TEACHER PRACTICES FOR CULTURALLY RESPONSIVE MATH MODELING IN GRADES K-2

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There is growing recognition that mathematical modeling can be a lever for equity in elementary mathematics classrooms. This study focuses on the impact of a professional development program focused on culturally responsive mathematical modeling on 8 kindergarten through 2nd grade teachers' practices in modeling lessons. We use a project developed observation tool to evaluate two video recorded modeling lessons from each teacher (16 total). Findings focus on patterns in the strengths and challenges in primary grade teachers' practices for teaching modeling, including how teachers' practices align with culturally responsive teaching. We discuss implications of our findings for the design and refinement of professional development.

Keywords: Instructional Practice; Culturally Relevant Pedagogy; Modeling; Elementary School

Introduction

Mathematical modeling (MM) is an iterative process involving problem posing, testing, validation, and revision of mathematical models to inform decisions (Lesh & Zawojewski, 2007; Pollak, 2012). There is growing recognition that mathematical modeling can be a lever for equity in elementary mathematics classrooms. Modeling encourages diverse student contributions and gives teachers opportunities to "recognize and reward a broader range of mathematical abilities than those traditionally emphasized" (Lesh & Doerr, 2003, p. 23). When modeling tasks are grounded in culturally responsive contexts, teachers are empowered to build on the knowledge and cultural resources that students bring to the classroom and students are empowered to draw on their identities and experiences to inform mathematical work and take action (Aguirre et al., 2019; Suh et al., 2018; Turner et al., 2017). By building multiple connections to self, family, community, other subjects, and the world, mathematical modeling tasks humanize mathematics teaching and learning (Anhalt et al., 2018; Gutierrez, 2018; Suh et al., 2018).

While professional development initiatives have begun to support teachers' learning of culturally responsive mathematics modeling (Turner et al., 2022a), research on how teachers learn to enact practices for teaching modeling is limited, particularly in primary grades (kindergarten to second). This is because modeling includes practices that are not typical in primary grade mathematics classrooms like making assumptions, and testing and revising models (Niss, Blum, & Galbraith, 2007). Therefore, the primary aim of this study is to understand the potential impact of a professional development program focused on culturally responsive mathematical modeling on primary grade teachers' practices in modeling lessons. The following research questions guide our study: To what extent does a professional development program support teachers to enact practices for culturally responsive mathematical modeling lessons?

- What strengths and challenges do we notice in teachers' practices?
- How do teachers' practices for modeling lessons support culturally responsive mathematics teaching?

Culturally Responsive Mathematical Modeling Instruction

Our framework for culturally responsive mathematical modeling draws on mathematics education research that centers equity, namely work on culturally responsive mathematics teaching (Bartell et al., 2017; Leonard et al., 2010). Zavala and Aguirre (in press) highlight three essential strands of culturally responsive mathematics teaching, including: a) Connections to Knowledge and Identities –building on students' experiences, mathematical understandings, and cultural/community-based funds of knowledge to support their mathematics learning; b) Rigor and Support - maintaining high cognitive demand while simultaneously providing access points for learning including affirming multilingualism; and c) Power and Participation - teachers enhance equitable participation by disrupting status, distributing intellectual authority, and supporting student ownership of ideas. We see mathematical modeling instruction as a way to advance culturally responsive teaching because the relevant and cognitively demanding nature of modeling tasks cultivates space for diverse ideas and opportunities for connections and action (Anhalt et al., 2018; Cirillo et al., 2016; Turner et al., 2021, 2022a; Zavala & Aguirre, in press).

Teaching Practices for Culturally Responsive Mathematical Modeling in Primary Grades

There is growing consensus that teachers can support primary grades students to engage in mathematical modeling (English, 2012; Fulton, 2021; Wickstrom & Aytes, 2018). Researchers specifically highlight the importance of modeling tasks grounded in culturally relevant contexts to help students draw on their lived experiences (Albarracín, 2021; Albarracín & Gorgorió, 2020; Dindyal, 2010; Wickstom & Aytes, 2018), and the value of collaboration and dialogue with peers (Bonnotto, 2009; Mousoulides & English; 2008). Teacher practices which support primary grade students' engagement in mathematical modeling, include posing a series of smaller questions to help students make sense of a broader task (Albarracín, 2021), and introducing constraints one at a time so that students could adjust and refine their models without starting over (Osana & Foster, 2021). Other productive teacher moves include pausing small group work to share student strategies, and offering physical tools, graphic organizers, and sentence starters to scaffold students' modeling building work (Carlson, 2016 et al., 2016; Fulton, 2021; Wickstron & Aytes, 2018; Author). Teachers of young students also play an important role in helping students make assumptions about unknown quantities (Leavy & Hourigan, 2021; Stankiewicz-Van Der Zanden, Brown & Leavy, 2021). Yet most of these studies report on intensive efforts with one or two individual teachers and not on how teachers learn practices for modeling via participation in professional development. Our study aims to address this gap.

