking using the Splitting Conic Solver (SCS). System dynamics are defined by:

$$\begin{aligned} \mathbf{z}_{t+1} &= \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{u}_t \\ \mathbf{x}_{t+1} &= \mathbf{C}\mathbf{z}_{t+1} \end{aligned} \quad (18)$$

The MPC feedback law solves the following optimization at each step:

$$\min_{u_t, z_t} \sum_{t=1}^{t=N_p} \left(\mathbf{Cz_t} - \mathbf{Cz_t^{ref}}\right)^\top Q \left(\mathbf{Cz_t} - \mathbf{Cz_t^{ref}}\right) + \left(\frac{\mathbf{u_t} - \mathbf{u_{t-1}}}{\Delta t}\right)^\top P \left(\frac{\mathbf{u_t} - \mathbf{u_{t-1}}}{\Delta t}\right) \quad (19a)$$

subject to

$$\mathbf{u_{min}} \leq \mathbf{u_t} \leq \mathbf{u_{max}} \;\; \forall t \quad (19b)$$

$$\mathbf{Cz}_0 = [x_0, y_0, v_0, \psi_0, \delta_0]^\top \quad (19c)$$

We penalize the error between the predicted and measured state by $([\mathbf{C}z_{ref} - \mathbf{C}z_{pred}])$, where $(\mathbf{C}z_{pred})$ is the predicted state of the robot. An additional penalty term penalizes sudden changes in control input that can cause instability. The state and control penalty matrix (P and Q) are positive semidefinite matrices.

5. RESULTS AND DISCUSSION

In this section, we compare and verify the path-tracking abilities of our newly proposed Koopman EDMD model and linear MPC feedback law with the standard benchmark kinematic bicycle model employing NMPC feedback law. The linear model and feedback control law are highlighted in section 4.2,4.4, respectively. Our test data contains random trajectories sampled from the Skidpad, Slalom and Eight maneuvers described in section 4.1 not previously used in training the KEDMD model. Fig. (2a-2f) compare the path-tracking capabilities of the two approaches for our selected maneuvers.

In the context of the Skidpad maneuver (Fig. (2a, 2d)) and the Eight maneuver (Fig. (2c, 2f)), our proposed KEDMD and linear MPC methodology exhibit performance on par with the kinematic model using NMPC. This observation can be attributed to the low excitement of dynamics experienced during the steady state conditions of these maneuvers. The distinction in performance becomes evident, particularly during the execution of the Slalom maneuver (Fig. (2b, 2e)), where the roll-plane and yaw-plane dynamics are continuously stimulated. Here, the superiority of our KEDMD model over the kinematic bicycle model is evident, indicating its successful capture of the underlying dynamics of the system.

The average MSE for tracking errors for a collection of skidpad, slalom, eight runs from the test dataset is provided in Table 2.

Table 2. Tracking performance of KEDMD model + linear MPC and kinematic bicycle model + NMPC across test maneuvers

Maneuver	Control	x-MSE (m)	y-MSE (m)	θ-MSE (rad)
Skidpad	Koopman model + linear MPC	1.7e-4	1.9e-3	1.09e-6
	Kinematic model + NMPC	4.9e-4	2.8e-4	0.08
Slalom	Koopman model + linear MPC	1.8e-5	5.8e-5	3.4e-5
	Kinematic model + NMPC	1.4e-4	0.010	0.0015
Eight	Koopman model + linear MPC	0.0024	0.0093	0.039
	Kinematic model + NMPC	1e-4	9.0e-4	1e-4

6. CONCLUSION

This study presents an analytical construction of lifting functions based on Lie brackets, which form the fundamental basis vectors for our lifted Koopman EDMD model. We postulate that utilizing Lie brackets-based STLC vector fields as basis vectors adequately account for the nonholonomic constraints inherent in our robotic system. The effectiveness of our hypothesis is confirmed through the feedback control performance of the Koopman EDMD model, which demonstrates superior tracking capabilities over the sampled trajectories.

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