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Letter

MONDified gravity

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ABSTRACT

A new class of modified gravity theories, made possible by subtle features of the canonical formulation of general covariance, naturally allows MOND-like behavior (MOdified Newtonian Dynamics) in effective space-time solutions without introducing new fields. A detailed analysis reveals a relationship with various quantum-gravity features, in particular in canonical approaches, and shows several properties of potential observational relevance. A fundamental origin of MOND and a corresponding solution to the dark-matter problem are therefore possible and testable.

1. Introduction

Applications of general relativity to cosmology at low curvature and, increasingly, to black holes in strong-field regimes have led to several unexplained phenomena, highlighting the need to find alternative gravitational theories for detailed comparisons with observations. The requirement that such theories be generally covariant is often taken as implying that they must be related to general relativity by introducing additional interactions from higher-curvature terms or from new fields of scalar, vector or tensor nature. However, many such theories are ruled out by the observational insight that the speed of gravitational waves is very close to the speed of light [1–4]. Moreover, higher-curvature actions often have instabilities caused by higher time derivatives [5].

Recent results in canonical gravity [6,7] have culminated in the conclusion that the usual road that leads to higher-curvature or scalar-vector-tensor actions is not the only one to alternative gravity theories, thanks to a subtle feature of general covariance: Its mathematical formulation, expressed canonically by conditions on fields on a foliation of space-time into spacelike hypersurfaces, does not take the same form as in the common picture of coordinate changes in a 4-dimensional space-time manifold. Conditions that ensure general covariance of the foliation have been constructed early on in canonical formulations of general relativity [8], requiring the imposition of constraints which generate hypersurface deformations as a gauge symmetry. It is well-known that algebraic properties of the constraints are rather complicated because they have Poisson brackets or commutators with structure functions that depend on the fundamental fields, in particular on the spatial metric on a hypersurface. (In an independent line of mathematical research, the relationship between this feature and an L_∞-structure that modifies the usual Jacobi identity has been analyzed in [9].)

Classically, the structure function equals the inverse of a fundamental spatial metric field used to define the theory, and all standard theories of modified gravity in metric form maintain this relationship. The opening to new classes of modified gravity consists in the observation that the structure function may well have a different relationship with the fundamental fields. Provided that its gauge transformations take the form required for coordinate transformations of an inverse spatial metric, the structure function then defines an *emergent* space-time line element distinct from the fundamental fields. With hindsight, the results of [6] show that there are indeed new non-trivial candidates for this new class of *emergent modified gravity*, at least in spherically symmetric models. Here, we provide a complete fundamental formulation as well as new applications.

In particular, we use our new formulation of the underlying canonical theory to show that several of its features can imply a natural relativistic realization of Modified Newtonian Dynamics (MOND, [10,11]) without introducing extra fields. Constructing such theories has proven difficult in manifestly covariant form in a standard space-time description, unless new non-geometrical fields are introduced [12–14]. The new theories discussed here may therefore help to analyze or explain long-standing cosmological puzzles such as dark matter. Their fundamental nature is highlighted by several examples provided here that show how terms required for emergent modified gravity may be implied by ingredients of canonical quantum gravity. We discuss properties of orbital motion and light deflection in emergent modified gravity in order to show how its implication can be confronted by observations.

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2. Emergent modified gravity

Infinitesimal deformations of spacelike hypersurfaces in space-time are known to have an unusual and challenging feature: While infinitesimal deformations $D(\vec{M})$ along a spatial vector field $\vec{M}$ tangential to a spacelike hypersurface have a simple commutator

$$[D(\vec{M}_1), D(\vec{M}_2)] = D(\mathcal{L}_{\vec{M}_1}\vec{M}_2) \tag{1}$$

given by a directional or Lie derivative $\mathcal{L}_{\vec{M}_1}\vec{M}_2$, the commutator of two deformations $H(N)$ normal to the hypersurface by a displacement function N,

$$[H(N_1), H(N_2)] = D(N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1)), \tag{2}$$

depends, through the gradient $\nabla^a N = q^{ab}\partial_b N$, on the metric q_{ab} induced on the hypersurface, with inverse q^{ab}. Since the hypersurfaces may be curved with varying spatial geometries, the commutator depends on the hypersurface to which it is applied. The generators $D(\vec{M})$ and $H(N)$ of gauge transformations are constrained to vanish on physical solutions of the theory. The presence of structure functions then complicates quantizations of these equations because it requires specific factor orderings of non-commuting metric and constraint operators. In the context of classical modified gravity, however, the presence of a structure function is powerful because it gives us a direct link from algebraic properties of gauge transformations to geometrical structures of space or space-time.

