REGGE THEORY AND THE PION FORM FACTOR*

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It is clear that the leading order perturbative QCD prediction is incompatible with the electromagnetic pion form factor at the energies which have so far been probed experimentally. As such, it is necessary to consider non-perturbative effects in its treatment. In this contribution, we consider various non-perturbative effects, in particular the Reggeization of the quark, and consider how their implementation affects the high-energy behavior of the pion form factor.

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1. Introduction

Leading order perturbative QCD (PQCD) predictions of the pion form factor are expected to describe the data well for sufficiently high energies due to the running of the strong coupling constant. However, data at such energies does not yet exist, and the PQCD predictions for the data that do exist are seen to be off by as much as a factor of three [1–3]. It is, therefore, evident that models which attempt to describe existing data must include non-perturbative effects.

In this work we model the EM pion form factor by the $\gamma \to q\bar{q} \to \pi\pi$ processes as shown in Fig. 1, and explore the behavior that comes from Reggeizing the exchanged quark. This model is motivated experimentally by the work of [4], in which it was observed that hadronic cross sections involving the exchange of a valence quark are larger than hadronic cross sections in which the valence quark exchange is not possible. This model has theoretical motivation as well. In the high-energy limit, we expect the momentum carried by the exchanged particle to be small, and hence the propagator will be largest when the mass of the exchanged particle is small (*i.e.* when the exchanged particle is a quark).

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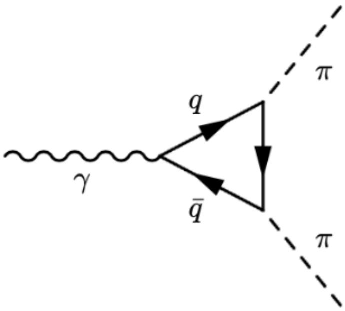

Fig. 1. Feynman diagram representing the $\gamma \to q\bar{q} \to \pi\pi$ process with the $q\bar{q} \to \pi\pi$ mediated by the exchange of a quark.

We start by considering the high-energy behavior of this model consisting of all scalar particles in the completely perturbative case. This will serve as a baseline for comparison of all subsequent calculations. After this, we modify the perturbative picture with non-perturbative effects which modify the propagator terms, gluon exchange at the pion vertices, and finally, by Reggeizing the exchanged quark. We then repeat the calculation with the quarks promoted to spinors and the photon promoted to a vector.

2. Form factor at high energies

We start by considering this model completely perturbatively with all of the particles treated as scalars. In this scenario, the amplitude can be written straightforwardly using the Feynman rules from the $g\phi^3$ interaction term

$$F(s) = \frac{g^3}{i} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left((k+p_2)^2 - m_q^2\right)\left((k-p_3)^2 - m_q^2\right)(k^2-\mu^2)}, \quad (1)$$

where p_2 and p_3 are the momenta of the outgoing pions, k is the momentum of the exchanged scalar quark, m_q is the mass of the quarks in the $q\bar{q}$ pair, and μ is the mass of the exchanged quark. It is most convenient to carry out this calculation using a dispersion relation. To calculate the discontinuity, we put the $q\bar{q}$ pair on shell according to Cutkosky's cutting rules, so that

$$\Delta F(s) = \frac{g^3}{i} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{(2\pi)^2\, \delta\left((k+p_2)^2 - m_q^2\right)\; \delta\left((k-p_3)^2 - m_q^2\right)}{k^2-\mu^2}. \quad (2)$$

The delta functions can be rewritten as functions of k_0 and $|\vec{k}|$. This allows us to perform two of the four integrals. Furthermore, there is no azimuthal angle dependence, so the azimuthal angle integration can be easily performed.

The result $\left(\text{using } s=(p_2+p_3)^2 \text{ and } \sigma_i=\sqrt{1-\frac{4m_i^2}{s}}\right)$ is

$$\Delta F(s)=\frac{g^3\sigma_q}{8\pi}\int_{-1}^{1}\mathrm{d}z\frac{1}{s(z\sigma_q\sigma_\pi-1)+2\left(m_\pi^2+m_q^2-\mu^2\right)}\,. \tag{3}$$

Noting that in the limit of large s we have $\sigma_i\to 1$, it is clear that the discontinuity is dominated by the region $z\simeq 1$. This corresponds to forward scattering and indicates that the transverse quark carries very little momentum.

