

A Generic Mixed-Integer Linear Model for Optimal Planning of Multi-Energy Hub

Mingze Li^{1,2}, Siyuan Wang³, *Member, IEEE*, Lei Fan^{1,2}, *Senior Member, IEEE*,,

Jian Shi^{1,2}, *Senior Member, IEEE*, and Zhu Han¹, *Fellow, IEEE*,

Dept. of Electrical and Computer Engineering¹, Engineering Technology², University of Houston, TX, USA

Dept. of Environmental Health and Engineering³, Johns Hopkins University, MD, USA

Abstract—With growing interdependence among various energy forms, such as electricity, heat, and cooling, multi-energy system (MES) is playing an increasingly important role. As a focal point in MES, energy hub (EH) needs to be properly planned for increased efficiency of energy utilization. This paper presents a generic mixed-integer linear model for EH planning to minimize the overall investment and operation cost given the demand and price of each energy. Our approach uses a graph with multiple rows and layers to represent the energy conversion topology in an EH. In this work, we formulate the EH optimal planning problem with a single combined model to implement the following two steps: 1) optimize the device investments represented in system topology graph; 2) manage the energy conversion and flow to meet the end-use demands meanwhile minimizing the cost. The numerical study shows the efficacy of our proposed model.

Index Terms—Multi-energy systems, mix-integer linear programming, optimal configuration, energy hub.

NOMENCLATURE

Parameters

$\eta_{dt}^{et,in}$	Charge efficiency of system component dt for energy et
$\eta_{dt}^{et,out}$	Discharge efficiency of system component dt for energy et
$\eta_{dt}^{et,in,et,out}$	Conversion efficiency of system component dt for transforming energy et_{in} to et_{out}
λ_t	Electricity price at period t
λ_{dt}^{ds}	Price of device dt with size ds
$\bar{p}_{in}^{dt,et,ds}$	Maximum input of device dt with size ds for energy et
$\bar{p}_{in}^{dt,et}$	Maximum input of device dt for energy et of All sizes
$\bar{p}_{out}^{dt,et,ds}$	Maximum output of device dt with size ds for energy et
$\bar{p}_{out}^{dt,et}$	Maximum output of device dt for energy et of All sizes
$\bar{p}^{et}$	Capacity of transmission lines or pipes for energy et
$\bar{s}_{in}^{dt,et,ds}$	Maximum storage of device dt for energy et with size ds
DS	The number of device size
$L_{t,s}^{et}$	The energy end-use demand
T	The set of one time loop

Sets

$\mathbb{D}$	The set of devices
$\mathbb{D}_c$	The set of conversion devices

$\mathbb{D}_p$	The set of placeholder devices
$\mathbb{D}_s$	The set of storage devices
$\mathbb{E}$	The set of energy types
$\mathbb{H}_{dt}$	The set of energy groups that device dt can convert et_1 to et_2
$\mathbb{R}_{et,in}$	The set of devices that can not produce energy type et
$\mathbb{R}_{et,out}$	The set of devices that can not receive energy type et
$\mathbb{S}_{et,in}$	The set of devices that can produce energy type et
$\mathbb{S}_{et,out}$	The set of devices that can receive energy type et

Decision Variables

$\hat{p}_{ij,t,s}^{dt,et,in}$	Input energy of device dt at position ij during time period t for energy type et
$\hat{p}_{ij,t,s}^{dt,et,out}$	Output energy of device dt at position ij during time period t for energy type et
$\tilde{p}_{ij,mn,t,s}^{et}$	The energy flow on the line energy transferring energy type et exist between location ij and mn
$p_{ij,t,s}^{et,in}$	Input energy of position ij during time period t for energy type et
$p_{ij,t,s}^{et,out}$	Output energy of position ij during time period t for energy type et
$s_{ij,t}^{dt,s}$	Current storage of device dt for energy et at time period t
$x_{ij,mn}^{et}$	Binary variable to indicate whether the line transferring energy type et exist between location ij and mn . 1 is true, 0, is false
y_{ij}^{dt}	Binary variable that indicates whether the device dt exist in location ij . 1 is true, 0, is false
z_{ds}^{dt}	Binary variable that indicates whether we select the size ds for device dt in system

I. INTRODUCTION

The utilization of diverse energy forms is experiencing an exponential rise in the contemporary era. For instance, the share of natural gas usage by the U.S. electric power sector grew from 32% in 2007 to 39% in 2020. Moreover, close to 70 gigawatts of Combined Heat and Power (CHP) generation capacity were scattered across the U.S., contributing to approximately 7% of the total generation capacity. This substantial use of natural gas-powered generation and highly efficient

CHP units affects the energy transmission and distribution like electricity, heating. As a result, challenges arise in short-term energy flow optimization and long-term asset configuration planning to ensure efficient energy utilization.