We analyze the specific form of the constraints for spherically symmetric line elements of the general form

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{(E^\varphi)^2}{E^x}(\mathrm{d}x + N^x\mathrm{d}t)^2 + E^x(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2) \tag{3}$$

in which all functions depend only on x and t. The functions N and N^x parameterize foliations of space-time into spacelike hypersurfaces, on which the induced metric is described by two functions, E^φ and E^x. These two functions, in a canonical formulation, are the momenta of components of extrinsic curvature, K_φ and K_x [15]. In canonical form, candidates for theories of gravity are given by phase-space functions D and H with Poisson brackets of the form (1) and (2), replacing the action functional. In general, the classical q^{ab} in $\nabla^a N$, or q^{xx} in spherical symmetry, is then some phase-space function $\tilde{q}^{xx}(E^x, E^\varphi, K_x, K_\varphi)$ that is not required to have a simple relationship with the fundamental canonical fields. A candidate theory defined like this is generally covariant if the structure function $\tilde{q}^{xx}$ is subject to gauge transformations consistent with coordinate changes of an inverse spatial metric. It can then be used to define a compatible emergent space-time line element, supplying a geometrical interpretation to solutions of the theory.

The covariance conditions can be evaluated in a canonical version of effective field theory, in which one starts with a general expression for the integrand of $H(N) = \int \mathrm{d}x NH$ as a local function of the canonical fields up to some order in spatial derivatives and a certain polynomial order in K_x, which has a non-zero density weight. The coefficients of spatial derivatives of E^x and E^φ and of the K_x-terms are initially free functions of E^x and K_φ (which both have zero density weight), but their form is strongly restricted by two conditions, that (i) the Poisson bracket $\{H(N_1), H(N_2)\}$ be of the form (2) with (ii) a structure function $\tilde{q}^{xx}$ that has gauge transformations consistent with coordinate transformations of an inverse spatial metric.

As shown in [7], completing earlier versions in [16,17], the most general such expression for a generator H of normal deformations takes the form

$$\begin{aligned} H = -\lambda_0\frac{\sqrt{E^x}}{2}E^\varphi\Bigg[& c_{f0} + \frac{\alpha_0}{E^x} + 2\frac{\sin^2(\bar{\lambda}K_\varphi)}{\bar{\lambda}^2}\frac{\partial c_f}{\partial E^x} + 4\frac{\sin(2\bar{\lambda}K_\varphi)}{2\bar{\lambda}}\frac{\partial q}{\partial E^x} \\ & + \frac{\alpha_2}{\bar{\lambda}^2 E^x}c_f - \frac{(E^\varphi)^2}{\lambda_0^2\bar{\lambda}^2 E^x}\left(\frac{\alpha_2}{E^x}\tilde{q}^{xx} + 2\frac{K_x}{E^\varphi}\frac{\partial \tilde{q}^{xx}}{\partial K_\varphi}\right) \\ & - \left(\frac{(E^x)'\left((E^\varphi)^{-2}\right)'}{2} + \frac{(E^x)''}{(E^\varphi)^2}\right)\cos^2(\bar{\lambda}K_\varphi)\Bigg] \end{aligned} \tag{4}$$

when it is expanded up to second order in spatial derivatives (using units such that the speed of light and Newton's constant equal one), where

$$\tilde{q}^{xx} = \left(\left(c_f + \left(\frac{\bar{\lambda}(E^x)'}{2E^\varphi}\right)^2\right)\cos^2(\bar{\lambda}K_\varphi) - 2q\bar{\lambda}^2\frac{\sin(2\bar{\lambda}K_\varphi)}{2\bar{\lambda}}\right)\lambda_0^2\frac{E^x}{(E^\varphi)^2} \tag{5}$$

is the structure function replacing $q^{xx} = E^x/(E^\varphi)^2$ in the classical bracket (2). Here, $\bar{\lambda}$ is a free constant, and λ_0, c_f, c_{f0}, α_0, α_2 and q are free functions of E^x not determined by the covariance condition alone. The free parameters take simple values zero or one in classical general relativity, which demonstrates the existence of a number of new modified gravity theories. (More precisely, we obtain the classical limit for $\lambda_0, c_f, \alpha_1, \alpha_2 \to 1$, $\bar{\lambda} \to 0$, and $c_{f0} \to -\Lambda$ if there is a cosmological constant.) Among the free functions, only c_f, λ_0 and q (for $\bar{\lambda} \neq 0$) appear in the emergent spatial metric and are therefore characteristic of emergent modified gravity, while c_{f0}, α_0 and α_2 parameterize freedom present in the classical spherically symmetric theory, akin to the free potential of dilaton gravity. The phase space is not enlarged, and therefore there are no additional fields such as those implied by higher time derivatives in a canonical formulation, or additional scalar, vector or tensor fields added to general relativity by hand.