Integrating Eq. (3) over z in a dispersion relation, we see that

$$F(s)=\frac{1}{\pi}\int\mathrm{d}s'\frac{\Delta F\left(s'\right)}{s'-s}=\frac{1}{\pi}\int\mathrm{d}s'\frac{\frac{\ln s'}{s'}}{s'-s}\,. \tag{4}$$

The denominator can be expanded, and to leading order $(s'-s)^{-1}\simeq s^{-1}$. Therefore, the only s' dependence comes from $\Delta F(s')$. The remaining integral is dominated by the region of integration $s'\sim s$, and so the result at large s is

$$F(s)\propto\frac{\ln^2 s}{s}\,. \tag{5}$$

This serves as a point of comparison for all subsequent calculations. The introduction of non-perturbative physics will add additional spatial dependence to the couplings. Thus, we should expect that, particularly for interactions at high energies, the behavior of the form factor should become softer relative to the perturbative calculation of Eq. (5).

The first modification we make is

$$g\to\frac{g}{\left(p^2-m_q^2\right)^{n-1}}\,, \tag{6}$$

where p is the momentum of the quark in the $q\bar{q}$ pair connected to the pion vertex. Such a change can be interpreted as some change to the propagator from higher-order diagrams in a way analogous to a self energy, or as a change to the $q\bar{q}\pi$ coupling. With this modification, the amplitude Eq. (1) picks up $n-1$ factors of the propagators for the $q\bar{q}$ pair

$$F(s)=\frac{g^3}{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{\left((k+p_2)^2-m_q^2\right)^n\left((k-p_3)^2-m_q^2\right)^n\left(k^2-\mu^2\right)}\,. \tag{7}$$

In this case, it is straightforward to proceed using the Feynman parameters. In the large-s limit, one can show both analytically and numerically that the leading s behavior changes as a function of n, $F(s) \sim s^{-n}$ for $n > 1$. This behavior is shown in Fig. 2. It is, therefore, possible to systematically modify the overall s dependence of the EM form factor at high energies by raising the power of the quark and antiquark pair momentum dependence.

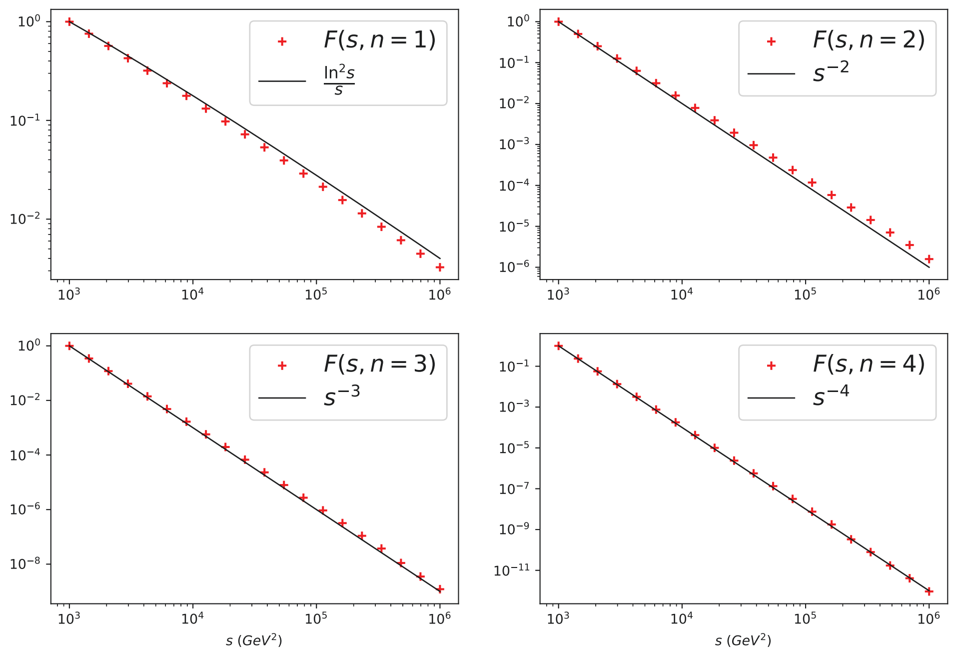

Fig. 2. Result of numerical integration of Eq. (7) as a function of s for several values of n.

