Efficient Dynamic Phasor Model of Distributed Photovoltaic Systems

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***Abstract*—The computational challenge in solving dynamic models of power distribution grids increases with the high penetration of distributed photovoltaic (PV) systems. The existing dynamic models of PV systems are overly detailed and computationally intractable for solving distribution grid dynamics with a large number of distributed PV systems. As IEEE-1547 requires smart PV inverters to support the grid voltage and frequency dynamically, this motivates to develop an accurate and computationally efficient dynamic model of distributed PVs. Therefore, in this work, a dynamic phasor model of smart PV inverters is developed and compared with an existing electromagnetic transient model (detailed model) in terms of accuracy and computational efficiency. The results show a remarkably fast solve time of the proposed dynamic phasor model compared to that of the detailed model (more than 30 times speed up), while sufficiently capturing necessary volt-var, volt-watt, and frequency-watt dynamics during the normal, and voltage ride-through and frequency ride-through events.**

***Index Terms*—Dynamic phasor model, smart PV inverters, frequency stability analysis, volt-var, volt-watt, frequency-watt**

I. Introduction

As the penetration of Photovoltaic (PV) generation is increasing in distribution systems, it is necessary for utilities to analyze the distribution systems with high PV penetration for transient events. The electromagnetic transient (EMT) type model provides the detailed and accurate transient response of the system; however, this approach would not be applicable to model realistic-sized power distribution grids with a large number of distributed PV inverters as the computational burden of the such model increases drastically. In this context, a state-space averaging model is developed in [1] which removes switching dynamics and provides some computational benefits; however, it is still computationally burdensome for system level studies.

Various simplification methods have been applied to the PV dynamic models connected to distribution feeder to ease the computational burden [2]. A small-signal model of PV inverter is developed in [3]; however, the model would be accurate around the operating points only. Studies in [4] and WECC model in [5] neglect the DC-side dynamics to simplify the model. However, the simplified models in [4], [5] are unable to represent DC-link and irradiance-driven dynamics.

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The EMT simulation of distribution grids with penetration of inverter-based resources (IBRs) would be computationally intractable. Therefore, in this regard, phasor-based model of inverters are developed as computationally efficient models of PV inverters for volt-var and volt-watt dynamic analysis at single frequency of interest. For example, the work in [6] employs phasor-based power flow solver (e.g., GridLAB-D) and integrates simplified inverter dynamic model to GridLAB-D. Although this is an acceptable approach for volt-var dynamic analysis, the errors obtained in [6] as compared to the inverter's detailed model are large. Another efficient and scalable phasor-based model of smart inverters is developed in our previous work [7], which is applicable for volt-watt and volt-var dynamics. Nevertheless, the models in [6], [7] are not applicable for dynamic frequency.

Another approach is to use dynamic phasor (DP) for capturing the frequency dynamics. DP modeling, as an averaging technique, is able to convert varying state variables into DC variables, which are the Fourier coefficients of the state variables. Therefore, it is a promising technique for reducing the computational burden and it maintains accurate simulations with larger time steps [8] compared to the EMT-type simulations. DP models of a single-phase inverter with constant DC-link voltage was developed in [9]. Although it is able to reduce the simulation time compared to the state-space averaging models, it neglects the Phase Locked Loop (PLL) considering constant grid frequency; therefore, this model can not be readily used for frequency dynamics at the system studies. In [10], the DP method for modeling an unbalanced radial distribution grid with an induction motor load and a single-phase PV inverter connected to the grid is developed. Another DP-based model of an unbalanced inverter-based Micro Grid with a single-phase PV was developed in [11], which showed good accuracy to represent the transient response and with a substantial reduction in the solve time compared to the EMT counterpart. The major drawback of the approaches in [10], [11] is the simplifications made on the PV inverter models that make them unsuitable to use for smart PV inverters. The works in [10], [11] neglected DC-side controller considering unity power factor mode for the PV system at maximum power point that makes the PV inverter unable to incorporate smart functionalities (i.e., volt-var, volt-watt).

