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An auction mechanism for platoon leader determination in single-brand cooperative vehicle platooning

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ABSTRACT

Cooperative vehicle platooning enabled by connected automated vehicle (CAV) technology has been identified to bring energy savings and driving-effort reduction. However, the intrinsic difference of gained benefits between the leading vehicle and the following vehicles hampers the spontaneous platooning via peer-to-peer coordination. This study proposes an auction mechanism that determines the leader–follower positioning together with the associated benefits, for facilitating the formation and maintaining the behavioral stability of vehicle platoons in a distributed way. We theoretically prove that there is no mechanism to achieve an efficient outcome in an ex post equilibrium, requiring individual rationality and budget balance. In this regard, we provide a truthful ϵ-approximate auction mechanism that deploys a linear transfer function, which guarantees that the implemented outcome is an efficient approximate dominant strategy equilibrium.

1. Introduction

The rapid development of connected automated vehicle (CAV) technology enables the realization of cooperative vehicle platooning, where a fleet of vehicles travel synchronously in a string with small headway. Vehicle platooning is anticipated to improve traffic safety performance and increase roadway capacity (Calvert et al., 2019; Axelsson, 2016), thereby boosting the overall efficiency for transportation systems. Meanwhile, vehicles in platoons attain the endogenous economic benefits in two aspects: improved fuel efficiency due to smoother aerodynamic movements (Tsugawa et al., 2016; Lammert et al., 2018) and reduced driving workload due to vehicle automation and assistance (Janssen, 2015). The promising benefits spark research interests in planning, managing, and commercializing of vehicle platooning systems, especially for truck platooning systems (Bhoopalam et al., 2018), as fuel and labor costs occupy 60% of the total operating cost in the trucking industry (Murray and Glidewell, 2019).

When examining the cost-saving mechanisms of vehicle platooning, one can easily find that the leading vehicle plays a key role. First, due to the aerodynamic drag reduction effect when vehicles travel in a platoon, the leading vehicle contributes most to the overall fuel efficiency improvement while experiencing less amount of fuel savings than its followers. The degree of improvement is influenced by vehicle shape and size (Hammache et al., 2002; Levedahl et al., 2010). An experimental study on a three-truck platoon indicates that the following vehicles can achieve an average 10% of energy savings while the leading vehicle achieves less than 1% (McAuliffe et al., 2017). Second, a leading vehicle's driver undertakes more driving effort than the following vehicles, as fully automated vehicles are not yet advanced enough to be implemented in platoons. Instead, partially automated vehicles equipped with cooperative adaptive cruise control (CACC) are expected to be used widely to facilitate vehicle platooning. With this technology, drivers on the following vehicles only need to conduct lateral control. Longitudinal control, including speed control and distance keeping, is delegated to the leading vehicle's driver (Shladover et al., 2015). With the increase of vehicle automation level, it can also be envisioned that drivers on the following vehicles can fully rest while the whole platoon is traveling (ACEA, 2018). Compared to fuel savings that largely depend on the engine performance, the benefit from effort reduction is more subjective and thereby is perceived differently by different individuals. This phenomenon is similar to the reduction in the value of travel time when using fully automated vehicles (van den Berg and Verhoef, 2016). Identified reduction factors include but are not limited to travel purposes, the type and duration of on-board activities, whether the vehicle is private or shared, and drivers' socioeconomic attributes (de Almeida Correia et al., 2019; Molin et al., 2020; Pudāne et al., 2018). Therefore, a platoon operator would intuitively choose the vehicle who brings the most energy savings and benefits on driving-effort reduction to lead the whole platoon. However, as the leading vehicle always

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benefits the least, drivers are reluctant to be a volunteer leader, if they are not compensated by others.

This underlying conflict triggers the idea of using profit sharing or benefit reallocation mechanisms to facilitate the formation of vehicle platoons. In practice, the mechanisms can be realized by monetary transfers among members in the formed platoon via telecommunications, such as vehicle-to-vehicle or vehicle-to-infrastructure communication. When assuming the individual drivers/vehicles are rational agents, a mechanism is useful only if it aligns with their compatibility in joining platoons. Otherwise, they are likely to switch positions with others or leave the assigned platoons for more significant benefits, making unstable platoons and disturbing traffic flow. The willingness for individuals to join and stay in a given platoon with designated monetary transfers is first delineated in our previous work (Sun and Yin, 2019) and referred to as *behavioral stability*. Moreover, the mechanism is also expected to achieve behavioral stability for a random platoon formation and, more importantly, for the optimum one with the maximum total benefit. In this regard, our previous work first solved a mixed-integer program for a fleet of "platoonable" vehicles to find the optimal platoon formation that maximizes the total benefit. Based on this result, we further applied a fair allocation mechanism to ensure that each vehicle obtains benefits proportional to their contribution to the system.

Our prior work assumes the existence of a central controller and explores a centralized benefit allocation scheme. In this study, we investigate a distributed benefit reallocation mechanism. Compared to the centralized approach, the distributed mechanism is not subject to the central coordination, so that it can be deployed more easily and flexibly, and is more scalable and adaptive to different platooning scenarios. However, the absence of a central controller also implies that vehicles have to perform as rational agents to make all decisions through peer-to-peer coordination. It then has the risk of generating an inefficient outcome because of the incompatible personalized utilities and the information credibility issue of the distributed networks (Jiang et al., 2017). Therefore, our study aims to theoretically address the incompatibility between individuals' utilities and system efficiency without leveraging a central coordinator, while ensuring the information credibility and mechanism fairness.

Our investigation focuses on single-brand vehicle platooning where vehicles are identical regarding their vehicle model and configuration. It implies that the gained fuel efficiency from platooning is indifferent to the sequence of vehicles. In other words, vehicles contribute the same to the total fuel efficiency, once the number of vehicles and the operational speed are determined. In addition, we assume that a single driver perceives the same driving effort in all following positions, which is always less than that at the leading position. Therefore, a platoon's total benefit is only determined by the leading vehicle being chosen. Under our mechanism, the leading vehicle will collect payments from its followers, as leading a platoon provides an effort-relief service. The other way around, the followers are the buyers of the service and pay the leader. When agents hold different willingness-to-pay (WTP) for this service, the agent with the minimum WTP is expected to be the leader to maximize the platoon's total utility. With this feature, an auction mechanism, named *platoon leader auction*, is proposed under which the leading vehicle and the monetary transfers are determined through a bidding process, which can be perfectly deployed in the peer-to-peer system.

Auction is the most well-studied mechanism in the literature, and the most commonly-used mechanism in practice. In transportation research, auctions have been widely used in the freight transportation service procurement to improve freight operations (Caplice and Sheffi, 2003; Lafkihi et al., 2019). In recent years, auctions have also been studied for parking management in urban areas, especially for the shared parking platform that utilizes supply from both public parking spaces and private parking lots (Xiao et al., 2018; Xiao and Xu, 2018). One of the fundamental properties required for auctions is *incentive compatibility*, meaning that agents maximize their utilities by bidding their values for the good truthfully. The well-known second-price auction, or Vickrey auction, is one of the mechanisms that can ensure dominant strategy incentive compatibility (Vickrey, 1961). Since each agent's WTP is their private information that is intangible to others, platoon leader auctions that are incentive compatible admit trustworthy information sharing and stable outcomes. The second commonly-seen requirement is *individual rationality*, meaning that auctions guarantee all that agents can achieve non-negative utility by joining the platoon. Consequently, incentive compatible and individual rational auctions ensure behavioral stability and maximize the total utility from all agents, which we refer to as achieving the *economic efficiency*. On top of these three requirements, we expect platoon leader auctions to be *budget balanced*, meaning that no external compensation is made to agents nor extra profits are collected from them.

The platoon leader auction is uniquely distinguished to other standard auction formats, as agents here are not clear whether they are the sellers who provides the platooning service, or the buyers who procure the service before the play begins. This is only discovered as a part of the auction outcome. Contrarily, well-studied auctions, including single-unit goods auctions, multi-unit goods auctions, or bilateral trade, clearly divide the agents and their behaviors into the buyer's side and the seller's side. Therefore, many well-stated theorems in auction theory cannot be directly applied in this study, leaving us space to fundamentally inspect the existence and the characterizations of the desired auction mechanism. We conduct our analysis mainly in the *ex post* environment, where the outcome depends only on the revealed bids, instead of any assumptions regarding any agents' beliefs on their companions' true WTP. If the desired auctions exist, they admit efficient outcomes for any realizations on agents' WTP, attaining the first-best solution. Unfortunately, our analysis in Section 3 reveals that there does not exist an ex post mechanism satisfying economic efficiency, incentive compatibility, individual rationality and budget balance at the same time. As a remedy, we propose an alternate mechanism types in Section 4, which employs linear monetary transfer functions regarding agents' WTP and results in an approximate equilibrium that assures behavioral stability. We then compare it with a posted-price mechanism with a threshold policy, which can be further optimized to achieve the second-best solution, to elaborate its simplicity and effectiveness in real-life implementations.

