The Bureau
of Meteorology

Interaction of a Kelvin wave with one or two hydraulically controlled sill flows in idealized simulations *(paper in preparation)*

Oceane Richet (oceane.richet@bom.gov.au—Bureau of Meteorology, Hobart, Australia), **Renske Gelderloos** (Department of Hydraulic Engineering, Delft University of Technology, Delf, The Netherlands / Department of Earth and Planetary Sciences, The John Hopkins University, Baltimore, MD, USA) and **Lawrence Pratt** (Woods Hole Oceanographic Institute (WHOI), Woods Hole, MA, USA)

GENERAL CONTEXT

The outflow of dense waters from the basins in which they are formed is often constrained by a narrow passage or passages in which hydraulic control occurs, the Denmark Strait and Faroe-Bank Channel composing one example. Hydraulic control of steady-state systems has been well studied, but how does control affect the ever present transients that are generated by variability in the formation rates of dense water? We investigate further by considering a fait of a Kelvin wave that carries a transport anomaly and that interacts with either one or two straits that drain an idealized basin.

REFLECTION COEFFICIENT THEORY

Pratt and Chechelnitsky's (1997) theory for quantifying the interaction of a Kelvin wave with a hydraulically controlling sill (see Fig. 1) in a 1.5-layer flow with uniform potential vorticity in a rotating channel with a rectangular cross section (gradually varying channel geometry).

$$R_{c,\mathrm{theory}} = \frac{(1-G)(1+F)}{(1+G)(1-F)}, \quad R_{v,\mathrm{theory}} = \frac{1-F}{1+F}R_{c,\mathrm{theory}} = \frac{1-G}{1+G}.$$

With F the Froude number of the steady and unperturbed flow and G the ratio of geostrophic transport and transport through the strait.

$$R_{c,\mathrm{model}} = \frac{\eta_{\mathrm{reflected}}}{\eta_{\mathrm{incident}}}, \qquad R_{v,\mathrm{model}} = \frac{V_{\mathrm{reflected}}}{V_{\mathrm{incident}}}.$$

TIME-SCALES

T_c Basin circumnavigation time of Kelvin wave

$T_{drain} = \frac{T_c}{1-R_V}$, Draining time scale for transport anomaly

RESEARCH QUESTIONS

$$R_{v,theory} = \frac{1-G}{1+G} = \frac{1-T_s^2}{1+T_s^2} = \frac{1-\tanh^2\left[\frac{W_s f}{(g'D_\infty)^{1/2}}\right]}{1+\tanh^2\left[\frac{W_s f}{(g'D_\infty)^{1/2}}\right]}.$$

1. Understand how the draining time scale depends on environmental conditions and strait geometry, and if and how that changes in the case of multiple draining straits

   1.a. Is it possible to diagnose and quantify the wave reflection process that is hypothesised to transmit information from the constricting strait and sill to the upstream basin?

   1.b. To what extent do the theoretical predictions for the reflection coefficient in an idealised system with gradually varying geometry agree with the observed magnitudes and parameter dependencies in a model system in which the geometry varies rapidly? Here relevant parameters include the stratification, the Coriolis parameter, the width of the strait and the depth of the sill.

   1.c. How does the presence of two draining straits—instead of one— impact the reflection coefficient and the draining time scale of the basin?

