Nonlinear Energy Arbitrage Models and Algorithms for Battery Energy Storage Systems in Electricity Market

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***Abstract*—Battery energy storage systems (BESSs) are gaining attention due to reduced costs and high flexibility, but developing accurate models for operation presents challenges. This paper introduces a model for the charging and discharging processes via a single current decision variable, approximates the relation between the open circuit voltage and the state of charge with linear functions, and presents an optimization model with bilinear constraints for identifying optimal BESS operational strategies. A transformation technique is introduced to manage the bilinear constraints, transforming the model into an exponential optimization problem with linear constraints. A new sequential linear and quadratic programming approach is developed, with proven convergence. Preliminary experiments demonstrate the efficacy and efficiency of this approach.**

***Index Terms*—Battery Energy Storage, Electricity Market, Energy Arbitrage Model, Nonconvex Quadratic Programming.**

I. INTRODUCTION

To maintain power system reliability and flexibility amidst the rise of renewable energy, energy storage has become crucial for managing dynamic changes caused by wind and solar energy [1]. Battery energy storage systems (BESSs) are favored options due to their versatility and advantages. Unlike geographically constrained options like pump storage and compressed air storage, batteries can be installed in both power generation and distribution systems. Moreover, the cost of battery energy storage has significantly decreased [2].

Research often uses an optimization model to describe battery storage operations, with separate variables for charging and discharging processes [3]. Complementarity conditions are applied to prevent simultaneous charging and discharging, modeled using bilinear constraints [4] or binary variables [5]. Eliminating these constraints for a convex optimization model was investigated in the literature, but the approaches can yield impractical, infeasible solutions [3].

Researchers [6] and [7] have explored detailed battery energy storage models, highlighting the dependency between open circuit voltage (OCV) and state of charge (SOC). The OCV, symbolizing the voltage difference at the open circuit, relies on the SOC. For this paper, we presume this relationship is governed by the open-circuit voltage dependency function (OCVDF). Numerous non-linear and non-convex OCVDF formulations are presented in the literature [6], [8]–[12], leading to complex non-convex BESS optimization models that pose significant computational challenges. However, the complementary conditions are neglected and the non-linearity and non-convexity introduced by the OCVDF are not addressed in the literature.

The main goal of this work is to address the above-mentioned major challenges in the BESS optimization models. To address the first major challenge, we will review the basic BESS optimization model widely used in the literature using two charging/discharging power variables. By exploring the physical principles in the process of battery charging and discharging, we propose to use only a single current variable (i_t) to model both the charging and discharging processes in the BESS. Specifically, $i_t > 0$ indicates that the battery is in the charging process and $i_t < 0$ indicates that the battery is in the discharging process. Consequentially, such a formulation ensures that both charging and discharging won't happen simultaneously, which provides a natural way to model the charging/discharging process of the battery. Mathematically speaking, this also allows us to get rid of the complimentary conditions (i.e., $p_t^c p_t^d = 0$) in most existing optimization models in the literature and thus help to substantially simplify these models.

To address the second challenge, we observe that one major nonlinearity and non-convexity stem from the OCVDF when the detailed BESS optimization model is applied [11]. However, as pointed out by [13], normally, the state of charge is operated in the range $[20\%, 80\%]$ because the lifetime of the battery will be reduced significantly when it is operated outside such a range. When the SOC is restricted to the range, we can use some linear functions to obtain a very good approximation to the original nonlinear OCVDF ($g(s)$). This can be verified from our experiment. For example, if we use the OCVDF defined in [14] as the original OCVDF and apply the least squares method in Banach space [15] to obtain the linear approximate function, then we need to solve the following optimization problem to estimate the two parameters c_0 and c_1 in the approximate linear function.

$$\begin{aligned} \min_{s_u, \bar{g}} \quad & L(\bar{g}) = \int_{0.2}^{0.8} (\bar{g}(s) - g(s))^2 \mathrm{d}s; \\ \text{s.t.} \quad & \bar{g}(s) = c_0 + c_1 s. \end{aligned}$$

The following figure shows the original OCVDF $g(s)$ and its linear approximation $\bar{g}(s)$.