In the remainder of this paper, Section 2 provides the basic assumptions and the utility functions used in modeling. The non-existence of the desired auctions and its proof are elaborated in Section 3, with an emphasis on the contradiction among incentive compatibility, budget balance, and individual rationality. Section 4 provides the two alternate mechanisms and analyzes the possible equilibrium outcomes in detail. Based on that, the practical implementation of these mechanisms are briefly discussed. Lastly, Section 5 concludes this paper and points out its limitations and future research directions.

2. The basic model

We consider a quasilinear environment where there is a set of agents. Each possesses private information on their WTP for platooning service and the corresponding quasilinear utility. Mathematically, the quasilinear environment can be characterized by a tuple $(\mathcal{M}, \Theta, \boldsymbol{X}, U)$, where

- $\mathcal{M} = \{1, 2, \ldots, m\}$ is the finite set of agents. Each agent $i \in \mathcal{M}$ represents a driver who is willing to participate in vehicle platooning through the auction.
- $\Theta = \Theta_1 \times \cdots \times \Theta_m$. Θ_i is the one-dimensional space of agent i's type, representing their WTP for the platooning service. Therefore, Θ is the joint type space for all agents. We assume that agents are *ex ante* symmetric, indicating that types come from the same space and no agents know the exact values of others' types before the

bidding process. Correspondingly, each type space, $\Theta_i, i \in \mathcal{M}$, can be formulated as a continuous space bounded by $\bar{\theta}$ above and $\underline{\theta}$ below, where $0 < \underline{\theta} \leq \bar{\theta} < \infty$.

- $\boldsymbol{X}$ is the space of outcomes. Each outcome $\chi = [x_1, x_2, \dots, x_m] \in \boldsymbol{X}$ represents a possible leader allocation. Specifically, x_i equals one if agent i is a follower, and zero otherwise.
- $U = [U_1, U_2, \dots, U_m]$ defines the utility functions:

$$U_i(\chi, \theta, p) = V_i(\chi, \theta) - p_i, \ \forall i \in \mathcal{M} \tag{2.1}$$

where

  - $V_i(\chi, \theta)$ is the valuation agent i obtained under the chosen outcome χ and the joint type θ. Mathematically, $V_i : \boldsymbol{X} \times \Theta \to \mathbb{R}$.
  - $p = p_1 \times \cdots \times p_m \in \mathbb{R}^m$ is the vector of monetary transfers.

In this environment, the platoon leader auction is defined by a tuple $\langle B, \chi, p\rangle$ where

- $B = B_1 \times B_2 \cdots \times B_m$. B_i is the one-dimensional continuous space of agent $i's$ bid, which is referred to as their action space.
- The outcome function χ: $B \to \boldsymbol{X}$ maps each joint action to an outcome. For instance, given an input $\hat{\theta} \in B$, $\chi(\hat{\theta}) = e_i$, if $\hat{\theta}_i < \hat{\theta}_j$, $\forall j \in \mathcal{M}, j \neq i$, where e_i is an $1 \times m$ vector with ith entry equals zero and others equal to one. Accordingly,

$$V_i(\chi, \theta) = V_i(\chi(\hat{\theta}), \theta)$$

In a platoon leader auction, we further defined the valuation function as

$$V_i(\chi(\hat{\theta}), \theta) = \frac{1}{m}\delta(m) + x_i(\hat{\theta})\theta_i \tag{2.2}$$

Here, the first term represents the fuel efficiency each agent obtained. $\delta(m)$ defines the energy-saving benefit of a platoon, which is equally shared by all platoon members. $\delta(m)$ is a increasing function of m and $\delta(1)$ equals zero. The second term shows the effort-reduction benefit, which is only achieved by agents in the following positions.
- The transfer function $p : B \to \mathbb{R}^m$ maps an action to an $1 \times m$ vector of monetary transfers. We expect that the summation of all m monetary transfers to be zero, meaning that no external authority will subsidize platoons' formation or gain profits from it. We call such a property as *ex post budget balance* (EPBB). It is a pivotal assumption for the platoon leader auction, allowing platoon formed more freely.

Note that, as all agents bid their WTP, this auction is also a direct mechanism, and the action space where the joint actions lie is identical to the type space. Therefore, $B \equiv \Theta$ in this study, and we use Θ to replace B for simplicity. Correspondingly, the outcome function χ and transfer function p can also be viewed as a mapping from space Θ to space $\boldsymbol{X}$ and that from space Θ to $\mathbb{R}^m$. Then, the quasilinear utility function can be reformulated as

$$U_i(\chi(\hat{\theta}), \theta, p) = V_i(\chi(\hat{\theta}), \theta) - p_i(\hat{\theta}), \ \forall i \in \mathcal{M} \tag{2.3}$$

Here, we distinguish the joint action $\hat{\theta}$ from the joint type θ, suggesting that agents have possibilities to misreport their types. For practical concerns, U_i is defined per unit distance, allowing a platoon leader auction to be implemented in real-time.

The auction design problem is to identify the transfer function p, such that for every realization of agents' type $\theta \in \Theta$, the outcome of the auction assigns the leading position efficiently. Mathematically, the problem can be formulated as follows:

$$\max_{p(\theta), \chi(\hat{\theta})} \quad \sum_i U_i(\chi(\hat{\theta}), \theta_i, p(\hat{\theta})) \tag{2.4a}$$

$$s.t. \quad \sum_{i \in \mathcal{M}} p_i(\hat{\theta}) = 0, \forall \hat{\theta} \in \Theta \tag{2.4b}$$

$$U_i(\chi(\theta_i, \theta_{-i}), \theta_i, p) \geq U_i(\chi(\hat{\theta}_i, \theta_{-i}), \theta_i, p), \ \forall i \in \mathcal{M}, \ \forall \hat{\theta}_i \in \Theta_i \tag{2.4c}$$

$$U_i(\chi(\theta_i, \hat{\theta}_{-i}), \theta_i, p) \geq 0, \ \forall i \in \mathcal{M}, \ \forall \hat{\theta}_i \in \Theta_i \tag{2.4d}$$

$$x_i(\theta_i) p_i(\hat{\theta}) \geq 0, \forall i \in \mathcal{M}, \ \forall \hat{\theta}_i \in \Theta_i \tag{2.4e}$$

$$\sum_{i \in \mathcal{M}} x_i(\hat{\theta}) = m - 1, \ \forall \hat{\theta} \in \Theta \tag{2.4f}$$

$$x_i(\hat{\theta}) \in \{0, \ 1\}, \forall i \in \mathcal{M}, \ \forall \hat{\theta} \in \Theta \tag{2.4g}$$

Here, the objective is maximizing the total utility of a platoon. Eq. (2.4b) is budget balanced constraint. With this constraint and the definition of valuation function in Eq. (2.2) and that of utility function in Eq. (2.3), Eq. (2.4a) can be simplified as

$$\delta(m) + \sum_i x_i(\hat{\theta})\theta_i \tag{2.5}$$

As the auction outcome satisfies *ex post budget balance*EPBB), the leader of a platoon obtains a valuation of $\frac{1}{m}\delta(m)$ and all the money transfers from all other followers, and a follower i obtains a valuation of $\frac{1}{m}\delta(m) + \theta_i$ but it pays a monetary transfer $p_i(\hat{\theta})$. The individual utility function is then refined as

$$U_i(\chi(\hat{\theta}), \theta_i, p) = \begin{cases} \frac{1}{m}\delta(m) + \sum_{j:j\neq i} p_j(\hat{\theta}), & \text{if } \hat{\theta}_i < \hat{\theta}_j, \ \forall j \neq i \\ \frac{1}{m}\delta(m) + \frac{|T|-1}{|T|}\theta_i + \frac{1}{|T|}\sum_{j:j\neq i} p_j(\hat{\theta}) - \frac{|T|-1}{|T|} p_i(\hat{\theta}), & \text{if } \exists j \neq i, \ s.t. \ \hat{\theta}_i = \hat{\theta}_j < \hat{\theta}_k, \ \forall k \neq i, j \\ \frac{1}{m}\delta(m) + \theta_i - p_i(\hat{\theta}), & \text{if } \exists j \neq i, s.t. \ \hat{\theta}_j < \hat{\theta}_i \end{cases} \tag{2.6}$$

Here, $|T|$ denotes the number of agents with the smallest bid. It suggests the tie-breaking rule: a random agent in the tie is chosen as the leader whenever there is a tie on the lowest bid.

