

Cooperative Control of a Hybrid Exoskeleton Using Optimal Time Varying Impedance Parameters During Stair Ascent

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Abstract—Potentially, cooperative control of functional electrical stimulation (FES) and electric motors in a hybrid exoskeleton can perform stair ascent while adapting to a user's locomotion. Towards this goal, it would be essential to determine the time varying impedance model parameters of each user while ensuring the stability of the closed loop system. While some previous studies address the stability problem when estimating time varying impedance model parameters, constraints on the parameters to their physiological values are not guaranteed. In this paper, we develop a model predictive control (MPC) based approach to prescribe physiologically constrained time varying stiffness and damping parameters for an impedance model. A terminal cost and controller for the stiffness and damping are designed to ensure the MPC problem is recursively feasible, satisfy physiological constraints, and is asymptotically stable. Another MPC-based cooperative control approach is then used to ensure that the knee joint follows the knee trajectory generated via the impedance model with optimized parameters. Simulations results show foot, knee joint, and impedance model tracking while allocating inputs between FES and motors during stair ascent and adequate foot clearance and placement.

I. INTRODUCTION

Spinal cord injuries (SCI) are debilitating and frequently cause a loss of essential lower limb functions, inhibiting activities such as walking, running, sitting-to-standing, etc. Function electrical stimulation (FES) can be incorporated into closed-loop control algorithms [1] to assist persons with SCI in performing activities such as walking or standing. FES applies artificial electrical currents to activate motor neurons and generate muscle contractions that produce a desired motion. However, its artificial nature makes it prone to muscle fatigue. The onset of fatigue causes a sharp decay in torque output thus limiting its effectiveness when performing long-term periodic motions.

In recent years, hybrid exoskeletons have been proposed [2]–[4], which allows the joint torque to be distributed between FES and motors instead of only relying on FES, thus reducing the potential impact of muscle fatigue while maintaining the therapeutic benefit of stimulation. In our previous

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studies a Model Predictive Control (MPC) approach was used to allocate optimal motor and FES inputs based on a person-specific fatigue model when performing knee extension and sitting to standing tasks [5], [6]. This cooperative approach enables the user to perform activities that require high power while simultaneously maintaining the therapeutic benefit of FES.

While hybrid exoskeletons can effectively perform sit-to-stand and walking [7], [8], the stair ascent problem remains unsolved and would potentially enable versatility in or outside homes. In comparison to level-ground walking, the two main challenges in performing stair ascent are 1) detecting each stair and designing prescribed joint angle trajectories to ensure foot clearance [9], [10] and 2) generating desired torque at the knee joint [11]. While, the stair detection problem has been explored in previous studies using methods such as deep learning [12] and dynamic mode primitives [13]–[15], there is a further need to investigate control methods which adapt to human locomotion during stair climbing. It is noted in [16] that the knee joint stiffness varies during each gait cycle phase during stair ascent and descent based on a Force-Length curve. Thus, there is a desire to develop a control approach using a hybrid exoskeleton for stair ascent which not only has the knee joint follow a desired trajectory but also estimates individualized time-varying impedance parameters based on which the controller adapts.

Recent studies investigated control methods with varying impedance model parameters, including system identification by learning from demonstration (LFD) [17]–[19], optimization and optimal control approaches [20], [21], and artificial intelligence [22], [23]. These methods often require offline data, are computationally expensive and do not guarantee closed-loop stability [24], which is especially necessary when considering user safety during exoskeleton use. Studies such as [24]–[27] considered the stability of time varying impedance parameters. [25] proposed a virtual tank-based approach to modify time-varying stiffness and damping. Constraints were placed on the virtual tank to limit the energy for impedance modifications while ensuring safety. However, this method relies on the energy state of the system and the constraints on the virtual tank. [26] developed a state-independent stability condition that relates the variable stiffness and damping to their rates of change. [24] designed online adaptation laws for damping and stiffness parameters