To fill the gap in the existing literature, this work proposes to develop DP-based model of a single-phase two-stage smart PV inverter useful for computationally efficient dynamic simulation incorporating smart features as per the IEEE 1547 (i.e., volt-var, volt-watt, and frequency-watt) to dynamically support the grid voltage and frequency. The proposed DP-based model is developed in two stages. In the first stage, the average model [7] of the smart PV inverter is simplified by removing the fast dynamics of the inner control loops and DC-side harmonic. filter but keeping slower dynamics along with the LCL filter dynamics (LCL filter maintains the dynamic interaction between the grid and the inverter). The accuracy and efficiency of this model have been validated in our previous work [12]. Then, in the second stage, DP model of the system is developed from the simplified average model [12], developing DP equation of LCL filter and replacing PLL with a frequency estimator that is able to work in the phasor domain which is necessary for the model to work under off-nominal frequency. It should be noted that the developed DP model is an efficient model for the dynamic simulation of a smart PV inverter-dominated distribution grid under fast volt-var, volt-watt, and frequency-watt dynamics.

The rest of the paper is organized as following. Section II describes the smart PV system power circuit and control configuration. The proposed dynamic phasor model is described in detail in section III. Section IV validates the proposed models through several case studies. The main conclusions and future direction are discussed in section V.

II. Smart PV Systems

Typical configuration of a two-stage single-phase smart PV system [13] with active and reactive power support functions (as per the IEEE standard 1547-2018) is shown in Fig. 1. Power circuit contains a PV array model, LC harmonic filter, boost converter, DC-link capacitor, full-bridge inverter and LCL filter. PV-side and Grid-side controllers are responsible for carrying out active and reactive power support of the grid.

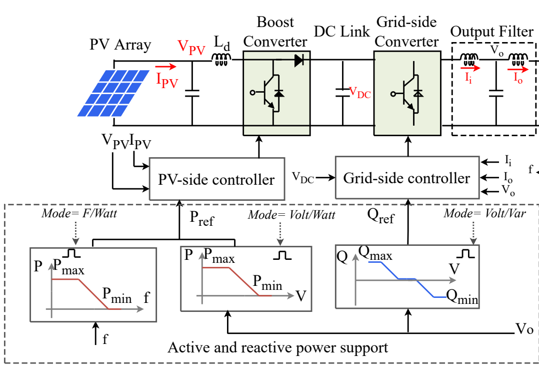

Fig. 1. Typical configuration and control of PV system [13], with active and reactive power support function as per the IEEE-1547.

The detailed model of the PV system is shown in Fig. 2. The PV-side controller operates either in the MPPT or constant power generation (CPG) mode. The later mode always makes PV to operate below the PV's inverter maximum power for the purpose of active power curtailment needed in implementing smart features (e.g., volt-watt, volt-var, and frequency-watt). The grid-side controller is based on the cascaded control architecture, the DC-link voltage and reactive power control loops as the outer loops, generate dq current references for the inner current control loop.

Such power electronic-based model is best represented using detailed switching model which represents the converters in full detail, including the precise, fast switching operations and non-linear nature. However, high frequencies of switching devices require the small simulation step size resulting in exceptionally high computation time [1]. Therefore, the existing average model in [1] replaces switching model of the converters with their average equivalent models which shows faster response, ignoring the switching dynamics and PWM blocks. However, it is still computationally burdensome for system level studies.

III. Proposed Model

A. Model Simplification

First, an efficient dynamic model of the smart PV inverter illustrated in Fig. 2 is derived from its average model where the switching devices are modeled by their equivalent one-cycle average models [7]. The average model is simplified by removing fast dynamics of the inner control loops and DC-side harmonic filter; while keeping slower dynamics along with the LCL filter dynamics (see Fig. 3). It should be noted that removing fast dynamic states may lead to modeling errors when compared to the detailed model; however, the results from our previous works [7], [12] show the remarkably fast performance of the proposed simplified model while sufficiently capturing the necessary dynamics as the errors are insignificant. Nevertheless, frequency dynamic simulation of inverter-dominated distribution systems has not been captured as this was developed for the fundamental frequency of interest (60Hz).

B. Dynamic Phasor Representation

Fourier series of a periodic signal $x(t)$ can be written as follows:

$$x(t) = \sum_{k=-\infty}^{+\infty} < x >_k (t) e^{jkwt}, \tag{1}$$

where w is the frequency, k is an integer value and $< x >_k (t)$ is k^{th} Fourier coefficient or k^{th} DP of the signal $x(t)$ which is a time-varying complex quantity and calculated by (2),

$$< x >_k (t) = \frac{1}{T} \int_{t-T}^{t} x(s) e^{-jkws} ds, \tag{2}$$

where $< x >_0 (t)$ is the DC component of the signal $x(t)$ and equivalent of the one-cycle average model of switching converters where state $x(t)$ is the switching signal [9]. $< x >_k (t)$ is rewritten in term of real and imaginary parts in (3), where index t is dropped for brevity.

$$< x >_k = < x >_k^r + < x >_k^i \tag{3}$$

Equations (2) and (3) will be used for the development of DP model in the section III-C.

