Dynamical Dark Energy and Infinite Statistics

Vishnu Jejjala[1*], Michael J. Kavic[2†], Djordje Minic[3‡], and Tatsu Takeuchi[3§]

[1] *Mandelstam Institute for Theoretical Physics, School of Physics, NITheP, and CoE-MaSS, University of the Witwatersrand, Johannesburg, WITS 2050, South Africa*

[2] *Department of Chemistry and Physics, SUNY Old Westbury, Old Westbury, NY 11568, U.S.A.*

[3] *Department of Physics, Virginia Tech, Blacksburg, VA 24061, U.S.A.*

Abstract

In the ΛCDM model, dark energy is viewed as a constant vacuum energy density, the cosmological constant in the Einstein–Hilbert action. This assumption can be relaxed in various models that introduce a dynamical dark energy. In this letter, we argue that the mixing between infrared and ultraviolet degrees of freedom in quantum gravity lead to infinite statistics, the unique statistics consistent with Lorentz invariance in the presence of non-locality, and yield a fine structure for dark energy. Introducing IR and UV cutoffs into the quantum gravity action, we deduce the form of Λ as a function of redshift and translate this to the behavior of the Hubble parameter.

* vishnu@neo.phys.wits.ac.za

† kavicm@oldwestbury.edu

‡ dminic@vt.edu

§ takeuchi@vt.edu

1 Introduction

Since the seminal discovery that the expansion of our Universe is currently accelerating [1,2], many models have been proposed to explain and understand this observation [3,4]. These include various dark energy models in which the acceleration is due to a currently small but non-zero vacuum energy density, quintessence models which incorporate inflaton-like scalar fields, and $f(R)$ modified gravity models in which late-time acceleration is due to subdominant terms in the action that become important at small curvature [5]. See also [6].

In the widely accepted ΛCDM cosmology, dark energy is provided by the time independent cosmological constant Λ [7]. However, recent disagreements between the values of the Hubble constant H_0 determined from early ($z \sim 1000$ [8]) and late ($z < 10$ [9–13]) Universe observations have rekindled interest in dynamical dark energy models in which the vacuum energy density is time dependent [14–22].

Furthermore, it has also been argued that the quantization of gravity may naturally lead to time dependent dark energy [23–25]. In particular, Ref. [24] points out that the matrix model formulation of non-perturbative quantum gravity proposed in Refs. [26–30] naturally leads to a time dependent $\Lambda(z)$ due to the doubling of spacetime that is required in this approach. There, the cosmological constant in the observable spacetime is given by the integration over the unobservable dual spacetime curvature, and consequently, the dynamical evolution of the latter leads to the time dependence of the former.

In this letter, we follow the line of thought of Ref. [24] and propose a concrete functional form for $\Lambda(z)$ by focusing on another feature of quantum gravity, namely infinite statistics [31–35]. While we work within one particular framework for definiteness, we emphasize that much of what we say is a generic feature of theories in which UV and IR degrees of freedom mix. This is natural in low energy effective field theories obtained from string theory, for example as a consequence of non-commutativity.

Infinite statistics is motivated by non-locality and Lorentz covariance, and is realized in large-N matrix models [36]. It is the statistics of the partonic degrees of freedom of the matrix theory from which spacetime is constructed, and suggests a form for the density of states for the partons from which $\Lambda(z)$ can be calculated. In other words, the functional form of $\Lambda(z)$ we propose is a manifestation of the infinite statistics that the partons must follow. This is similar to other macroscopic statistical manifestations of quantum mechanics such as blackbody radiation.

This letter is organized as follows. In Section 2, we briefly review the new approach to quantum gravity based on "quantum relativity" [26–30] and there we outline the general argument for the variation of $\Lambda(z)$ with time. In Section 3, we review the relevance of infinite statistics to quantum gravity. Infinite statistics is realized in large-N matrix models [36] and thus it is particularly appropriate for the new matrix model like approach reviewed in section 2. In Section 4, we derive an explicit formula for $\Lambda(z)$, and the resulting modification to the evolution of the Hubble parameter $H(z)$ is discussed in Section 5, This, in principle, can be compared with actual observation of $H(z)$. Section 6 concludes with a few remarks.