With advancements in various energy conversion technologies, different forms of energy in Multi-Energy Systems (MESs) are becoming increasingly interconnected. A MES isn't just a combination of multiple energy sources; it's a complex, holistic system that can accommodate various energy inputs and outputs in different forms, thus efficiently utilizing and converting different types of energy at a system-wide level. The aims of MES design include optimal energy usage and cost-effectiveness. By adopting a synergistic approach that taps into multiple energy sources, it meets a building's power, heating, and cooling needs. The MES is designed to satisfy the demands of end-users for electricity, heat, cooling, gas, and other forms of energy. In addition, MES utilizes the combined strength of diverse resources such as wind, solar, hydro, coal, natural gas, within an expansive integrated energy base. This facilitates the development and operation of a complementary multi-energy system, integrating wind, solar, water, and thermal storage to maximize resource utilization.

An Energy Hub (EH) [1] is a fundamental unit within an MES, acting as a focal point where production, conversion, storage, and consumption of diverse energy carriers occur. Viewing it systemically, an EH hosts the input, output, conversion, and storage of multiple energy systems within a functional unit. The transformation and redistribution of energy within an EH are achieved through the deployment of numerous distributed technologies. This includes technologies such as Combined Heat and Power (CHP), Electric Boilers (EB), Electric Heat Pumps (EHP), Auxiliary Boilers (AB), Compression Electric Refrigerator Groups (CERG), and Water Absorption Refrigerator Groups (WARG) [2]. These technologies ensure efficient and effective energy distribution, thereby aiding the transition towards a sustainable energy future. What distinguishes an EH from a traditional energy system is that the loads within a hub can be serviced by multiple carriers to minimize total costs. The diverse energy carriers within a hub, characterized by their cost or availability, provide numerous options for load supply.

Traditional methods for planning the expansion of energy infrastructures strive to identify the best scale, location, and timeframe for new resources within a defined planning period. These conventional strategies frequently focus on optimizing a single type of energy, neglecting the consideration of its interplay with other energy systems. However, the challenge arises when these traditional and separated planning tools cannot harness the advancement of energy conversion technologies for effective long-term planning of EHs with strong energy interdependence. A co-optimization planning strategy fundamentally links multiple energy conversion devices and enables transactions between various energy systems in Energy Hubs (EHs). The increasing interconnectivity among different energy systems provides an opportunity to enhance a comprehensive expansion planning strategy that encompasses multiple

energy infrastructures. This shift in approach paves the way for more efficient and sustainable energy planning and utilization.

Optimization techniques play a key role in both short-term and long-term planning issues. In the short term, these techniques can schedule energy network operations with the aim of minimizing energy consumption and costs, taking into account real-time demand, supply conditions, and system constraints. In the long term, optimization assists in designing and planning energy infrastructure. It aids in determining the ideal topology and size of energy generation and conversion devices, as well as the layout of energy transmission and distribution networks [3]. Recently, energy hub (EH) planning and optimization have received considerable attention in the scientific community. Notably, [4] presents a transformative approach to optimizing smart Multi-Energy Systems (MESs), showing significant improvements in energy efficiency and cost reduction through strategic integration of transportation, natural gas, and active distribution networks.

In addition, [5] introduces a novel two-stage mixed-integer linear programming approach for district-level MES planning. This provides a comprehensive solution for optimizing equipment selection and system configuration, thereby improving efficiency in multi-energy systems. [6] has delved into the potential environmental and financial benefits of a network of EHs, demonstrating through two case studies that the inclusion of distributed energy systems and energy interaction can lead to significant cost savings, CO₂ emission reductions, and decreased natural gas consumption in more complex networks. In the same vein, [7] presents a novel two-stage mixed-integer linear programming approach for optimizing both equipment selection and system configuration in multi-energy systems. This approach effectively enhances energy utilization efficiency and supports distributed renewable energy integration. [8] introduces an innovative and cost-effective approach for strategically placing devices across a network to reduce overall expenses. Building on their research, we propose a more flexible optimization method for device placement in the network to minimize the cost.