The rather involved relationship (5) is unambiguously determined by the Poisson bracket of two modified Hamiltonian constraints of the form (2). The fact that $\tilde{q}^{xx}$ is not a simple combination of the basic phase-space functions demonstrates that these theories are not easily related to action principles formulated for a fundamental space-time metric, as done in standard modified gravity. Nevertheless, dynamical and gauge properties, and therefore physical predictions, are uniquely determined by Hamilton's equations generated by the constraints.

Given the canonical structure and the covariance condition, it is possible to evaluate the space-time dynamics implied by H and the corresponding emergent line element with inverse spatial metric (5). The freedom of choosing a suitable space-time slicing allows us to impose additional conditions in order to simplify the calculations or to focus attention on specific space-times such as static ones close to the standard Schwarzschild solution. For the latter, we choose $E^x = x^2$ and $N^x = 0$. (According to (3), the radial coordinate x then determines the area of symmetric spheres by the usual equation $A = 4\pi x^2$.)

The second choice, $N^x = 0$, implies that the equations of motion are given by Hamilton's equations of NH (rather than the full $H(N) + D(N^x)$), where N is the same space-time function that appears in (3). The first choice, $E^x = x^2$, implies a consistency condition because E^x is then time-independent and the corresponding Hamilton equation must be zero. Given the general Hamiltonian, this condition reads $\dot{E}^x = -N\partial H/\partial K_x = 0$, or $\tan(2\bar{\lambda}K_\varphi) = -2\bar{\lambda}q/(c_f + \bar{\lambda}^2x^2/(E^\varphi)^2)$. A simple solution of $K_\varphi = 0$ is obtained if we choose a modified dynamics with the classical value $q = 0$. The other curvature $K_x = 0$, then also vanishes as a consequence of $D = 0$. The constraint $H = 0$ implies

$$0 = (\ln(E^\varphi)^2)' + \frac{(E^\varphi)^2}{x}\left(c_{f0} + \frac{\alpha_0}{x^2}\right) - \frac{\alpha_2 + 2}{x}, \tag{6}$$

which for specific choices of c_{f0} and α_2 can be solved for E^φ. The remaining Hamilton equations are consistent with a static solution, $\dot{E}^\varphi = 0$, and a final condition,

$$(\ln(NE^\varphi))' = \frac{\alpha_2}{x} - (\ln\lambda_0)', \tag{7}$$

is implied by $\dot{K}_\varphi = 0$ because $K_\varphi = 0$. The condition $\dot{K}_x = 0$ does not result in an independent equation.

Equation (6) may be solved by making an ansatz $E^\varphi(x) = x/\sqrt{1 - f_\varphi(x)}$ with a function f_φ subject to the equation

$$0 = xf_\varphi' + \alpha_2 f_\varphi + \alpha_0 - \alpha_2 + c_{f0}x^2. \tag{8}$$

If the free functions are such that the classical limit with a vanishing cosmological constant is obtained for large x, the solution to this equation is asymptotically of the form $f_\varphi(x) \sim 2M/x$ with a constant M that is interpreted as the central mass, such as a black hole, that gives rise to our curved space-time. The solution of (7) for N then takes the form

$$N(x) = \frac{\sqrt{1-f_\varphi(x)}}{\mu(x)\lambda_0(x)}\,, \tag{9}$$

provided

$$(\ln\mu)' = \frac{1-\alpha_2(x)}{x}\,. \tag{10}$$

These solutions simplify (5) and imply an emergent space-time line element

$$\mathrm{d}s^2 = -\frac{\left(1-f_\varphi(x)\right)}{\mu(x)^2\lambda_0(x)^2}\mathrm{d}t^2 + \frac{\mathrm{d}x^2}{\lambda_0(x)^2\left(c_f(x)+\bar{\lambda}^2\left(1-f_\varphi(x)\right)\right)\left(1-f_\varphi(x)\right)} + x^2\mathrm{d}\Omega^2 \tag{11}$$

which is clearly different from the general form of the Schwarzschild line element, implying new effects. A characteristic feature of (11) is that $g_{xx} \neq -g_{tt}^{-1}$ in general. Several new phenomena can be expected from this inequality, for instance in the behavior of horizons, but most of them depend on details of the specific modification functions c_f and λ_0 and the corresponding solutions f_φ and μ. We will focus instead on a more generic phenomenon that may show implications even in non-relativistic regimes. To this end, we will first derive an effective gravitational potential for geodesic motion of massive objects in a space-time with line element (11).