We can consider modifying the behavior of the exchanged quark propagator in a similar manner. However, based on Eq. (3), we know that the exchanged quark carries very little of the total momentum, and therefore we do not expect it to depend strongly on s. Indeed, one can show analytically and numerically that raising the transverse quark propagator to an arbitrary, positive power does not affect the overall s behavior at high energies.

The formation of pions from two quarks requires the exchange of gluons. Therefore, we need to study the effects of adding gluon exchange at the pion vertices. Figure 3 shows a single gluon exchange at one of the pion vertices. Rather than calculate a two-loop diagram, we simply calculate the contribution from the gluon loop, and then determine if this contribution changes the dependence of the remaining loop on the quark and antiquark pair momentum. From this, much as we did for the previous case where we modified the propagator term, we can determine if the overall s behavior of the form factor changes.

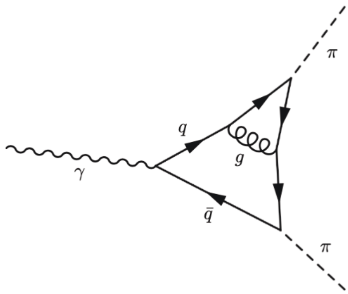

Fig. 3. Single-gluon exchange at the quark–pion vertex.

Assuming Eq. (6), the contribution of the gluon loop is

$$F_g\left(p_2^2, s\right) = \int \frac{\mathrm{d}^4\ell}{(2\pi)^4} \frac{1}{\ell^2\left(\left(p_1+k\right)^2 - m_q^2\right)^n \left(\left(k+\ell\right)^2 - m_q^2\right)}, \tag{8}$$

where $p_1 = k + p_2$. Investigation of this term leads to the discovery that F_g is suppressed at most by a factor of $\frac{1}{p_1^2}$ regardless of the value of n, as is seen in Fig. 4. Comparing this to the previous analysis of Eq. (7), we see that by

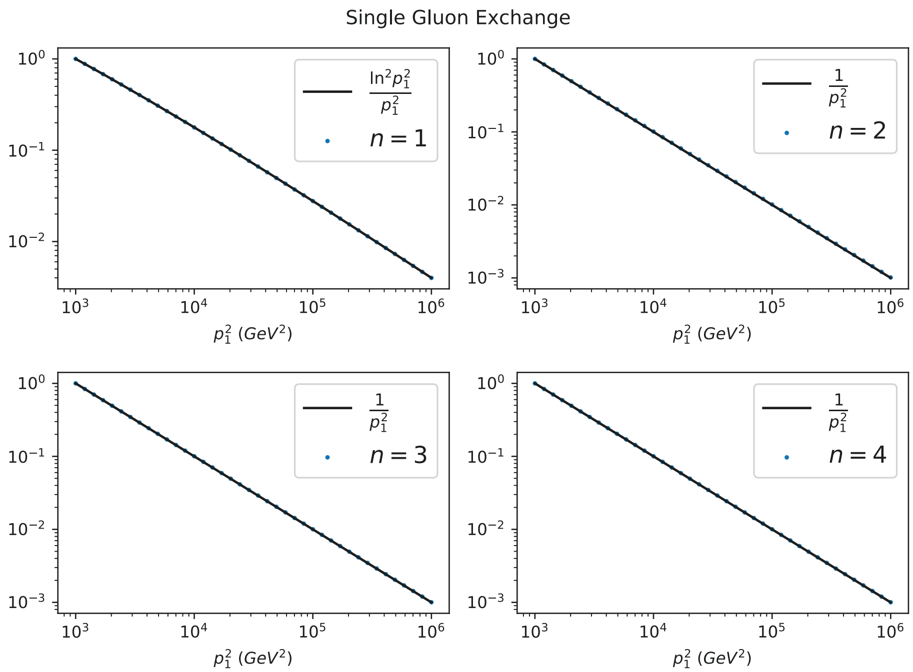

Fig. 4. Result of numerical integration of Eq. (8) as a function of s for several values of n.

adding gluons the form factor can be made as soft as $\frac{1}{s}$, but no softer. This effectively means that diagrams involving the gluon exchange shield the s dependence of the form factor from the non-perturbative modifications which appear within the gluon loop.

In reality, the photon is a spin-one particle and the quarks have a spin of 1/2. This will make the calculations more cumbersome by introducing Dirac matrices in the numerator of each propagator and in the couplings. However, despite the complications, the calculations are handled in much the same way, and the behavior in the limit of large s of the form factor is unchanged.