Research on the Impact of Professional Development on Teacher Practice

Project developed measures that relate to the specific focus of the professional development program are often used evaluate teacher practice, such as teachers' use of student thinking in instruction (Jacobs et al., 2007), or practices for facilitating classroom discourse (Cavanna, 2014). While studies sometimes find connections between ideas learned in professional development and teachers' subsequent classroom practice (Chen et al., 2020; Jacobs et al., 2007), others note impacts on teacher beliefs and perspectives, but not practices (Shirrell, Hopkins & Spillane, 2019). Despite these differences, there is consensus that understanding which practices teachers take up in their classrooms, and why, is essential, as it can inform revisions to professional learning programs (Caswell, Esmonde & Takeuchi, 2011; Franke et al, 2001).

Observation Tools for Measuring Teacher Practice in Mathematical Modeling Lessons

Observational tools to measure mathematics teaching practice tend to focus broadly on standards-based, or problem-solving oriented instruction (Walkowiak et al, 2014). A tool with a specific focus on mathematical modeling is important because scores would be interpreted as capturing aspects relevant and specific to teaching mathematical modeling. Developing an observation tool that attends, in substantive ways, to mathematical modeling and culturally responsive teaching has been a goal of our current project. We previously described (Turner et al., 2022b) our multi-step process for the initial development of this tool, including synthesizing key outcomes from relevant literature, generating initial validity evidence from an expert panel review, and testing the tool in diverse K-5 classrooms (Bostic et al, 2019). In this study, we use the tool to understand the strengths and challenges in primary grade teachers' practices for teaching modeling, and to explore how their practices align with culturally responsive teaching.

Methods

Professional Development Context

This study is part of a broader research and professional development program focused on culturally responsive mathematical modeling in elementary grades. Teachers participated in a year-long, professional development program that included both monthly in person sessions and asynchronous activities to deepen learning. In person sessions introduced frameworks for culturally responsive mathematics teaching (Zavala & Aguirre, in press), and included time to explore modeling tasks and routines, collaboratively plan activities, and reflect on classroom enactments. Asynchronous materials included readings, videos of modeling activities in K-5 classrooms, and collaborative reflection prompts. Teachers also had access to digital materials (modeling tasks, student work samples) to support classroom enactments.

Participants

This study focused on 8 primary grade teachers (kindergarten through grade 2) who participated in our broader professional development program at one of three research sites. Two of the teachers taught kindergarten, five taught first grade, and one taught 2nd grade. 6 of the teachers worked in schools that served racially and linguistically diverse students from underserved communities. Classrooms included migrant and refugee students from diverse countries of origin, and significant numbers of multilingual students. 2 teachers taught in predominantly white schools with a small but growing population of multilingual students.

Data Sources

Data sources included two video-taped modeling lessons from each teacher's classroom (16 lessons total). One lesson included a "snack sharing" modeling task that focused on making a plan to share snack items with classmates across one or more days. This lesson was recorded in the fall or winter of the school year. A second lesson focused on a "making" modeling task in which students generated a plan for making a set of items (e.g., picture frames, bird feeders,) for a school or community purpose. This lesson was recorded later in the school year. Lessons ranged in length from 54 to 187 minutes (average of 88), and often occurred over two days.

Classroom Observation Tool for Modeling Lessons

Each lesson was scored using the project developed classroom observation tool for culturally responsive mathematical modeling lessons (Turner et al., 2022b). The tool includes eight dimensions that focus on teaching practices for specific phases of the modeling process. Each dimension describes four levels of teacher practice (not present (0), emerging (1), proficient (2) and advanced (3). These levels are distinguished by the extent to which teachers use culturally responsive practices in that phase of the modeling cycle, such as maintaining high cognitive demand while simultaneously providing supports (Rigor and Support), soliciting diverse student ideas and allowing student ideas to drive decisions (Knowledge and Identities; Power and Participation). A ninth dimension focuses on culturally responsive practices that apply across the modeling process (i.e., connections to students' experiences and cultural/community contexts).

