Classical and Quantum Gravity

PAPER

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To cite this article: Antonios Kontos and [illegible] 2023 *Class. Quantum Grav.* **40** 155014

IOP Publishing Classical and Quantum Gravity
Class. Quantum Grav. 40 (2023) 155004 (11pp) https://doi.org/10.1088/1361-6382/ac

Clipping losses from a piecewis mirror

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Received 18 March 2023; revised 23 May 2023
Accepted for publication 6 June 2023
Published 29 June 2023

Abstract
The properties of optical coatings play an import ments. Gravitational-wave detectors not only rec ings but also ones with relatively large diameters ous material can be scaled up easily but for now exh noise characteristics. Crystalline coatings sho thermal noise properties but cannot easily scale this paper we explore the possibility of a piecewi amorphous and crystalline material. Specifically losses of such a piecewise coating as a function o face between the two coatings. The calculation sho to the total losses, as other surface imperfectio the final result, but are not considered here. Fina of the scattering losses from a piecewise coating of realizing such a design.

Keywords: crystalline coating, thermal noise, gravitational wave detector, clipping losses, scattering

(Some figures may appear in colour only in the onlin

1. Introduction

Gravitational-wave (GW) detectors use highly reflective to detect space-time metric [1, 2]. These HR mirrors are usually made ou polished low-absorption substrate with either an amorp ing placed on its surface [3–6]. For the purposes of GW detectors, coatin exhibit exceptionally low optical losses, namely absorp

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the mechanical loss of the coating material is a crucial p thermal noise characteristics [3, 7]. This is coating thermal noise (CTN rently a significant noise source for GW detectors at the [8– 10]. On the other hand, scattering and absorption on the t impact in both the sensitivity of the detectors and their [11–13].

The Laser Interferometer Gravitational-wave Observat vatory use amorphous coatings made with alternating lay (Ta_2O_5) and low index (SiO_2) material. Efforts to find amorphous material t current mirrors have resulted in interesting new possib ity of the required specifications [14–20]. To date, AlGaAs/GaAs, a crystallin the best CTN characteristics [4], where it meets the required noise levels, ar losses. Crystalline coatings however may need to addres gent noise or increased scattering and absorption [21, 22]. crystalline coating that can currently be fabricated is crystalline coatings to the size required for current (such as the Cosmic Explorer [23] or Einstein Telescope [24] (mirrors of 35 cm radius) is n trivial. To take advantage of the low thermal noise of cry amorphous coatings, one possibility is to fabricate a pi from crystalline material, while the outer part of the mi Assuming the beam power is mostly contained in the inner p will be determined by the crystalline coating propertie that the optical losses from clipping are in principle m on the properties of the coatings at the interface and how Here we perform an analytical calculation that relates t surface mismatch at the interface of the two coatings. Ev

In section 2, we present the basis of the calculation, which i culation [25]. Section 3 describes the considered geometry of the section 4, we show how the standard clipping calculation can b that would result from a mismatch in the interface of the losses of a TM00 beam on a piecewise mirror, while in section 5, section 6 we presented perform a similar calculation for the TM10 mode, which mi 7 details an experimental measurement of the scattering f to explore further challenges that might arise from this

2. Standard clipping calculation

When a single-mode beam reflects off a mirror with finite s lost by scattering to other modes. This is often describe size of a mirror are calculated from the overlap integra and a clipped beam. The integral can be performed numeric solutions offer efficiency, convenience and accuracy, es is to explore a given parameter space, as is the case here. and equations [25]. Here we reproduce much of the calculation f the reader is encouraged to refer to [25] for more details. It is important to note assumes axially symmetric geometry and therefore only us impact of breaking the assumption of axial symmetry in t discussed in section 5.

Suppose a beam that is in eigenmode :

$$\Psi(\ ,\ ,\ ) = \Psi(\ ,\ ,\ ) \qquad (1)$$

where $\Psi(\ ,\ ,\ )$ is the field at point $(\ ,\ ,\ )$. The clipped beam can then be modeled

$$\Psi_{\mathrm{clip}}(\ ,\ ,\ ) = \Psi(\ ,\ ,\ ) \times \ _0(\ ,\ ;\ ). \qquad (2)$$

where $_0(\ ,\ ;\ )$ represents a mask that is zero for all the points out... case where the beam is centered on the mirror with radius :

$$_0(\ ,\ ;\ ) = \begin{cases} 1 & ^2 + \ ^2 \leqslant \ ^2 \\ 0 & \text{elsewhere} \end{cases} \qquad (3)$$