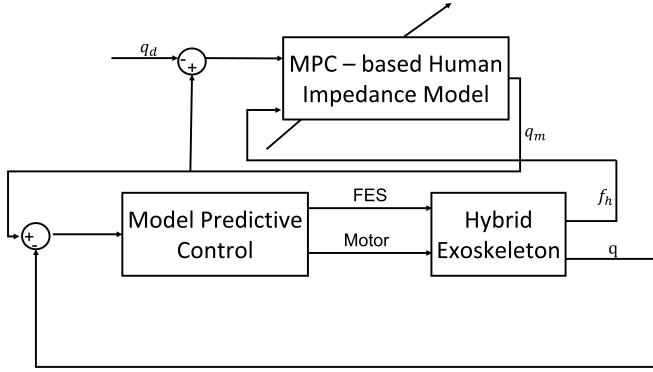


Figure 1. Overall two-loop control design to adapt to varying human stiffness and damping parameters during swing phase

to account for modeling uncertainties in the human-robot interaction dynamics while ensuring closed-loop stability without the requirement of direct interaction forces between the human and robot. While the stability analysis shows that the tracking error and the error between the estimated and true impedance parameters are uniformly ultimately bounded (U.U.B), it is not guaranteed that the impedance parameters are within constraints on their actual physiological values.

In this paper, a new MPC scheme is proposed to obtain optimal time-varying stiffness and damping subject to physiological constraints of the knee joint. A terminal control was designed on each time horizon for the stiffness and damping parameters to ensure recursive feasibility and asymptotic stability. The optimal stiffness and damping are then used with an admittance control strategy to allocate effort between motors and FES while ensuring the knee joint follows the impedance model angle. Simulations were performed to show the knee joint's impedance parameter estimation and tracking performance during the stair ascent task.

II. CONTROL DESIGN

The proposed control system consists of two control loops. The first loop is an inner control loop that uses MPC to determine motor and FES inputs such that the exoskeleton trajectory matches a desired impedance model. The second loop is an outer loop that determines optimal stiffness and damping parameters of the impedance model using MPC while also ensuring the impedance model follows a desired stair ascent knee trajectory. A control diagram of the proposed control scheme is shown in Fig. 1.

A. MPC to Determine Time-Varying Impedance Model Parameters

The impedance model of the knee joint is given as

$$M\ddot{q}_m + B(t)\dot{q}_m + K(t)q_m + \tau_s(t) = \tau_h \quad (1)$$

where $q_m \in \mathbb{R}$ is the impedance model angle, $M \in \mathbb{R}^+$ is the constant inertia of the knee joint, $B(t), K(t) \in \mathbb{R}^+$ are time varying damping and stiffness parameters, $\tau_s(t) \in \mathbb{R}$ is a stabilizing torque and $\tau_h \in \mathbb{R}$ is a measured interaction

torque between the user and the exoskeleton. The goal of the outer control loop is to determine the parameters optimal stiffness, damping, and stabilizing torque while ensuring that q_m follows a desired angle, $q_d \in \mathbb{R}$. The impedance model in (1) can be written in a linearly parameterized form as

$$M\ddot{q}_m + \begin{bmatrix} \dot{q}_m & q_m & 1 \end{bmatrix} \begin{bmatrix} B(t) \\ K(t) \\ \tau_s \end{bmatrix} = M\ddot{q}_m + Y(t)\theta(t) = \tau_h \quad (2)$$

where $Y \in \mathbb{R}^{1 \times 3}$ is a known regressor vector and $\theta \in \mathbb{R}^{3 \times 1}$ is the impedance parameter vector along with the stabilizing torque. To facilitate the goal of determining the optimal stiffness and damping while simultaneously tracking the desired joint angle, the following error terms are defined

$$e_1 = q_m - q_d \quad (3)$$

$$e_2 = \dot{e}_1 + \alpha_1 e_1 \quad (4)$$

where $\alpha_1 \in \mathbb{R}^+$ is a positive gain. The optimal stiffness and damping are determined using an MPC scheme that solves the following optimization problem on the prediction horizon $[t_k, t_k + T]$ where T is the prediction horizon length

$$\min_{\theta} J(z_{t_k}, \theta_{t_k}) = \int_{t_k}^{t_k+T} l(z(\tau), \theta(\tau)) d\tau + V(z_{t_k+T}) \quad (5)$$