2 Quantum Spacetime and Quantum Gravity

In this section, we outline the non-perturbative formulation of quantum gravity in terms of a doubled matrix model quantum theory proposed in Refs. [26–30], *i.e.* the metastring. In this description, everything is built out of partonic degrees of freedom represented by the entries of the quantum gravitational matrix model, and, in the leading term in the expansion involving the fundamental length, dark energy is realized as a dynamical geometry of dual spacetime.

The starting point of the metastring formalism is the following worldsheet action [37,38], which is chiral, doubles the degrees of freedom (*i.e.* works in phase space), and is manifestly invariant under Born reciprocity/T-duality:

$$S_{2d} = \frac{1}{4\pi}\int_{\Sigma}\Big[\,\partial_\tau \mathbb{X}^A(\eta_{AB}+\omega_{AB}) - \partial_\sigma \mathbb{X}^A\, H_{AB}\Big]\partial_\sigma \mathbb{X}^B\ . \tag{1}$$

Here Σ is the worldsheet, the doubled target space variables $\mathbb{X}^A = (x^a/\lambda, \tilde{x}_a/\lambda)$ combine the sum ($x = x_L + x_R$) and the difference ($\tilde{x} = x_L - x_R$) of the left- and right-movers on the string ($a, A = 0, 1, \cdots, d-1 = 25$, for the critical bosonic string), and $\lambda = 1/\epsilon = \sqrt{\alpha'}$ is the string length scale [39]. The mutually compatible dynamical fields $\omega_{AB}(\mathbb{X}), \eta_{AB}(\mathbb{X})$, and $H_{AB}(\mathbb{X})$ are respectively: the antisymmetric symplectic structure ω_{AB}, the symmetric polarization (doubly orthogonal) metric η_{AB}, and the doubled symmetric metric H_{AB}, which together define a Born geometry [26, 40, 41]. See also [42, 43].

Quantization renders the doubled "phase-space" operators $\hat{\mathbb{X}}^A = (\hat{x}^a/\lambda, \hat{\tilde{x}}_b/\lambda)$ inherently non-commutative [30]:

$$\big[\,\hat{\mathbb{X}}^A,\, \hat{\mathbb{X}}^B\,\big] \,=\, i\omega^{AB}\ . \tag{2}$$

In this formulation, all effective fields must be regarded a priori as bi-local $\phi(x, \tilde{x})$ [28], subject to Eq. (2), and therefore inherently non-local (yet covariant) in the conventional x^a-spacetime. Such non-commutative field theories [44,45] generically display a mixing between the ultraviolet (UV) and infrared (IR) physics, with continuum limits defined via a double-scale renormalization group (RG) and self-dual fixed points [29, 45]. In the current case, the UV and IR mixing occurs between the observable x^a-spacetime and the unobservable $\tilde{x}_a$-spacetime.

The metastring offers a new view on quantum gravity by noting that the world-sheet can be made modular in our formulation, with the doubling of τ and σ, so that $\hat{\mathbb{X}}(\tau, \sigma)$ can be in general viewed as an infinite dimensional matrix (the matrix indices coming from the Fourier components of the doubles of τ and σ) [36,46]. Then the corresponding metastring matrix model action should look like

$$S \sim \int d\tau\, d\sigma\ \mathrm{Tr}\left[\partial_\tau \hat{\mathbb{X}}^A \partial_\sigma \hat{\mathbb{X}}^B (\omega_{AB} + \eta_{AB}) - \partial_\sigma \hat{\mathbb{X}}^A\, H_{AB}\, \partial_\sigma \hat{\mathbb{X}}^B\right]\ , \tag{3}$$

where the trace is over the infinite matrix indices. The matrix entries become the natural partonic degrees of freedom of quantum spacetime. The non-perturbative formulation of quantum gravity is obtained by replacing ∂_σ in the above worldsheet action with a commutator involving one extra $\hat{\mathbb{X}}^{26}$:

$$\partial_\sigma \hat{\mathbb{X}}^A \quad \to \quad \left[\hat{\mathbb{X}}^{26}, \hat{\mathbb{X}}^A\right]\ , \qquad\qquad A = 0, 1, \cdots, 25\ . \tag{4}$$