This study introduces a mixed-integer linear programming (MILP) model tailored for EH planning. This model integrates strategic planning of an EH, effectively accommodating a range of energy carriers including electricity, natural gas, heat, and cooling. The main goal is to attain the least-cost planning of energy infrastructures within certain constraints, with the aim of meeting the energy needs of the hub throughout a specific planning period. Significantly, our method enables various forms of energy conversion, providing superior flexibility to handle future energy load needs. We achieve this through a one-stage mixed-integer programming approach. The multifaceted energy planning problem is structured to identify the most suitable energy conversion and storage devices, along with their network topology, to align with anticipated load demands and system limitations. We also introduced scholastic optimization model strategy and scenario production for the unpredictable demands in different seasons by one-stage mixed-integer programming.

TABLE I
RATED CAPACITIES OF EACH CANDIDATE SYSTEM
COMPONENT

	Type 1	Type 2	Type 3
CHP	Elec: 350 kW Heat: 525 kW	Elec: 700 kW Heat: 1050 kW	Elec: 1050 kW Heat: 1575 kW
CERG	400 kW	800 kW	1200 kW
WARG	400 kW	800 kW	1200 kW
EB	450 kW	900 kW	1350 kW
EHP	400 kW	800 kW	1200 kW
Heat Storage	200 kW 400kWh	400 kW 800kWh	600 kW 1200kWh

II. TOPOLOGY NETWORK BASED MODELING FRAMEWORK

In this work, we will design the topology structure of Energy Hub (EH) to meet the end-use demands of user meanwhile minimizing the cost of injected energy. Traditionally, EH planing has two main steps: 1) optimize the device investments and topology device interconnections; 2) optimize the energy flow to minimum the cost. Different from most papers, we use a single mixed-integer programming problem to implement the two steps simultaneously, which could improve the optimality of planning decisions.

In this paper, we assume the EH as an $M \times N$ matrix, with each position having the potential to accommodate a single device. These devices are interconnected through lines representing different types of energy. We utilize two binary decision variable matrices, y_{ij}^{dt} and $x_{ij,mn}^{et}$, to define the topology network. Here, y_{ij}^{dt} indicates whether device dt is positioned at location ij , and $x_{ij,mn}^{et}$ signifies if there is a line between ij and mn capable of transporting energy type et . In our study, we assume $n = j + 1$, suggesting that the line can only be positioned between two adjacent columns.

All elements within the matrix are binary variables, defined such that $y_{ij}^{dt} = 1$ if system component dt is at position ij within the EH; otherwise, $y_{ij}^{dt} = 0$, meaning the system component is not present in position ij of the EH.

Furthermore, we employ a binary decision variable matrix, z_{ds}^{dt} , to determine the size of each device, where ds represents the device type. As displayed in Table I, we have categorized each device into three types, each with different capacities and prices.

Next, we propose two $M \times N$ matrices to represent the input and output energy of system components. These matrices are formulated as follows:

$$p_{ij,t,s}^{et,in} = \begin{pmatrix} p_{11,t,s}^{et,in} & p_{12,t,s}^{et,in} & \cdots & p_{1n,t,s}^{et,in} \\ p_{21,t,s}^{et,in} & p_{22,t,s}^{et,in} & \cdots & p_{2n,t,s}^{et,in} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1,t,s}^{et,in} & p_{m2,t,s}^{et,in} & \cdots & p_{mn,t,s}^{et,in} \end{pmatrix}$$

$$p_{ij,t,s}^{et,out} = \begin{pmatrix} p_{11,t,s}^{et,out} & p_{12,t,s}^{et,out} & \cdots & p_{1n,t,s}^{et,out} \\ p_{21,t,s}^{et,out} & p_{22,t,s}^{et,out} & \cdots & p_{2n,t,s}^{et,out} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1,t,s}^{et,out} & p_{m2,t,s}^{et,out} & \cdots & p_{mn,t,s}^{et,out} \end{pmatrix}$$

The variable s represents different scenarios, taking into account various energy demands that may arise due to changes in