2. Discuss the results in the context of the GISR

MODEL

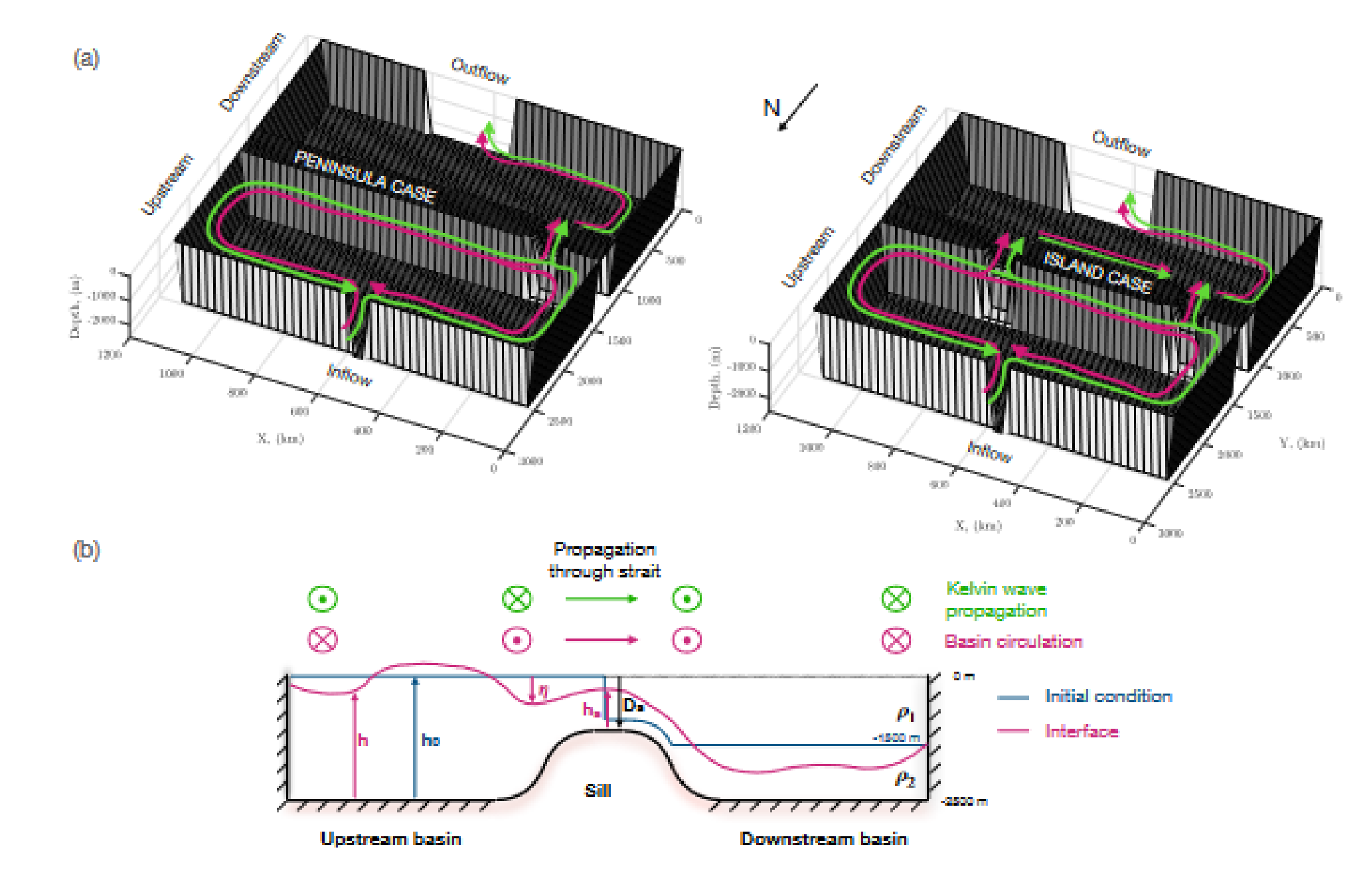


Fig. 1—Bathymetry of the PENINSULA and ISLAND models and sketch of the initial condition (blue) and steady state (pink) for the interface depth. A schematic of the steady state circulation is represented on the bathymetry by the pink arrows and the Kelvin wave propagation by the green arrow.

- 1.5-layer reduced gravity model—active lower layer
- Shallow water equations on a f-plane
- Horizontal resolution 5km
- Vertical walls, flat bottom
- Free slip and no-normal flow

BACKGROUND CIRCULATION

- Constant 1.5-Sv inflow prescribed through the northern boundary
- Initial Kelvin waves generated by the inflow and the dam break (Fig. 1)
- Spinup end at 50 days = quasi-steady state—initial Kelvin waves are fully transmitted to the downstream basin
- Anticyclonic boundary flow in the upstream basin (Fig. 2)
- Separated flow in the channel (Fig. 2)
- Hydraulically controlled flow in the strait