Therefore, as with the relationship between M-theory and type IIA string theory, a fully interactive and non-perturbative formulation of metastring theory is given in terms of a matrix model form of the above metastring action (with $a, b, c = 0, 1, 2, \cdots, 25, 26$)

$$S \sim \int d\tau\ \mathrm{Tr}\left(\partial_\tau \hat{\mathbb{X}}^a \left[\hat{\mathbb{X}}^b, \hat{\mathbb{X}}^c\right] \eta_{abc} - H_{ac}\left[\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b\right]\left[\hat{\mathbb{X}}^c, \hat{\mathbb{X}}^d\right] H_{bd}\right)\ , \tag{5}$$

where the first term is of the Chern–Simons form, the second term is of the Yang–Mills form, and η_{abc} contains both ω_{AB} and η_{AB}. In general, we do not need an overall trace if we think of quantum gravity as a pure quantum theory. Thus, the following matrix model becomes a pure quantum formulation of quantum gravity

$$\mathbb{S}_{ncM} \ = \ \frac{1}{4\pi}\int_\tau \left(\partial_\tau \hat{\mathbb{X}}^i \left[\hat{\mathbb{X}}^j, \hat{\mathbb{X}}^k\right] g_{ijk} - \left[\hat{\mathbb{X}}^i, \hat{\mathbb{X}}^j\right]\left[\hat{\mathbb{X}}^k, \hat{\mathbb{X}}^\ell\right] h_{ijk\ell}\right)\ , \tag{6}$$

with 27 bosonic $\hat{\mathbb{X}}$ matrices.[5] Within this formulation, both matter and gravitational sectors emerge from the dynamics of the partonic quanta of quantum spacetime.

[5] In this formulation, supersymmetry and its avatars are not fundamental but emergent [29].

In particular, in Ref. [24] it has been argued that the generalized geometric formulation of string theory discussed above, Eq. (6), provides an effective description of dark energy, and a de Sitter spacetime. This is due to the theory's chirality and non-commutatively, as in Eq. (2), doubled realization of the target space, and the stringy effective action on the doubled non-commutative spacetime $(x^a, \tilde{x}_a)$, which leads to the effective action

$$S_{\text{eff}}^{nc} \;=\; \int_x \int_{\tilde{x}} \text{Tr}\sqrt{g(x,\tilde{x})}\,\Big[\, R(x,\tilde{x}) + L_m(x,\tilde{x}) + \cdots \Big] \,, \tag{7}$$

where the ellipses denote higher-order curvature terms induced by string theory, and L_m is the matter Lagrangian put in by hand. This result can be understood as a generalization of the famous calculation by Friedan [47]. Owing to Eq. (2), we have

$$\big[\,\hat{x}^a,\, \hat{\tilde{x}}_b\,\big] \;=\; 2\pi i\,\lambda^2\,\delta^a_b\,, \qquad \big[\,\hat{x}^a,\, \hat{x}^b\,\big] \;=\; \big[\,\hat{\tilde{x}}_a,\, \hat{\tilde{x}}_b\,\big] \;=\; 0\,, \tag{8}$$

where λ denotes the fundamental length scale, such as the Planck scale, and $\epsilon = 1/\lambda$ is the corresponding fundamental energy scale, while the string tension is $\alpha' = \lambda/\epsilon = \lambda^2$. Thus S_{eff}^{nc} expands into numerous terms with different powers of λ, which upon $\tilde{x}$-integration, and from the x-space vantage point, produce various effective terms. To lowest (zeroth) order of the expansion in the non-commutative parameter λ of S_{eff}^{nc} takes the form:

$$\begin{aligned} S_{d=4} \;&=\; -\int_x \int_{\tilde{x}} \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}\,\Big[\,R(x) + \tilde{R}(\tilde{x})\,\Big] \\ &=\; -\int_x \sqrt{-g(x)}\,\left[R(x)\int_{\tilde{x}}\sqrt{-\tilde{g}(\tilde{x})} + \int_{\tilde{x}}\sqrt{-\tilde{g}(\tilde{x})}\;\tilde{R}(\tilde{x})\right]\,, \end{aligned} \tag{9}$$

a result which first was obtained almost three decades ago, effectively neglecting ω_{AB} by assuming that $[\,\hat{x}^a,\, \hat{\tilde{x}}_b\,] = 0$ [48]. In this leading limit, the $\tilde{x}$-integration in the first term of (9) defines the gravitational constant G_N,

$$1/G_N \;\sim\; \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}\,, \tag{10}$$

and in the second term produces a *positive* cosmological constant $\Lambda > 0$ (dark energy)

$$\Lambda/G_N \;\sim\; \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}\,\tilde{R}(\tilde{x})\,. \tag{11}$$