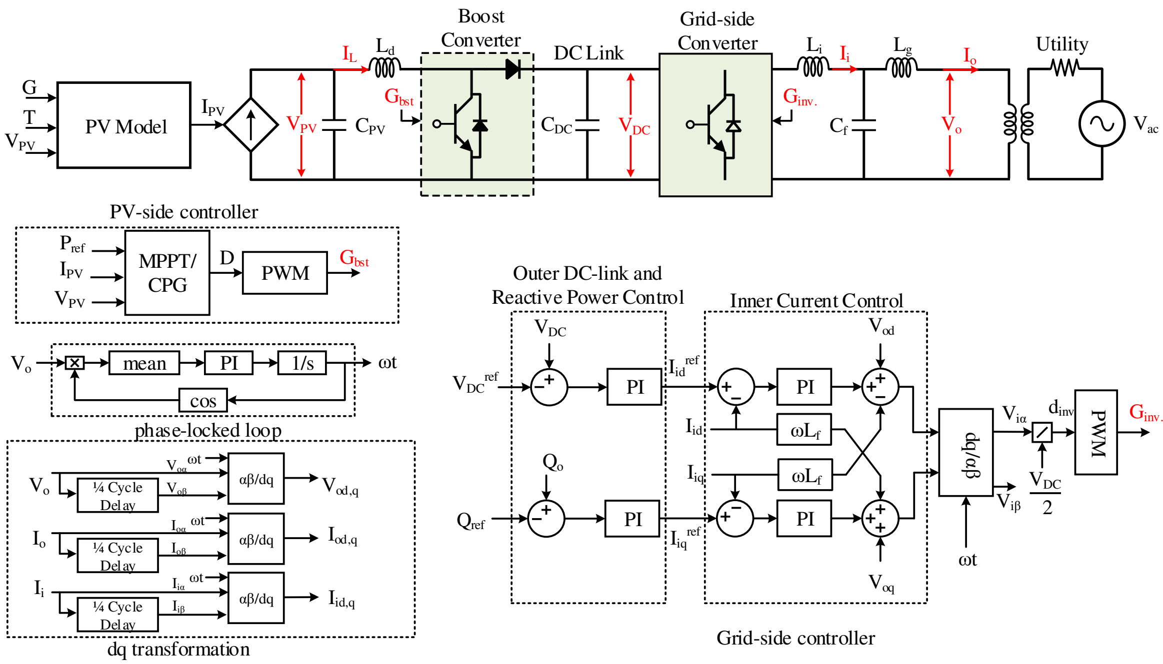

Fig. 2. Detailed model of smart inverter [7].

C. Development of Dynamic Phasor Model

The DP model of the PV system as shown in Fig. 3 is derived from the simplified model [12]. DP method is an averaging method based on demodulating the waveform $x(t)$ to its Fourier components over the interval $(t-T, t]$ [9]. The value of k determines the accuracy and number of differential algebraic equations (DAE) of the DP model. Larger k leads to better accuracy, while it increases the computational burden. Therefore, choosing the right value for k is crucial. In this regard, the existing signals of the simplified model in [12] are classified in four groups, which are handled differently in the proposed DP model as following,

- PV array V_{pv} and I_{pv} are purely DC variables.
- DC-link voltage V_{DC} is a DC signal along with a double fundamental frequency component (ripple) . In general, DC-link capacitor is designed to keep the V_{DC} ripple less than 1.5%. Thus, the double fundamental frequency ripple can be ignored and its Fourier series DC coefficient $< V_{DC} >_0$ is sufficient for the DP model which is calculated as (4) and implemented using a mean value as shown in Fig. 3.
$$< x >_0 = \frac{1}{T}\int_{t-T}^{t} x(s)ds \tag{4}$$
- Duty cycles of boost converter D and DC-AC inverter d_{inv} are one-cycle average of their periodic switching signals. The switching frequency of the converters are much higher than the grid frequency. Thus, the DC coefficients of the switching signals are sufficient for the DP model. Based on (4), the one-cycle average model of the switching converters, D and d_{inv} which are employed in simplified average model in [12] are the DC coefficients of the Fourier series. Therefore, the same approach is used in the DP model.
- AC signals of the LCL filter I_i, V_{cf}, I_o, and V_o are periodic at f (PV inverter frequency). The first Fourier coefficients (fundamental components) of the AC signals $< x >_1$ for showing the AC variables are accurate estimations which results in the equations (5) and (6) as the dynamic phasor model of the LCL filter.