3. Quark Regge trajectory

In this contribution, we only consider the scalar quark Regge trajectory. However, it is possible to construct a Regge trajectory for a spinor particle by considering a ladder of spinor–vector interactions mediated by the quark exchange in the u-channel as was described by [5]. We model the scalar Regge trajectory by first considering a Reggeon $A(s)$ in the s-channel which is constructed by the sum of all ladder diagrams as shown in Fig. 5 [6].

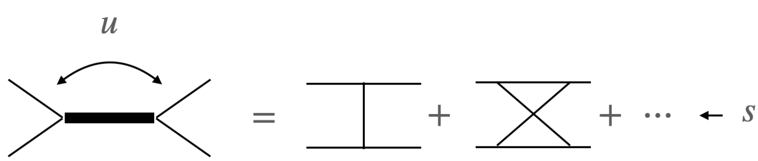

Fig. 5. Diagrammatic representation of our model for the quark Reggeon, which is constructed by summing over ladder diagrams $A_\ell = A_\ell^1 + A_\ell^2 + \ldots$

The first contribution to the ℓ^{th} partial wave of the amplitude A_ℓ is

$$A_\ell^1(s) = \int \mathrm{d}z_s \frac{g_0^2}{\lambda^2 - u(s, z_s)} P_\ell(z_s)\,, \tag{9}$$

where λ is the mass of the exchanged scalar particle, g_0 is the coupling constant at the vertices, z_s is the cosine of the scattering angle in the s-channel, and $P_\ell(z)$ are Legendre polynomials. We calculate higher-order contributions to the partial wave amplitude by unitarity, *i.e.*

$$A_\ell^2(s) = \frac{1}{\pi} \int \mathrm{d}s' \frac{A_\ell^1(s')\,\rho(s')\,A_\ell^1(s')}{s' - s}\,, \tag{10}$$

where $\rho(s) = \frac{\lambda^{1/2}(s,m^2,m^2)}{16\pi s}$, m is the mass of the external particles, and $\lambda(a,b,c)$ is the Källén function. Defining $N_\ell(s) = g_0^2(\ell+1)A_\ell^1(s)$ and

$D_\ell(s) = -1 + \frac{sg_0^2}{\pi}\int \frac{\mathrm{d}s'}{s'(s'-s)} N_\ell(s')\rho(s')$, we write

$$A_\ell(s) = \frac{g_0^2 N_\ell(s)}{\ell - D_\ell(s)}\,. \tag{11}$$

The factor of $(\ell+1)$ was introduced in order to regularize the pole which is present in Eq. (9) at $\ell = -1$. It is clear that Eq. (11) has the form of a simple Regge pole where

$$A_\ell(s) = \frac{\beta(s)}{\ell - \alpha(s)}\,, \quad \alpha(s) = D_{\alpha(s)}(s)\,, \quad \beta(s) = g_0(s)^2 \frac{N_{\alpha(s)}(s)}{1 - D_{\alpha(s)}(s)}\,. \tag{12}$$

Unitarity gives the imaginary part of the trajectory $\mathrm{Im}\,\alpha(s) = \rho(s)\beta(s)$ above the production threshold, and we therefore require that $\beta(s)$ is real above the threshold. This is not automatically satisfied by Eq. (12), but can be corrected by making the change

$$\frac{g_0(s)^2}{1 - D_{\alpha(s)}(s)} \to c\,\mathrm{e}^{-\frac{s}{\pi}\int_{s_{\mathrm{th}}}^{\infty}\mathrm{d}s'\frac{\phi(s)}{s'(s'-s)}}\,, \tag{13}$$

where $\phi(s)$ is the phase of $N_\ell(s)$ when represented in polar coordinates. This allows us to write a set of integral equations for the Regge trajectory as a once subtracted dispersion relation in terms of $\nu = \frac{s}{4m^2} - 1$ and $r = \frac{\lambda^2}{2m^2}$

$$N_{\alpha(\nu)}(\nu) = \frac{\sqrt{\pi}}{2^{\ell+1}(\nu+r)^{\ell+1}} \sum_{n=0}^{\infty} \frac{\Gamma(2n+\ell+1)}{\Gamma(n+\ell+\frac{3}{2})\Gamma(n+1)} \left(\frac{\nu}{2(\nu+r)}\right)^{2n}\,,$$