Thus the weakness of gravity is determined by the size of the canonically conjugate dual $\tilde{x}$-space, while the smallness of the cosmological constant is given by its curvature $\tilde{R}$. Ref. [24] also discusses a see-saw formula for the cosmological constant, as well as its radiative stability in the underlying general framework of a non-commutative generalized geometric phase-space formulation of string theory [26–30], which is non-local but covariant.

To summarize, a non-perturbative formulation of quantum gravity can be given in terms of a doubled matrix model, Eq. (6), in which everything is built out of partonic degrees of freedom represented by the entries of doubled matrices $\hat{\mathbb{X}}$. In the leading term of the effective spacetime description, dark energy is realized as a dynamical geometry of the dual spacetime, and consequently, is inherently time dependent.

3 Quantum Gravity and Infinite Statistics

Matrix models, in the limit of large matrix size, can be directly related to infinite statistics [36]. Given that the metastring action, Eq. (6), formulates non-perturbative quantum gravity as a matrix model, in which dark energy is realized as the dynamical geometry of dual spacetime in the commutative limit, we argue in this section that infinite statistics can be used to model the fine structure of dark energy [34]. In particular, the partonic degrees of freedom of the matrix model, out of which both spacetime and matter degrees of freedom emerge, obey infinite statistics, and thus, infinite statistics controls the fine structure of dark energy. The idea here is that by using the general statistical arguments, we can illustrate the time dependence of Λ based on a dynamical dual spacetime geometry without appealing to any particular models of that dynamics.

The proposal that quantum statistical effects are essential in the macroscopic realizations of quantum gravity has been made in the past. First, it was argued in Ref. [31] that black hole statistics is infinite statistics [32, 49]. (See also, Ref. [50]). Also, in Ref. [51] a statistical argument was used to argue for probable values of the cosmological constant. More recently, such statistical arguments were used in Ref. [52] to analyze black hole spin in gravitational wave observations.

Given our proposal regarding the realization of dark energy in a fundamentally non-local but Lorentz covariant formulation of quantum gravity, on a purely quantum level one should consider the statistics of quanta from which dual spacetime emerges at large distances. If one remembers that only one statistics is consistent with non-locality and Lorentz symmetry, both of which underpin this approach to quantum gravity, one is led to infinite statistics [31, 32] and a fine structure for dark energy. Therefore the natural implementation of the physical effects associated with infinite statistics in the context of dark energy should be sought in this generic non-commutative formulation of string theory.

If dark energy originates from the curvature of the dual space, then in the context of quantum gravity it possesses fine structure. That fine structure can be deduced from the infinite statistics of the quanta of dual spacetime. The virtue of the metastring action is that supplies a mechanism for UV/IR mixing. If we simply assume this mixing *ab initio*, our conclusions are generic.

In Ref. [34] (see also [35]) we have presented a general argument for the relevance of infinite statistics for the fine structure of dark energy. We remind the reader that infinite statistics is defined in terms of the Cuntz algebra

$$\hat{a}_i \hat{a}_j^\dagger \;=\; \delta_{ij} \;, \tag{12}$$

which can be viewed as the $q = 0$ deformation of the q-deformed commutation relations

$$\hat{a}_i \hat{a}_j^\dagger - q \hat{a}_i^\dagger \hat{a}_j \;=\; \delta_{ij} \;. \tag{13}$$

The case $q = 1$ corresponds to Bose–Einstein statistics, and $q = -1$ to Fermi–Dirac statistics. Unlike the bosonic and fermionic statistics, infinite statistics realizes any permutation (not just even or odd) of the associated $SU(N)$ Young tableaux for N particles. In particular, infinite statistics governs the master fields of large-N matrix models [36], and thus it is appropriate for our approach to quantum gravity based on a non-perturbative matrix model formulation. For example, the master field of the quadratic single matrix model is given as $\hat{a} + \hat{a}^\dagger$, with $\hat{a}$ and $\hat{a}^\dagger$ satisfying the Cuntz algebra $\hat{a}\hat{a}^\dagger = 1$ [36].