The modal decomposition of this clipped beam is given by

$$\Psi_{\mathrm{clip}}(\ ,\ ,\ ) = \sum \ \Psi(\ ,\ ,\ ) \qquad (4)$$

where $\Psi(\ ,\ ,\ )$ again are the LG modes, and the coefficients are giv...

$$= \iint \Psi^*(\ ,\ ,\ )\Psi_{\mathrm{clip}}(\ ,\ ,\ )\,\mathrm{d}\ \mathrm{d}\ . \qquad (5)$$

A more convenient notation, which is to define the coefficient of the LG mode, of the ... assuming that the ... orthogonal. We then have:

$$= \ = \iint \Psi^*(\ ,\ ,\ )\Psi(\ ,\ ,\ )\ _0(\ ,\ ;\ )\,\mathrm{d}\ \mathrm{d}\ . \qquad (6)$$

Inserting an LG mode Ψ and noting that the LG modes are really ... by 2 indices, Vajente performed the integral analytically ... overlap from the input LG mode to the output LG mode

$$_{,} = \delta \sqrt{!\,!(\ +|\ |)!(\ +|\ |)!} \sum_{=0}^{\min(\ ,\ )} \frac{\rho^{2|\ |+4+2}\ _1 \ _1\left[\begin{matrix} +\ +|\ |+1 \\ |\ |+2+2 \end{matrix}\right](-\rho^2)}{!(\ -\ )!(\ -\ )!(|\ |+\ )!(|\ |+2+1)!} \qquad (7)$$

where $\rho = \sqrt{2}\ /\ $, with the beam width on the mirror. For example, t... losses of a TEM_{00} mode on a finite size mirror, one needs to calculat...

$$= 1 - |\ _{0,00}|^2. \qquad (8)$$

3. Piecewise mirror geometry

Figure 1 shows the basic geometry of the two-coating mirror ... AlGaAs/GaAs coating, and radius of the entire mirror. Given the ... the material and the difficulties in the production of su... mismatch between the levels of the surfaces of the two co... as well as a radial separation between the two coatings, ... referred to as mismatch and δ mismatch.

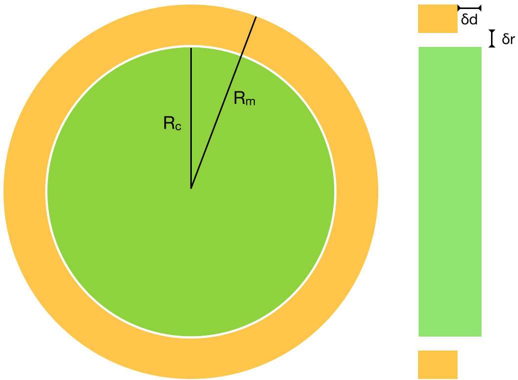

Figure 1. Face (left) and side (right) view of the concentric
ring is the amorphous coating, while the inner circle is
ring in between the two coatings represents the substra

Aside from fabrication challenges, the surface mism
mirror transmission and coating-stack design. Assumi
$Al_{0.92}Ga_{0.08}As$/GaAs coating will always be thicker than the Ta_2O_5/SiO_2 coating, due to the Ti:T
smaller index of refraction difference of $Al_{0.92}Ga_{0.08}As$ and GaAs ($\approx 3.48 - 3.00$) between Al
048) and between Ta_2O_5 and SiO_2 ($\approx 2.06 - 1.45 = 0.61$). Specifically, the total th
ness of a mirror with 5 ppm transmission (typically used
detector cavity arm), will be 6684 nm for the crystallin
amorphous coating will be $6060 = 623$ nm, for a total mismatch,
the clipping losses for such a large mismatch would be pr

This means, that the coating stack design will need to
types have the same thickness within a few nanometers, a
the case of 5 ppm transmission and a quarter-wave stack
the transmission of the amorphous coating so that its th
That happens for $T_{amorphous} = 1.23$ ppm. Assuming that this is a cavity end
mismatch in transmissions might be manageable. This ap
rors with higher desired transmission, or cavity input
in this paper can be used to determine the acceptable thi
coatings.