subject to:

$$\begin{aligned} M\ddot{q}_m(t_k) + Y(t_k)\theta(t_k) &= \tau_h(t_k) \\ \theta(t_k) &\leq \bar{\theta} \\ z_{t_k+T} &\in \Omega_T \end{aligned} \quad (6)$$

where $z \in \mathbb{R}^{2 \times 1}$ is an error vector defined as $z = [e_1 \ e_2]^T$, $\bar{\theta} \in \mathbb{R}$ is a physiological constraint on the impedance model parameters, Ω_T is a terminal region defined as

$$\|z_{t_k+T}\| \leq \delta \quad (7)$$

where $\delta \in \mathbb{R}^+$ is a bound designed later to ensure recursive feasibility, $l(z)$ and $V(z_{t_k+T})$ are defined as

$$l(z) = z^T Q z + \theta^T R \theta, \quad (8)$$

$$V(z_{t_k+T}) = \frac{1}{2}e_1^2 + \frac{1}{2}M e_2^2 \quad (9)$$

respectively, where $Q \in \mathbb{R}^{2 \times 2}$ and $R \in \mathbb{R}^{3 \times 3}$ are positive definite symmetric matrices. The optimal control problem can be solved on each time horizon $[t_k, t_k + T]$ using a gradient projection approach [28] to obtain optimal impedance model parameters and stabilizing force while satisfying their physical constraints.

1) Recursive Feasibility of MPC Scheme:

Theorem 1. *If the terminal stiffness and damping are chosen as*

$$\theta = \tilde{Y}^T (\tilde{Y} \tilde{Y}^T + \epsilon)^{-1} (M \alpha_1 \dot{e}_1 + k_1 e_2)$$

where $k_1 \in \mathbb{R}^+$ is a positive gain, $\tilde{Y} \in \mathbb{R}^{1 \times 3}$ is a regressor error vector, $\epsilon \in \mathbb{R}^+$ is a positive constant designed such that $(\tilde{Y} \tilde{Y}^T + \epsilon)^{-1}$ is not zero (e.g. by choosing ϵ as the

spectral radius of $\tilde{Y}\tilde{Y}^T$), and $\gamma_3 \in \mathbb{R}^+$ is a positive gain that which satisfies

$$\gamma_3 > \frac{\epsilon\rho^2(\sqrt{\frac{\gamma_2}{\gamma_1}}\|z(t_0)\|)}{2k_2} + \beta$$

where $\gamma_1, \gamma_2 \in \mathbb{R}^+$ are bounds on $V(z_{t_k+T})$, $k_2 \in \mathbb{R}^+$ is a positive gain, $\rho(\|z\|) \in \mathbb{R}^+$ is a positive monotonic bounded function and α_1, k_2, k_1 are chosen such that $\alpha_1 > \frac{1}{2}, k_1 - \frac{\epsilon k_2}{2} > \frac{1}{2}$, and β is a defined constant, the terminal state is invariant and the MPC is recursively feasible. Further, the terminal region in (7) should be designed as

$$\delta \leq \Phi_1$$

where Φ_1 is defined as $\Phi_1 = \frac{\bar{\theta}}{\Upsilon(M\alpha_1 + M\alpha_1^2 + k_1)}$ where Υ bounds $(\tilde{Y}\tilde{Y}^T + \epsilon)^{-1}$ as $\|\tilde{Y}\tilde{Y}^T(\tilde{Y}\tilde{Y}^T + \epsilon)^{-1}\| \leq \Upsilon$.

Proof: The proof to Theorem 1 is available upon request. \blacksquare

B. Inner Loop Cooperative Control of the Hybrid Exoskeleton

Once the optimal stiffness and damping parameters for the impedance model are determined using MPC, the inner control loop will determine the motor torque and FES stimulation input required to ensure that the actual knee joint follows the impedance model angle.