$$\phi(\nu) = \arg\left(N_{\alpha(\nu)}(\nu)\right)\,,$$

$$\beta(\nu) = c\,\nu^{\alpha(\nu)} \exp\left(i\phi(\nu) - \frac{\nu+1}{\pi}\int_0^\infty \mathrm{d}\nu' \frac{\mathrm{Im}\,\alpha(\nu')\ln\nu' + \phi(\nu')}{(\nu'+1)(\nu'-\nu)}\right) |N_{\alpha(\nu)}(\nu)|\,,$$

$$\mathrm{Im}\,\alpha(\nu) = \rho(\nu)\beta(\nu)\,,$$

$$\mathrm{Re}\,\alpha(\nu) = a + \frac{\nu+1}{\pi}\int_0^\infty \mathrm{d}\nu' \frac{\rho(\nu')\beta(\nu')}{(\nu'+1)(\nu'-\nu)}\,. \tag{14}$$

This set of equations can be solved iteratively and depend on the parameters a, c, and r. As an example, Fig. 6 shows iteration over for the parameters $a = -0.1$, $c = 1$, and $r = 2$.

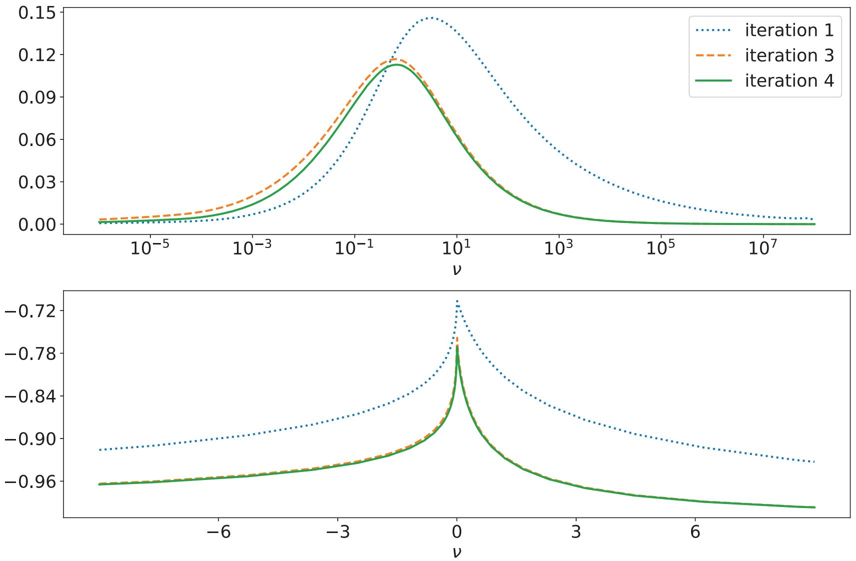

Fig. 6. Imaginary (top panel) and real (bottom panel) parts of the Regge trajectory of a scalar particle produced using Eq. (14) with $r = 1$ and $c = 10$. The parameter a is fixed such that $\mathrm{Re}\,\alpha(\nu) \to -1$ as $\nu \to -\infty$.

The leading order behavior of the sum of ladder diagrams of Fig. 5 near the pole $\ell = \alpha(s)$ is $A(s,u) \sim u^{\alpha(s)}$. This can be used in the context of our model for the pion form factor by substituting A_ℓ for the $q\bar{q} \to \pi\pi$ portion of the diagram, so that for large s, including all non-perturbative effects considered so far

$$F(s) = \frac{g^3}{i} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{s^{\alpha(u)}}{\left((k+p_2)^2 - m_q^2\right)^n \left((k-p_3)^2 - m_q^2\right)^n} . \tag{15}$$

By varying a and c of (14), this gives us further control over the s dependence of the pion form factor. By comparing an expression like Eq. (15) but with particles of correct spin to data, one can extract values for n, a, and c, and determine what this implies about the nature of QCD.

4. Summary and outlook

In this work, we have determined a method for systematically altering the high-energy behavior of the EM pion form factor by introducing non-perturbative interactions. This is achieved through modifying the pion interaction vertex and introducing a Regge pole for the exchanged quark. This will allow the pion form factor some flexibility in describing the data, and will hopefully yield some insight into the nature of QCD.

What still needs to be done is to construct a Regge trajectory for the quark in which the quantum numbers of the Regge trajectory are those of a quark, *i.e.* a spinor particle. Once this is accomplished, a meaningful comparison to data can be done, and the parameters of the Regge trajectory and the non-perturbative pion vertex can be extracted.

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