More concretely, infinite statistics is quantum Boltzmann statistics, and thus in the quantum context, it is of the Wien type [34]. This turns out to be crucial in our application of infinite statistics to the fine structure of dark energy.

4 Infinite Statistics and Dark Energy

In this section, we use the insight that infinite statistics controls the fine structure of dark energy in quantum gravity. Infinite statistics is the quantum statistics of distinguishable partons of the quantum gravitational matrix model, and as such it is essentially just the Boltzmann statistics, or equivalently, the Wien statistics of quantum spacetime partons. Given the dual relationship between the observed x^a-spacetime and dual $\tilde{x}_a$-spacetime, quantum gravity is endowed with both UV and IR cut-offs, and thus, the Wien distribution of spacetime quanta/partons responsible for the fine structure of dark energy comes with an explicit cut-off.

Therefore, motivated by the above general reasoning about the role of infinite statistics in quantum gravity, let us examine the dark energy spectral function of the quantum Boltzmann (or Wien) type in the dual energy $\tilde{E}$-space [34]:

$$\rho_{\text{dark energy}}(\tilde{E}, E_0) \;=\; A\,\tilde{E}^3\, e^{-B\tilde{E}/E_0}\;, \tag{14}$$

where A and B are dimensionless constants. From Eq. (11), we have

$$\rho_{vac}(\tilde{E}_{UV}) \;=\; \frac{\Lambda(\tilde{E}_{UV})}{8\pi G_N} \;=\; \int_0^{\tilde{E}_{UV}} d\tilde{E}\;\rho_{\text{dark energy}}(\tilde{E}, E_0)\;, \tag{15}$$

where $\rho_{vac}(\tilde{E}_{UV})$ is the effective vacuum energy density in the observable spacetime, while $\tilde{E}_{UV}$ is the UV cutoff in the unobservable dual spacetime. Due to the UV/IR correspondence between the two spacetimes, we have

$$\tilde{E}_{UV} E_{IR} \;=\; \mu\;, \tag{16}$$

where E_{IR} is the IR cutoff in the observable spacetime, and μ is an invariant associated with the doubly orthogonal group of transformations in the metastring approach [26–30]. We expect E_{IR} to be governed by the size of the observable Universe, thus

$$E_{IR} \;=\; \frac{E_0}{a} \;=\; E_0(1+z)\;, \tag{17}$$

where a is the scale factor of the Friedmann–Lemaître–Robertson–Walker metric, z is the redshift, and we identify E_0 as the current ($z = 0$) IR cutoff. Thus,

$$\tilde{E}_{UV} \;=\; \frac{\mu}{E_{IR}} \;=\; \frac{\mu a}{E_0} \;=\; \frac{\mu}{E_0(1+z)}\;, \tag{18}$$

and we find

$$\rho_{vac}(z) \;=\; \frac{\Lambda(z)}{8\pi G_N} \;=\; \int_0^{\tilde{E}_{UV}} d\tilde{E}\;\rho_{\text{dark energy}}(\tilde{E}, E_0) \;=\; \rho_*\Big[1 - b(\xi)\Big]\;, \tag{19}$$

where

$$\rho_* \;=\; \frac{6A}{B^4}E_0^4\;, \qquad b(\xi) \;=\; \left(1 + \xi + \frac{\xi^2}{2} + \frac{\xi^3}{6}\right) e^{-\xi}\;, \tag{20}$$

and

$$\xi \;=\; \frac{B\tilde{E}_{UV}}{E_0} \;=\; \frac{B\mu}{E_0^2(1+z)} \;=\; \frac{\xi_0}{1+z}\;, \qquad \xi_0 \;=\; \frac{B\mu}{E_0^2}\;. \tag{21}$$

The proportionality of ρ_* to E_0^4 is analogous to the derivation of the Stefan–Boltzmann T^4 law from the Wien distribution [34]. The functional form of $b(\xi)$ is shown in Figure 1(a). Therefore, in our proposal, $\Lambda(z)$ evolves as

$$\frac{\Lambda(z)}{\Lambda(0)} \;=\; \frac{1 - b(\xi)}{1 - b(\xi_0)}\;. \tag{22}$$

