Optimization of Spiral Coil Design for WPT Systems using Machine Learning

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***Abstract*—This paper presents a spiral coil design for wireless power transfer (WPT) systems using a machine learning (ML)-based optimization method. The designed model allows us to obtain the optimal values of the number of turns N and coil pitch size p, when other coil design parameters such as the coil outer diameter D_o, the wire thickness w_t, and the operating frequency f_s, are given based on the application environment. The proposed ML-based spiral coil design method is assessed by using two metrics: the top-k accuracy and the intersection over union (IoU) factor. The first metric shows that the quality Q factor of the coil, a critical parameter to determine the efficiency of a WPT system, has an error rate less than 4% with respect to the value of the true top-1 Q factor in the proposed method. The second metric showcases that the IoU of the proposed method is more than 78% even when a small amount of the available data, less than $1,000$ samples, is used to train the model. The performance of the proposed method is also demonstrated by means of fabricated spiral coils in two application environments. Specifically, for each environment, seven spiral coils are fabricated to find the optimum design point for various values of p and N with fixed values of D_o, w_t, f_s. It is observed that the measured optimal p and N values are identical to the values output by the proposed ML-based optimization method.**

***Index Terms*—Spiral Coil Design, High-Frequency Wireless Power Transfer, Machine Learning.**

I. Introduction

Recently, battery-powered movable devices, such as drones, robots, and automated guided vehicles, have drawn a lot of attention to move toward electrification [1]–[3]. These devices work diligently and actively, demanding convenient and agile charging methods, and a wireless charging system, without any plug-in connections, has the potential to improve work performance. One of the essential components of a wireless power transfer (WPT) system is the coupling coil, providing the electromagnetic coupling between the transmitter and the receiver. Such a coil is usually designed by taking into account the geometrical structure of the device (e.g., the size of the coil should not exceed the available space) and the application environment (e.g., electromagnetic properties) [4]. Towards this end, several analysis methods and simulators such as Ansys-Q3D, HFSS, or COMSOL, are used to find the optimal design structure [5], [6]. However, such simulators and analysis, in general, require a long time, $2 \sim 5$ minutes for one coil design in Ansys-Q3D, as they need to compare several designs with various parameters. Also, they might intermittently estimate inaccurate results due to their non-linear characteristics [7]–[9]. Therefore, it becomes of fundamental importance to explore new methods that speed up the design, while offering performance guarantees. Driven by this observation, in [5], we proposed a machine learning (ML) approach to characterize the quality Q factor, a figure-of-merit of the coil performance, as a function of the geometrical parameters of the spiral coil. In particular, we developed a well-trained feedforward neural network (FNN) model, that accurately with 96% accuracy estimates the Q factor. Although our proposed method in [5] significantly reduces the computation time and analysis complexity, it does not offer insights on how to design and optimize the coil based on the application environment of interest. Therefore, with the goal to fill the aforementioned gap, we here propose a design method with a searching algorithm based on ML to find the optimal values of the number of turns N and of the pitch size p for the highest Q value, when other design parameters are given [10]. In particular, for given parameters such as the coil outer diameter D_o, the wire thickness w_t, and the operating frequency f_s, the optimal pair, N and p, results in the largest Q factor, estimated by using the method we proposed in [5]. Our method also provides the top-k region, which consists of the interpolation (obtained by using the Clough-Tocher scheme [11], [12]) of the (p, N) pairs producing a k^{th}-highest Q factor, among the Q factors generated by all considered (p, N) pairs, where $k \in \{1, 2, \ldots\}$. Based on the top-k region, we numerically evaluate the proposed method by using two metrics, namely the top-k accuracy and the intersection over union (IoU). The top-k accuracy measures how probable the output of the proposed method is to be at least a true[1] top-k design. The IoU measures the extent of overlap between two regions, namely the top-k region output by our method and the true top-k region, obtained from the dataset. Our evaluations showcase the effectiveness of our proposed method: (i) the top-k accuracy shows that the Q factor of the coil designed by our method has an error rate less than 4% with respect to the value of the true top-1 Q factor, and (ii) the IoU is more than 78%. The effectiveness of our method is also demonstrated through experiments, which consist of the actual fabrication of some

[1] We say that a top-k design is true if: (i) it yields a Q factor that is among the k largest ones; and (ii) it belongs to the dataset generated using the Ansys-Q3D simulator.