$$C_f \frac{d}{dt} < v_{C_f} >_1 = - < i_o >_1 + jw < v_{C_f} >_1 + < i_i >_1 \tag{5}$$

$$L_g \frac{d}{dt} < i_o >_1 = - < v_{C_f} >_1 + jw < i_o >_1 - < v_o >_1 \tag{6}$$

where $w = 2\pi f$ is the angular frequency of PV inverter. f is calculated by the frequency estimation block which is the replacement for the PLL block. As shown in Fig. 3, f is calculated using the derivative of the inverter output voltage phase angle variation $\Delta\Theta$ with respect to the phasor at the grid's frequency f_n. High frequency components caused by the derivative block is filtered out by a low-pass filter with time constant T (it is equal to 1 in the PLL block). As mentioned earlier, DP model of the PV inverter is developed for f; however, the grid model may not be readily available in dynamic phasor rather on single frequency of interest, i.e., f_n. Therefore, to connect the DP model of PV inverter to the the grid model, the first Fourier coefficient of the PV inverter output current $< I_o >_1$ should be transformed into the grid's using (7). Conversely, the first Fourier coefficient of the PV inverter output voltage $< V_o >_{f_n 1}$ should be transformed into the inverter frequency f using (8). This approach facilitates integration of the proposed dynamic phasor model of PV inverters to any grid model in phasor domain even if the grid is not modelled using the dynamic phasor.

$$< I_o >_{f_n,1} = e^{j\Delta\Theta} < I_o >_1 \tag{7}$$

$$< V_o >_1 = e^{-j\Delta\Theta} < V_o >_{f_n,1} \tag{8}$$

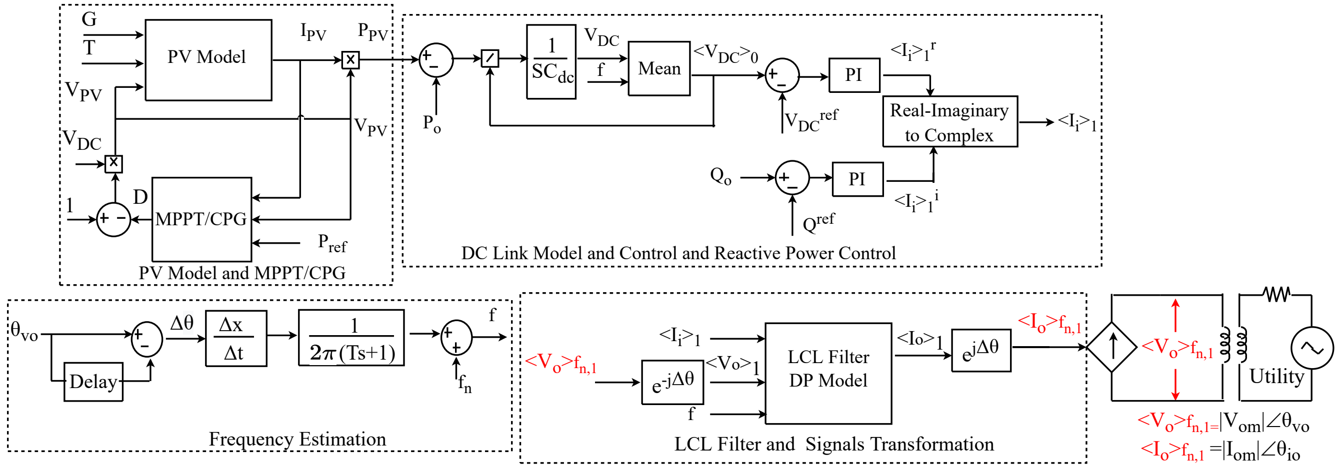

Fig. 3. Dynamic Phasor (DP) model of PV smart inverter.