4. Mismatch loss calculation

To perform the calculation of the clipping losses for th
ent mask than the one used in the clipping calculation ab
described in figure 1 where:

$$(\, , \,) = \begin{cases} 1 & {}^2 + {}^2 \leqslant {}^2 \\ 4\pi \; /\lambda & {}^2 < {}^2 + {}^2 \leqslant (\; + \delta \;)^2 \\ 4\pi\delta \; /\lambda & (\; + \delta \;)^2 < {}^2 + {}^2 \leqslant \\ 0 & {}^2 + {}^2 > \end{cases} \quad (9)$$

where we assume that the effect of a surface mismatch δ is to add $4\pi\delta/\lambda$ phase to the mismatch beam, which accounts for the round trip phase delay. Thi for small δ. In addition, the mask includes a partially refec with parameter $= 0.2$, the refectivity of an interface, the $_2$/SiO thickness of the crystalline coating. To simplify the presentation o we fix at the value that gives average losses, and note that due to this parameter is approximately 2%. Also note, that the re a coating also introduces a phase, which is only zero at , deviates from that for other transmission values. We don small for our calculations, and is equivalent to less th mismatch.

To take advantage of the analytical formulas derived by function in terms of $_0$, the mask function clipping calculation see that the mask for this coating shown in figure 1

$$(,) = {}_0(, ;) + {}^{4\pi\delta/\lambda}({}_0(, ;) - {}_0(, ; + \delta)) \qquad (10)$$

$$+ {}^{4\pi/\lambda}({}_0(, ; + \delta) - {}_0(, ;)). \qquad (11)$$

Now we can take advantage of equation (7) to find the different mask:

$$_{,} = {}_{,}( ) + {}^{4\pi\delta/\lambda}[{}_{,}( ) - {}_{,}(+ \delta)] \qquad (12)$$

$$+ {}^{4\pi/\lambda}[{}_{,}(+ \delta) - {}_{,}( )]. \qquad (13)$$

Using equations (7), (8) and (7) we can now calculate the total losses for a wise mirror.

5. Results for TM00 input

Assuming that the cavity mode is $= 6.2$ cm, TM00 with maximum crystalline coating radius $= 10$ cm, and the total $= 17$ cm (the radius is the radius of the LIGO mirrors), we calculate the total losses due to th both the radial mismatch (δ) and the surface mismatch (δ). This is shown in figure 2. The - and - axis is in the figure are in terms of the beam wavelengt lines correspond to 20 ppm increments in total losses. T the total losses are much more sensitive to the surface m mismatch. In order to keep total losses lower than 100 ppm, δ must be kept smaller than about 10 nm. On the other hand, similar l of the order of 1 mm. Note that without the outer amorphous coatin crystalline coating radius and 1 mm width, the total los 1000 ppm.

The above calculation assumes that the beam is perfectl wise mirror. To understand the impact of a strong misalig of a TM00 mode with $\Delta = 2$ cm offset, as a function of the radial and We found that the overall trends are similar, only the los amount of offset.

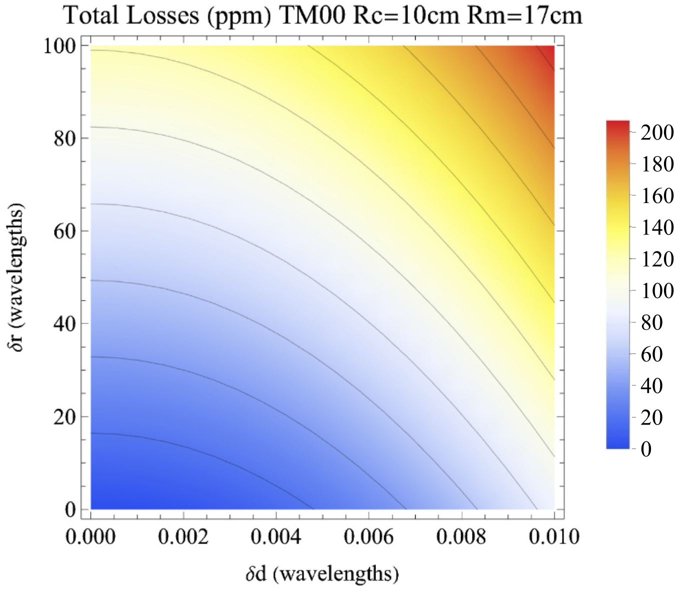

Figure 2. Total losses for a TM00 resonating mode as a functio mismatch parameters. The axes in the figure are in terms of the beam (1064 nm), and the contour lines correspond to 20 ppm in