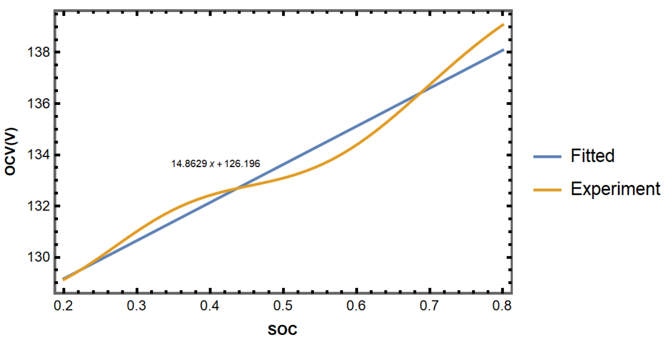

Fig. 1: The OCVDF and its linear approximation

As we'll demonstrate later, even a linear OCVDF leads to a non-convex quadratic programming with linear and bilinear constraints (BLCQP) model for BESS optimization, posing significant computational challenges. To manage these bilinear constraints, we propose a novel nonlinear transformation, converting the original BLCQP into an equivalent exponential optimization problem with solely linear constraints (LCEO). We adapt the conventional sequential quadratic programming method to devise a sequential linear/quadratic programming (SLQP) for the reformulated LCEO model and establish convergence. Numerical experiments validate the effectiveness of our new BESS model and the efficiency of the SLQP algorithm.

This paper is organized as follows. In Section II, we first describe the basic BESS optimization model in the literature and simplify the model. In Section III, we introduce a nonlinear transformation technique to reformulate the model as an equivalent LCEO. In Section IV, we describe the SLQP approach for the LCEO and establish the convergence of SLQP. In Section V, we present a case study to demonstrate the efficacy of our new models and techniques. In Section VI, we conclude the paper by summarizing the topics discussed.

II. Battery Arbitrage Optimization Model in Electricity Market

In the literature [3], [6], [8], the essential part of BESS power arbitrage model (PAM) can be extracted as follows:

$$(PAM) \min_{p_t^d, p_t^c, s_t} \sum_{t \in \mathcal{T}} \lambda_t (p_t^c - p_t^d) \tag{1a}$$

$$\text{s.t.} \quad p_t^c p_t^d = 0, \forall t \in \mathcal{T}; \tag{1b}$$

$$0 \le p^c \le P_t^c, \forall t \in \mathcal{T}; \tag{1c}$$

$$0 \le p^d \le P_t^d, \forall t \in \mathcal{T}; \tag{1d}$$

$$s^l \le s_t \le s^u, \forall t \in \mathcal{T}; \tag{1e}$$

$$ECs_{t+1} = ECs_t + p_t^c \eta^c \Delta - \frac{p_t^d}{\eta^d} \Delta, \forall t \in \mathcal{T}. \tag{1f}$$

In the electricity market, the BESS owners use the PAM to help them make decisions. In this PAM, the objective function is to minimize the costs (or maximize the profits equivalently) of the BESS in the electricity market. This PAM includes the complementary constraints (1b), the charging/discharging rates limits (1c-1d), the operational ranges constraints (1e) and the state of charge balance (1f), where EC is the capacity and Δ is the time granularity. This PAM has been widely used in different power system applications such as unit commitment, economic dispatch, transmission planning, etc. However, this model ignores the relationship between the state of charge and voltage of the storage and thus significantly simplifies the battery energy storage physical operation process. As pointed out by Arroyo et al. in [3], this simplified model may lead to infeasible dispatch strategies for the field operations of the BESS.

A. Physical Model

In this section, we propose a brand new BESS schedule model based on equivalent circuit model by using the fundamental physical laws. In the following, we always assume the time span we study is $\mathcal{T} = \{1, 2, 3, \ldots, T\}$.