1) Knee Extension Dynamic Model: The knee dynamics during the swing phase with motor and FES inputs [6] is given as

$$J\ddot{q} + \tau_p + G(q) = u_1 + \rho(q, \dot{q})\varphi a, \quad (10)$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}$ represent angular position, velocity, and acceleration of a limb joint respectively, u_1 is the motor input, $J \in \mathbb{R}^+$ is the moment of inertia of the leg, $G(q) = mgl\sin(q)$ is the gravitational torque where $m \in \mathbb{R}^+$ is the mass of leg, $g \in \mathbb{R}^+$ is the gravitational acceleration constant, $l \in \mathbb{R}^+$ is the distance from the knee joint to the center of mass and τ_p is a passive torque defined as

$$\tau_p = d_1(-q) + d_2\dot{\phi} + d_3e^{d_4\phi} - d_5e^{d_6\phi} \quad (11)$$

where $\phi \in \mathbb{R}$ is defined as $\phi = \frac{\pi}{2} - q$ and $d_1 \dots d_6 \in \mathbb{R}^+$ are person specific parameters. The torque contribution from FES is given by the $\rho(q, \dot{q})\varphi a$ term where $\rho(q, \dot{q})$ is a force-length and force velocity relationship defined as

$$\rho(q, \dot{q}) = (c_2\phi^2 + c_1\phi + c_0)(1 + c_3\dot{\phi})a \quad (12)$$

and $a \in [0, 1]$ is the quadriceps muscle activation which evolves based on the solution to the following differential equation

$$\dot{a} = \frac{u_2 - a}{T_a} \quad (13)$$

where $u_2 \in [0, 1]$ is the normalized FES current or pulse width input and $T_a \in \mathbb{R}^+$ is the activation time constant. Additionally, φ is the FES-induced muscle fatigue modeled as

$$\dot{\varphi} = w_f(\varphi_{min} - \varphi)a + w_r(1 - \varphi)(1 - a), \quad (14)$$

where $w_f, w_r \in \mathbb{R}^+$ are time constants for fatigue and recovery of the muscle and φ_{min} is the minimum fatigue value for each person. Since a is bounded as $a \in [0, 1]$, the solution for (14) at the bounds $u_f = 0$ and $u_f = 1$ result in φ being bounded as $\varphi \in [\varphi_{min}, 1]$ where a fatigue value of 1 means the muscle is fully rested and a fatigue value of φ_{min} means the muscle is completely fatigued.

2) MPC to Allocate FES and Motor Inputs: A MPC approach is used to determine the appropriate FES and motor inputs to apply to the user and exoskeleton in order to have the knee joint angle track follow the impedance model. To track the impedance model, the following error terms $e_m, r \in \mathbb{R}$ are defined as

$$e_m = q_m - q \quad (15)$$

$$r = \dot{e}_m + \alpha_2 e_m \quad (16)$$

where $\alpha_2 \in \mathbb{R}^+$ is a positive constant. A backstepping error $e_x \in \mathbb{R}$ is defined as

$$e_x = a - a_d \quad (17)$$

where a_d is a virtual control input defined as $a_d = \varphi^{-1}(M_{mp}(\ddot{q}_m + \alpha_2 e_m) + L_p)$ where M_{mp}, L_p are defined in [29]. As seen in [29], the closed loop error system then becomes

$$M_{mp}\dot{r} = -\varphi e_x - \frac{u_1}{\rho} \quad (18)$$

where ρ is defined in (12) and u_1 is the motor input. Further taking the derivative of (17) gives

$$\dot{e}_x = -\frac{a}{T_a} + \frac{1}{T_a}u_2 - \dot{a}_d \quad (19)$$

where u_2 is the normalized FES input. Designing u_2 as $u_2 = T_a(v + \dot{a}_d + \frac{a}{T_a} + \varphi r)$ gives

$$\dot{e}_x = v + \varphi r \quad (20)$$

where $v \in \mathbb{R}$ is the unknown FES input to be optimized by the MPC. By defining a nominal state $x \in \mathbb{R}^{3 \times 1}$ as $x = [e_m \ r \ e_x]^T$ and control input $u \in \mathbb{R}^{2 \times 1}$ defined as $u = [u_1 \ v]^T$ the system in (10) can be converted to an error system written in state space form as

$$\begin{bmatrix} \dot{e}_m \\ \dot{r} \\ \dot{e}_x \end{bmatrix} = \begin{bmatrix} r - \alpha_2 e_m \\ \frac{1}{M_{mp}}(-\varphi e_x - \frac{u_1}{\rho}) \\ v + \varphi r \end{bmatrix} = f(x, u) \quad (21)$$