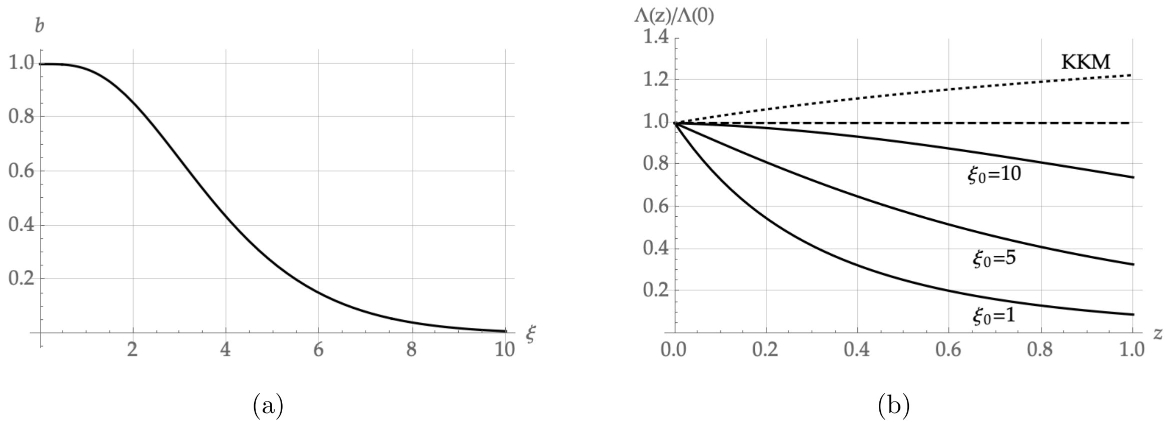

Figure 1: (a) Behavior of the function $b(\xi)$. (b) The behavior of $\Lambda(z)/\Lambda(0)$ in our proposal for $\xi_0 = 1$, $\xi_0 = 5$, and $\xi_0 = 10$ compared against constant Λ (dashed) and that from Ref. [25] (dotted).

In the limit $z \to \infty$ (early Universe), we have $\xi = \xi_0/(1+z) \to 0$, $b(\xi) \to 1$, and $\Lambda(z)/\Lambda(0) \to 0$. The z dependence of this ratio in the range $z \leq 1$ is shown for several values of ξ_0 in Figure. 1(b).

What can the value of the invariant μ be? We can identify $E_{IR} = E_0$ with the current vacuum energy scale. If we set the curent $\tilde{E}_{UV}$ to the Planck mass $E_P = G_N^{-1/2}$, c.f. Eq. (10), we have

$$
\begin{aligned}
\mu &= E_{IR}\tilde{E}_{UV} = E_0 E_P \\
&= \left(2.24 \times 10^{-12}\ \text{GeV}\right)\left(1.22 \times 10^{19}\ \text{GeV}\right) = 2.73 \times 10^{7}\ \text{GeV}^2\,.
\end{aligned}
\tag{23}
$$

This means $\sqrt{\mu} \simeq 5.23$ TeV, more or less the scale at which the LHC operates. However, this is only a very special choice, and in general, μ is a parameter we should fit.

5 Evolution of the Hubble Parameter

Let us see how the z dependence of $\Lambda(z)$ will affect the evolution of the Hubble parameter $H(z)$. $H(z)$ evolves in a spatially flat ($k = 0$) matter dominated ($z \lesssim 3000$) universe as [53]

$$
H^2(z) \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\Big[\Omega_m(1+z)^3 + \Omega_\Lambda(z)\Big]\,, \tag{24}
$$