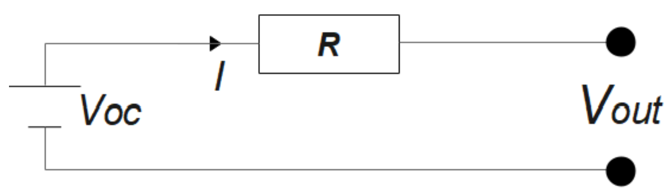

Fig. 2: Equivalent Circuit

Figure 2 shows the circuit of BESS. Let R be the equivalent resistance. When BESS is in the discharging status, we have the following equations.

$$v_t^{oc,d} = v_t^{out,d} + i_t^d R;$$
$$p_t^d = v_t^{out,d} i_t^d.$$

When BESS is in the charging status, we have the following equations.

$$v_t^{oc,c} = v_t^{out,c} - i_t^c R;$$
$$p_t^c = v_t^{out,c} i_t^c.$$

Now, we use general formulation to characterize both the charging and discharging status. The positive value of current i represents that the battery is charging. Then negative value of current i represents that the battery is discharging. Then we have.

$$v_t^{oc} = v_t^{out} - i_t R; \tag{2a}$$
$$p_t^{out} = v_t^{out} i_t. \tag{2b}$$

Using this general formulation, we may express the objective function (1a) in PAM as $\sum_{t \in \mathcal{T}} \lambda_t p_t^{out} = \sum_{t \in \mathcal{T}} \lambda_t (v_t^{oc} i_t + i_t^2 R)$, and eliminate the complimentary constraint (1b). By using the equivalent circuit model (2a) - (2b), we can reformulate the PAM as the voltage (V) current (I) arbitrage model (VIAM) as follows.

$$(VIAM) \min_{v_t^{oc}, i_t, s_t} \sum_{t \in \mathcal{T}} \lambda_t (v_t^{oc} i_t + i_t^2 R) \tag{3a}$$

$$\text{s.t.}\quad ECs_{t+1} = ECs_t + v_t^{oc} i_t \Delta, \forall t \in \mathcal{T}; \quad (3b)$$

$$v_t^{oc} = g(s_t), \forall t \in \mathcal{T}; \quad (3c)$$

$$s_1 = s_{T+1} = \bar{s}; \quad (3d)$$

$$s^l \leq s_t \leq s^u, \forall t \in \mathcal{T}; \quad (3e)$$

$$-I^d \leq i_t \leq I^c, \forall t \in \mathcal{T}. \quad (3f)$$

Notice in this VIAM model, we consider the dependency of OCV on SOC by constraint (3c). We also replace the limits of charging/discharging power rates with the limits of charging/discharging current rates, this corresponds to constraint (3f). We note in VIAM model, in order to compare the solution quality, we assume that the SOC levels s equal at the beginning and the end of the studied time interval $\mathcal{T}$, i.e., $s_1 = s_{T+1}$. A specific choice is $s_1 = s_{T+1} = \bar{s} = 0.5$. Under such an assumption, the profit comes only from the trading of electricity.

As shown in the survey papers [9]–[12] and the references therein, the OCVDF $g(s)$ is usually highly nonlinear and non-convex. The non-linearity and non-convexity in both the constraint functions and the objective function pose a tremendous computational challenge to even obtain a local optimal solution. To tackle such a challenge, we propose to approximate the OCVDF $g(s)$ by some relatively simple functions. In this paper, we will consider the scenario where the OCVDF $g(s)$ is approximated by a linear function and discuss how to solve the resulting bilinear optimization model in the following section.

III. A Non-convex Quadratic Programming Model and Reformulation Techniques

In this section, we consider a special case of VIAM model (3a) where the OCVDF $g(s)$ is a linear function. This leads to a non-convex quadratic programming model with linear and bi-linear constraints (BLCQP). We then introduce a novel nonlinear transformation to reformulate the original BLCQP model as another equivalent optimization problem with linear constraints and exponential objective function (LCEO).

A. A Quadratic Programming Model Based on Linear Approximation of OCVDF

In this subsection, we discuss how to solve model (3a) where the OCVDF is linear or approximated by an linear function. Let $g(s) = c_0 + c_1 s$ denote the linear approximate function. By applying the relation defined by constraint (3b) to the function $g(s)$, we obtain a new constraint as follows

$$\begin{aligned} g(s_{t+1}) &= g(s_t + \frac{\Delta}{EC} v_t^{oc} i_t) \\ &= g(s_t) + c_1 \frac{\Delta}{EC} v_t^{oc} i_t \\ &= g(s_t) + c_1 \frac{\Delta}{EC} g(s_t) i_t. \end{aligned}$$