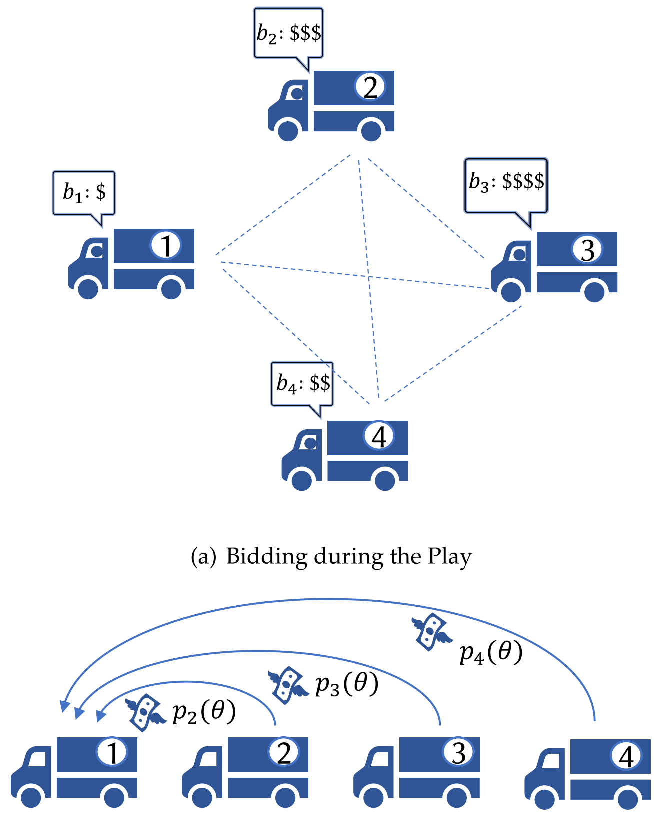
In addition, *ex post incentive compatibility* (EPIC) defined in Eq. (2.4c) assures that all agents achieve the maximum utility if they bid truthfully. Mechanisms with ex post incentive compatibility essentially implement the outcome function in Nash equilibria. *Ex post individual rationality* (EPIR) defined in Eq. (2.4d) means that all agents achieve non-negative utilities by platooning. With these two constraints, no agent would like to switch positions or leave the platoon for a greater utility, which naturally leads to the behavioral stability we introduced previously. Eq. (2.4e) further confines the transfer from each follower to non-negative. This constraint stems from the sense of fairness in auctioning, as it ensures that the leader (service seller) collects non-negative payments and the followers transfer non-negative payments. Moreover, it is a necessary condition of EPIC, which proof is given under Lemma 3.3. We explicitly state the condition here to ease the proofs in Section 3.

In the following context, we refer to the constraints in Eqs. (2.4d) and (2.4e) as individual rationality. Finally, Eqs. (2.4f) and (2.4g) restrict the deterministic outcome where only $m-1$ agents are assigned to be followers. Clearly, if an auction satisfies all the constraints, the outcome function χ admits that economic efficiency (EE) is automatically achieved, since all agents participate in the platoon, and the agent with the real smallest type is assigned as the leader.

For the illustration purpose, Fig. 1 provides an example of four vehicle agents participating in the platoon leader auction. It can be seen that agent 1 raises the smallest bid, making itself the platoon leader after the play. Agents 2, 3, 4 naturally become the follower and pay agent 1. Moreover, they treat the following positions as identical good though they raise different bid.

3. The impossible result

Ideally, we expect to find an optimal solution for problem Eq. (2.4), making the auction mechanism economically efficient, ex post budget



(a) Bidding during the Play

(b) Outcome after the Play

Fig. 1. A toy example of platoon leader auction.

balanced, ex post incentive compatible, and ex post individual rational, simultaneously. However, directing solving the optimization problem is intractable due to the dependence of p, χ and θ. We formally state the impossibility to achieve all these properties simultaneously and offer the proof.

Proposition 3.1. *For a platoon leader auction, there is no mechanism that can be truthfully implemented in an ex post equilibrium to achieve economic efficiency under the constraints of ex post budget balance and ex post individual rationality.*

The way we prove this impossibility is by examining whether a platoon leader auction can achieve one of the four properties when assuming the other three hold (Fig. 2); for instance, whether a platoon leader auction is ex post individual rational if it is ex post incentive compatible, budget balanced, and economic efficient. Notice that when a platoon leader auction is ex post incentive compatible and individual rational, all agents obtain maximum non-negative utilities by truth-telling. It automatically results in an efficient outcome. Therefore, we do not need to consider the case when an auction is ex post incentive compatible, individual rational, and budget balanced. If such an auction exists, it must be efficient. The proof is decomposed into three parts, elaborated in Lemmas 3.1, 3.2, 3.4. In the first two parts, we use Groves Theorem, which conceptually characterizes this class of mechanisms (Groves, 1973) to check how individual rationality and budget balance contradict with each other, under an ex post incentive compatible and economically efficient platoon leader auction. In the third part, we assume the platoon leader auction is ex post individual rational and budget balance, and examine the "degree" of incentive compatibility it can achieve. Without loss of generality, we assume that the ordered sequence of types follows

$$\theta_1 < \theta_2 < \cdots < \theta_m. \tag{3.1}$$

throughout this section.

3.1. Achieving ex post budget balance

Groves theorem characterizes the structure of transfer functions that ensures mechanisms to be economically efficient and ex post incentive compatible. To check whether an ex post incentive compatible and ex post individual rational mechanism is ex post budget balanced, we only need to check whether any ex post individual rational Grove mechanism are ex post budget balanced.

Lemma 3.1. *Ex post individual rational Grove mechanisms do not satisfy ex post budget balance under any platoon leader auctions.*

Proof. We prove it by contradiction. Assume there exists a mechanism in the Groves mechanism family such that it is EPBB and EPIR.

We first suppose the mechanism is the well-known Vickrey–Clark–Groves (VCG) auction/mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973), which is a special case in the Groves mechanism family. It characterizes the transfer function as

$$p_i(\theta) = -[\sum_{j\neq i} V_j(\chi^*(\theta),\theta) - \sum_{i\neq j} V_j(\chi^*(\theta_{-i}),\theta_{-i})] \tag{3.2}$$

Here, $\chi^*(\theta)$ denotes the efficient outcome for a platoon leader auction joined by all agents in $\mathcal{M}$ when all of them are truth-telling. Considering the type order in Eq. (3.1), $\chi^*(\theta) = [0, 1, \ldots, 1]$, implying that agent 1 being the leader is the efficient outcome. $\chi^*(\theta_{-i})$ denotes the efficient outcome for a platoon leader auction joined by all agents in $\mathcal{M}$ *except agent i* when all of them are truth-telling. If i equals to 1, $\chi^*(\theta_{-i})$ admits agent 2 being the leader; otherwise, $\chi^*(\theta_{-i}) = \chi^*(\theta)$. We now derive the exact valuation function and transfer functions as follows:

$$\begin{aligned}
\sum_{j\neq 1} V_j(\chi^*(\theta),\theta) &= \sum_{j\neq 1}\theta_j + \frac{m-1}{m}\delta(m)\\
\sum_{j\neq 1} V_j(\chi^*(\theta_{-i}),\theta_{-i}) &= \begin{cases}\sum_{j\neq 1,2}\theta_j + \delta(m-1), & \text{if } i=1,\\ \sum_{j\neq 1}\theta_j + \delta(m-1), & \text{otherwise.}\end{cases}\\
p_1(\theta) &= -\theta_2 + \frac{m-1}{m}\delta(m) - \delta(m-1)\\
p_j(\theta) &= \frac{m-1}{m}\delta(m) - \delta(m-1), \forall 2\leq j\leq m
\end{aligned}$$

The sum of all transfers is $-\theta_2 + (m-1)\delta(m) - m\delta(m-1)$, which is likely to be nonzero for the general case, so that it violates the EPBB constraint. Even worse, when the energy-saving benefit is relatively small compared to the driving-effort reduction, the total budget is negative, meaning that an external authority must subsidize the platoon. Therefore, this mechanism cannot be a VCG mechanism.