IV. Numerical Studies

In this section,the efficiency and accuracy of the proposed DP model of a 5 kW two-stage single-phase smart PV system is evaluated through three different case studies and comparing its results with ones from detailed and phasor models developed in [12]. It should be mentioned that the phasor model [12] is developed based on the algebraic equations of fundamental frequency phasor of the AC states. Therefore, it is a fast model able to capture volt-var and volt-watt dynamics; while it fails to work under off-nominal frequency conditions. The specification of the PV system are taken from [12]. Settings of the volt-var, volt-watt and frequency-watt droops shown in Fig. 1 are provided in Table I.

TABLE I
VOLT-VAR, VOLT-WATT AND FREQUENCY-WATT DROOP SETTINGS

Droop type	Parameter	Value
volt-var	V_N Q_{max}, V_1,V_2 Q_{min}, V_3,V_4 P_{min}, V_2	277 (V) 2.2 (kVAr), 0.92V_N, 0.98V_N -2.2 (kVAr), 1.02V_N, 1.08V_N 0, 1.1V_N
volt-watt	P_{max}, V_1 P_{min}, V_2	5 (kW), 1.06V_N 0, 1.1V_N
frequency-watt	P_{max}, P_{min} Droop Slope	5 (kW), 0 0.05

A. Low Voltage Ride-Through (LVRT) in Volt-VAr Mode

According to the IEEE 1547, LVRT capability means that a smart inverter continues injecting current even when the voltage drops into the low voltage ride-through operating region (i.e., 0.5-0.88 p.u). Fig. 4 depicts the PV system performance during low voltage event and normal voltage range. The grid voltage is varied according to Fig. 4(a). As shown in Fig. 4(b), the smart inverter is injecting reactive power when the voltage drops. Conversely, it absorbs reactive power when the voltage rises. This case study verifies the proper LVRT performance and volt-var functionality of the designed smart PV inverter using the DP model. It also represents that the results from the proposed DP model and phasor model closely match the ones from the detailed model. To quantify the accuracy, the relative error of reactive power between DP and phasor model with respect to the detailed model are calculated. The maximum

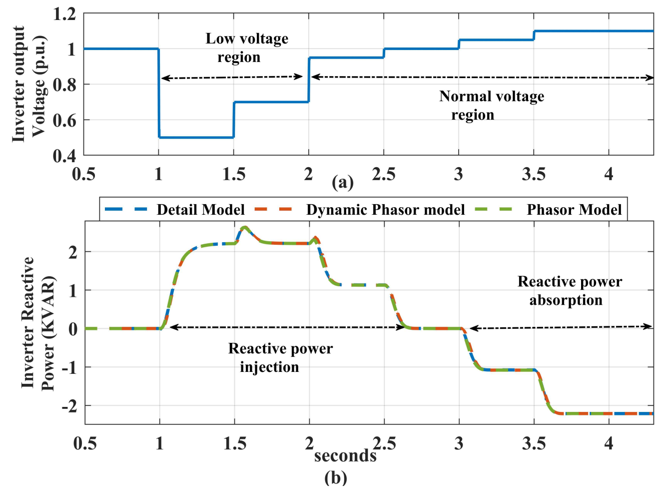

Fig. 4. Low Voltage Ride-Through: (a) grid voltage, (b) reactive power.

relative error between DP and detailed model during transients is 2.1×10^{-3}%, while it is 7.5×10^{-3}% between phasor and detailed model. Although these error values are negligible, it shows better accuracy of DP model.

B. Volt-Watt Performance

In this case the smart PV system's performance in volt-watt operation mode is evaluated, and the performance is shown in Fig. 5(a) and (b). At 1 s, when the bus voltage rises from 1 p.u., the active power is curtailed to the values obtained by the volt-watt droop. Furthermore, Fig. 5 (b) shows the good agreement between active power from DP model, phasor model and detailed model. To make it clearer, active power response's maximum relative error between DP model and detailed model is 7.6×10^{-4}%; while it is 1.1×10^{-3}% between phasor and detailed model.