TABLE II
DISTRIBUTED TECHNOLOGIES

Device	Input	Output
CHP	Gas	Electricity, Heat
AB	Gas	Heat
CERG	Electricity	Cooling
WARG	Heat	Cooling
EB	Electricity	Heat
EHP	Electricity	Heat
Heat Storage	Heat	Heat

weather or other circumstances. We aim to ensure our system can adapt to and fulfill these varied demands. In the subsequent two scenarios, $p_{ij,t,s}^{et,in}$ and $p_{ij,t,s}^{et,out}$ represent the energy input and output at the system position ij , respectively. Furthermore, another set of variables characterize the energy input and output of the devices themselves:

$$\hat{p}_{ij,t,s}^{dt,et,in} = \begin{pmatrix} \hat{p}_{11,t,s}^{dt,et,in} & \hat{p}_{12,t,s}^{dt,et,in} & \cdots & \hat{p}_{1n,t,s}^{dt,et,in} \\ \hat{p}_{21,t,s}^{dt,et,in} & \hat{p}_{22,t,s}^{dt,et,in} & \cdots & \hat{p}_{2n,t,s}^{dt,et,in} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{p}_{m1,t,s}^{dt,et,in} & \hat{p}_{m2,t,s}^{dt,et,in} & \cdots & \hat{p}_{mn,t,s}^{dt,et,in} \end{pmatrix}$$

$$\hat{p}_{ij,t,s}^{dt,et,out} = \begin{pmatrix} \hat{p}_{11,t,s}^{dt,et,out} & \hat{p}_{12,t,s}^{dt,et,out} & \cdots & \hat{p}_{1n,t,s}^{dt,et,out} \\ \hat{p}_{21,t,s}^{dt,et,out} & \hat{p}_{22,t,s}^{dt,et,out} & \cdots & \hat{p}_{2n,t,s}^{dt,et,out} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{p}_{m1,t,s}^{dt,et,out} & \hat{p}_{m2,t,s}^{dt,et,out} & \cdots & \hat{p}_{mn,t,s}^{dt,et,out} \end{pmatrix}$$

Based on the given variables and parameters, we can derive the following relationship:

$$p_{ij,t,s}^{et,in} = \sum_{dt \in \mathbb{S}_{et,in}} \hat{p}_{ij,t,s}^{dt,et,in}, \quad (1)$$

$$p_{ij,t,s}^{et,out} = \sum_{dt \in \mathbb{S}_{et,out}} \hat{p}_{ij,t,s}^{dt,et,out}. \quad (2)$$

Naturally, $\mathbb{S}_{et,in}$ represents the set of devices that can accept energy of type et as input. Take gas for instance, devices such as CHP and AB can utilize gas as an input and transform it into other forms of energy. Therefore, we can express this relationship as $p_{ij,t,s}^{gas,in} = \hat{p}_{ij,t,s}^{CHP,gas,in} + \hat{p}_{ij,t,s}^{AB,gas,in}$. In this instance, we have neglected the aspect of placeholder positioning. Detailed input and output parameters for the various energy forms can be found in table II.

In the following two circumstances, $\hat{p}_{ij,t,s}^{dt,et,in}$ or $\hat{p}_{ij,t,s}^{dt,et,out}$ equates to zero: 1) system component dt is not located at ij ; 2) system component dt does not possess input/output of energy type et . As such, $\hat{p}_{ij,t,s}^{dt,et,in}$ and $\hat{p}_{ij,t,s}^{dt,et,out}$ are typically sparse matrices.

In our model, we further introduce a variable $\tilde{p}_{ij,mn,t,s}^{et}$ to characterize the energy type et flow on line ij,mn at time period t .

III. A ONE STAGE MIXED-INTEGER LINEAR MODEL FOR ENERGY HUB PLANNING

A. Objective Function

The energy input is composed of gas fuel and electricity. Here, the gas fuel price remains constant, while the electricity

price changes with time. For instance, when gas prices are low, an Absorption Chiller (AB) is employed to generate heat at a reduced operational cost. Conversely, when electricity prices are low, the AB is shut down, and an Electric Heat Pump (EHP) is selected. With these conditions in mind, we can establish the objective function:

$$\min \sum_{s \in \mathbb{S}} \sum_{et \in \mathbb{E}} \sum_{t=1}^T \left[\lambda_t \sum_{i=1}^n p_{i1,t,s}^{et,in} \right] + \sum_{dt \in \mathbb{D}} \sum_{k=1}^{DS} z_{ds}^{dt} \lambda_{ds}^{dt}. \quad (3)$$

B. System Component Modeling

The mathematical models for each type of system component are provided below. We utilize the initial letters to represent each energy type.