We then move to the general form of Grove mechanisms:

$$p_i(\theta) = -\sum_{j\neq i} V_j(\chi^*(\theta),\theta) + h_i(\theta_{-i})$$

The difference between VCG mechanism and a general Grove mechanism is that the latter adopts a generic function $h_i : \theta_{-i} \to \mathbb{R}, \forall i \in \mathcal{M}$, which is a function of i that the input does not depend on $i's$ type. Clearly, in VCG mechanism, $h_i(\theta_{-i}) = \sum_{j\neq i} V_j(\chi^*(\theta_{-i}),\theta_{-i})]$. The transfer functions now can be generalized as

$$p_1(\theta) = -[\sum_{k\in\mathcal{M},k\neq 1}\theta_k + \frac{m-1}{m}\delta(m)] + h_1(\theta_{-1}) \tag{3.3a}$$

$$p_j(\theta) = -[\sum_{k\in\mathcal{M},k\neq j,1}\theta_k + \frac{m-1}{m}\delta(m)] + h_j(\theta_{-j}),\ \forall j\in\mathcal{M}, j\neq 1 \tag{3.3b}$$

By assumption, the leader always collects a positive amount of payment, and the followers always pay positive amounts, regardless

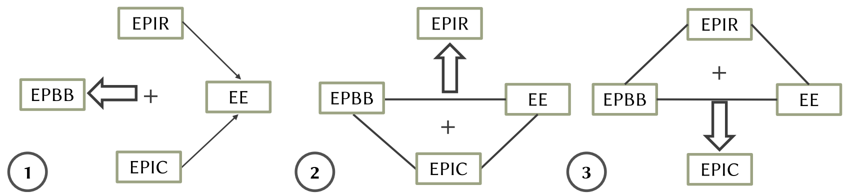


Fig. 2. Logic flow of the proof sketch.

of whether the position allocation is efficient. These conditions can be mathematically expressed as

$$\sum_{k\in m,k\neq 1}\theta_k+\frac{m-1}{m}\delta(m)-h_1(\theta_{-1})\geq 0 \tag{3.4a}$$

$$\sum_{k\in M,k\neq j}\theta_k+\frac{m-1}{m}\delta(m)-\theta_1-h_j(\theta_{-j})\leq 0,\ \forall j\in\mathcal{M},j\neq 1 \tag{3.4b}$$

$$\sum_{k\in M,k\neq 1}\theta_k-\theta_2+\frac{m-1}{m}\delta(m)-h_1(\theta_{-1})\leq 0 \tag{3.4c}$$

$$\sum_{k\in\mathcal{M},k\neq j}\theta_k+\frac{m-1}{m}\delta(m)-h_j(\theta_{-j})\geq 0,\ \forall j\in\mathcal{M},j\neq 1 \tag{3.4d}$$

Specifically, Eqs. (3.4a) and (3.4b) describe the payment when all agents bid truthfully; Eq. (3.4c) states that if agent 1 misreports and becomes a follower, it will pay; while Eq. (3.4d) states that if any other agent misreports and becomes the leader, it will collect.

Rearranging the above constraints leads to the following results:

$$\sum_{k\in\mathcal{M},k\neq 1}\theta_k-\theta_2+\frac{m-1}{m}\delta(m)\leq h_1(\theta_{-1})\leq\sum_{k\in\mathcal{M},k\neq 1}\theta_k+\frac{m-1}{m}\delta(m) \tag{3.5a}$$

$$\sum_{k\in\mathcal{M},k\neq i}\theta_k-\theta_1+\frac{m-1}{m}\delta(m)\geq h_j(\theta_{-j})\geq\sum_{k\in\mathcal{M},k\neq i}\theta_k+\frac{m-1}{m}\delta(m),\quad \forall j\in\mathcal{M},j\neq 1 \tag{3.5b}$$

Combining equation sets (3.3) and (3.5) results in the transfers' upper and lower bounds:

$$-\theta_2\leq p_1(\theta_{-1})\leq 0 \tag{3.6a}$$

$$0\leq p_j(\theta_{-j})\leq\theta_1,\ \forall j\in\mathcal{M},j\neq 1 \tag{3.6b}$$

Combining Eq. (3.6b) and EPBB leads to

$$p_1(\theta_{-1})=-\sum_{j\in\mathcal{M},j\neq 1}p_j(\theta_{-j})\in[-(m-1)\theta_1,0],$$

indicating that agent 1's payment depends on their type, which contradicts the assumption that p_1 is a only function of θ_{-1}. The mechanism we assumed does not exist when there exists one smallest type. ☐

3.2. Achieving ex post individual rationality

Lemma 3.2. *Ex post budget-balanced Grove mechanisms in platoon leader auctions are not ex post individual rational.*

Proof. To prove this, we refer to proposition 7.10 in Börgers and Krahmer (2015). The proposition says that the necessary and sufficient condition for an ex post budget-balanced Grove mechanism to implement an efficient outcome is that there is a function $f_i:\Theta_{-i}\to\mathbb{R}$ for every $i\in\mathcal{M}$ such that

$$\sum_{i\in\mathcal{M}}U_i(\chi(\theta),\theta_i,p(\theta))=\sum_{i\in\mathcal{M}}f_i(\theta_{-i}) \tag{3.7}$$

Moreover,

$$h_i(\theta_{-i})=(m-1)f_i(\theta_{-i}),\forall i\in\mathcal{M} \tag{3.8}$$

with h_i defined in Eq. (3.3). We know that under the efficient outcome,

$$\sum_{i\in\mathcal{M}}U_i(\chi(\theta),\theta_i,p(\theta))=\delta(m)+\sum_{\forall i\in\mathcal{M},i\neq 1}\theta_i \tag{3.9}$$

Besides, EPIR can be characterized by the following system of equations:

$$-p_1(\theta)=\sum_{\forall j\in\mathcal{M},j\neq 1}p_j(\theta)\geq 0,$$

$$p_j(\theta)=-\sum_{k\in\mathcal{M},k\neq i,1}\theta_k-\frac{m-1}{m}\delta(m)+(m-1)f_j(\theta_{-j})\geq 0,\ \forall j\in\mathcal{M},j\neq 1,$$

$$\frac{1}{m}\delta(m)+\theta_j-p_j(\theta_{-j})\geq 0,\ \forall j\in\mathcal{M},j\neq 1.$$

Together with Eq. (3.7), Eq. (3.8) and (3.9), it leads to the bounds of f_i:

$$\frac{1}{m-1}\sum_{k\in\mathcal{M},k\neq i,1}\theta_k+\frac{1}{m}\delta(m)\leq f_i(\theta_{-i})\leq\frac{1}{m-1}\sum_{k\in\mathcal{M},k\neq 1}\theta_k+\frac{1}{m-1}\delta(m),\ \forall\, i\in\mathcal{M},i\neq 1,$$

The only type of functions that satisfy these conditions are

$$f_i(\theta_{-i})=\frac{1}{m-1}\sum_{k\in\mathcal{M},k\neq 1,k\neq i}\theta_k+\frac{1}{x}\delta(m)$$

with $m-1\leq x\leq m$ be a constant.

The payment for any agents other than agent 1 can be derived as

$$p_i=(\frac{1}{x}-\frac{1}{m})(m-1)\delta(m)\leq\frac{1}{m}\delta(m),\ \forall i\in\mathcal{M},\ i\neq 1 \tag{3.10}$$

It suggests that the transfer functions are determined only by $\delta(m)$, the energy-saving benefit, instead of θ, the agents' types.

Denote $(\frac{1}{x}-\frac{1}{m})(m-1)$ as $\frac{1}{s}$ for simplicity with $m\leq s$. Since all agents are incentive compatible, they obtain the maximum utilities by truth-telling:

$$u_1(\theta_1,\theta_{-1})=(\frac{1}{m}+\frac{m-1}{s})\delta(m)\geq\theta_1-\frac{1}{s}\delta(m)=u_1(\hat\theta_1,\theta_{-1})|_{\hat\theta_1>\theta_2}$$

$$u_i(\theta_i,\theta_{-i})=\theta_i-\frac{1}{s}\delta(m)\geq(\frac{1}{m}+\frac{m-1}{s})\delta(m)=u_i(\hat\theta_i,\theta_{-i})|_{\hat\theta_i<\theta_1},\quad\forall i\in\mathcal{M},i\neq 1$$

$$\Rightarrow$$

$$\theta_1\leq(\frac{m}{s}+\frac{1}{m})\delta(m)\leq\theta_i,\ \forall i\in\mathcal{M},i\neq 1 \tag{3.11}$$

This condition implies that x is a function of θ, which contradicts the previous conclusion that the transfer function is indifferent to agents' types. Therefore, we cannot achieve individual rationality in budget-balanced Grove mechanisms. ☐

3.3. Achieving ex post incentive compatibility

We first provide a definition to characterize the different strategies for agents if they are buyers and sellers under any given outcome. The term *incentive compatibility* is used loosely in this section for concise descriptions.

Definition 3.1 (*Two-Sided Ex Post Incentive Compatible*)**.** Ex post incentive compatibility for all platoon members is denoted as **two-sided ex post incentive compatible** (two-sided EPIC) emphasizing that the mechanism implements the efficient outcome in the ex post equilibrium for both the seller (leader) and the buyer (follower) sides. Then for any given outcome, if only the agents on the buyer side can maximize their utilities by truth-telling, we refer to it as **buyer-sided ex post incentive compatible**. Similarly, if only the agents on the seller side can maximize their utilities by truth-telling, we refer to it as **seller-sided ex post incentive compatible**.