The problem formulation for the nominal MPC is given as

$$\min_{u_{t_k}} J(x_{t_k}, u_{t_k}) = \int_{t_k}^{t_k+T} l_2 dt + V(x(t_k + T)) \quad (22)$$

subject to:

$$\begin{aligned} \dot{x} &= f(x, u) \\ u &\in \mathcal{U} \\ x_{t_k+T} &\in \Omega_T \end{aligned} \quad (23)$$

where \mathcal{U} is a set of control inputs that bounds u . l_2 and the terminal cost $V(x(t_k + T))$ are given as

$$l_2(x, u) = x^T Q x + u^T R u$$

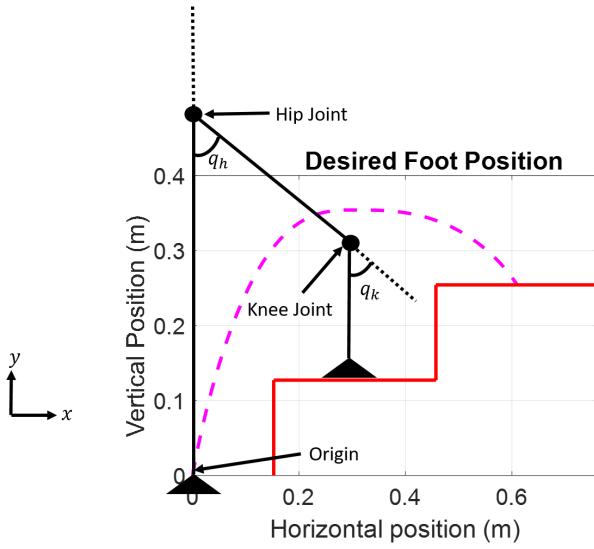


Figure 2. Exoskeleton model defined in the x-y plane where the origin is the foot of the user's stance leg along with the desired foot position to ensure that the user's foot clears the stairs.

$$V(x(t_k + T)) = \frac{1}{2}e_m^2 + \frac{1}{2}M_{m\rho}r^2 + \frac{1}{2}e_x^2$$

where $Q \in \mathbb{R}^{4 \times 4}$ and $R \in \mathbb{R}^{2 \times 2}$ are positive definite, symmetric matrices. The MPC approach in [29] additionally derives a feedback controller to guarantee robustness to modeling uncertainties along with a terminal controller and state region which guarantees recursive feasibility and that the control inputs stay within their bounds.

III. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed MPC framework in (5) to obtain optimal damping and stiffness during stair ascent, simulations were performed using the knee flexion/extension dynamics in (10). The desired trajectory for the knee joint was computed using inverse kinematics of a two-link model with orientation shown in Fig. 2. The desired trajectory ensured that the user's swing foot clears the stair with constant height of .13 meters and width of .30 meters. Fig. 3 shows the desired foot placement trajectory in both the horizontal and vertical directions along with the corresponding knee and hip joint trajectories. In this simulation, it is assumed that the hip angle can be controlled to follow its desired trajectory using a stabilizing controller. During the swing phase, the desired foot trajectory in vertical and horizontal directions is designed using a third order polynomial to ensure foot clearance while also impacting the stair at its midpoint. Once the foot is planted, the stance phase hip and knee trajectories are designed to drive the hip and knee angles to their zero position in preparation for the subsequent step. The human interaction torque between the user's shank and the exoskeleton was designed as a periodic signal based on the desired knee joint trajectory with an additional Gaussian white noise. Overall, the simulation was performed for 96 seconds during which the user climbed a total of 12 steps. Fig. 4 shows tracking performance of