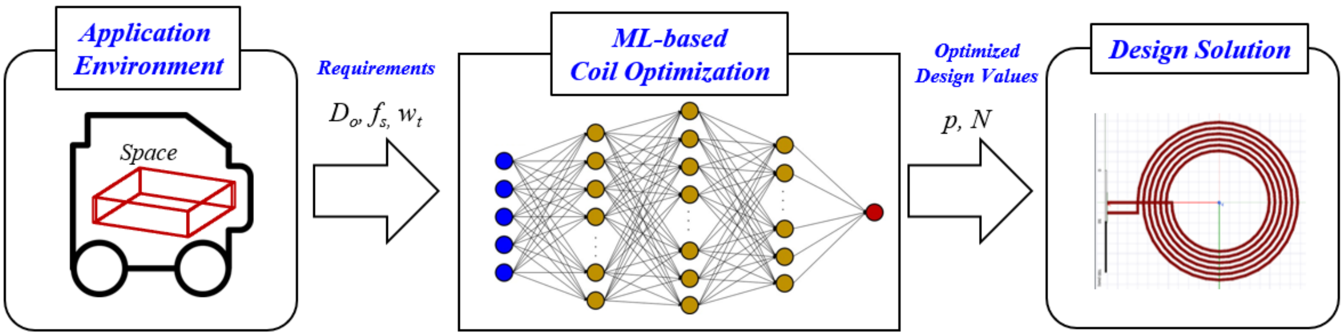


Fig. 1: Block diagram of the spiral coil design using ML-based optimization.

samples of the spiral coil in Section III.

II. ML-Based Optimization Method

In this section, we describe our proposed ML-based spiral coil design method, as illustrated in Fig. 1. First, we introduce the FNN that enables us to develop a fast and accurate spiral coil design method. Then, we summarize the ML-based Q factor estimator [5] for spiral coils, and we leverage it to propose our spiral coil design method. Finally, we evaluate the proposed method using two different metrics, namely the top-k accuracy and the IoU.

A. Feedforward Neural Networks

A feedforward neural network (FNN) is one of the basic neural networks in deep learning and ML [10], [13], [14]. An FNN is structured as a graph in which every neuron (i.e., a node) in each layer is connected to every neuron in the next layer through weighted edges (i.e., links). Such a structure of connections between two nearest layers is repeated in every layer of an FNN. All of the neurons in an FNN perform almost the same operation, i.e., they first calculate a weighted summation over all values received through the incoming edges, and then they apply a non-linear function, referred to as the activation function, to the result of the summation. The inputs to a neuron are the results of the activation functions in the previous layer. The weights of every edge are parameters that can be learned from a dataset during the training phase, and the activation functions used by the neurons can vary according to the application of the FNN.

In this work, we consider the same FNN model as the one we used in [5] to estimate the Q factor using the five parameters N, p, D_o, f_s, and w_t for the design of the spiral coil. In particular, the considered FNN has a total of 3 hidden layers with $64, 128$ and 32 neurons in each layer. The activation functions for every neuron are set to be the rectified linear unit (ReLU) activation function, i.e., $\mathsf{ReLU}(x) = \max\{0, x\}$.

B. Spiral Coil Q Factor Estimation

We here briefly summarize our recent work in [5], one of the key components of our proposed design method. Our ML-based Q factor estimation in [5] provides the value of the estimated Q factor for a specific spiral coil with fixed values of N, p, D_o, f_s, and w_t. We highlight that accurately estimating the Q factor is important as this estimate would shed light on the performance of the coil before its actual fabrication. To build such an estimator, in [5], we trained an FNN model based on a dataset consisting of $19,874$ sample values of the Q factor generated from $19,874$ sample values of N, p, D_o, f_s, and w_t. In particular, we obtained these measurements from a 3D quasi-static electromagnetic field simulator. Our results in [5] show that our trained ML-based Q factor estimator accurately, 96% level of accuracy on average, estimates the true Q value of the coil designed according to the five input parameters N, p, D_o, f_s, and w_t.

Compared to the estimator designed in [5], that only provides an estimated Q factor for a coil with fixed parameters, our proposed ML-based spiral coil design method brings the best design parameters for a coil under some constraints depending on the application of interest. The main distinction is that our method predicts the best design in terms of the Q factor among some given design parameters. Moreover, it provides not only the best design parameters for a spiral coil but also a number of candidates worth considering with their associated Q factor values.