Lemma 3.3. *Under any given outcome, if a platoon leader auction is ex post budget balanced and ex post individual rational, the buyers are truth-telling only if they all pay a uniform price, p, independent of any of their types.*

Proof. This can be proved by contrapositive. Denote the set of followers as $\mathcal{F}$ under any arbitrary outcome. We suppose that a transfer function is discriminatory, meaning that any p_i with $i \in \mathcal{F}$ can differentiate from each other. Therefore, several cases can be considered:

1. If p_i is a function of θ_i: Once the function is known, all followers would like to misreport to pay $p^* = \min\{p_i,\ i \in \mathcal{F}\}$ under the *ex post* environment, hence not EPIC. As an example, suppose $\partial p_i/\partial\theta_i > 0$. Consequently, all followers can bid $\theta_1 + \epsilon$ with $\epsilon > 0$ being a very small positive value to pay less than that if they are truth-telling. Alternatively, if $\partial p_i/\partial\theta_i < 0$, all agents can bid $\bar{\theta}$ to pay less than that they supposed to.
2. If p_i is not a function of θ_i but a function of θ_{-i}, denoted as $p_i(\theta_{-i})$. Under the EPBB constraint,
$$p_i(\theta_{-i}) = -\sum_{j\neq i, j\in\mathcal{M}} p_j(\theta_{-j}),\ \forall i \in \mathcal{M} \tag{3.12}$$
If there exists an agent j with function $p_j(\theta_{-j})$ where (θ_{-j}) includes θ_i, then p_i is a function of θ_i as well. It leads the analysis back to the first case, which does not ensure EPIC. In this sense, for all followers i, $p_i(\theta_{-i})$ should only be a function of $\theta_k, \forall k \in \mathcal{M}/\mathcal{F}$. In other words, the followers' payment is a uniform price and is independent of any followers' types.
3. Suppose p_i is not a function of $\theta \in \Theta$. Put differently, p_i is indifferent to any agents' types. It suggests that p_i must be a uniform price. □

Lemma 3.4. *A platoon leader auction cannot be incentive compatible if it is ex post budget balanced and ex post individual rational. However, there exists transfer functions that achieve either the buy-side ex post incentive compatibility or the seller-side ex post incentive compatibility for this auction.*

Proof. The proof starts with the conditions that characterize the buyer-side EPIC. We still use $\mathcal{F}$ to indicate the set of following vehicles under an arbitrary outcome. With Lemma 3.3, no agent in $\mathcal{F}$ has an incentive to misreport if misreporting cannot change the leader allocation. Therefore, the only case that must be considered is when leader allocation is changed by misreporting. Suppose that each follower pays p, the leader will collect $(m-1)p$ under EPBB. EPIC requires that

$$\theta_i - p \geq (m-1)p \Rightarrow p \leq \frac{1}{m}\theta_i, \forall i \in \mathcal{F}$$

Therefore,

$$p \leq \frac{1}{m}\min\{\theta_i,\ i \in \mathcal{F}\} \tag{3.13}$$

Similarly, the seller-side EPIC can be characterized. If the leader has no incentive to misreport for being a follower, the following condition must be satisfied:

$$(m-1)p \geq \theta_i - p \Rightarrow p \geq \frac{\theta_i}{m},\ \text{if agent } i \text{ is the leader.}$$

As EPIR suggests that all the m agents participate in platooning, the *set* $\mathcal{F}$ contains $m-1$ agents, and the set $\mathcal{M}/\mathcal{F}$ contains only one agent, the leader. Under the efficient outcome when agent 1 serves as the leader, we have p bounded by

$$\frac{\theta_1}{m} \leq p \leq \frac{\theta_2}{m} \tag{3.14}$$

Lemma 3.3 demonstrates that p cannot be a function of θ_2 under buyer-side EPIC, so that p is a function of θ_1, satisfying Eq. (3.14). There are multiple ways to create such a function, for instance,

$$p = \frac{\theta_1}{m}.$$

As θ_1 is unknown before the auction, p is further set to be $\frac{\hat{\theta}_1}{m}$ where $\hat{\theta}_1$ is agent 1's reported type. In this case, one can easily see that agent 1 has the incentive to report $\hat{\theta}_1$ greater than θ_1 to increase its utility. It suggests that under the buyer-side EPIC, the seller-side EPIC cannot be achieved.

Similarly, if p is set to be a function of θ_2, agent 1 has no incentive to misreport, meaning that the seller-side EPIC is achieved. But as Lemma 3.3 suggests, agent 2 has the incentive to misreport, therefore the buyer-side EPIC fails this time. □

Remark. One may notice that our assumption in the beginning of this section excludes the case that more than one agent has the smallest type. We avoid this discussion for two reasons. First, in mechanism design, we usually do not consider the case when $\exists i, j \in \mathcal{M}$ such that $\theta_i = \theta_j$. Second, we prove in Section 4 that the mechanism under the linear transfer function proposed achieves all desired properties if the tie-breaking rule is applied when more than one agent has the smallest type.

4. The transfer function and its implementation

In general, the results in Section 3 suggest that no incentive compatible equilibrium achieves first-best efficient outcomes for all joint type realizations. Therefore, we discuss some mechanisms that either can be implemented in an approximate equilibrium, or result in second-best solutions.

4.1. Linear transfer functions

Corollary 4.1 (*First-Price Platoon Leader Auction*)**.** *Suppose that agents in $\mathcal{M}$ bid $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m$. Denote the payment by any follower as p_F and that by the leader as p_L, a linear transfer function:*

$$p_F = \frac{1}{m}\theta^*,\ p_L = -\frac{m-1}{m}\theta^*, \tag{4.1}$$

with $\theta^* = \min\{\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m\}$ ensures behavioral stability for all agents in a platoon leader auction. As the function only depends on the smallest bid, we denote it as the first-price platoon leader auction.

Proof. In this auction mechanism, an arbitrary agent i with type θ_i and bid $\hat{\theta}_i$ has a utilities

$$u_i = \begin{cases} \frac{1}{m}\delta(m) + \theta_i - \frac{1}{m}\theta^*, & \text{if } \hat{\theta}_i > \theta^* \\ \frac{1}{m}\delta(m) + \frac{1}{T}\frac{m-1}{m}\theta^* + \frac{T-1}{T}(\theta_i - \frac{\theta^*}{m}), & \text{if } \hat{\theta}_i = \theta^* \end{cases} \tag{4.2}$$

Here, T indicates the number of agents with the smallest bid θ^*. Clearly, all agents have non-negative utility in this auction mechanism, so that they do not have the incentive to leave the platoon.

We then prove that no agents would like to switch positions.

Assume agents 1 and 2 hold the smallest and the second smallest types: $\theta_1 < \theta_2 \leq \theta_j, \forall j \geq 3$. Suppose all agents except agent 1 are followers under the outcome of first-price platoon leader auction, Lemma 3.3 suggests that they have no willingness to switch positions with each other. We then show that they are also not willing to be the leader.

If all agents are incentive compatible, agent j has a utility of

$$u_j(\theta) = \frac{1}{m}\delta(m) + \theta_j - \frac{1}{m}\theta_1$$

by truth-telling. Here, the small capital letter $u = [u_1, u_2, \ldots, u_m]$ represents a function set mapping action space Θ to $\mathbb{R}^m_+$ giving an outcome of χ. If agent j misreports $\hat{\theta}_j < \theta_1 < \theta_j$ and becomes the leader, their "forged" utility would be

$$u_j(\hat{\theta}_j, \theta_{-j}) = \frac{1}{m}\delta(m) + \frac{m-1}{m}\hat{\theta}_j$$

Comparing the two utilities, it is easy to see that

$$\begin{aligned} u_j(\hat{\theta}_j, \theta_{-j}) &= \frac{1}{m}\delta(m) + \frac{m-1}{m}\hat{\theta}_j < \frac{1}{m}\delta(m) + \frac{m-1}{m}\theta_1 \\ &< \frac{1}{m}\delta(m) + \theta_j - \frac{1}{m}\theta_1 = u_j(\theta), \end{aligned}$$

which proves followers are EPIC. Here, $\theta = (\theta_j, \theta_{-j})$, $\forall j \in \mathcal{M}$.