1) *Energy Conversion Component*: For energy conversion components which facilitate the transformation of one form of energy to another (for instance, an Auxiliary Boiler can convert gas to heat), we model this process as follows:

$$\hat{p}_{ij,t,s}^{dt,et,out,out} = \eta_{dt}^{et_{in},et_{out}} \hat{p}_{ij,t,s}^{dt,et_{in},in}, \quad \forall i, j, t, s, (et_{in}, et_{out}) \in \mathbb{H}_{dt}, \quad \forall dt \in \mathbb{D}_c \quad (4)$$

where $\eta_{dt}^{et_{in},et_{out}}$ can be either a constant efficiency coefficient or a function of the working condition of the component. In the planning project, we set a constant value for each device. The input(/output) constraints is:

$$0 \leq \hat{p}_{ij,t,s}^{dt,et,in} \leq y_{ij}^{dt} \bar{p}_{in}^{dt,et}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_c, \quad (5)$$

$$0 \leq \hat{p}_{ij,t,s}^{dt,et,out} \leq y_{ij}^{dt} \bar{p}_{out}^{dt,et}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_c, \quad (6)$$

$$0 \leq \hat{p}_{ij,t,s}^{dt,et,in} \leq \sum_{t=1}^{DS} z_{ds}^{dt} \bar{p}_{in}^{dt,et,ds}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_c, \quad (7)$$

$$0 \leq \hat{p}_{ij,t,s}^{dt,et,out} \leq \sum_{t=1}^{DS} z_{ds}^{dt} \bar{p}_{out}^{dt,et,ds}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_c. \quad (8)$$

where $\bar{p}_{ij}^{dt,et}$ is the maximum value of the input of device dt .

2) *Placelholding Connection Component*: Placelholding connection components have been incorporated into the EH. It is important to note that the inclusion of these placelholding connections serves solely to facilitate the construction of the planning model, and does not represent the installation of actual transmission lines or pipelines. If system component ij is a placelholding connection component, the relationship between its input and output is

$$\hat{p}_{ij,t,s}^{dt,et,out} = \hat{p}_{ij,t,s}^{dt,et,in}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_p \quad (9)$$

$$0 \leq \hat{p}_{ij,t,s}^{dt,et,in} \leq y_{ij}^{dt} M, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_p \quad (10)$$

where M is a significantly large positive number to ensure that there are no constraints on the amount of power transmitted, considering that the placelholder connection components don't represent actual transmission lines or pipes.

3) *Storage*: Storage components function as single input and single output entities. The correlation between the input and output for storage component ij , in relation to energy type et , is defined as follows:

$$s_{ij,t}^{dt,s} = s_{ij,t-1}^{dt,s} + \eta_{dt}^{et,in} \hat{p}_{ij,t,s}^{dt,et,in} - \hat{p}_{ij,t,s}^{dt,et,out} / \eta_{dt}^{et,out}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_s, \quad (11)$$

The capacity constraints are:

$$0 \leq s_{ij,t}^{dt,s} \leq \sum_{t=1}^{DS} z_{ds}^{dt} \bar{s}_{in}^{dt,et,ds}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_s \quad (12)$$

$$0 \leq s_{ij,t}^{dt,s} \leq y_{ij}^{dt} \bar{s}_{in}^{dt,et}, \quad \forall i, j, t, s, et, ds, \quad \forall dt \in \mathbb{D}_s \quad (13)$$

$$s_{ij,0}^{dt,s} = s_{ij,|T|}^{dt,s}, \quad \forall i, j, s, et, ds, \quad \forall dt \in \mathbb{D}_s, \quad (14)$$

Equations (11)-(14) constrain the energy flow within the storage component. In (11), $\eta_{dt}^{et_{in},et_{out}}$ represents the conversion rate of energy storage and output. Equations (12) to (13) dictate the storage component's capacity. Equation (14) ensures that the state of the storage component at the beginning and end of a given time period is the same, thereby simplifying the model. It should be noted that storage components are also subject to conversion constraints given by (5)-(8). To represent storage resources that doesn't physically allow simultaneously charge and discharge, such as battery, our model can be easily generalized as in [9], [10].