Hydraulic control causes the Kelvin wave transmission to be incomplete, with some transport "reflected" back into the upstream basin

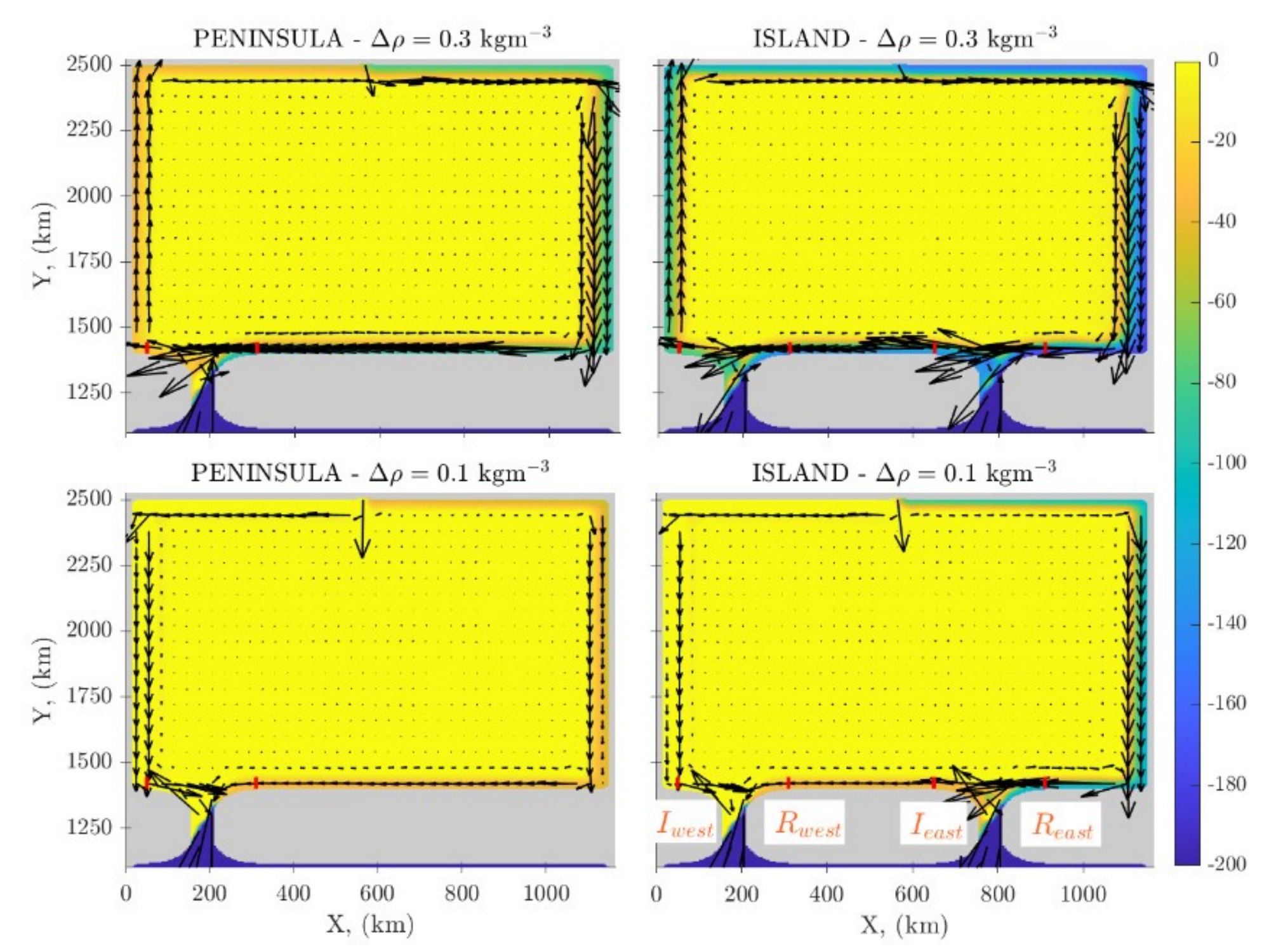
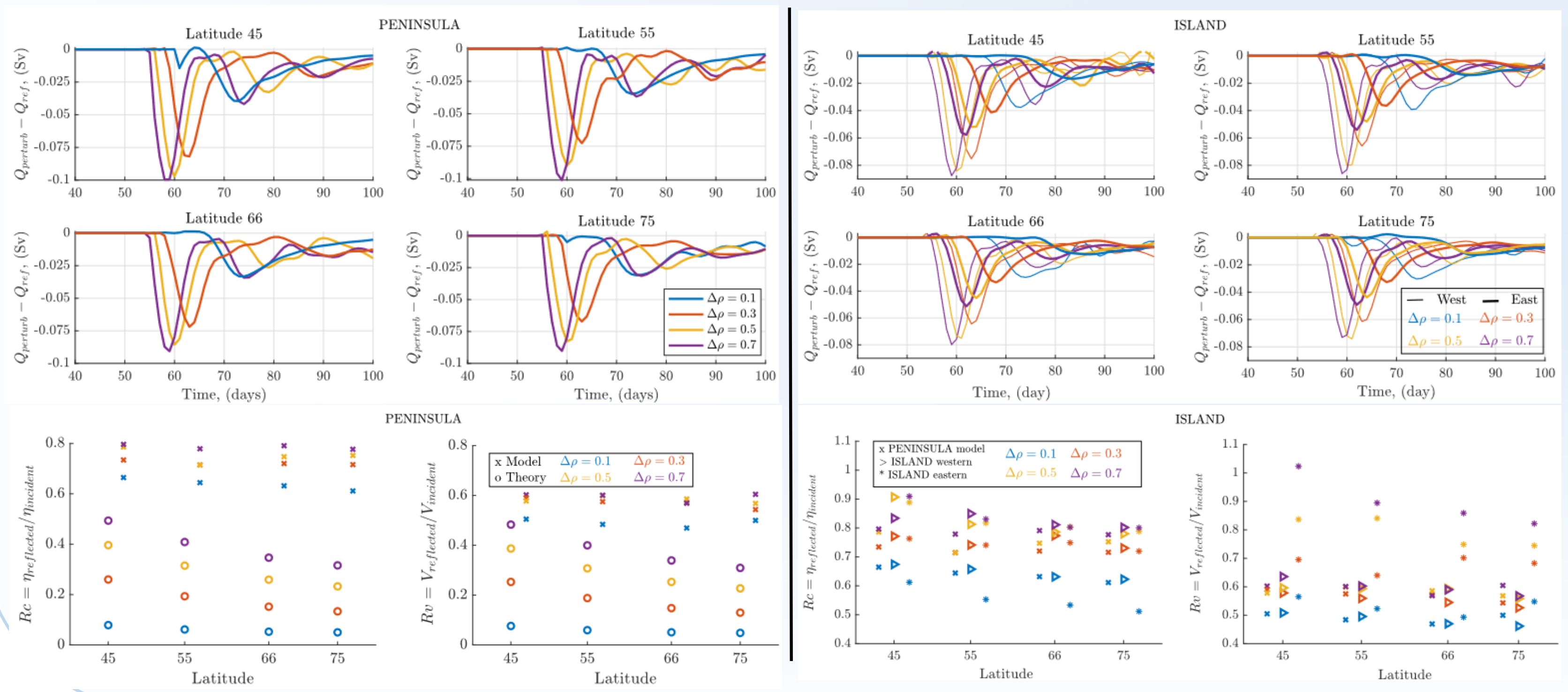