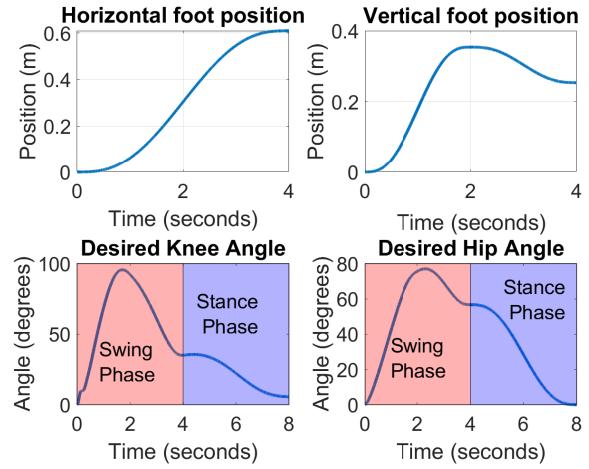


Figure 3. Desired foot position trajectories during the swing phase along with the corresponding hip and knee joint angles during both the swing and stance phase.

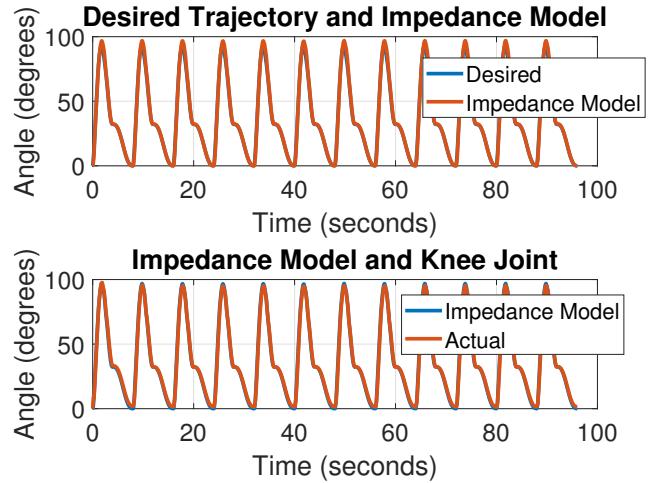


Figure 4. Desired knee joint trajectory along with the impedance model angle and the actual knee joint trajectory. The RMSE between the desired trajectory and impedance model angle is 1.48 degrees while the RMSE between the impedance model and actual knee joint angle is 1.82 degrees. Overall RMSE between the desired and actual trajectory is 3.13 degrees.

both the control loops. The root mean squared error (RMSE) between the desired knee trajectory and the impedance model angle is 1.48 degrees and the RMSE between the impedance model and the actual knee joint angle is 1.82 degrees. The overall RMSE between the actual knee joint angle and desired trajectory is 3.13 degrees. Fig. 5 shows the average damping and stiffness parameters of the impedance model computed by the MPC scheme during each step along with its constrained value marked by the horizontal line. It is seen that the damping and stiffness parameters satisfy the prescribed constraints. Fig. 6 shows the motor and normalized FES inputs computed by the allocation MPC framework in (22) along with the respective FES torque computed using (12). Fig. 7 shows the actual foot placement for each step calculated using forward kinematics from the actual

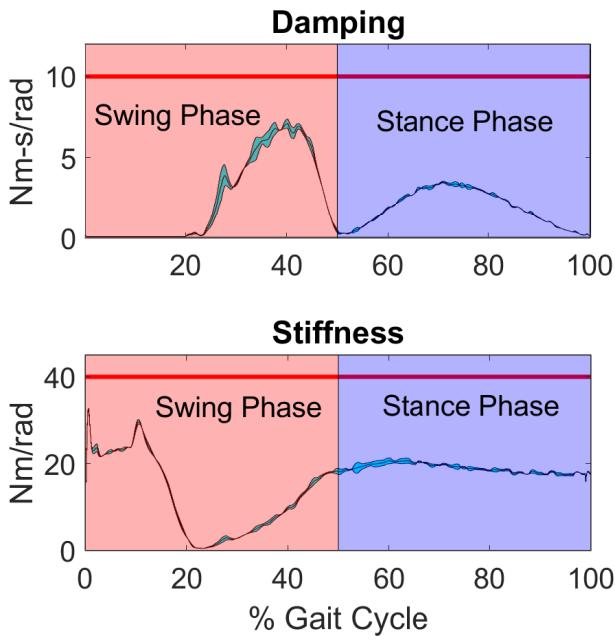


Figure 5. Average impedance model parameters computed by the MPC with shaded standard deviation during each of the 12 steps. The horizontal line represents the physiological stiffness and damping constraints, which are satisfied by the MPC.