C. ML-based Spiral Coil Design Method

Given the application of interest, our proposed method takes values of D_o, f_s, and w_t as input, and it provides as output the other coil design factors (i.e., p and N), which yield the largest Q factor. In other words, given the values of some spiral coil design parameters, i.e., D_o, f_s, and w_t, the method estimates the remaining design parameters p and N that result in the highest Q factor of the coil. In this paper, we incorporated the Q factor estimator proposed in [5] and an exhaustive search algorithm. Moreover, by leveraging an interpolation technique, specifically the Clough-Tocher scheme [11], [12], the proposed method provides not only the top design parameters but also other design candidates as well as the behavior of the Q factor along such parameters. Our proposed method is graphically shown in Fig. 2, and it consists of two main building blocks, namely a trained FNN model and a searching algorithm with an interpolation technique.

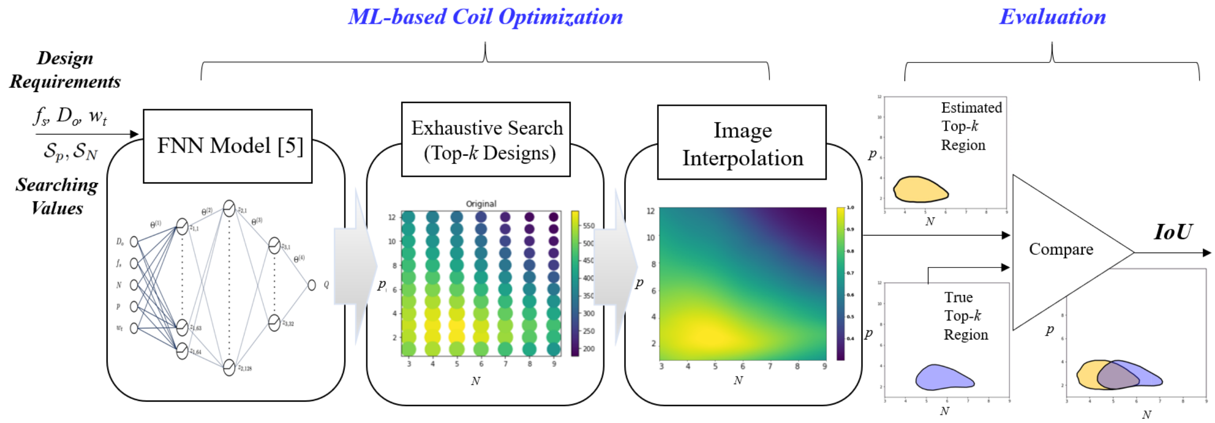


Fig. 2: Detailed configuration and evaluation of the ML-based coil optimization.

Moreover, a high-level view of our method in terms of a pseudo-code is given in Algorithm 1.

Algorithm 1 ML-based spiral coil design method

Input: $D_o, f_s, w_t, \mathcal{S}_p, \mathcal{S}_N, \mathcal{S}_{f_s}$
Output: $\hat{p}, \hat{N}, Q_{\max}$ and $\mathcal{I}$
Initiate $Q_{\max} = 0$, $\hat{p} = 0$, and $\hat{N} = 0$
for $p \in \mathcal{S}_p$ and $N \in \mathcal{S}_N$ **do**
  $Q \leftarrow \text{FNN}(D_o, f_s, w_t, p, N)$
  Save (N, p, Q) in $\mathcal{M}$ ▷ This is for interpolation
  if $Q \geq Q_{\max}$ **then**
    Update $Q_{\max} \leftarrow Q$
    Update $\hat{p} \leftarrow p$ and $\hat{N} \leftarrow N$
$\mathcal{I} \leftarrow \text{Interpolation}(\mathcal{M})$