C. System Constraints

1) *Topology Constraints*: We also have some constraints on system level, to decide how to place and connect these devices.

$$\sum_{dt \in \mathbb{D}} y_{ij}^{dt} = 1, \quad \forall i, j \quad (15)$$

In our system, each node can only possess one component.

$$\sum_{ij} y_{ij}^{dt} = 1, \quad \forall dt \quad (16)$$

the system can at most possess one of the component size ds .

$$\sum_{i=1}^{DS} z_{ds}^{dt} = 1, \quad \forall i, j, dt \quad (17)$$

Equation (17) restricts the choice of device size. For each device, one must choose one type and incorporate it into the system.

Taking into account connection rules, it is clear that transmission lines or pipelines of a specific energy type can only connect with certain devices. For example, if $et = electricity$, then for $\mathbf{x}_{ij,mn}^{et}$, $dt \in \{CHP, battery, holding\}$, for $\mathbf{y}_{ij}^{dt}$, only these devices could generate electricity. Similarly, for the receivers, if $et = electricity$, for $\mathbf{x}_{s,ij,mn}^{et}$, then $dt = EHP, CERG, battery, holding$, for $\mathbf{y}_{s,ij}^{dt}$, only these devices can receive electricity. Therefore, the following holds:

$$x_{ij,mn}^{et} \leq \sum_{dt \in \mathbb{S}_{et,out}} y_{ij}^{dt}, \quad \forall i, j, m, n, et, \quad (18)$$

$$x_{ij,mn}^{et} \leq \sum_{dt \in \mathbb{S}_{et,in}} y_{mn}^{dt}, \forall i, j, m, n, et, \quad (19)$$

$$1 - x_{ij,mn}^{et} \geq \sum_{dt \in \mathbb{R}_{et,out}} y_{ij}^{dt}, \forall i, j, m, n, et, \quad (20)$$

$$1 - x_{ij,mn}^{et} \geq \sum_{dt \in \mathbb{R}_{et,out}} y_{mn}^{dt}, \forall i, j, m, n, et, \quad (21)$$

where $\mathbb{S}_{et,in}$ represents the set of devices that can accept energy et , and $\mathbb{S}_{et,out}$ signifies the set of devices that can produce energy et . In Equation (16), we posit that if ij, mn contains a transmission line or pipeline that carries energy et , then location ij must have a device capable of producing et , and in Equation (17) we posit that location mn must house a device capable of accepting et . Equations (18) and (19) suggest that if there's a line carrying et , the start location cannot contain a device that cannot produce et and the end location cannot house a device that cannot accept et . These constraints ensure every line is connected to the correct device.

In a similar fashion, we consider the device type. If $dt = \text{CHP}$, we find:

$$1 - y_{ij}^{dt} \geq 1 - \sum_{et} x_{ij,mn}^{et}, \quad (22)$$

for $et \in \{\text{electricity}, \text{heat}\}$. And

$$1 - y_{ij}^{dt} \geq 1 - x_{m(j-1),ij}^{et}, \quad (23)$$

for $et \in \{\text{gas}\}$. CHP must have these input and output connections. Other devices have similar patterns.

2) *Budget Constraints*: Considering the energy flow, we have:

$$p_{ij,t,s}^{et,in} = \sum_{k=1}^m \tilde{p}_{m(j-1),ij,t,s}^{et}, \forall i, j, s, t, et, \quad (24)$$

$$p_{ij,t,s}^{et,out} = \sum_{k=1}^m \tilde{p}_{ij,m(j+1),t,s}^{et}, \forall i, j, s, t, et, \quad (25)$$

$$0 \leq \tilde{p}_{ij,m(j+1),t,s}^{et} \leq x_{ij,m(j+1)} \bar{p}^{et}, \forall i, j, s, t, et, \quad (26)$$

As the energy produce by device could be sent to the next layer by multiple lines, and device can also receive energy from multiple lines.

$$\sum_{i=1}^m p_{iN,t,s}^{et,out} = L_{t,s}^{et}, \forall i, s, t, et, \quad (27)$$

The output of the last layer must meet the demand of end users.

$$\underline{p}^{et,in} \leq \sum_{j=1}^n \hat{p}_{1j}^{et,in} \leq \overline{p}^{et,in}, \forall j, s, t, et, \quad (28)$$

where $\underline{p}^{et,in}$ and $\overline{p}^{et,in}$ is the minimum and maximum supplement in a time period from market. Whatsmore, if $dt = \text{holding}$,

$$\hat{p}_{i1,t,s}^{et,in} = 0, \forall et \in \{\text{heat}, \text{cooling}\} \quad (29)$$

$$\hat{p}_{iN,t,s}^{et,out} = 0, \forall et \in \{\text{gas}\} \quad (30)$$

Because we cannot get heat and cooling supplement from market and cannot send gas to end user.