Table 1: EQ-STEMM Classroom Observation Tool for Elementary Modeling Lessons

Dimension	Focus of Dimension	Variation in Levels of Practice
1: Making Sense of the Context / Situation	Teachers offer supports to help students make sense of the context, solicit students' ideas or questions about the task context, and focus students on key considerations related to context.	•Presence of supports •Intensity of teacher solicitation •Focus on key considerations
2. Posing Problems	Teachers build on student ideas to pose the modeling problem. Teachers support students to ask and analyze mathematical questions.	•Student ownership of problem posing •Support for asking math questions
3. Identifying Important Quantities	Teachers support students to identify key quantities and to decide on a specific value for one or more quantities. Teacher asks students to explain the relevance of key quantities.	•Student ownership of quantities •Allowing variation in quantities •Support for explaining relevance
4. Making Assumptions	The teacher supports students to make /state assumptions, and to justify the relevance and reasonableness of assumptions.	•Student ownership of assumptions •Support for explaining relevance
5. Constructing and Operating on Models	The teacher facilitates student work as students create and operate on models, soliciting student ideas and supporting students to justify work.	•Student ownership of models •Support for justification
6. Analyzing or Interpreting Models	Teacher provides structures to support analyzing models or solutions and supports student participation so student ideas influence discussion.	•Presence of analysis supports •Student ownership of analysis •Support for justification
7. Revising Models	The teacher supports students in revising, ensuring that student contributions play a central role in model revisions, and supporting students to justify.	•Student ownership of revision •Support for justification
8. Reporting Out	The teacher supports students/groups to report and explain their work. The teacher provides students with options for reporting out their results.	•Student ownership of report out •Support for student choice
9: Connections to Students' Out-of-Class Experiences and Cultural and Community Contexts	Teachers support students to make connections to out-of-class experiences and/or cultural and community contexts. Connections inform modeling work - influencing decisions made or actions taken in any phase of modeling process.	•Connections present through the modeling cycle •Student ownership of connections •Connections inform modeling

Each lesson was scored by at least two members of the research team using a scoring method adapted from prior projects (Foote et al., 2020; Walkowiak et al., 2014). We watched lesson videos in eight-to-ten-minute segments, scripting teacher and student talk and actions to produce detailed lesson logs. After each segment, we noted evidence related to each dimension (including time stamps and examples) on a coding sheet. This process was repeated until the end of the lesson video. We reviewed all evidence against the dimension descriptors to assign a final score by dimension. Groups of two to four researchers reviewed scores for each lesson, and differences were resolved via discussion. Analysis focused on patterns of strengths and challenges by dimension. Patterns were examined within lessons (i.e., strengths and challenges in the snack sharing modeling lessons) and across lessons. Finally, we reviewed the lesson notes and coding sheet evidence to explore the specific teacher practices connected to each pattern.

Findings

Overview of Teacher Practice Scores

Table 2 includes average teacher practice scores for selected dimensions for each set of modeling lessons. For the purposes of this report, we focus on three dimensions that reflect key patterns related to our research questions, including consistent areas of strength (dimension 1), persistent areas of challenge (dimension 4) and dimensions where teacher practice seemed to shift between the first and second set of lessons (dimension 3).

Table 2: Teacher Practice Scores on Selected Dimensions

Dimension	Average Score on Snack Sharing Modeling Lessons	Average Score on Making Modeling Lessons
1. Making Sense of the Context	2.75	2.63
3. Identifying Important Quantities	1.63	2.38
4. Making Assumptions	0.88	0.75

Areas of Strength: Supporting Students to Make Sense of Real-World Contexts

We found that teachers enacted multiple strategies to help students make sense of the real-world context of the modeling tasks, including sharing images and realia related to the context, and inviting students to share observations, wonderings, and relevant experiences (e.g., “Have you ever had this snack? How do we usually share snacks at school?”). This strength was evident across all eight teachers, and across both sets of lessons observed. As an example, Ms. T, a first-grade teacher, launched the snack sharing modeling lesson by holding up two containers of small Madeline cakes and inviting students to share what they noticed. Students began by describing the physical attributes of the cakes (e.g., “they look like sweet bread”), which the teacher revoiced and recorded on a class chart. Students then posed questions about how the cakes were made, how large they were, whether the class would be able to eat them, and whether there were the same number of cakes in both boxes. Students passed around the containers as they observed and wondered, until most children in the class had the opportunity to share. After about ten minutes, the teacher noticed one student looking around the room and counting her classmates. The teacher paused the discussion and asked the class to reflect on her thinking.

Ms. T: I want to go back to Deena’s thinking. Because she was looking at my box and then she was like… counting (gestures to show how she was counting the children) … then she looked at the box. What was she thinking?
Student 1: Maybe she was wondering if there was less or more cakes than the children.
Ms. T: That’s a really good math question, is that what you were wondering?
(Deena confirmed that she was thinking about whether they would have enough for everyone, and the teacher then drew on this idea to pose the modeling problem.)