The above analysis holds when $\hat{\theta}_1 < \theta_j, \forall j \geq 2$. Therefore, we prove that agent 1, who has the smallest type, has no incentive to be the follower. Suppose agent 1 reports $\hat{\theta}_1$ and makes agent 2 the leader instead, agent 1's 'forged' utility would be

$$u_1(\hat{\theta}_1, \theta_{-1}) = \frac{1}{m}\delta(m) + \theta_1 - \frac{1}{m}\theta_2$$

Since $\theta_2 > \theta_1$, the forged utility is no greater than the truth-telling utility:

$$u_1(\hat{\theta}_1, \theta_{-1}) < \frac{1}{m}\delta(m) + \theta_1 - \frac{1}{m}\theta_1 = u_1(\theta)$$

It suggests that although agent 1 has the willingness to misreport, they will never raise their bid above θ_2 and become a follower.

Lastly, we consider the trivial case, where a number of t agents have the smallest type, say $\underline{\theta}'$. The set of such agents is denoted as T. According to the tie-breaking rule, all the agents in T have the same expected utility if they are truth-telling:

$$\begin{aligned} u_i(\underline{\theta}', \theta_{-i}) &= \frac{1}{m}\delta(m) + \frac{1}{t}\frac{m-1}{m}\underline{\theta}' + \frac{t-1}{t}(\underline{\theta}' - \frac{\underline{\theta}'}{m}) \\ &= \frac{1}{m}\delta(m) + \frac{m-1}{m}\underline{\theta}' \end{aligned}$$

If one of them, say agent i, misreports θ', the utility function would be either

$$u_i(\theta', \theta_{-i}) = \frac{1}{m}\delta(m) + \frac{m-1}{m}\theta' < u_i(\underline{\theta}', \theta_{-i})$$

if $\theta' < \underline{\theta}'$ makes agent i the leader, or

$$u_i(\theta', \theta_{-i}) = \frac{1}{m}\delta(m) + \underline{\theta}' - \frac{1}{m}\underline{\theta}' = u_i(\underline{\theta}', \theta_{-i})$$

if $\theta' > \underline{\theta}'$ makes agent i a follower. In sum, no agent in the set T has an incentive to misreport to switch their position or leave the platoon, thereby ensuring behavioral stability. This proof shows that individual rationality and incentive compatibility is only a sufficient condition for behavioral stability, but not the necessary condition. □

Remark. One familiar with multi-unit auctions can easily observe that the first-price platoon leader auction is a variation of multi-unit goods Vickrey auctions (Krishna, 2009). To see this, consider forming a platoon with m vehicles as selling the platooning service for $m-1$ units. Those who win the auction pay an amount that is determined by the one who loses. However, an auctioneer (mechanism designer) in the multi-unit goods Vickrey Auction collects the payment. No such person exists in the first-price platoon leader auction making ex post budget balance a necessary condition.

4.2. The approximation equilibrium outcome

As stated before, the mechanism is not ex post incentive compatible since agent 1, who has the smallest bid, can misreport. Now let us check the utility functions to see whether agent 1 has a best response when all other agents bid truthfully. Notice that when $\hat{\theta}_1 < \theta_2$, $u_1(\hat{\theta}_1)$ is a monotonically increasing function of $\hat{\theta}_1$, so that agent 1 would like to increase their bid for a larger utility. However, if $\hat{\theta}_1 = \theta_2$, the tie-breaking rule is applied, meaning that agent 1 has a half probability of being the leader and half of being the follower:

$$\begin{aligned} u_1(\hat{\theta}_1, \theta_{-1})|_{\hat{\theta}_1=\theta_2} &= \frac{1}{m}\delta(m) + \frac{1}{2}(\theta_1 - \frac{\theta_2}{m}) + \frac{1}{2}(m-1)\frac{\theta_2}{m} \\ &= \frac{1}{m}\delta(m) + \frac{1}{2}\theta_1 + \frac{m-2}{2m}\theta_2 \end{aligned}$$

Clearly,

$$u_1(\hat{\theta}_1, \theta_{-1})|_{\hat{\theta}_1=\theta_2} < u_1(\hat{\theta}_1, \theta_{-1})|_{\hat{\theta}_1\to\theta_2^-} = \frac{1}{m}\delta(m) + \lim_{\hat{\theta}_1\to\theta_2^-}\frac{\hat{\theta}_1}{m} = \frac{1}{m}\delta(m) + \frac{\theta_2}{m}$$

On the other hand, if agent 1 reports $\hat{\theta}_1 > \theta_2$,

$$u_1(\hat{\theta}_1, \theta_{-1})|_{\hat{\theta}_1>\theta_2} = \frac{1}{m}\delta(m) + \theta_1 - \frac{\theta_2}{m}$$

$$u_1(\hat{\theta}_1, \theta_{-1})|_{\hat{\theta}_1=\theta_2} - u_1(\hat{\theta}_1, \theta_{-1})|_{\hat{\theta}_1=\theta_2} = \frac{1}{2}(\theta_1 - \theta_2) < 0$$

As illustrated by Fig. 3(a), the utility function of agent 1 is discontinuous. Hence there is no best response for agent 1 nor pure Nash equilibrium for the first-price platoon leader auction. Nevertheless, we can indicate that the first-price platoon leader auction is an approximate truthful mechanism. To do so, we borrow the concept of $\epsilon-$ approximate equilibrium (Nisan et al., 2007) and provide two definitions below.

Definition 4.1 (*ϵ-Approximate Equilibrium*). Let Δ_i denote the space of all possible probability distribution over θ_i, $\forall i \in \mathcal{M}$. Then a vector of mixed strategy $\sigma^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_m^*) \in \Delta = \Delta_1 \times \Delta_2 \times \cdots \times \Delta_m$ is an $\epsilon-$ *approximate equilibrium* if

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i', \sigma_{-i}^*) - \epsilon, \ \forall \sigma_i' \in \Delta_i, \ \forall i \in \mathcal{M} \tag{4.3}$$

with ϵ be a non-negative real number.

Definition 4.2 (*Truthful ϵ-Approximation Mechanism*). A mechanism $\langle B, \chi, p\rangle$ is said to be a truthful $\epsilon-$approximation mechanism if it implements the outcome rule $\hat{\chi} : \Theta \to \boldsymbol{X}$ in $\epsilon-$ approximate equilibrium, and if the $\epsilon-$ approximate equilibrium b^* of $\langle B, \chi, p\rangle$ satisfies $\chi(b^*(\theta)) = \hat{\chi}(\theta)$.

Proposition 4.1. *The first-price platoon leader auction is a truthful $\epsilon-$approximation mechanism that achieves the economically efficient outcome under ex post budget balance and ex post individual rationality constraints.*

Proof. Assume agents 1 and 2 hold the smallest and the second smallest types: $\theta_1 < \theta_2 \leq \theta_j, \forall j \geq 3$. We now prove that joint strategy $\sigma^* = \theta$ is an $\epsilon-$ approximate equilibrium. Note that Δ is identical to Φ since the pure action space Φ is compact. Therefore, we still use Φ for consistency. The proof of Corollary 4.1 indicates that the utilities of all agents other than agent 1 satisfy Eq. (4.3) with $\epsilon \geq 0$. We now prove that agent 1's utility satisfies Eq. (4.3) with a properly chosen ϵ. Fig. 3(a) suggests that $\sup\{u_1(\theta', \theta_{-1}) | \theta' \in \Theta_1\}$ equals $\frac{1}{m}\delta(m) + \frac{m-1}{m}\theta_2$. Then $u_1(\theta_1, \theta_{-1}) \geq u_1(\theta', \theta_{-1}) - \epsilon$ for all $\theta' \in \Theta$ when $\epsilon = \frac{m-1}{m}(\theta_2 - \theta_1)$. It concludes the proof. □

Additionally, one could check that truth-telling is also an ϵ-approximate dominant strategy, which is summarized as below:

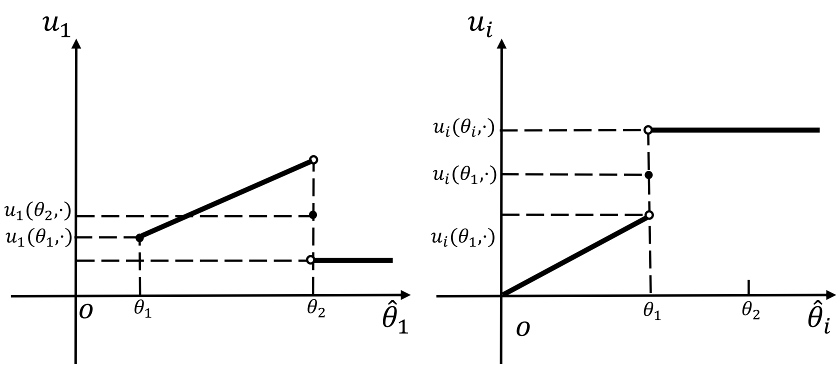
Corollary 4.2. *The first-price platoon leader auction is ϵ-approximate dominant strategy incentive compatible, meaning that*

$$u_i(\theta_i, \theta_{-i}') \geq u_i(\theta_i', \theta_{-i}') - \epsilon, \ \forall \theta_i' \in \Theta_i, \ \forall \theta_{-i} \in \Theta_{-i}, \forall i \in \mathcal{M},$$

when agents have types θ_1, θ_2, ..., θ_m.