Let $g^l = c_0 + c_1 s_l$ and $g^u = c_0 + c_1 s_u$ be the lower and upper bound for the linear OCVDF $g(s)$, then we can replace the box constraint (3e) on SOC by that of OCVDF $g(s)$. For notational convenience, let

$$\tau = c_1 \frac{\Delta}{EC}, \quad \bar{g} = g(\bar{s}), \quad g_t = g(s_t), \quad \forall t \in \mathcal{T}.$$

Then we obtain the following quadratic programming model with linear and bi-linear constraints (BLCQP):

$$(BLCQP) \min_{i_t, g_t} \sum_{t \in \mathcal{T}} \lambda_t (g_t i_t + i_t^2 R) \quad (4a)$$

$$\text{s.t.}\quad g_{t+1} = g_t + \tau g_t i_t, \forall t \in \mathcal{T}; \quad (4b)$$

$$g_1 = g_{T+1} = \bar{g}; \quad (4c)$$

$$g^l \leq g_t \leq g^u, \forall t \in \mathcal{T}; \quad (4d)$$

$$-I^d \leq i_t \leq I^c, \forall t \in \mathcal{T}. \quad (4e)$$

We have

Theorem 1. *Model (4) is equivalent to model (3).*

Proof. The sufficiency follows from our deduction of the BLCQP model and thus it remains to prove the necessity. Suppose that g_t is a feasible solution to model (4a). Since $g(s)$ is a linear bijection, its inverse function g^{-1} exists and is also linear. By applying g^{-1} to both sides of constraint (4b), we obtain

$$g^{-1}(g_{t+1}) = g^{-1}(g_t + \tau g_t i_t) = g^{-1}(g_t) + \frac{\tau}{c_1} g_t i_t,$$

which is equivalent to constraint (3b). Therefore, we obtain a solution $s_t = g^{-1}(g_t), \forall t \in \mathcal{T}$ to model (3a). This completes the proof of the theorem. □

Note that in model (4a), there are some non-convex quadratic (or bilinear) terms in both objective function and constraint (4b). This implies that it is still nontrivial to solve model (4a) due to the presence of the non-convex quadratic functions.

B. A Nonlinear Transformation

In this subsection we introduce a nonlinear transformation to simplify the bilinear constraints in model (4a). For this, we note that in the constraint (4b), if we take the logarithm on both sides, we obtain:

$$\log(g_{t+1}) = \log(g_t) + \log(1 + \tau i_t), \forall t \in \mathcal{T}.$$

Let

$$y_t = \log(g_t), z_t = \log(1 + \tau i_t), \forall t \in \mathcal{T}, \quad (5)$$

we can rewrite the constraint (4b) as

$$y_{t+1} = y_t + z_t, \forall t \in \mathcal{T}.$$

Moreover, constraint (4c) can be rewritten as $y_1 = y_{T+1}$ equivalently. Recall that $g_t = e^{y_t}$ and $i_t = \frac{e^{z_t} - 1}{\tau}$, we reformulate the objective function by using constraint (4b), and new variables y_t, z_t,

$$\begin{aligned} \lambda_t (g_t i_t + i_t^2 R) &= \lambda_t (\frac{g_{t+1} - g_t}{\tau} + i_t^2 R) \\ &= \frac{\lambda_t}{\tau} (e^{y_{t+1}} - e^{y_t}) + \frac{\lambda_t R}{\tau^2} (e^{z_t} - 1)^2 \end{aligned}$$

Let

$$\bar{y} = \log(g_1) = \log(g_{T+1}) = \log(\bar{g});$$
$$y^l = \log(g^l), y^u = \log(g^u);$$
$$z^l = \log(1-\tau I^d), z^u = \log(1+\tau I^c).$$

Using the new variables y_t and z_t as defined in (5), we can recast model (4a) as the following optimization problem with linear constraints (LCEO):

$$(LCEO) \min_{y_t, z_t} \sum_{t\in\mathcal{T}} \frac{\lambda_{t-1}-\lambda_t}{\tau} e^{y_t} + \frac{\lambda_t R}{\tau^2}(e^{z_t}-1)^2 \tag{6a}$$
$$\text{s.t.} \quad y_{t+1} = y_t + z_t, \forall t \in \mathcal{T}; \tag{6b}$$
$$y_1 = \bar{y}; \tag{6c}$$
$$y_{T+1} = \bar{y}; \tag{6d}$$
$$y^l \le y_t \le y^u, \forall t \in \mathcal{T}; \tag{6e}$$
$$z^l \le z_t \le z^u, \forall t \in \mathcal{T}. \tag{6f}$$