Ms. T: We have two boxes. I was wondering this question. You wanted to know if there was enough for all of us. What if I said: how many days will snacks last for our class? If we have this for snack every day, could we find out how many days it's going to last us?

This excerpt reflects patterns in the culturally responsive practices that teachers used to support students as they made sense of the modeling context. Teachers provided multiple supports for sense making, and repeated opportunities for students to share their ideas, both with partners and in whole group discussion. Teachers used targeted prompts to focus students' attention on key features of the context (i.e., in this case, Deena's idea that comparing the number of children to the number of cakes might be useful). Teachers also ensured students had ownership over ideas, recording their names alongside their comments, and explicitly connecting the modeling problem to their ideas. As a result, by the time teachers posed the modeling problem, students had generated a broad range of ideas about the context.

Areas of Shift: Identifying Important Quantities

The dimension where teacher practices shifted most from the first to the second set of lessons was Dimension 3, Identifying Important Quantities. In the snack sharing modeling lessons, half of the teachers evidenced emergent levels of practice (4 teachers), with fewer teachers scoring proficient (3 teachers) or advanced (1 teacher). Common patterns in teacher practices included providing time for students to discuss what they already knew that could help them with the modeling problem, and what they might need to find out or decide. For example, another first-grade teacher, Ms. B, posed a modeling problem related to how many boxes of hot chocolate they would need for all the first grade classes at a winter celebration. After posing the task, she asked students: "What might we need to know if we were going to do this project. Stand up and talk to your talking partner. We have some things we know and some things we need to decide." Following the partner talk, Ms. B solicited and recorded students' ideas. Students quickly identified one quantity that would be relevant– the number of students in each first-grade class. However, when students started to generate a range of ideas about this quantity ("I am estimating 16 or 19 kids in our class", and "We could count how many chairs there are or count the lockers") Ms. B responded by narrowing the conversation and directing students towards a specific value. She stated, "Could we decide that there are 16 kids in our class and the all the other classes too? I think that's fair. Let's write it down - '16 kids in every first-grade class.'"

This practice of encouraging students to list relevant quantities, but then funneling the class towards a single set of values so that all students worked with the same numbers as they built and operated on their models was common in this first set of lessons. These practices closed off space for variation and therein, limited students' opportunities to explain and justify their decisions, which reduced the cognitive demand. In other words, teachers' practices reflected tensions related to components of culturally responsive teaching – teachers provided supports, but these supports reduced the cognitive demand and minimized students' ownership over ideas.

In contrast, in the second set of modeling lessons – the making tasks – all teachers scored at the proficient (5 teachers) or advanced levels (3 teachers). A shift we noted is that teachers were more adept at efficiently focusing students' observations on relevant quantities, and importantly, on how quantities might vary. Teachers used strategically selected images or prompts that drew students' attention to variation. For example, Ms. J., a kindergarten teacher, taught a lesson about planning a set of materials for making a valentine craft (heart creatures) with their table groups. She launched the task by sharing images of heart creatures with different features. After students shared initial observations, Ms. J asked followed up with a more specific prompt to focus students on variation in quantities relevant to the problem (What is the same? What is different?)

Ms. J: (shows an image of different heart creatures) What is the same and what is different?
Student 1: (pointing to specific creatures) Big, little, little
Ms. J: So you mean the size is different, good.
Student 2: all of them are different
Ms. J: Ok, and how are they different?
Student 2: Eyes, and faces, and arms, and legs
Student 3: They all have heart heads.
Student 4: This one has 4 hearts on the arms and legs.
Student 3: They don't have the same eyes.
Ms. J: The same number of eyes? So they're different.

Students continued to identify consistencies and variations in the heart creatures, which Ms. J recorded on a class chart. Before she sent students off to work, she revisited the quantities they identified and the decisions they would have to make as they planned for the materials. This increased emphasis on variation and more consistent invitations for students to make decisions about quantities were notable patterns in the second set of lessons. These practices reflected components of culturally responsive mathematics teaching as teachers maintained high cognitive demand by holding space for variation and supporting students to explain and justify ideas.