We omit the proof because of its simplicity.

When there is no tie on the smallest type, ϵ depends on the smallest two types' realizations, which can be either large or small. We then check how large ϵ is on average. We further assume that the realization of each agent's type follows a common cumulative distribution function $\Phi(\theta)$, which has a probability density function $\phi(\theta)$, $\underline{\theta} \leq \overline{\theta}$. Recall that θ_1 and θ_2 represent the smallest and the second-smallest random numbers



(a) Agent 1's utility function (b) Agent i's utility functions, $2 \leq i \leq m$

Fig. 3. Misreporting strategies and utility functions when $p = \frac{\theta_1}{m}$.

in m numbers, we can find the independent distribution functions for θ_1 and θ_2.

$$Pr(\theta_1 \leq x) = 1 - (1 - \Phi(\theta))^m$$

$$Pr(\theta_2 \leq y) = \sum_{r=2}^{m} \binom{m}{r} \Phi(y)^r (1 - \Phi(y))^{m-r}$$

Then the joint distribution of θ_1 and θ_2 can be derived as

$$\begin{aligned} G(x, y) &= Pr(\theta_1 \leq x, \theta_2 \leq y) \\ &= Pr(\theta_2 \leq y) - Pr(\theta_1 \geq x, \theta_2 \leq y) \\ &= \sum_{r=2}^{m} \binom{m}{r} \left[\Phi(y)^r - (\Phi(y) - \Phi(x))^r\right] [1 - \Phi(x)]^{m-r}, \text{ if } x \leq y. \end{aligned}$$

The difference of θ_1 an θ_2, $\theta_2 - \theta_1$, has a distribution characterized by the probability density function $f(\gamma)$:

$$f(\gamma) = \int_{\underline{\theta}}^{\overline{\theta}} g(x, x + \gamma) dx$$

where

$$g(x, y) = \frac{d^2 G(x, y)}{dx dy}.$$

Notice that

$$\frac{m}{m-1}\epsilon = \theta_2 - \theta_1 \in [0, \overline{\theta} - \underline{\theta}],$$

its expectation can be derived as

$$\mathbb{E}\epsilon = \frac{m-1}{m} \int_0^{\overline{\theta}-\underline{\theta}} \gamma f(\gamma) d\gamma \tag{4.4}$$

4.3. Variations on linear transfer functions

Clearly, the transfer function of first-price platoon leader auction, Eq. (4.1), is not the only linear function that satisfies the conditions stated in Eq. (3.14). Many other linear transfer functions can be applied to generate alternative auction mechanisms. As first-price platoon leader auction depends on the smallest type, we can derive a *second-price* platoon leader that has the transfer function as follows:

$$p_F = \frac{\hat{\theta}_2}{m}, p_L = -\frac{m-1}{m}\hat{\theta}_2$$

In this case, agent 1 has no incentive to misreport (Fig. 4(a)). Agent 2 would like to decrease its reported value until that equals θ_1. As presented in Fig. 4(b), its utility function is also discontinuous. Moreover, all agents with a type no less than θ_2 has the same utility function. No equilibrium exists under this transfer function as well; similarly, an $\epsilon-$ approximate equilibrium could be found following the analysis in Proposition 4.1's proof.

Another type of second-price platoon leader auction has the transfer function:

$$p_F = \frac{\hat{\theta}_2}{m-1}, \; p_L = -(m-2)p_F.$$

To achieve ex post incentive compatible and ex post budget balance, this mechanism enforces agent 2 to leave the platoon after all agents revealed their bids. However, an amount of $\delta(m) - \delta(m-1) + \theta_2$ of surplus will be lost by doing so. This mechanism is inspired by McAfee (1992)'s trade reduction rule for double auction, under which the surplus loss is approaching zero when both supply and demand approach infinity. For this reason, it is not suitable for the platooning problem, as the number of agents is relatively small, making the surplus loss non-negligible.

The last platoon leader auction use both the smallest and the second-smallest type when designing the transfer function:

$$p_F = \frac{\hat{\theta}_1}{m}, p_L = \frac{m-1}{m}\hat{\theta}_2$$

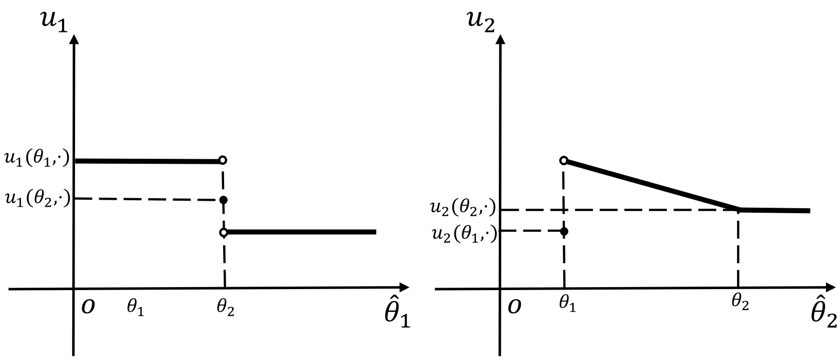
Clearly, the auction is ex post incentive compatible now, but not ex post budget imbalance. Indeed, there is a budget deficiency of $\frac{m-1}{m}(\hat{\theta}_2 - \hat{\theta}_1)$. Interestingly, it equals the "ϵ" defined for the first-price platoon leader auction, meaning that the latter actually internalizes the deficiency from outside of the system to the agent with the smallest type. In addition, the last two auctions suggest that when loosing some of the constraints, ex post incentive compatible mechanisms exist.

4.4. The posted-price mechanism

Posted price is often used in auctions, providing the minimum price that a seller is willing to accept for selling an item. In the single-unit goods auction, posted price ensures ex post incentive compatible: a buyer only buys the item if their valuation of the item is greater than the posted price so that they always obtain non-negative utility; a seller only sells the item if the posted price is higher than their valuation, making their utility non-negative as well. However, the posted-price mechanism does not lead to promising results for platoon leader allocation.

Giving a posted price for being the followers in a platoon, each agent correspondingly signals either *in* or *out* to others, leading to outcomes varying by the leader allocation and the number of agents in the platoon. We specifically consider the model when there are only two agents, numbered 1 and 2, to evaluate each outcome's efficiency. Assume the two agents have types θ_1 and θ_2, respectively. The posted price is p with $\underline{\theta} \leq p \leq \overline{\theta}$. In addition, platooning brings them energy-saving benefits δ. Therefore, the action taken is determined by a threshold policy. It means that for the arbitrary agent $i, i \in \{1, 2\}$, if $\theta_i < p - \delta$, agent i signals *out*; otherwise, agent i signals *in*. We now characterize each type of outcome in detail:

1. When $\theta_1, \theta_2 < p - \delta$, both agents signal *out* and obtain zero utility in correspondence with ex post individual rationality.



(a) Agent 1's utility function (b) Agent i's utility function, $2 \leq i \leq m$

Fig. 4. Misreporting strategies and utility functions when $p = \frac{\theta_2}{m}$.