We remark that compared with the original bilinear optimization model (4), model (6) has only linear constraints with an exponential objective function. Notice that if

$$\lambda_{t-1} \ge \lambda_t, \quad e^{z_t} \ge \frac{1}{2}, \quad \forall t \in \mathcal{T},$$

then model (6) becomes a convex optimization problem. Particularly, because $e^{z_t} \ge e^{z_l} = 1 - \tau I^d$, the relation $e^{z_t} \ge \frac{1}{2}$ is usually satisfied in our problem setting due to the fact that τI^d is much smaller than 1. It is also worth mentioning that when the prices remain invariant, i.e., $\lambda_1 = \lambda_2 = \cdots = \lambda_T$, then model (6) has a trivial optimal solution $(y_1 = y_2 = \cdots = y_T, z_1 = z_2 = \cdots = z_T = 0)$. This implies that a flat price dynamic will lead to less profit in the BESS.

IV. A Sequential Linear/Quadratic Programming Approach

In this section, we propose a sequential linear/quadratic programming approach (SLQP) for the reformulated LCEO model introduced in the previous section and establish its convergence.

To start, we mention that the solution $(y^{(0)}, z^{(0)}) = (y_1 = y_2 = \cdots = y_{T+1}, z_1 = z_2 = \cdots = z_T = 0)$ is feasible for the LCEO and it strictly satisfies the two box constraints.

Suppose at the current iterate k, a feasible solution $(y^{(k)}, z^{(k)})$ is available. For every $t \in \mathcal{T}$, let

$$f_1(y_t) = \frac{\lambda_{t-1}-\lambda_t}{\tau} e^{y_t}, \quad f_2(z_t) = \frac{\lambda_t R}{\tau^2}(e^{z_t}-1)^2;$$
$$f_1'(y_t) = \frac{\lambda_{t-1}-\lambda_t}{\tau} e^{y_t}, \quad f_2'(z_t) = \frac{\lambda_t R}{\tau^2}(2e^{2z_t}-2e^{z_t});$$
$$f_1''(y_t) = \frac{\lambda_{t-1}-\lambda_t}{\tau} e^{y_t}, \quad f_2''(z_t) = \frac{\lambda_t R}{\tau^2}(4e^{2z_t}-2e^{z_t}).$$

As pointed out in the previous subsection, the function $f_2(z_t)$ is convex w.r.t. z_t as $e^{z_t} \ge \frac{1}{2}$ under the setting for the problem in this paper. Therefore, we can approximate $f_2(z_t)$ in a neighborhood of z_t^k via its Taylor expansion as follows:

$$\bar{f}_2(z_t^k + d_z^{t,k}) = f_2(z_t^k) + f_2'(z_t^k) d_z^{t,k} + \frac{f_2''(z_t^k)}{2}(d_z^{t,k})^2.$$

Since the function $f_1(y_t)$ is only convex when $\lambda_{t-1} \ge \lambda_t$, and non-convex when $\lambda_{t-1} < \lambda_t$, we suggest to approximate $f_1(.)$ in some neighborhood of $y_t^{(k)}$ via the following function

$$\bar{f}_1(y_t^k + d_y^{k,t}) = \begin{cases} f_1(y_t^k) + f_1'(y_t^k) d_y^{k,t} + \frac{f_1''(y_t^k)}{2}(d_y^{k,t})^2 & \text{If } \lambda_{t-1} \ge \lambda_t; \\ f_1(y_t^k) + f_1'(y_t^k) d_y^{k,t} & \text{Otherwise.} \end{cases}$$