Areas of Challenge: Making Assumptions

We found that supporting students to make assumptions was one of several dimensions that reflected challenges in teachers' emerging practices for mathematical modeling. In a few instances, teachers skipped conversations about assumptions, and instead directed students towards established quantities (e.g., there are this many students in our class) without consideration of how assumptions about the context could impact decisions. More often, teachers prompted students to consider potential assumptions, but students' role was limited to answering questions while the teacher-maintained ownership over stating and justifying the assumption. For example, in the snack sharing lessons, a second-grade teacher, Ms. F, asked students whether everyone in the class eats snack each day (focusing students' attention on potential assumptions).

Ms. F: But wait, do all of you eat the snack each day? How are we going to figure out how much we need to give out? It's [only] 18 if everybody eats the snack.
Student 1: maybe Ms. T [will want snack] too.
Ms. F: Ms. T is not here during snack time.
Student 2: How many people are missing [absent]?
Ms. F: There are 5 people missing.
Ms. F: So not everyone is going to eat snack. Do some of you bring snack from home? Raise your hand if you bring snack from home. (several students raise hands.)
Student 3: Only sometimes [do I bring snack from home]
Student 4: Since I really like those things that she brought I would eat that snack and save mine for tomorrow.
Student 5: I'd eat snack from home.
Ms. F: So we had 18 minus 1 person who won't eat snack, so we are going to say 17 people. And we know that is not true, some days people will be missing, but we are just going to say 17 people on most days.

While students responded to Ms. F's question with multiple ideas that could have supported assumptions (e.g., whether other teachers should be included, whether absent students should be counted, whether students who brought snack from home would always prefer the home snack), Ms. F directed the assumption making process and stated an assumption for students. We found a similar pattern in other lessons; students considered how different features of the context could impact the problem - they did not avoid this complexity – but teachers responded by making assumptions for students that narrowed the complexity, sometimes significantly. In other words, teachers' initial practices did not reflect key components of culturally responsive teaching. Students did not have ownership of ideas, as the teacher did not distribute intellectual authority. The cognitively demanding work of explaining assumptions was directed by the teacher.

Discussion and Conclusion

Enacting culturally responsive mathematical modeling in the elementary grades is ambitious teaching that has potential to humanize the teaching and learning of mathematics with young students. Practices for rehumanizing mathematics (Gutiérrez, 2018), such as helping students to make sense of and connect to the contexts of mathematical problem are common teaching practice in primary grade classrooms. We suspect that participating teachers already had many strategies for supporting this kind of sense making, which supported the strengths observed in dimension 1. Additionally, the professional development program provided multiple examples of visuals, tools and discussion prompts that teachers could use to introduce modeling contexts and foster sense making. While teachers' practices in this dimension reflected multiple components of culturally responsive mathematics teaching, there were also areas for growth. In particular, teachers' efforts to help students make sense of contexts did not include invitations for students to share related experiences from outside of school. We suspect this reflected the tasks themselves (which were grounded in school or classroom scenarios) but also points to an area of refinement for our professional development program.

Teaching culturally responsive mathematical modeling requires a new set of pedagogical skills. We found that teachers allowed more variation in quantities as they became comfortable with the openness and student decision making that characterizes modeling tasks. We suspect that the shifts in teacher practice in the important quantities dimension may have been related to two factors. First, in typical primary grade curriculum tasks, quantities are explicitly stated for students. In modeling tasks, teachers have a key role in supporting students to identify relevant quantities, and to make sense of potential variation (Anhalt, 2014). Given that all our participants were new to teaching modeling, these were new practices that teachers needed time to develop (English et al., 2005). Second, across the year we introduced various routines for supporting students to identify key quantities during professional development sessions. We also began to prompt teachers to reflect on the different decisions that student had to make in each modeling lesson. We recognized that this would be a challenging space for teachers and designed targeted tools and prompts to support these practices, which may have supported the shifts observed.

Finally, we suspect that teaching practices around assumptions were challenging for multiple reasons. The need to make assumptions to inform decisions is a unique feature of modeling (Galbriath, 2013; Suh et al., 2021), and as such, supporting students to make assumptions was a new teaching practice for all the teachers in our study. The fact that teachers entered this practice even in their first modeling lessons is promising. Teachers' initial attempts - which involved drawing students' attention to features of the context that might guide their decisions - suggest an emerging practice that could be further developed in professional development sessions. In other words, teachers likely need to play an active role in supporting students to state and explain assumptions, given that grappling with ambiguous or undefined information can be challenging for young children (English et al., 2005). Strategic prompts to focus children on the need for assumptions seem appropriate (i.e., What about people who bring snack from home, should we consider that?). However, professional development sessions could support teachers to respond to the diverse ideas that children generate in ways that distribute intellectual authority and maintain student ownership over ideas. Supporting teachers' culturally responsive practices for making assumptions is a key area of refinement for our professional development program.

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