2. When $p - \delta < \theta_1 < 2p$, agent 1 signals *in*. There are three cases, depending on the value of θ_2:
   i. Suppose $\theta_2 < p - \delta$. Agent 1 becomes the follower and pays p to agent 2 accordingly. Both agents obtain positive utilities. Agent 1 would not deviate to signal *out* since doing so will lead to zero utility. If Agent 2 deviates to signal *in*, they will be the leader with a probability of one-half. The expected utility is $u_2' = \frac{1}{2}(\theta_2 + \delta - p) + \frac{1}{2}(\delta + p)$, which is less than their current utility, $\delta + p$. Therefore, agent 2 will not deviate to signal *in*, and (*in*, *out*) is an ex post equilibrium.
   ii. Suppose $p - \delta \leq \theta_2 < 2p$. Both agents signal *in*, and each of them is selected as the leader with equal probability, making $u_1 = \frac{\theta_1}{2} + \delta$ and $u_2 = \frac{\theta_2}{2} + \delta$. Because $p > \theta_1, \theta_2$, both agents have incentives to deviate and signal *out* to increase their utility to $p + \delta$. Once one of them deviates to signal *out*, the other would not signal *out* because doing so will lead to zero utility instead of a positive one. Hence (*in*, *in*) is not an ex post equilibrium. Agents will deviate to one of the two different equilibria: (*in*, *out*) and (*out*, *in*). Each of them can achieve half economic efficiency on average.
   iii. Suppose $\theta_2 \geq 2p$, which requires $p \leq \frac{\bar{\theta}}{2}$. Similar to case ii, both agents signal *in*, and each of them becomes the leader with half probability. Agent 1 then has the incentive to deviate to *out*, because by doing so, they can obtain $p + \delta$, which is greater than $\frac{\theta_1}{2} + \delta$. If agent 1 deviates, agent 2 will still signal *in* because by doing so, they obtain $\theta_2 - p + \delta \geq \frac{\theta_2}{2} + \delta$. Therefore, (*in*, *in*) is not an ex post equilibrium; agents will deviate to (*out*, *in*), which is an ex post efficient equilibrium.
3. When $\theta_1 \geq 2p$, agent 1 signals *in*. There are also three cases depending on the value of θ_2:
   i. $\theta_2 < p - \delta$: Agent 2 signals *out*. Accordingly, agent 1 becomes the follower and pays p to agent 2, and both agents obtain positive utilities. Since deviating to other strategies will decrease both agents' utilities, (*in*, *out*) is an ex post efficient equilibrium.
   ii. $p - \delta \leq \theta_2 < 2p$: This case is symmetric to that when $\theta_2 \geq 2p$ and $p - \delta \leq \theta_1 < 2p$. From the previous analysis, it is known that signaling (*in*, *in*) is not an ex post equilibrium. Agent 2 will deviate to signal *out*, resulting an efficient equilibrium (*in*, *out*).
   iii. $\theta_2 \geq 2p$: Both agents signal *in* and each of them becomes the leader with half probability. No agents will deviate to signal *in*, as doing so will deteriorate their utility. As a consequence, (*in*, *in*) is an ex post equilibrium but only achieves half efficiency on average.

Accordingly, we can also characterize the case ($\theta_1 < p - \delta$, $p - \delta < \theta_2 < 2p$) and ($\theta_1 < p - \delta$, $\theta_2 \geq 2p$) due to symmetry. The complete set of outcomes is provided in Fig. 5. One can see that even for the simplest case with two agents, the posted-price mechanism cannot achieve efficiency under ex post equilibrium for all outcomes. As a remedy, p needs to be optimized to maximize the expected economic efficiency with interim budget balance and interim individual rationality constraints. Due to these complexities, we believe the first-price auction are more appropriate to be deployed in practice.

5. Discussions and conclusions

Cooperative vehicle platooning enables vehicles to drive closely together, yielding improved fuel efficiency and reduced driving effort. However, when looking into the details, one can find that the leading vehicles – the essential component of vehicle platoons – usually achieves the least benefit. Therefore, if no proper benefit allocation is performed, vehicles are reluctant to platoon together. Even if they happen to do so, they may want to switch positions or leave the platoon for greater benefits, resulting in potential behavioral instability.

In this study, we proposed an auction mechanism to redistribute benefits in the single-brand vehicle platooning scenario. All participants are identical in their vehicle model and configuration but are different in their willingness-to-pay for reducing driving effort. Since the promise is to be used in a distributed fashion through peer-to-peer coordination, the auction is expected to be individual rational, incentive compatible, economic efficient, and budget balanced. However, we proved that such a desired auction does not exist. To address this impossibility, we discussed a first-price auction, under which the monetary transfers are linear functions of the smallest type of all agents. The first-price auction is proved to be $\epsilon - approximate$ dominant strategy incentive compatible. As a comparison, we also discussed the posted-price mechanism and pointed out its inferiority.

Though we investigate the auction mechanism in the context of vehicle platooning, the mechanism indeed can be applied in other formats of shared mobility that possess similar attributes with vehicle platooning. For example, the peer-to-peer ride-sharing service (Tafreshian et al., 2020), financially incentivized team formation, and competitions in ride-sourcing market (Ye et al.), where agents are normally viewed as identical or symmetric agents. Next, we conclude the proposed auction mechanism regarding the practical applications and possible theoretical extensions.

5.1. Auction implementations

In this study, we separate the benefit reallocation from the decision-making in the planning level, namely vehicle routing and time scheduling. On the contrary, several other studies under the same topic apply an integrated model, which first optimizes vehicle schedules then

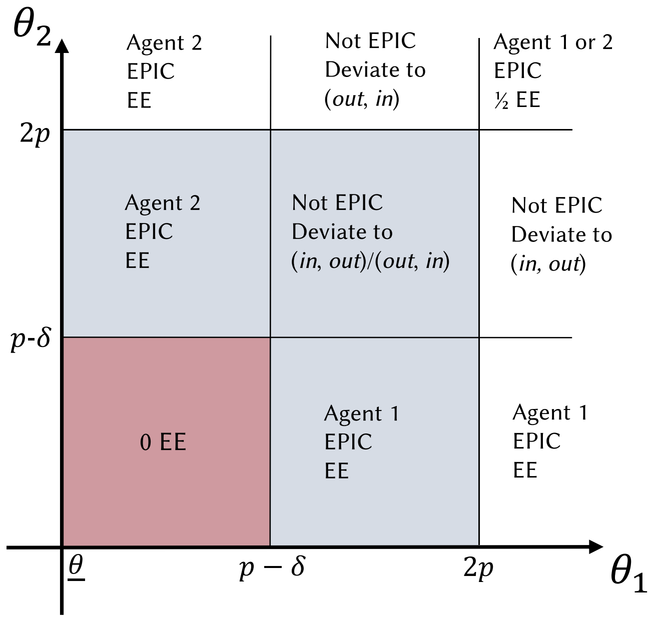


Fig. 5. Outcome under the posted-price mechanism for two-agent scenario.

conducts the benefit reallocation for the coordinated trips in the planning horizon (Johansson and Mårtensson, 2019; Stehbeck, 2019). This top-down approach is comprehensive since benefit-sharing includes disutility from delay or detour. In this regard, it relies on a central controller to gather travel information and provide travel itineraries and the associated costs for vehicles. Such an approach is also less adaptive to schedule changes and travel time uncertainty since centralized optimization takes considerable computational time. Alternatively, the platoon leader auction is simple enough that it allows direct peer-to-peer collaborations without a central controller so that it is flexible and can be applied in real time at different locations.

Vehicles can conduct an auction to determine the platoon leader at a hub or a parking lot, or, perhaps more practically, in motion via vehicle-to-vehicle communications. The auction can also be performed dynamically, in correspondence to different vehicle routes. If new vehicles come to an already-formed platoon spontaneously, or some platoon members have to leave for their destinations, the auction will be performed again to determine the new leadership and the payment. If two formed platoons meet spontaneously and are willing to form a larger platoon, they only need to bid the leaders' types. A platoon will not merge with others if doing so does not increase all its agents' utilities. It is likely to happen when the marginal energy-saving benefit is less than the average one. In this sense, a platoon will not grow infinitely long.

5.2. Theoretical extensions

In this study, we also make a few assumptions and simplifications in the model building to make the analysis tractable. These limitations can be addressed in future research.

First, we emphasize on single-brand vehicle platooning scenario where only the leader–follower positioning matters to the total benefits in a formed platoon. However, the energy consumption, the influence from the surrounding traffic at the platoon forming stage, have not been explicitly considered. Furthermore, total benefits in the formed platoon are also affected by all vehicle sequence and traveling speed when vehicles are multi-brand, making the auction mechanism ineffective. The applicable distributed mechanisms for multi-brand vehicle platooning are addressed in another parallel study Sun and Yin (2021).

Second, our study focuses on the analysis in the ex post environment. It avoids the discussion of Bayesian incentive compatible mechanisms, which are studied based on the assumption that agents know their own types and only have subjective probability distributions over possible values of other agents' types. In general, the feasible space of Bayesian or interim incentive compatible mechanisms is larger than the ex post one. However, Bayesian or interim incentive compatible mechanisms deploy a Bayesian Nash equilibrium that only guarantees efficient outcomes on average, instead of for every realization of agents' types. Moreover, our preliminary analysis reveals that it is very likely that no Bayesian Nash equilibrium exists for the platoon leader auction.

Third, we presume that the platoon length can be infinitely long in our model. If we impose a platoon length limit, $\overline{L}$, at least $\lceil m/\overline{L} \rceil$ number of leaders should be determined for m vehicles. Moreover, when multiple platoons exist, the competition among them must be considered since agents will join the one that maximizes their individual utilities. One may find some insights from the relevant literature on the competition among auctions (Haruvy et al., 2008).

Finally, we also exclude the consideration that two or more agents are making decisions collectively. It is likely to happen in truck platooning when one freight carrier operates more than one truck capable of platooning and expects to maximize the group utility. We leave this problem to future research.

CRediT authorship contribution statement

Xiaotong Sun: Conceptualization, Methodology, Formal analysis, Writing – original draft. **Yafeng Yin:** Conceptualization, Formal analysis, Writing – review & editing, Supervision.

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