3. Gravitational potential

A standard procedure derives geodesic motion using spherical and time translation symmetry of our space-time, together with the mass condition $||\mathbf{p}||^2 = g_{\mu\nu}p^\mu p^\nu = -m^2$ for the 4-momentum $\mathbf{p} = m\mathrm{d}\mathbf{x}/\mathrm{d}\tau$ of an object of mass m, using proper time τ. Killing vectors for the symmetries provide the conserved energy $E = (1-f_\varphi)\mu^{-2}\lambda_0^{-2}\mathrm{d}t/\mathrm{d}\tau$ and angular momentum $L = x^2\mathrm{d}\varphi/\mathrm{d}\tau$, and the mass condition takes the form

$$0 = \frac{1}{2}m\left(\frac{\mathrm{d}x}{\mathrm{d}\tau}\right)^2 + \frac{\tilde{q}^{xx}}{2}\left(m + \frac{L^2}{mx^2} - \frac{E^2}{mN^2}\right) \tag{12}$$

of a Newtonian-style energy balance, where $\tilde{q}^{xx}$ is the inverse spatial metric and $-N^2$ the time component in (11). Geodesic motion of massive objects can therefore be expressed by the effective potential

$$V(x) = \frac{\lambda_0^2}{2}\left(c_f + \bar{\lambda}^2\left(1-f_\varphi\right)\right)\left(\left(1-f_\varphi\right)\left(m+\frac{L^2}{mx^2}\right) - \frac{\mu^2\lambda_0^2E^2}{m}\right). \tag{13}$$

In the classical case, in which the x-dependent functions μ, c_f and λ_0 equal one while $\bar{\lambda} = 0$ and the solution to (8) is $f_\varphi(x) = 2M/x + \Lambda x^2$, (13) contains Newton's gravitational potential, the centrifugal potential, and a long-distance correction from Λ. Depending on the modification functions, the potential may deviate from the two classical power laws of x^{-1} and x^2. In a region with small and constant deviations from the classical values for the remaining parameters, $\alpha_2 \approx 1$, $\alpha_0 \approx 1$, $c_{f0} \approx -\Lambda$, and some λ_0, the solution of (8) stays close to its classical form,

$$f_\varphi(x) \approx \frac{2M}{x^{\alpha_2}} + \frac{\alpha_2-\alpha_0}{\alpha_2}x^{\alpha_2} + \frac{\Lambda}{3}\frac{1}{\alpha_2+2}x^{2+\alpha_2}\,, \tag{14}$$

and μ depends only weakly on x as a consequence of (10). The Newtonian power law may therefore change, but not significantly enough for MOND-like effects that would require a logarithmic contribution to f_φ in order to result in a $1/x$ force law on intermediate scales.

Logarithmic MOND effects can, however, appear in the gravitational potential via more dramatic modifications in at least two ways. The first is by choosing $\alpha_0 = \alpha_2 = 1$ and $c_{f0} = -\Lambda + (c_1/E^x)\ln\left(e^2E^x/c_2\right)$, with constants c_1 and c_2 with vanishing c_1 in the classical limit, such that (8) is solved by $f_\varphi = 2M/x + \Lambda x^2/3 - c_1\ln\left(x^2/c_2\right)$ despite having $c_{f0} \approx -\Lambda$ at large scales. The second, simpler way that we will mainly focus on is by simply adding a logarithmic contribution to the function c_f, which directly appears in the gravitational potential. Our new theories of emergent modified gravity allow logarithmic terms in these functions while preserving general covariance. More importantly, as we will now show, there are several reasons for quantum-gravity effects to imply logarithmic contributions to c_f, which appears directly in the gravitational Hamiltonian and the structure function. Such terms would be relevant on intermediate scales, between the Newtonian one dominated by $2M/x$ and the cosmological one dominated by $\Lambda x^2/3$. This is the extra-solar or galactic range of MOND.