As shown in Fig. 2 and Algorithm 1, the method requires a fixed triplet (f_s, D_o, and w_t) and two sets of feasible values for p and N (denoted by $\mathcal{S}_p$ and $\mathcal{S}_N$, respectively). The method provides the best (i.e., that produces the largest Q factor) values for the design parameters p and N among every pair of elements in $\mathcal{S}_p$ and $\mathcal{S}_N$. In particular, the method first initiates the variables $Q_{\max} = 0, \hat{p} = 0$, and $\hat{N} = 0$ that represents the maximum value of the Q factor, the best pitch, and the best number of turns, respectively. In order to find the best parameters $\hat{p}$ and $\hat{N}$ among all of the elements in $\mathcal{S}_p$ and $\mathcal{S}_N$, the method iteratively runs the Q factor estimator designed in [5] for every possible pair (p, N) (i.e., exhaustive search over $\mathcal{S}_p$ and $\mathcal{S}_N$). At every iteration, the Q factor estimator evaluates the quality of the coil according to $(D_o, f_s, w_t, p_i, N_j)$ for all (i, j), where[2] $p_i \in \mathcal{S}_p,\ i \in \{1, \ldots, |\mathcal{S}_p|\}$ and $N_j \in \mathcal{S}_N,\ j \in \{1, \ldots, |\mathcal{S}_N|\}$. The estimated Q factor is then compared with $Q_{\max}$, and if it is larger than $Q_{\max}$, then the triplet $(\hat{p}, \hat{N}, Q_{\max})$ is updated such that $\hat{p} = p_i, \hat{N} = N_j$, and $Q_{\max} = Q$. Moreover, the triplet (p_i, N_j, Q) from the Q factor estimator is always stored in $\mathcal{M}$ so that later we can evaluate the quality of the coil with parameters (p, N) not belonging to $\mathcal{S}_p$ and $\mathcal{S}_N$ through an interpolation. After running all of the iterations in the searching algorithm, the triplet $(\hat{p}, \hat{N}, Q_{\max})$ provides the best parameters and the corresponding estimated Q factor, which is the maximum over all of the estimated Q factors, resulting from all $p \in \mathcal{S}_p$ and $N \in \mathcal{S}_N$. One can further obtain any arbitrary pair of candidates (p, N) for the coil design from $\mathcal{I}$, which is the result of the interpolation of the elements in $\mathcal{M}$.

[2]For a set $\mathcal{A}$, the notation $|\mathcal{A}|$ indicates the cardinality of $\mathcal{A}$.

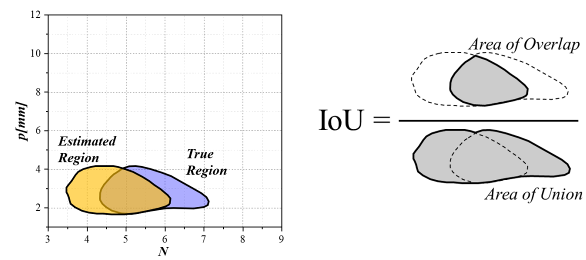
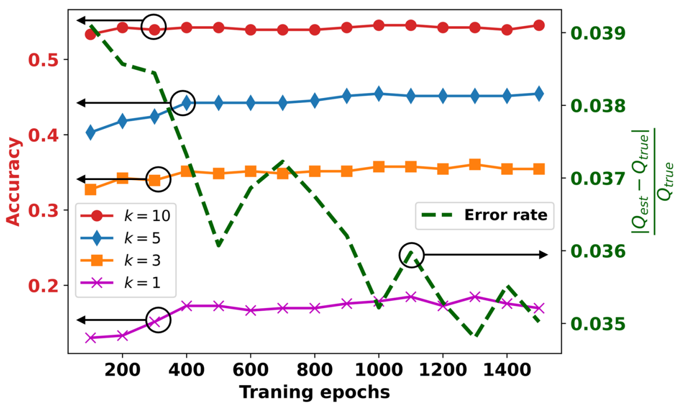


Fig. 3: Definition of the intersection over union (IoU) factor [15].