Fig. 2—Snapshots of the interface displacement (colors) and of the velocity field (arrows) at 50 days for the PENINSULA and the ISLAND cases at latitude 66◦N and for Δrho = 0.3 kg/m3 and Δrho = 0.1 kg/m3

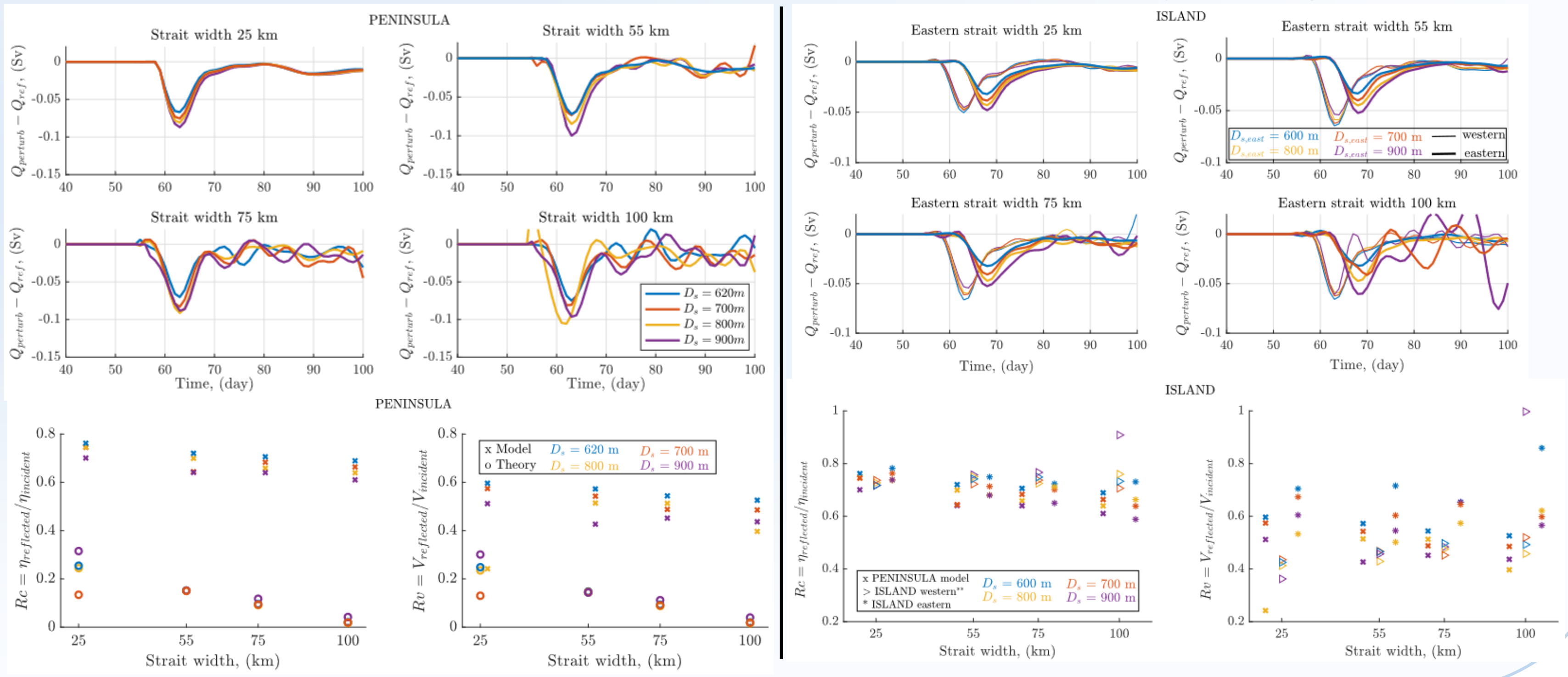
RESULTS—KELVIN WAVE GENERATED BY AN INFLOW INCREASE OF 0.5 Sv FOR 5 DAYS

Each curve show the transport anomaly measure at a sill and triggered by the temporary 0.5 Sv increase in transport introduced upstream at t=50 days. The multiple oscillations are caused by multiple transits of partially reflected Kelvin waves propagating around the basin. Each time the wave encounters a channel entrance, some of its transport escapes downstream. Then the reflection coefficients are presented for each case and compared with theory (circles) for the PENINSULA cases.