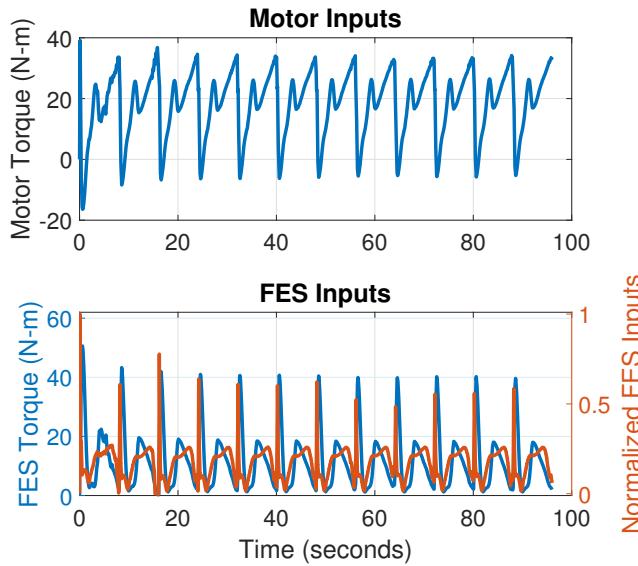


Figure 6. Motor and FES inputs to perform tracking of the desired impedance angle. The FES torques are calculated using the Force-Length and Force-Velocity relationships described in (12).

knee joint angle and the hip angle obtained from inverse kinematics. The average difference in location from where the foot impacts the stair to the desired impact location is .036 meters. Thus, the developed control method for stair ascent is effective in modulating time varying stiffness and damping to perform a stair ascent task.

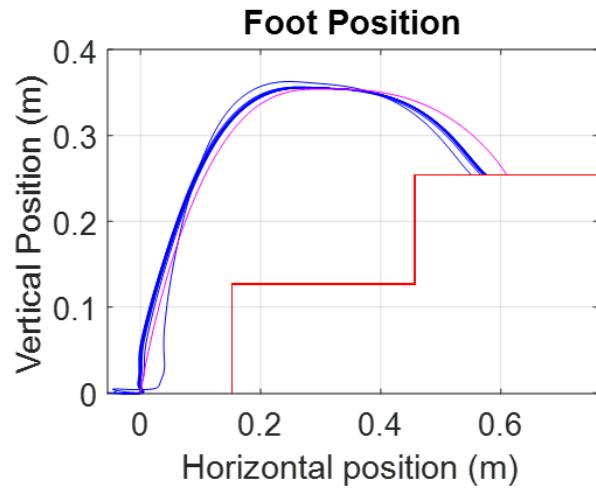


Figure 7. Foot placement of the user in the x-y plane for all 12 steps computed using forward kinematics of the actual knee joint angle.

IV. CONCLUSION

In this paper, an MPC approach is developed to determine optimal damping and stiffness parameters during stair ascent. The MPC incorporated a terminal impedance estimate to ensure recursive feasibility and exponential stability of the closed loop system. The optimal stiffness and damping were then incorporated into an admittance control framework to ensure that the knee joint angle tracks the impedance model while allocating inputs between motors and FES. Preliminary simulation results show that the proposed approach can be used to effectively perform a stair ascent task. In the future, the proposed control architecture can be incorporated with stair detection technology to perform stair ascent on stairs with variable width and heights. Additionally, different FES stimulation approaches such as distributed stimulation can be incorporated into the control to generate more power from FES to perform the tasks while relying less on the motor torque.

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