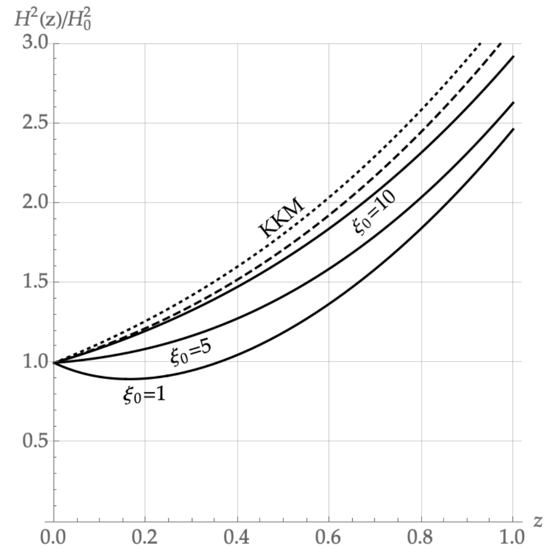

Figure 2: The behavior of $H^2(z)/H_0^2$ in our proposal for $\xi_0 = 1$, $\xi_0 = 5$, and $\xi_0 = 10$ compared against the constant Λ case (dashed) and that from Ref. [25] (dotted).

where $\Omega_\Lambda(z)$ and Ω_m are respectively the density fractions of vacuum energy and matter normalized to the present critical density $\rho_{c,0} = 3H_0^2/8\pi G_N$:

$$\Omega_\Lambda(z) \;=\; \frac{\rho_\Lambda(z)}{\rho_{c,0}} \;=\; \frac{\Lambda(z)}{3H_0^2}\;, \qquad \Omega_m \;=\; \frac{\rho_m}{\rho_{c,0}}\;. \tag{25}$$

The Hubble constant H_0 is the value of the Hubble parameter $H(z)$ at redshift $z = 0$. Consistency of Eq. (24) requires

$$\Omega_m + \Omega_\Lambda(0) \;=\; 1\;. \tag{26}$$

Observations yield $\Omega_M = 0.3$ and $\Omega_\Lambda(0) = 0.7$ [8]. Therefore,

$$\frac{H(z)^2}{H_0^2} \;=\; \Omega_M(1+z)^3 \;+\; \Omega_\Lambda(0)\left[\frac{\Lambda(z)}{\Lambda(0)}\right]\;. \tag{27}$$

Substituting Eq. (22) into this expression will give us the z dependence of $H(z)$. The behavior of $H^2(z)/H_0^2$ is shown for several values of ξ_0 in Figure 2, compared against the constant Λ case. Note that in our proposal, the $\Lambda(z)$ contribution to $H(z)$ vanishes when $z \gg 1$. Also shown in Figure 2 is the result of Kitamoto, Kitazawa, and Matsubara (KKM) in Ref. [25] in which the authors compute a β-function for $g = G_N H^2$ in Einstein gravity in four dimensional de Sitter space to obtain (their formula (5.39))

$$\left[\frac{H(z)^2}{H_0^2}\right]_{\rm KKM} \;=\; \Omega_m\,(1+z)^3 + \Omega_\Lambda(0)\log\Big[e + \log\,(1+z)\Big]\;. \tag{28}$$

While the KKM model makes certain assumptions about a conformally coupled scalar field to ensure the running of the coupling g and identifies the behavior of the Hubble constant with quantum IR effects, we motivate the time evolution of $\Lambda(z)$ from infinite statistics and the dynamical mixing of UV and IR degrees of freedom. The evolution of the Hubble parameter in the context of the model proposed here differs significantly from either the constant Λ case or the KKM model for most values of ξ_0. Thus there exists the potential for a definitive prediction to be made of the evolution Hubble parameter by fitting the model proposed here to current cosmological data. We will discuss the phenomenology of the above formulæ for $H^2(z)/H_0^2$, and whether they have any relevance to the H_0 tension elsewhere [54].

6 Concluding Remarks

In this paper we have discussed the relation between infinite statistics and dynamical dark energy based on the recent proposal for the origin of dark energy from the curvature of dual spacetime [24] in the context of the new approach to quantum gravity of Refs. [26–30]. Specifically, following the general arguments for the relevance of infinite statistics in quantum gravity [34], we have derived a formula for $\Lambda(z)$ in a particular example based on the quantum statistical effects (due to infinite statistics) within this general approach.

Note that we have not included the matter sector explicitly in the above discussion. However, the dual part of the matter sector can be naturally related to the dark matter sector that is sensitive to dark energy [55] which illustrates the unity of the description of the entire dark sector based on the properties of the dual spacetime, as predicted by the above generic non-commutative formulation of string theory/quantum gravity [26–30].

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