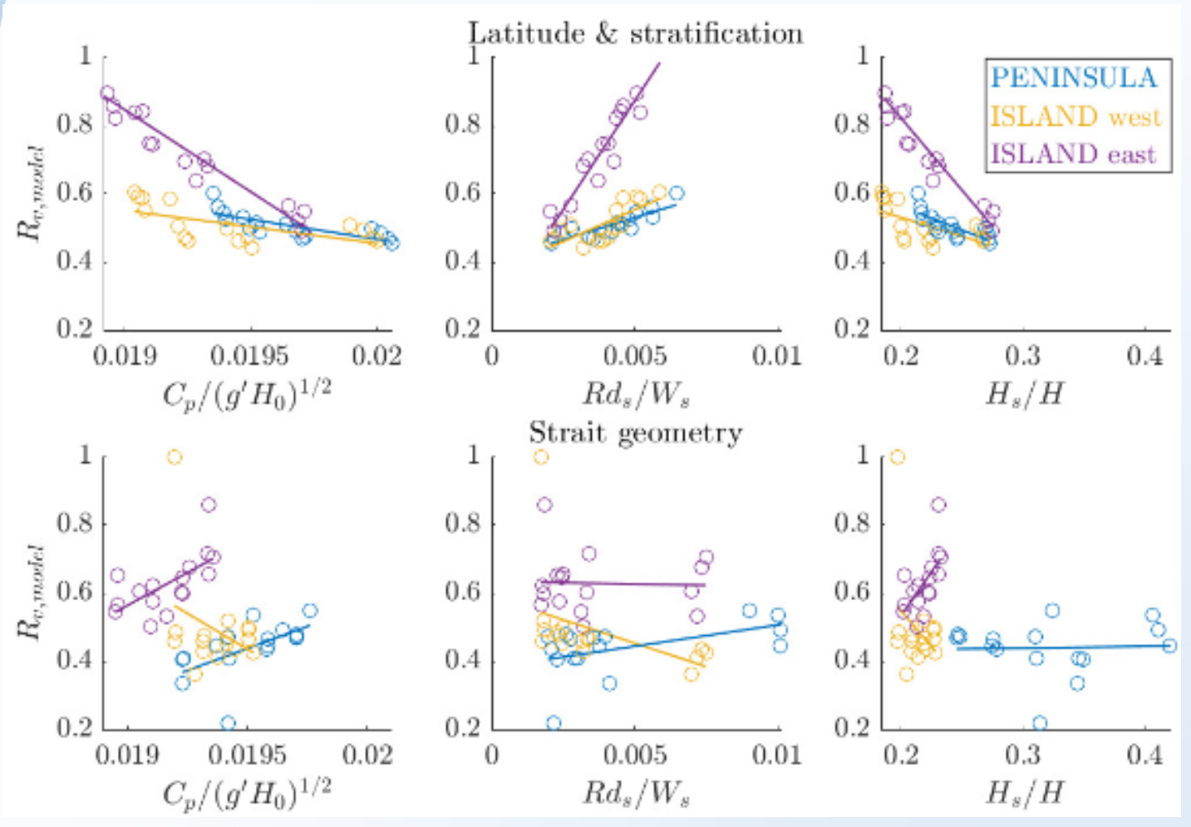
IMPACT OF STRATIFICATION AND LATITUDE (Ds = 620 m & strait width = 55 km)



IMPACT OF STRAIT WIDTH AND SILL DEPTH (Latitude 66°N & Δρ = 0.3 kg/m^3)