IV. CASE STUDY

A. Test System Assumptions

We explain the boundary conditions in our test, which include load demands and energy prices.

1) *End-use Energy Demand*: The demand patterns for electricity, heat, and cooling are from a hospital site in a Mediterranean region, as illustrated in [2]. There are rich body of demand forecast methods for a given time period. The heat and cooling demands, for instance, are approximated by calculating the temperature difference between indoors and outdoors. As an example, when the ambient temperature falls below 16 °C, the heat demand is estimated by determining the difference between the outdoor ambient temperature and 16 °C. The cooling demand is computed in a similar fashion, presuming that cooling is necessary when the outdoor ambient temperature exceeds 26 °C [5]. The electricity demand is approximated based on the forecasted maximum load (167.6 MW) [8]. In this paper, we utilize the estimated data from [2] as input for our experiments.

Load scenarios are characterized by different patterns for summer, spring/autumn, and winter seasons with corresponding probabilities.

2) *Energy Price*: The price of the electricity purchased from the grid was divided into three different categories as a function of the period of the day. The price is cheap (F3) from 11 p.m. to 6 a.m. (night period), while it is expensive (F1) from 8 a.m. to 8 p.m. (day period), and normal (F2) from 6 a.m. to 8 a.m. and from 8 p.m. to 11 p.m.

B. Optimal Result

This study uses Julia with Gurobi solver to conduct the MES planning. All experiments are implemented with Julia 1.5 on a standard i9 3.4-GHz computer.

Considering the number of EH layers is four, the optimization result of the MES configuration planning is presented in Fig. 3. As the optimization result shows, a 700 kW wind farm, 8400kW EHP, 450 kW AB, 80kW CERG and a total of 400 kWh heat storage components were chosen as the optimal planning scheme to satisfy different scenarios. The electricity demand is satisfied by the purchased electricity.