To find a search direction (d_y^k, d_z^k), we propose to solve the following convex quadratic optimization problem

$$\min_{d_y^{t,k}, d_z^{t,k}} \sum_{t\in\mathcal{T}} \bar{f}_1(y_t^k + d_y^{t,k}) + \bar{f}_2(z_t^k + d_z^{t,k}) \tag{7a}$$
$$\text{s.t.} \quad d_y^{t+1,k} = d_y^{t,k} + d_z^{t,k}, \forall t \in \mathcal{T}; \tag{7b}$$
$$d_y^{1,k} = 0; \tag{7c}$$
$$d_y^{T+1,k} = 0; \tag{7d}$$
$$y^l \le y_t^k + d_y^{t,k} \le y^u, \forall t \in \mathcal{T}; \tag{7e}$$
$$z^l \le z_t^k + d_z^{t,k} \le z^u, \forall t \in \mathcal{T}. \tag{7f}$$

Theoretically, the above problem is polynomially solvable. Once a search direction is identified, we need to find a step size α satisfying the following relation to ensure sufficient descent:

$$\sum_{t\in\mathcal{T}} f_1(y_t^k + \alpha d_y^{k,t}) + f_2(z_t^k + \alpha d_z^{t,k}) \le \sum_{t\in\mathcal{T}} f_1(y_t^k) + f_2(z_t^k) + \frac{\alpha}{2}(f_1'(y_t^k) d_y^{t,k} + f_2'(z_t^k) d_z^{t,k}). \tag{8}$$

We point out that in the LSP algorithm 1, α is first set as the maximal allowable step size α_{max}. In our experiments, we choose $\alpha_{max} = 1$. We have

Algorithm 1 Line Search Procedure (LSP)

Input: f_1, f_2, f_1', f_2', y^k, z^k, d_y^k, d_z^k, $\epsilon > 0$, $0 < \eta < 1$.
Output: Stepsize α^k.
Begin
Set $\alpha = \alpha_{max}$.
while $f_1(y^k + \alpha d_y^k) + f_2(z^k + \alpha d_z^k) > f_1(y^k) + f_2(z^k) + \frac{\alpha}{2}(f_1'(y^k) d_y^k + f_2'(z^k) d_z^k)$ **do**
$\alpha = \eta \cdot \alpha$.
end while
Output $\alpha^k = \alpha$.
End

Theorem 2. *The line search algorithm 1 can always find a step size α satisfying the inequality (8).*

Now, we are ready to describe the following sequential linear/quadratic programming (SLQP) approach for model (6) as shown in Algorithm 2. To explore the conditions under which the solution generated by the SLQP method is a local minimum, we need to rewrite the two box constraints (6e-6f) in the LCEO model as convex quadratic constraints. We can also eliminate the decision variables $\{z_t : t \in \mathcal{T}\}$ in the LCEO model via using the relation $z_t = y_{t+1} - y_t$. This leads to the optimization model (9a).

Algorithm 2 Sequential Linear/Quadratic Programming

Input: Parameters: $\lambda_t, \tau, R, \bar{y}, y^l, y^u, z^l, z^u, \epsilon > 0$;
Output: Solution $(\hat{y}, \hat{z})$.
Begin
Set $k = 0$;
Set $y_t^k = \bar{y}, \forall 1 \le t \le T+1$;
Set $z^k = 0, \forall 1 \le t \le T$;
while $k == 0$ or $\left\|y^k - y^{k-1}\right\|^2 + \left\|z^k - z^{k-1}\right\|^2 \ge \epsilon$ **do**
Solve model (7) for (d_y^k, d_z^k);
Use Algorithm 1 to find a suitable step size $\alpha^{(k)}$;
Update $(y^{k+1} = y^k + \alpha^{(k)} d_y^k, z^{k+1} = z^k + \alpha^{(k)} d_z^k)$;
$k = k + 1$;
end while
Output $(\hat{x}, \hat{y}) = (x^{(k)}, y^{(k)})$.
End

$$\min_{y_t} \sum_{t \in \mathcal{T}} \frac{\lambda_{t-1} - \lambda_t}{\tau} e^{y_t} + \frac{\lambda_t R}{\tau^2} (e^{y_{t+1} - y_t} - 1)^2 \tag{9a}$$

$$\text{s.t.} \quad y_1 = y_{T+1} = \bar{y} \tag{9b}$$

$$(y_t - y^l)(y_t - y^u) \le 0, \forall t \in \mathcal{T}; \tag{9c}$$

$$(y_{t+1} - y_t - z^l)(y_{t+1} - y_t - z^u) \le 0, \forall t \in \mathcal{T}. \tag{9d}$$