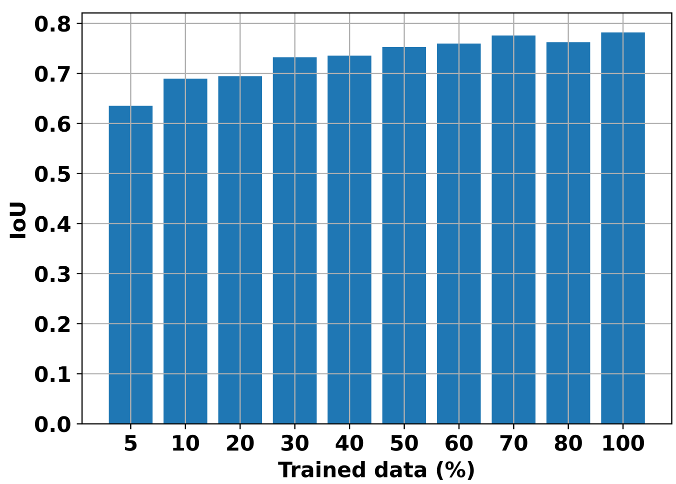
Beyond estimating one pair of optimal parameters for the spiral coil (i.e., $\hat{p}$ and $\hat{N}$), our method also provides a top-k region (i.e., range) of p and N for each fixed triplet (f_s, D_o, w_t). Specifically, this region consists of the interpolation (obtained by using the Clough-Tocher scheme [11], [12]) of all (p, N) pairs that produce a Q factor that is among the k^{th} largest Q factors generated by all $p \in \mathcal{S}_p$ and $N \in \mathcal{S}_N$. Examples of the estimated top-k regions with $k \in \{1, 3, 5, 10\}$ can be found in Fig. 5, and they will be discussed later. Any pair (p, N) in the top-k region guarantees that the corresponding spiral coil has a Q factor that is at least among the top-k Q factors.

D. Evaluation

We evaluated our proposed model in terms of two different metrics, the top-k accuracy and the IoU. The *top-k accuracy*



(a)



(b)

Fig. 4: Numerical evaluation of the proposed model. (a) The top-k accuracy and the error rate between the Q values of the estimated best coil (Q_{est}) and of the true best coil (Q_{true}). (b) Average IoU versus amount of training data when $k = 10$. The FNN model was trained over $1,000$ epochs.

measures the empirical probability that the top-1 design from the proposed method belongs to one of the true top-k designs. In other words, it measures how probable the output of the proposed method is to be at least a true top-k design. The *IoU* is defined as the ratio between the area of the overlap and the area of the union [15]. As shown in Fig. 3, the IoU increases with the size of the overlap area. The evaluation results, shown in Fig. 4, are based on fixed $\mathcal{S}_p$ and $\mathcal{S}_N$, and on 330 different values of f_s, D_o, and w_t. Therefore, the IoU evaluates the accuracy of the entire design from the proposed method, whereas the top-k accuracy assesses the model with a focus on the best design from the method.

Fig. 4a shows the top-k accuracy of the proposed method for $k \in \{1, 3, 5, 10\}$. As shown in this figure, although the top-k accuracy is not very high (e.g., 54% for $k = 10$), the Q factor of the coil designed by using the estimated top-1 parameters from the proposed method (denoted by Q_{est}) has an error rate less than 4% with respect to the value of the true top-1 Q factor (denoted by Q_{true}). These results indicate that the proposed method is very helpful for finding the best coil design. Also, the top-k region involves design parameters in $\mathcal{S}_p$ and $\mathcal{S}_N$ for which the coil has (at least) a top-k Q value. In particular, for a fixed triplet (f_s, D_o, and w_t), the *estimated* (respectively, the *true*) top-k region is built by interpolating all of the (N, p) pairs with their Q values from the proposed method (respectively, the dataset) and by collecting the top-k pairs from the interpolation result; the result of such an interpolation can be viewed as a contour plot. The computation time to obtain the top-k region was less than a second (i.e., 0.92 second), whereas Ansys-Q3D took $2 \sim 5$ minutes to test a single design. We also highlight that, thanks to the interpolation, our method provides not only the top-k designs but also the Q factor values for any design parameters (even real-valued) and thus, a manufacturer can control the coil design in a sophisticated way. The evaluation of the IoU for $k = 10$ is illustrated in Fig. 4b, from which we observe a maximum IoU of 78.19% and our method performs well (i.e., IoU is more than 60%) even when the amount of training data less than $1,000$ samples (i.e., 5% of the total $20,000$ samples).