A well-known source of logarithmic terms in quantum field theories is given by renormalization. In the present case of a canonical theory, this process requires Hamiltonian renormalization in a background-independent manner, which is still being developed; see for instance [18] or the moment derivation of the Coleman–Weinberg potential in [19], to which renormalization can be applied as in [20]. Treating gravitational models in this way remains a challenge, but there are independent results from [21] that explicitly show a non-classical logarithmic term in the Hamiltonian constraint of a quasiclassical spherically symmetric model, resulting from a solution for quantum fluctuations of the metric components. Importantly, these canonically derived logarithms need not be Lorentz invariants, as in standard renormalization theory. They cannot be implemented in standard higher-curvature actions but may find a new home in emergent modified gravity.

We are also able to present an independent calculation that leads to logarithmic terms in c_f, based on our new methods for the gravitational constraints and the covariance condition [7]. As suggested in [22], canonical quantization of a constrained system simplifies if one can eliminate the structure function, given by $\tilde{q}^{xx}$ in spherically symmetric models. This may be possible by considering suitable linear combinations of the constraints with phase-space dependent coefficients. The construction given in [22] can be generalized significantly, while also making sure that the resulting theories remain local and generally covariant. An example of such a solution obtained from a covariant linear combination of the constraints without structure functions, assuming the classical values $\alpha_0 = 1$ and $c_{f0} = -\Lambda$, is given by

$$c_f(E^x) = 1 + \frac{\bar{\lambda}^2}{2}\left(\Lambda E^x - \ln(E^x/c_0)\right) \tag{15}$$

where c_0 is a positive integration constant unrelated to the other modification functions.

Since this modification is more significant on intermediate scales than the constant α-dependence in the function (14), we can approximate $f_\varphi(x)$ by its classical form. Up to a slowly-changing factor of $\mu^{-1}\lambda_0^{-1}$, the lapse function (9) then also has its classical form. Nevertheless, the geometry is non-classical because c_f appears in the emergent line element (11), now given by

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{x}-\frac{\Lambda}{3}x^2\right)\frac{\mathrm{d}t^2}{\mu^2\lambda_0^2} + \left(1+\bar{\lambda}^2\left(1-\frac{2M}{x}-\ln\left(\frac{x}{\sqrt{c_0}}\right)+\frac{\Lambda}{6}x^2\right)\right)^{-1}\frac{\lambda_0^{-2}\mathrm{d}x^2}{1-2M/x-\Lambda x^2/3} + x^2(\mathrm{d}\vartheta^2+\sin^2\vartheta\,\mathrm{d}\varphi^2)\,. \tag{16}$$

Independently of the origin of (15), our results show that there is a modified gravity theory in canonical form which has this emergent line element as a solution.

At distances large enough to neglect the Newtonian potential, but not so large that the cosmological term is relevant, we have an effective potential dominated by the logarithmic term. For non-relativistic objects with $E \approx m$, we have

$$V \approx \frac{m\lambda_0^2}{2}\left(1+\bar{\lambda}^2\left(1-\ln\left(\frac{x}{\sqrt{c_0}}\right)\right)\right)\left(\frac{L^2}{m^2x^2}+1-\mu^2\lambda_0^2\right). \tag{17}$$

The combination of a negative logarithmic and a quadratic potential implies new stable circular orbits at a radius x_0, which we may use for an estimate of the relevant distance and velocity scales. At a local minimum of the potential, the angular momentum is given by

$$\frac{L^2}{m^2} = \frac{\bar{\lambda}^2}{2}\left(\mu^2\lambda_0^2 - 1\right)\left(1 - \frac{\bar{\lambda}^2}{2}\left(\ln\left(\frac{x^2}{c_0 e^2}\right) - 3\right)\right)^{-1} x^2 . \tag{18}$$

In this condition, a stable circular orbit is then seen to exist only if $\mu^2\lambda_0^2 > 1$, such that, to leading order in the $\bar{\lambda}$ expansion, it has the radius $x_0 = L(\bar{\lambda}m)^{-1}\sqrt{2/\mu^2\lambda_0^2 - 1}$ and velocity $v_0^2 = \frac{1}{2}\bar{\lambda}^2(\mu^2\lambda_0^2 - 1)$. The Tully-Fisher relation [23] $v_0^4 \approx a_0 M$, where $a_0 \approx \sqrt{\Lambda/3}/(2\pi)$ is Milgrom's universal acceleration [24], requires a mass-dependent $\mu^2\lambda_0^2 - 1 = 2\sqrt{a_0 M}/\bar{\lambda}^2$. In our derivation, μ is part of the lapse function which usually depends on the mass in static solutions, and λ_0 is expected to depend on M if its deviation from the classical value is implied by renormalization effects in quantum gravity. Notice also that λ_0 multiplies the gravitational Hamiltonian, just like Newton's constant does if gravity is coupled to matter, and may therefore be renormalized. The expression (18) implies that further MOND effects may be seen at even larger scales. However, because $\bar{\lambda}$ is expected to be small, such scales may be cosmological and Λ effects would have to be taken into account too.