Now let us consider the following Lagrangian function for the above problem:

$$\begin{aligned}\mathcal{L}(y_t, \gamma_t, \rho_t, \nu_t) = &\sum_{t \in \mathcal{T}} \frac{\lambda_{t-1} - \lambda_t}{\tau} e^{y_t} + \frac{\lambda_t R}{\tau^2} (e^{y_{t+1} - y_t} - 1)^2 \\ &+ \gamma_1 (y_1 - \bar{y}) + \gamma_{T+1} (y_{T+1} - \bar{y}) \\ &+ \sum_{t \in \mathcal{T}} \rho_t (y_t - y^l)(y_t - y^u) \\ &+ \sum_{t \in \mathcal{T}} \nu_t (y_{t+1} - y_t - z^l)(y_{t+1} - y_t - z^u).\end{aligned}$$

where $(\gamma_t, \rho_t, \nu_t)$ are the Lagrangian multipliers for constraints (9b), (9c) and (9d), respectively. We introduce the following regularity condition.

Condition 1 (Regularity)**.** *We say the solution (y^*) satisfies the regularity condition, if*

$$H = \nabla_y^2 L(y, \gamma, \rho, \nu) \succ 0,$$

where H is a tri-diagonal matrix whose elements are defined as follows:

$$\begin{aligned}H_{t,t} = &\frac{2Re^{-2(y_{t-1} + y_t)}}{\tau^2} [\lambda_{t-1} e^{3y_t} (2e^{y_t} - e^{y_{t-1}}) \\ &+ \lambda_t e^{2y_{t-1} + y_{t+1}} (2e^{y_{t+1}} - e^{y_t})] \\ &+ 2\nu_{t-1} + 2\nu_t + 2\rho_t + \frac{(\lambda_{t-1} - \lambda_t) e^{y_t}}{\tau}\end{aligned}$$

$$H_{t,t+1} = H_{t+1,t} = \frac{2\lambda_t Re^{y_{t+1} - 2y_t} (e^{y_t} - 2e^{y_{t+1}})}{\tau^2} - 2\nu_t$$

We have the following result.

Theorem 3. *The SLQP algorithm 2 will generate a sequence $\{(y_t^k, z_t^k)\}$ converging to a stationary point (y^*, z^*) of the LCEO problem (6). Moreover, if (y^*, z^*) satisfies the regularity condition 1, then (y^*, z^*) is a local minimum of LCEO model (6).*

V. Experimental Results

In this section, we first describe the data set used in our experiments, then run the simulation to validate the efficacy of our new model and algorithms under different scenarios. Finally, we report all the simulation results. We use the 5-minute electricity price in year 2020 from MISO real-time market. We use the battery data from Berrueta's paper [14].

A. VIAM VS LECO Simulation Results

In this subsection, we compare the performance of the LCEO model, the VIAM model with a linear OCVDF (VIAM-L) and the VIAM model with a nonlinear OCVDF (VIAM-NL). In our experiments, the scale of the problem, measured by the cardinality of $\mathcal{T}$, is determined by the number of days and price data resolution. For example, consider a scenario where the time span is 7 days and the price data resolution is 5-minutes, we have $T = 7 * 24 * 12 = 2016$. To assess the effect of the fluctuation in electricity price on the profit, we solve the daily profits through the year 2020, and calculate the mean and the standard deviation of the daily profits from each model. Table I compares the performance of three models (i.e., VIAM-L, VIAM-NL, and LCEO) in terms of the profit based on the control strategy derived from the obtained solution to the corresponding optimization model, where the average profits and the standard deviations are computed for a normalized battery with the capacity of 1 MW per day.