III. Experimental Results

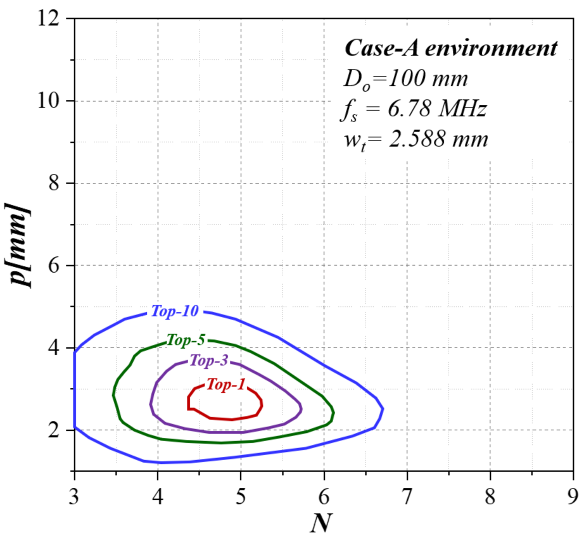
We assessed and verified the performance of the proposed ML-based spiral coil method through experiments with the actual fabrication of some samples of the spiral coil. The actual spiral coils were fabricated using the magnetic wire and the winding bobbin. The winding bobbin was printed by a 3D printer using polylactide (PLA) plastic materials and did not include any magnetic materials. The additional PCB board and the SMA connector were added to the nodes of the spiral coil to connect with the measurement equipment. We considered two different environments, with different operating frequencies and outer diameters of the coil. Specifically, we considered the following two cases and fabricated seven coils for each of the two cases:

- *Case-A:* D_o = 100 mm, f_s = 6.78 MHz, w_t = 2.588 mm.
- *Case-B:* D_o = 160 mm, f_s = 20.34 MHz, w_t = 2.588 mm.

These fabrications have different values of p and N, based on the estimated top-k region obtained from our ML-based optimization design, as shown in Fig. 5. For completeness, we also measured the Q factor of each of the fourteen fabricated coils, by using the Vector Network Analyzer (E5061B from Keysight Technologies). In order to minimize the measurement error due to the ratio between the resistance and the reactance, the resistance was measured in series resonance with an additional capacitor at the operating frequency ,and the total Q factor was extracted by using the measured resistance R_{eq} and reactance X_{eq} [5], [16].

Case-A. The estimated Q factor plot obtained by our proposed ML-based design method is illustrated in Fig. 6b. Our ML-based optimization method provides the best design around $p = 3$ mm and $N = 5$, the star point in Fig. 6b. Based on this result, we fabricated seven spiral coils with various values of N and p. In particular,

- A1: $N = 3$ and $p = 3$ mm,



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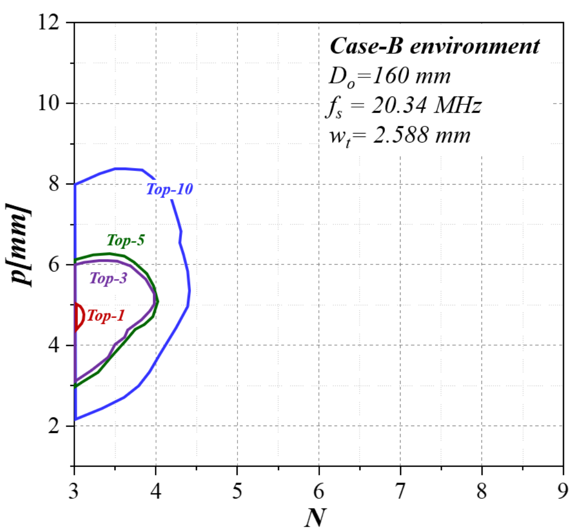


(b)

Fig. 5: Estimated top-k regions from the proposed ML-based optimization model. (a) Case-A;D_o = 100 mm, f_s = 6.78 MHz, w_t = 2.588 mm. (b) Case-B;D_o = 160 mm, f_s = 20.34 MHz, w_t = 2.588 mm.

- A2: $N = 4$ and $p = 3$ mm,
- A3: $N = 5$ and $p = 3$ mm,
- A4: $N = 6$ and $p = 3$ mm,
- A5: $N = 5$ and $p = 1$ mm,
- A6: $N = 5$ and $p = 5$ mm,
- A7: $N = 5$ and $p = 7$ mm,

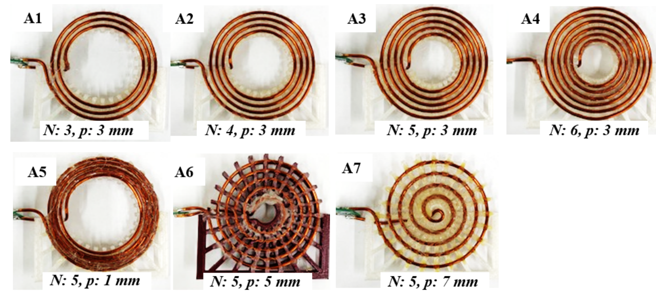
where the other parameters, $D_o = 100$ mm, $w_t = 2.588$ mm, and $f_s = 6.78$ MHz, are identical. Our measurement results show that A3 has the highest Q factor, equal to 367, among the seven fabricated coils. Also, this design falls within the top-k region of the proposed optimization method.