Since our theories of emergent modified gravity are fully relativistic and generally covariant, it is straightforward to derive additional physical effects such as modifications of the bending of light, which may be used to subject the modification functions to further observational bounds. For instance, the deflection angle of light moving around a central mass now takes the form

$$\Delta\phi \approx \frac{\pi}{\lambda_0}\left(1 - \frac{\sqrt{a_0 M}}{2} + \frac{\bar{\lambda}^2}{2}\ln\left(\frac{2b}{\sqrt{c_0}e}\right)\right) \tag{19}$$

to leading order in a_0 and $\bar{\lambda}$, with the impact parameter b of the light ray. Unlike the usually expected deflection angle in the MOND literature which is independent of the impact parameter, there is a logarithmic dependence in this relativistic extension and this could lead to a negative deflection angle repelling the light. In a semiclassical regime, however, we expect $\sqrt{a_0 M} < \bar{\lambda}^2$, so the overall correction to the deflection angle is positive for large enough impact parameters.

We briefly discuss the alternative MOND-like modification from $c_{f0} = -\Lambda + (c_1/E^x)\ln(e^2 E^x/c_2)$. From a fundamental perspective, this option is less preferred because it requires two undetermined constants rather than only one. Moreover, because c_{f0} does not appear in the emergent spatial metric, it can be obtained also by choosing a suitable dilaton potential. Phenomenologically, it also has more complicated and ambiguous results. Following a similar procedure as above it results in the angular momentum expression

$$\frac{L^2}{m^2 x^2} = \frac{c_1\left(1 + \lambda^2 + \lambda^2\left(1 - \mu^2\lambda_0^2\right) + 2c_1\lambda^2\ln(x^2/c_2)\right)}{c_1^- + \lambda^2\left(2c_1^- - 1\right) + c_1\left(1 + 2\lambda^2 c_1^-\right)\ln(x^2/c_2) + c_1^2\lambda^2\ln(x^2/c_2)^2} \tag{20}$$

substituting (18) and using the shortcut $c_1^- = 1 - c_1$. The Tully-Fisher relation can be obtained in several ways. For example, the choice $c_1 = \sqrt{a_0 M}$ (which needs renormalization for c_1) reproduces the Tully-Fisher relation to zeroth order in $\bar{\lambda}$. Alternatively, the constant μ can be used in several different ways and the expansions are ambiguous; one such realization is $1 - \mu^2\lambda_0^2 = \sqrt{a_0 M}/(c_1\bar{\lambda}^2)$, which reproduces the Tully-Fisher relation to zeroth order in both c_1 and $\bar{\lambda}$. The deflection angle of light is instead, to leading order in c_1 and $\bar{\lambda}$, given by

$$\Delta\phi \approx \frac{\pi}{\lambda_0}\left(1 - \bar{\lambda}^2\left(c_1 + \frac{1}{2}\right) - \frac{c_1}{2}\left(1 + \frac{\bar{\lambda}^2}{2}\right)\ln\left(\frac{4b^2}{c_2 e^2\mu^2\lambda_0^2}\right)\right) . \tag{21}$$

If we take $c_1 = \sqrt{a_0 M}$ then this result differs from (19) by a factor of approximately two when taking the limit $\bar{\lambda} \to 0$ with fixed a_0. Furthermore, depending on the value of c_2, at large enough impact parameter the light is only repelled.

These discrepancies can be used to distinguish between the two alternatives using the data on deflection angles. It is also possible to couple various matter fields to gravity in our canonical formulation, and thereby investigate the stability of compact objects. Unlike previous proposals such as [12–14], our theory of emergent modified gravity makes it possible to formulate MOND-like effects in a generally covariant form without introducing additional fields or higher time derivatives. Stability is therefore easier to ensure, and since there is a single emergent line element that determines the propagation of all massless objects, gravitational waves and light travel at the same speed. The space-time properties of the horizon and the interior region are well-behaved, and it is possible to obtain a global structure with singularity resolution for certain values of the constant c_0. We therefore have a promising class of new observationally viable alternatives to general relativity.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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