6. Results for TM10 input

It is interesting to consider the effect of the mismatch mode resonates in the cavity. In this case, the calculate two-coating interface coincides with a minimum of the mo TM10 LG mode. The results and conclusions can be easily ap

First, we calculate the total losses as a function of t TM00 and TM10, keeping the mirror radius constant at $R_m = 17$ cm, radial mismatch at $\delta r = 10$ µm, and surface mismatch $\delta d = 0$ µm. Figure 3 shows that indeed there is a m imum for the losses at the intensity minimum for the TM10 width of $R_c = 4.4$ cm. Of course, this is true for the case where the mirror. As we detail in 5, any mismatch will increa losses and eliminate the minimum. The total losses for the $R_c = 4.4$ cm are elevated and never reach zero because the TM10 mode e ond the mirror radius of a LIGO mirror. These are depicted simply $\delta = 0$. A slightly greater mirror radius of 18 or 19 cm e clipping losses for the TM10 losses.

We replicate figure 2 for TM10, only for $R_c = 4.4$ cm. Any other radius is unin teresting since it will definitely result in worse perfor of the shape of the mode. The results shown in figure 4 indicate that by matc the inner coating radius to the intensity minimum causes reasonable radial mismatch values. However, the surface phase mismatch now integrates over a much larger area of radial mismatch has become less relevant, the surface mi if we want the total losses lower than 200 ppm.

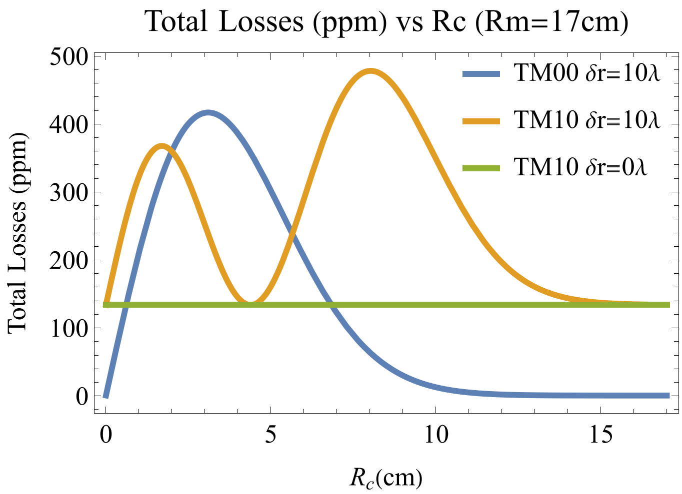

Figure 3. Total losses for TM00 and TM10 as a function of the cr , keeping the mirror radius constant at = 17 cm, radius of ... = 10 μm, ... and surface mismatch at $\delta m = 0\,\mu$m.

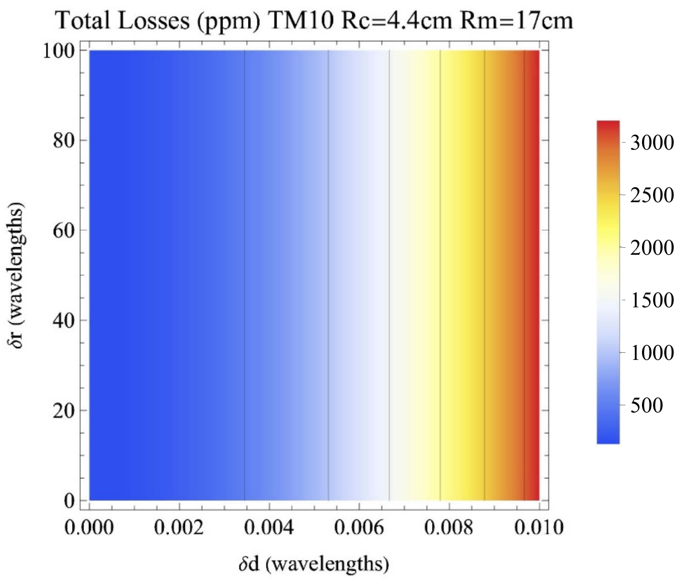

Figure 4. Total losses for a TM10 resonating mode as a functio mismatch parameters. For this calculation, the radius o to match the intensity minimum of the TM10 LG mode. The x are in terms of the beam wavelength (1064 nm), and the c 500 ppm increment in total losses.