Case-B. The estimated Q factor plot obtained by our proposed ML-based design method is illustrated in Fig. 7b. Our ML-based optimization method provides the best design around $p = 5$ mm and $N = 3$, the star point in Fig. 7b. Based on this, we fabricated seven spiral coils with various values of N and p. In particular,

- B1: $N = 3$ and $p = 5$ mm,
- B2: $N = 5$ and $p = 5$ mm,
- B3: $N = 6$ and $p = 5$ mm,
- B4: $N = 8$ and $p = 5$ mm,
- B5: $N = 3$ and $p = 1$ mm,
- B6: $N = 3$ and $p = 3$ mm,
- B7: $N = 3$ and $p = 8$ mm,

where the other parameters, $D_o = 160$ mm, $w_t = 2.588$ mm, and $f_s = 20.34$ MHz, are identical. Our measurement results show that B1 has the highest Q factor, equal to 484, among the seven fabricated coils. Also, this design falls within the top-k region of the proposed optimization method. □

Our experimental results demonstrate the effectiveness of the proposed ML-based coil optimization method. The estimated top-k regions shown in Fig. 6 and Fig. 7 from our ML-based optimization method suggested the best design for the fabrication of the spiral coil successfully. We note that the values of the experimental Q factor and the estimated Q



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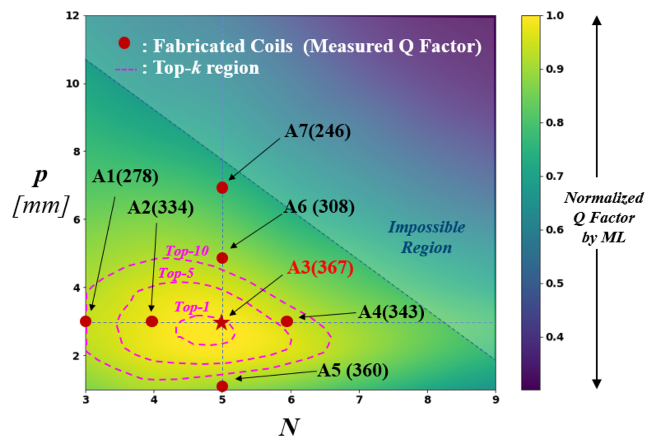

(b)

Fig. 6: Fabricated coils and comparison with the ML-based coil optimization. (a) Fabricated coils. (b) Estimated Q factor plot, top-k region, and measured results of the fabricated coils; D_o = 100 mm, f_s = 6.78 MHz, w_t = 2.588 mm.