C. Frequency Ride-Through (FRT) Capability

According to the IEEE standard 1547, FRT capability in the frequency-watt mode means that during over and under-frequency events (i.e., 61.2-62 Hz and 57-58.8 Hz), the smart inverter requires to stay connected to the grid and modulate the active power according to frequency-watt droop. Fig. 6(a) depicts over and under-frequency events. Pre-disturbance power

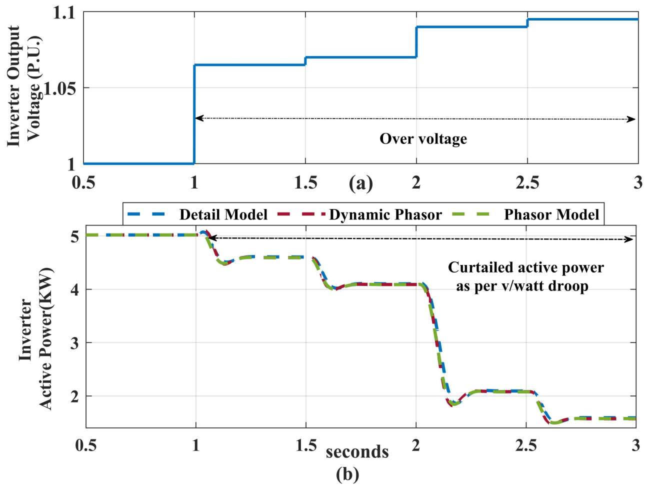

Fig. 5. Volt-Watt Performance: (a) grid voltage, (b) inverter active power.

is set to 35% to allow frequency-watt droop to increase the active power during under-frequency. It should be noted that since the phasor model [12] is unable to work under off-nominal frequency condition, only detailed model is used for the comparison purpose. Fig. 6(b) shows that the PV inverter reduces the active power during over frequency. Similarly, it increases the active power during under frequency and also rides through during mandatory operation regions highlighted in Fig. 6(b). The active power from the DP and detailed models match well, where the maximum relative error in transient is $4.5 \times 10^{-5}\%$.

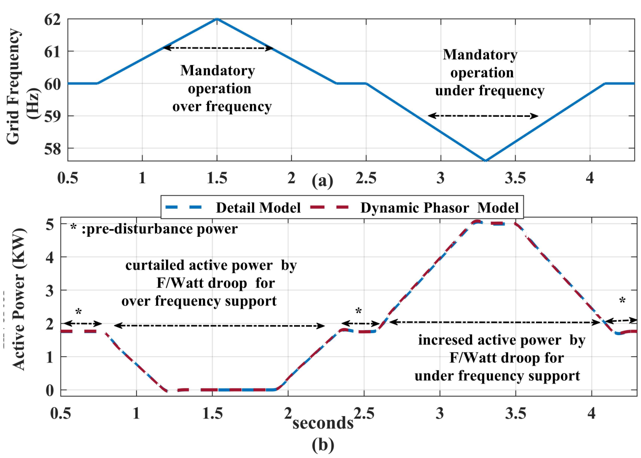

Fig. 6. FRT: (a) system frequency, (b) active power, (c) DC-link voltage.

D. Computational Efficiency

The execution time of LVRT and FRT case studies (for 4.3 s of simulation) for the detailed, DP, and phasor models is compared in Table II. The simulation is performed in MTLAB/Simulink R2019b using a PC with 2.9 GHZ CPU and 16 GB RAM. As summarized in Table II, DP model is more than thirty times faster than the detailed EMT model in both LVRT and FRT cases. On the other hand, the phasor model is almost two times faster than the DP model in LVRT case. Though the phasor model is computationally efficient, it has limitations that it is applicable for volt-watt and volt-var dynamics only and unable to work under off-nominal frequency. Therefore, the presented DP model can be regarded as a fast model able to capture both voltage and related dynamic of large-scale inverter-dominated distribution grids.

TABLE II
EXECUTION TIME OF DETAILED, DP, AND PHASOR MODELS.

Model	Step size, μs	Execution time of LVRT, s	Execution time of FRT, s
Detailed	1	266.2	266.5
Dynamic Phasor	100	8.3	8.4
Phasor	100	4.2	—

V. CONCLUSION AND FUTURE WORK

This work developed computationally efficient and accurate dynamic phasor (DP)-based model of smart PV inverters. The DP model is evaluated in terms of accuracy and speed using comparison of its results with the one from the detailed and phasor model models for various scenarios including LVRT, volt-watt, and FRT events. The results show significantly fast performance of the proposed DP model, while sufficiently capturing necessary volt-var, volt-watt and frequency-watt dynamics. It was also shown that the phasor model is a fast model while it can not capture frequency-related dynamics, thus the phasor-based model is limited in application. As the future work, the application of the proposed DP model on dynamic simulation of large-scale distribution feeder with high penetration of PV inverters will be studied.

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