To enable a more direct comparison between the TM00 and T the calculation shown in figure 4, we increased the mirror radius to b so that we eliminate the clipping losses due to the finite

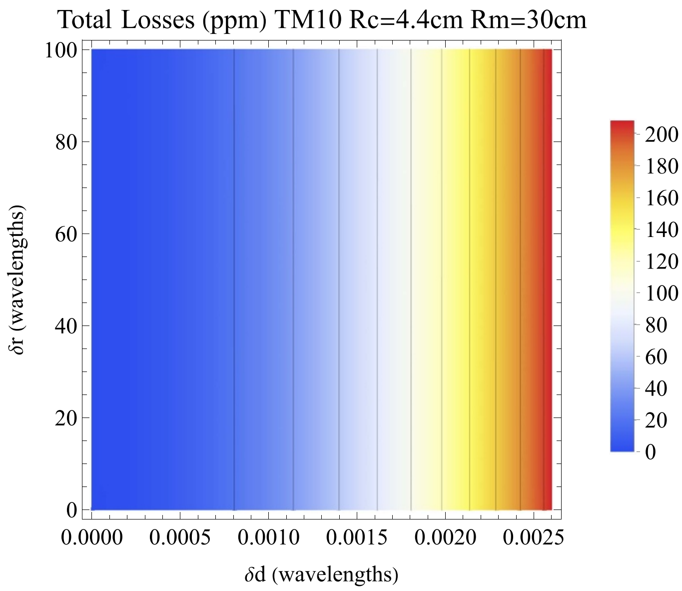

Figure 5. Total losses as a function of the mismatch paramet ing mode. As in figure 4, the radius of the crystalline coating is c intensity minimum of the TM10 LG mode. However, the rad so we eliminate the effect of the mirror size to the los been shortened compared to figure 4, so that the total losses do not exceed facilitate comparison with figure 2.

figure 5, in which the x-axis scale has been adjusted so that the total ppm. This allows for a more direct comparison between figures 2 and 5.

7. Scattering measurements of piecewise sample

The above calculations rely on various idealized assumpt limit to the losses expected if such a mirror is used. To s arise, we measured the scattering losses from two protot was half coated with amorphous Ta_2O_5/SiO_2 and half with crystalline ma (AlGaAs/GaAs), as shown in figure 6. The geometry of the piece is different tha one considered in the calculations, but the objective i rather to reveal potential additional issues at the inte the two coatings μm, much larger than the surface (200) and further studi samples with smaller separation.

The scattering measurements were performed in [26]. The samples with these were placed in two orientations, horizontally and verti on the interface. A camera recorded the scattered light a angles. The right panel of figure 6 shows an image of one of the samples taken f° from the normal. We can see from the scattering image that the the rest of the sample. It is clear that the scatter is do have a density of $1/100\,\mu m^{-1}$. Further analysis of the two orientati the amorphous side scatters more light, though this coul

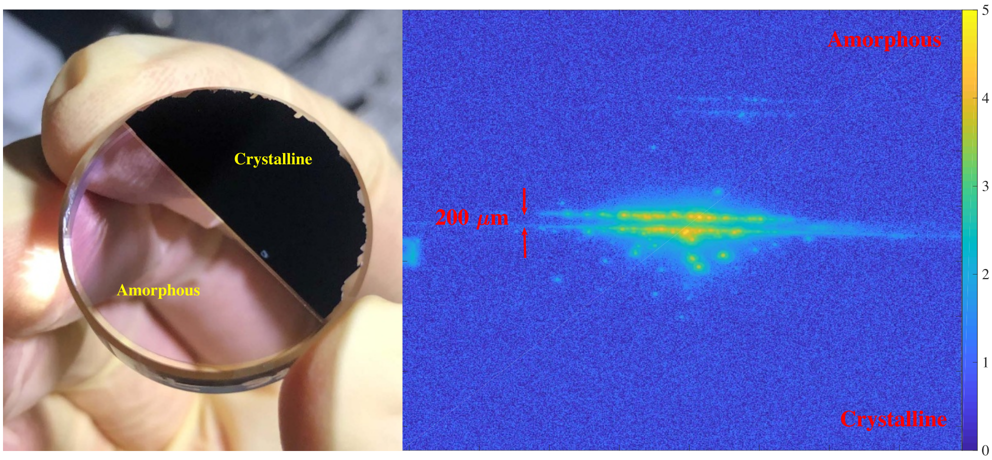

Figure 6. Left: A photo of one of the prototype mirror samples study. The amorphous coating consisted of Ta_2O_5/SiO_2 layer pairs, while the crystalli of AlGaAs/GaAs layer pairs. Right: A scattering image t The two edges at the interface is visible, as well as mul beam on target is 1.6 mm. The color scale is logarithmic