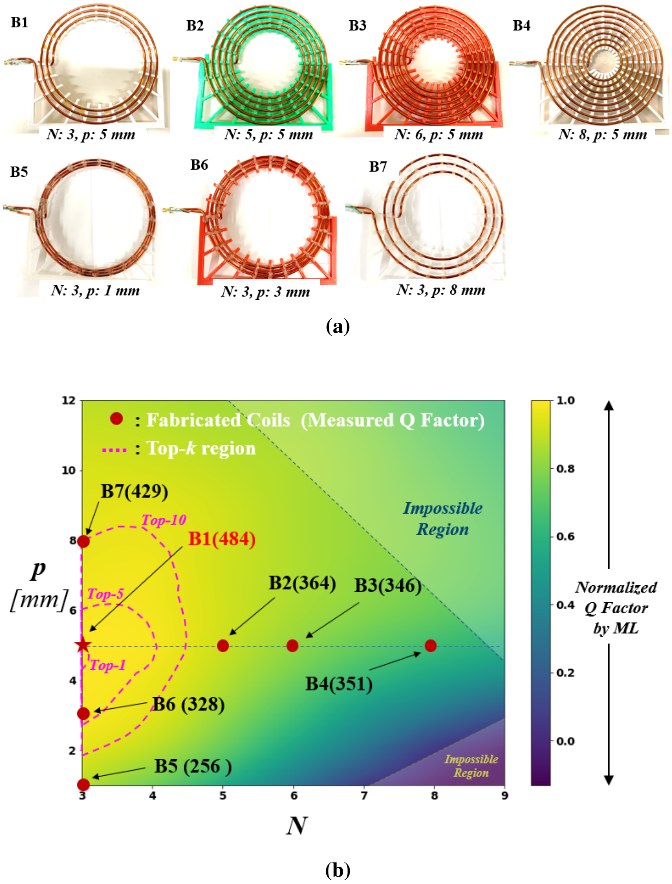

Fig. 7: Fabricated coils and comparison with the ML-based coil optimization. (a) Fabricated coils. (b) Estimated Q factor plot, top-k region, and measured results of the fabricated coils; D_o = 160 mm, f_s = 20.34 MHz, w_t = 2.588 mm.

factor by our method in [5] were different. We suspect that this difference is mainly due to: (i) an additional resistance caused by soldering; and (ii) a contact resistance in the measurement phase. Such a difference is not due to the training dataset collected from the Ansys-Q3D simulator. In fact, even if we increase the mesh of the design in Ansys-Q3D, the training data does not improve significantly. We point out that, although of practical relevance, comparisons between the experimental and estimated Q factors are not within the scope of this paper. Our main goal here is rather to verify whether the estimated top-k region and the experimental best coil design match, and this is indeed the case. Thus, in this study, considering a normalized version of the Q values is sufficient to find the optimal design. For a specific value of the Q factor, its normalized version is obtained by dividing it by the largest Q factor among all those obtained from $p \in \mathcal{S}_p$ and $N \in \mathcal{S}_N$.

Another appealing feature of our proposed ML-based spiral coil design method considers its computation time in providing suitable guidelines for the design and fabrication of a coil. As shown in Fig. 8, the proposed method takes around 1 sec (= 0.92 sec) to derive the top-k region for a fixed triplet $(f_s, D_o,$ and $w_t)$ and 100 pairs of (N, p). Whereas using simulation tools, such as Ansys-Q3D and Ansys-HFSS, takes around 2 minutes (Ansys-Q3D) and 16 minutes (Ansys-HFSS) for each pair (N, p). This means that it takes around 200 minutes (Ansys-Q3D) and 26 hours (Ansys-HFSS) when we search over 100 pairs (N, p) for optimization. Therefore, our proposed spiral coil provides notable time savings for the design and optimization of a spiral coil.

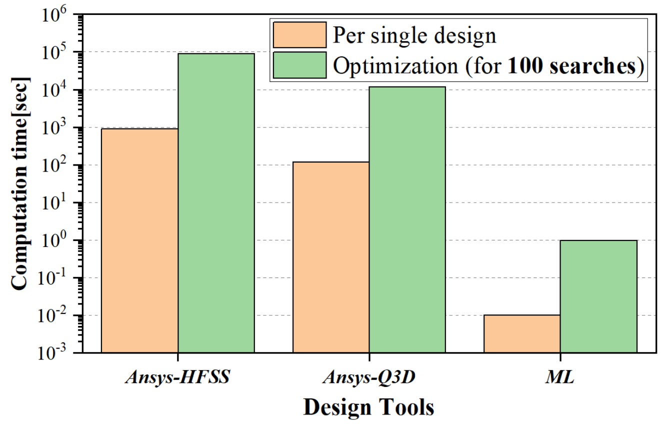

Fig. 8: Comparison of the computation time for the spiral coil design.

IV. Conclusion

This paper presented a spiral coil design for WPT systems using an ML-based optimization method. The proposed ML-based optimization method provides the top-k region to extract the best design of the coil in a specific environment. In particular, given some coil design parameters, f_s, D_o, and w_t, the proposed method was used to obtain the values of N and p that should be used for an effective coil design with a large Q factor. The proposed optimization method was evaluated under 330 requirement cases and showed a 78.19% overlap between the estimated and the true top-k regions. Moreover, actual coils were fabricated to verify the performance of the proposed optimization method; the optimal p and N values of the fabricated coils were identical to a top-k pair provided by our ML-based optimization. Numerical and experimental results showcase the effectiveness of the proposed method.

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