TABLE I: Solutions from VIAM and LECO Models

Days	T	Model	AVG Profit	SD of Profit
1	288	VIAM-L	59.2439	66.6185
		VIAM-NL	59.3645	66.8112
		LCEO	59.2439	66.6186
7	2016	VIAM-L	59.7943	33.5843
		VIAM-NL	59.9172	33.6756
		LCEO	59.7943	33.5844
30	8640	VIAM-L	60.2586	26.3356
		VIAM-NL	60.3769	26.3337
		LCEO	60.2532	26.2712
90	25920	VIAM-L	60.2586	13.4307
		VIAM-NL	60.3823	13.4425
		LCEO	60.2532	13.426
180	51840	VIAM-L	60.2614	13.5676
		VIAM-NL	60.3881	13.565
		LCEO	60.2532	13.5666
366	105408	VIAM-L	59.5409	NA
		VIAM-NL	59.6659	NA
		LCEO	59.5409	NA

From Table I, we can observe that on average, the same amount of profit can be made based on the optimal solutions of both the LCEO and the VIAM-L models, while a slightly higher profit can be made based on the optimal solution to the VIAM-NL model. This shows that the usage of the linear approximation to the nonlinear OCVDF caused very few losses in profit.

Figure 3 shows the average CPU time used to solve each model versus the sizes of the solved instances in Julia. As we can see from Figure 3, the VIAM-NL model always requires the longest CPU time to solve, which indicates that the linear approximation to the nonlinear OCVDF does help to reduce substantially the difficulty of the underlying optimization model. We also observe that in Figure 3, while the CPU time used to solve each model increases as the problem size increases, and the proposed SLQP approach for the LCEO enjoys the slowest growth rate of the CPU time. Particularly, for the largest instance with over 300,000 decision variables, the CPU time used by the SLQP for the LCEO is less than $\frac{1}{3}$ of the CPU time used by IPOPT to solve the VIAM-L. This indicates that the SLQP is more efficient for large scale instances.

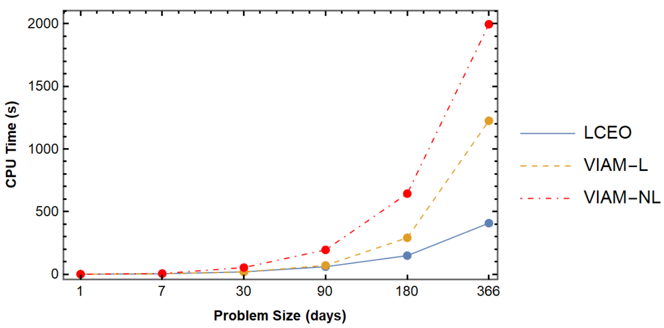

Fig. 3: Average CPU Time for VIAM and LECO Models

B. Impact of Price Dynamics on LECO Solutions

In this subsection, we discuss the impact of price dynamics on the LCEO model solutions and profits. Figure 4 shows the daily profits of a normalized battery under MISO and the daily price dynamic in 2020. As shown in Figure 4, there is a high volatility in the daily profits throughout the year.

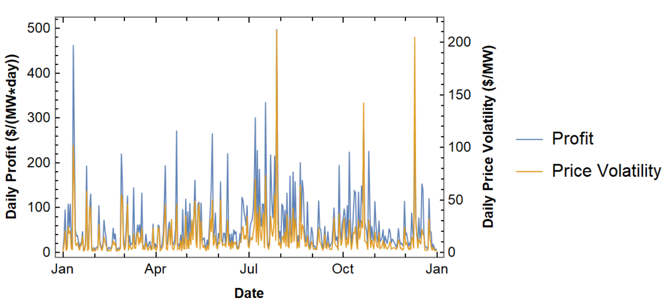

Fig. 4: Daily Arbitrage Profits in 2020

VI. Conclusion

In this paper, we proposed a new BESS optimization model which naturally addresses the simultaneous charging/discharging problem by using a single current decision variable. We also proposed to use some linear functions to approximate the highly nonlinear OCVDF to simplify the underlying BESS optimization model, which further leads to the new BLCQP model. To deal with the bilinear constraints in the BLCQP model, we introduced a novel reformulation technique to recast it as another equivalent LCEO with only linear constraints. A new SLQP approach is proposed for the LCEO model and its convergence is established. Our preliminary experiments illustrate that the optimal solution from the new model provides a very good approximation to the optimal solution of the BESS optimization model with the original nonlinear OCVDF, and the proposed SLQP algorithm is competitive with state-of-the-art optimization software such as IPOPT.

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