While the experiment does not measure the total integr of magnitude estimate by integrating the power over the the integrated loss was 1400 ppm, 14%. It is important to note that there i muthal asymmetry to the scattering pattern. These losse point defects and roughness, and not due to the clipping

We can translate this measurement to an estimate of the s were present in the interface [1]. For the clipping, we normalize our measur account for the different intensities in the two scenar approximately 1400 ppm. The comparison between this meas ping losses gives a broader perspective of the challenge

8. Conclusions

We calculated the effect of a mismatch in a piecewise two the total losses are very sensitive to a mis-match between direction). Specifically, if we require losses lower than be greater than about 10 nm. While this might present a fab an important consideration when designing the mirror sta to have nearly identical thicknesses. On the other hand, and can be as great as [illegible] µm. Additionally, using higher order LG mod take advantage of the intensity minima of the coatings do match is more important than the radial mismatch. By intr a TM00 beam and the coating, we estimated that the losses ative significance of the surface and radial mismatch rem measurements on a prototype piecewise mirror show that a located on the edges of the interface, and for this appro addressed. Therefore, these clipping calculations shou

Data availability statement

All data that support the fndings of this study are incluc
mentary fles).

Acknowledgments

The authors would like to thank Steve Penn from Hobart & Wi
discussions, and Garrett D Cole from Thorlabs for provid

Funding

Bard Research Fund; Bard Summer Research Institute.

Conflict of interest

The authors declare no confdicts of interest.

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References

[1] Aasi J (The LIGO Scientifc Collaboration) 2015 Advanced LIG
32 074001
[2] Acernese F 2014 Advanced virgo: a second-generation interfer
detector 32 024001
[3] Steinlechner J 2018 Development of mirror coatings for grav
A 376 20170282
[4] Cole G D, Zhang W, Martin M J, Ye J and Aspelmeyer M 2013 Tenfo
noise in high-refectivity optical coatings 7 644–50
[5] Cole G D 2016 High-performance near- and mid-infrared crystal 3 647–56
[6] Winkler G 2021 Mid-infrared interference coatings with exces
8686–96
[7] Reid S and Martin I W 2016 Development of mirror coatings fo
661
[8] Buikema A 2020 Sensitivity and performance of the advanced l
observing run D 102 062003
[9] Acernese F 2018 Status of advanced virgo 1802.03
[10] Martynov D V 2016 Sensitivity of the advanced ligo detectors at t
wave astronomy D 93 112004
[11] Brooks A F 2021 Point absorbers in advanced LIGO 60 4047–63
[12] Billingsley G, Yamamoto H and Zhang L 2017 Characterization
(LIGO DCC) p P1700029 (available at https://dcc.ligo.org/LIGO-P17
public)
[13] Soni S 2021 Reducing scattered light in LIGO’s third observi
38 025016
[14] Birney R 2018 Amorphous silicon with extremely low absorptior
gravitational astronomy 121 191101
[15] Tait S C 2020 Demonstration of the multimaterial coating conc
gravitational-wave detectors 125 011102

[16] Granata M 2016 Mechanical loss in state-of-the-art amorphous ... D 93 012007
[17] Cumming A V 2015 Measurement of the mechanical loss of prototyp... coatings for future gravitational wave detectors ... 32 035002
[18] Piergiovanni F 2020 The research on amorphous coatings for f... 14 6081 2216
[19] Vajente G 2021 Low mechanical loss TiO_2:GeO_2 coatings for reduced thermal noise ... itational wave interferometers ... 127 071101
[20] Granata M 2020 Amorphous optical coatings of present gravitat... 37 095004
[21] Yu J 2022 Excess noise in highly reflective crystalline mirro... (2210.15671)
[22] Cole G D 2023 Substrate-transferred GaAs/AlGaAs crystalline ... detectors 122 110502
[23] Evans M 2021 A horizon study for cosmic explorer: science, ob... (arXiv:2109.09882)
[24] ET Editorial Team 2020 Design report update 2020 for the Eins... https://www.gwoptics.org/research/et/ET-0007B-20_ETDesignRepo...
[25] Vajente G 2022 Analytical expressions for the clipping of la... circular apertures A 39 352–7
[26] Kontos A, Loglia B, King B and Dziubelski N 2021 Near speckle-... studies of mirror coatings 29 40122–3