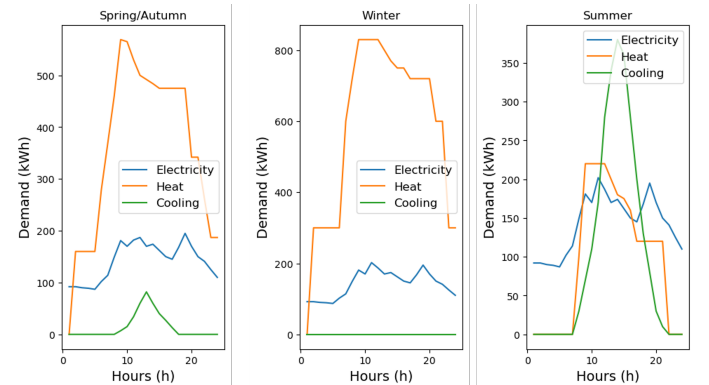


Fig. 1. The selected representative load scenarios

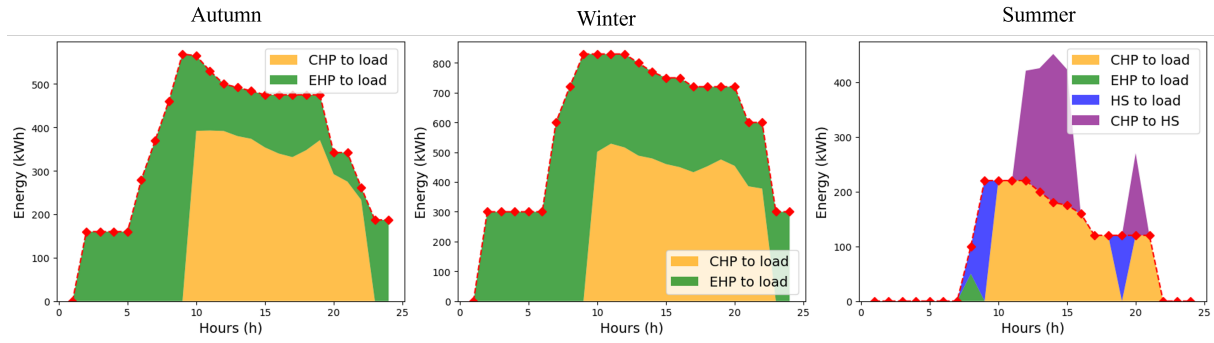


Fig. 2. Hourly Heat Energy Flows in the 3 Scenarios

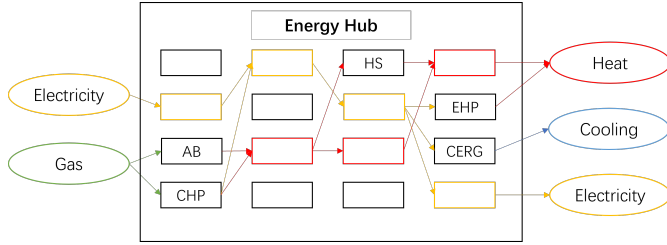


Fig. 3. The EH Configuration for a 4×4 Case



Fig. 4. Heat Storage Variation

Fig. 2 shows that Combined Heat and Power (CHP) and Electric Heat Pumps (EHP) generate heat to fulfill the base load in autumn and winter scenarios. In contrast, the summer scenario reveals a high cooling demand, prompting the CHP to generate electricity which is then transformed into cooling energy by the Cooling, Energy Recovery, and Generation (CERG) unit. The excess heat energy is stored in storage.

The heat storage (HS) strategy is optimized to reduce the heating expense by withholding heat when the electricity costs are low. This correlation between heat storage and electricity price is illustrated in Fig. 4. The heat storage system retains heat energy during periods of low electricity price and subsequently discharges it when the electricity price rises, in order to decrease the cost. It's important to note that all cooling demands are served by the CERG unit.

V. CONCLUSION

In this study, we demonstrated the effectiveness of our model in computing optimal results, which subsequently enhanced energy usage efficiency. Particularly, our approach effectively solves the model in a single step. Future work will focus on examining the performance of our model under varying demand scenarios. For instance, we plan to study the model's response during winter months when the demand for cooling decreases to zero and the requirement for heat significantly rises.

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REFERENCES

- [1] M. Geidl and G. Andersson, "Optimal power flow of multiple energy carriers," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 145–155, Jan. 2007.
- [2] G. Chicco and P. Mancarella, "Matrix modelling of small-scale trigeneration systems and application to operational optimization," *Energy*, vol. 34, no. 3, pp. 261–273, Mar. 2009.
- [3] C. Unsihuay-Vila, J. W. Marangon-Lima, A. Z. De Souza *et al.*, "A model to long-term, multiarea, multistage, and integrated expansion planning of electricity and natural gas systems," *IEEE Transactions on Power Systems*, vol. 25, no. 2, pp. 1154–1168, Feb. 2010.
- [4] S. Xie, Z. Hu, J. Wang *et al.*, "The optimal planning of smart multi-energy systems incorporating transportation, natural gas and active distribution networks," *Applied Energy*, vol. 269, p. 115006, Jul. 2020.
- [5] W. Huang, N. Zhang, J. Yang *et al.*, "Optimal configuration planning of multi-energy systems considering distributed renewable energy," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 1452–1464, Oct. 2017.
- [6] A. Maroufmashat, S. T. Taqvi, A. Miragha *et al.*, "Modeling and optimization of energy hubs: A comprehensive review," *Inventions*, vol. 4, no. 3, Dec. 2019.
- [7] W. Huang, N. Zhang, J. Yang *et al.*, "Optimal configuration planning of multi-energy systems considering distributed renewable energy," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 1452–1464, Oct. 2019.
- [8] Y. Wang, N. Zhang, Z. Zhuo *et al.*, "Mixed-integer linear programming-based optimal configuration planning for energy hub: Starting from scratch," *Applied Energy*, vol. 210, pp. 1141–1150, May 2018.
- [9] S. Wang, G. Geng, and Q. Jiang, "Robust co-planning of energy storage and transmission line with mixed integer recourse," *IEEE Transactions on Power Systems*, vol. 34, no. 6, pp. 4728–4738, Nov. 2019.
- [10] S. Wang, G. Geng, Q. Jiang *et al.*, "Generation expansion planning considering discrete storage model and renewable energy uncertainty: A bi-interval optimization approach," *IEEE Transactions on Industrial Informatics*, vol. 19, no. 3, pp. 2973–2983, May 2022.