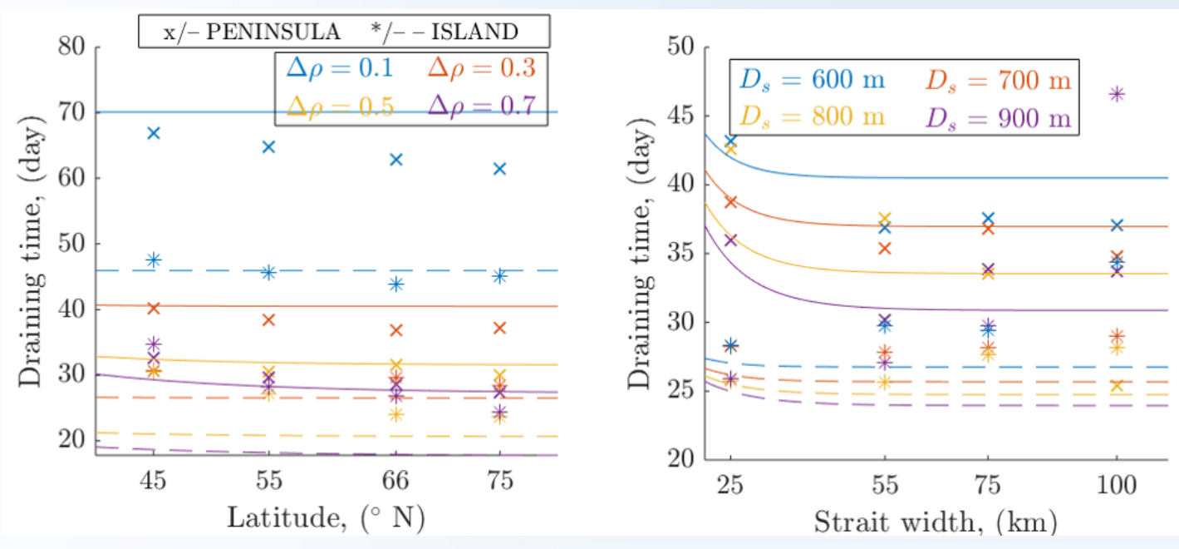


POSSIBLE PARAMETRIZATION



- Should depend on the width and depth contractions, and should approach unity if the contraction is complete.
- might preserve some structural similarity with theory (similar trend)
- Should approach zero when both the flow is unconstrained by width or depth contractions.
- γ is expected to be a rational number and that α is a tuneable parameter that contains info about the particular geometry of the channel entrance.

$$R_V = \frac{1-\alpha\left(\frac{\delta H}{H_b}\right)^{\gamma} tanh\left(\frac{fW_S}{2(g'\delta H)^{1/2}}\right)}{1+\alpha\left(\frac{\delta H}{H_b}\right)^{\gamma} tanh\left(\frac{fW_S}{2(g'\delta H)^{1/2}}\right)},$$



Parameter and direction	important quantity and direction	effect on draining time scale T_{drain}
$\Delta\rho\downarrow$	$C_p\downarrow$	$T_{drain}\uparrow$
$W_S\downarrow$	$W_S/R_d{}^*\downarrow\Rightarrow R_V\uparrow$	$T_{drain}\uparrow$
$\Delta\rho\uparrow$	$W_S/R_d{}^*\downarrow\Rightarrow R_V\uparrow$	$T_{drain}\uparrow$
$f\downarrow$	$W_S/R_d{}^*\downarrow\Rightarrow R_V\uparrow$	$T_{drain}\uparrow$
$H_S\downarrow$	$H_S/D_\infty{}^*\downarrow\Rightarrow R_V\uparrow$	$T_{drain}\uparrow$

PARAMETRIZATION & DRAINING TIMES

- Best fit to model results with α = 1.25 and γ = 1 (plain and dashed curves)
- Parametrization can be tuned through α to fit the model results better

CONCLUSIONS ON REFLECTION COEFFICIENTS

MODEL VS. THEORY

- Similar trend
- Larger values

IMPACT OF STRATIFICATION & LATITUDE

- Stronger reflection with stratification due to the increase of the Kelvin wave Rossby radius of deformation (larger wave-width-to-channel-width ratio)
- Weaker reflection with latitude due to the decrease of the Kelvin wave Rossby radius of deformation (smaller wave-width-to-channel-width ratio)

IMPACT OF STRAIT WIDTH AND SILL DEPTH

- Weaker reflection with strait width due to smaller wave-width-to-channel-width ratio
- Weaker reflection with sill depth due to depth contraction decreasing when sill depth increases

PENINSULA VS. ISLAND

The general trend of the varying eastern strait reflection coefficient is similar to the PENINSULA reflection coefficient. However, the presence of an island leads, in some cases, to values of Rc and Rv for the western strait that are elevated compared to the PENINSULA case.

Part of this work was supported by the National Science Foundation under Grant OCE-2148838, the Centre for Southern Hemisphere Oceans Research at CSIRO and the Woods Hole Summer School.