

# Hybrid FES-Exoskeleton Control: Using MPC to Distribute Actuation for Elbow and Wrist Movements

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## 2 ABSTRACT

3 Individuals who have suffered a cervical spinal cord injury prioritize the recovery of upper  
4 limb function for completing activities of daily living. Hybrid FES-exoskeleton systems have  
5 the potential to assist this population by providing a portable, powered, and wearable device;  
6 however, realization of this combination of technologies has been challenging. In particular, it  
7 has been difficult to show generalizability across motions, and to define optimal distribution of  
8 actuation, given the complex nature of the combined dynamic system. In this paper, we present a  
9 hybrid controller using a model predictive control (MPC) formulation that combines the actuation  
10 of both an exoskeleton and an FES system. The MPC cost function is designed to distribute  
11 actuation on a single degree of freedom to favor FES control effort, reducing exoskeleton power  
12 consumption, while ensuring smooth movements along different trajectories. Our controller was  
13 tested with 9 able-bodied participants using FES surface stimulation paired with an upper limb  
14 powered exoskeleton. The hybrid controller was compared to an exoskeleton alone controller,  
15 and we measured trajectory error and torque while moving the participant through two elbow  
16 flexion/extension trajectories, and separately through two wrist flexion/extension trajectories. The  
17 MPC-based hybrid controller showed a reduction in sum of squared torques by an average of  
18 48.7% and 57.9% on the elbow flexion/extension and wrist flexion/extension joints respectively,  
19 with only small differences in tracking accuracy compared to the exoskeleton alone controller.

20 **Keywords:** functional electrical stimulation, upper limb exoskeleton, model predictive control, hybrid control, movement assistance

## 1 INTRODUCTION

21 There are approximately 291,000 people in the United States living with spinal cord injuries, and the  
22 majority of these are cervical level injuries, resulting in tetraplegia (NSCISC, 2019). Injuries at such a  
23 high level of the spinal cord create severe arm and hand disabilities, resulting in an inability to complete  
24 Activities of Daily Living (ADLs). As a result, 71% of individuals with tetraplegia currently require  
25 assistance with ADLs (Collinger et al., 2013). Given this, it is not surprising that restoration of arm

26 and hand function is a top priority among people with tetraplegia due to cervical spinal cord injuries  
27 (SCI) (Anderson, 2004). With scarce rehabilitation and assistive technology options, these individuals are  
28 largely dependent on full-time caregivers for feeding, grooming, and many other activities of daily living.  
29 Regaining the ability to perform these tasks independently will reduce requirements on caregivers and  
30 increase opportunities for individuals to return to social participation in their communities, both of which  
31 are highly correlated to quality of life (Dijkers, 1997).

32 Recovery of arm and hand function through rehabilitation can be achieved for individuals with some  
33 residual muscle capability (Dietz et al., 2002; Beekhuizen and Field-Fote, 2005), and there are promising  
34 results that show that the same intensive robotic rehabilitation that has been successful for inducing  
35 plasticity and recovery following stroke (Reinkensmeyer et al., 2000; Charles et al., 2005; Blank et al.,  
36 2014; Lum et al., 2012) can be effective for SCI (Kadivar et al., 2012; Fitle et al., 2015; Francisco et al.,  
37 2017; Yozbatiran and Francisco, 2019; Frullo et al., 2017). For those *without* residual motor capability,  
38 however, or for those for whom rehabilitation interventions have not been able to restore functional  
39 movement, assistive technologies are a more viable option for replacing lost function. Such approaches  
40 incorporate mechanical devices that are attached to the limb and have the capability to move the limb  
41 or hand, or approaches that electrically stimulate the existing muscles, causing muscle contraction and  
42 inducing motion of the upper limb.

43 Functional electrical stimulation (FES) is a promising assistive technology to restore arm and hand  
44 function. By activating a person's own paralyzed muscles via surface electrodes placed on the skin or  
45 surgically implanted electrodes, limb movements can be generated. This approach requires very low energy  
46 consumption and exhibits high embodiment by the person; however, FES cannot produce sufficient torques  
47 to enable whole-arm reaching movements in people with tetraplegia, as many muscles are unresponsive to  
48 FES (Mulcahey et al., 1999; Peckham et al., 1976). Further, general multi-joint motions are notoriously  
49 hard to control with FES even with the most advanced systems (Ajiboye et al., 2017), often resulting in  
50 fine-tuned feed-forward implementations due to the physiological delays in muscle response to applied  
51 stimulation, and difficulty in accurately modeling the response to muscle activation. Augmenting FES with  
52 an assistive robot offers additional torque to support whole arm reaching while also offering improved  
53 movement accuracy, but this comes at the expense of increased bulkiness and decreased wearability of the  
54 combined FES-robotic system. An optimal combination of FES and an assistive robot would maximize the  
55 contribution of FES to minimize size and power requirements of the robot (Dunkelberger et al., 2020).

56 This combination of FES with robotic devices is starting to gain traction, and is termed hybrid FES-robot  
57 (or FES-exoskeleton) control. A conceptual representation of using FES with a robot is shown in Figure  
58 1, where both robotic and FES action can complement each other to assist in the completion of activities  
59 of daily living. Many of the early approaches to bring this concept to reality did not truly combine and  
60 coordinate the actuation strategies for upper limb movements (Dunkelberger et al., 2020). Instead, each of  
61 the actuation types was used to achieve separate functions. For example, robotic devices have been used to  
62 lock degrees of freedom (Klauer et al., 2014; Ambrosini et al., 2017) or as gravity compensation (Cannella  
63 et al., 2016) enabling the muscles to relax and preventing fatigue. Other works have used robotic support  
64 devices to actuate one set of degrees of freedom, while FES is used to actuate another set (Ajiboye et al.,  
65 2017; Schulz et al., 2011; Varoto et al., 2008). Typically the robot controls motions that need precision or  
66 require larger torques and forces to support, such as elbow flexion and extension, while FES is used for  
67 coarse movements, such as grasping. For upper limb motions with coupled degrees of freedom, such as  
68 shoulder, elbow, and wrist movements, these existing control strategies pit FES against a robot-imposed  
69 locked-joint, gravity, or single-joint motion constraint, essentially wasting the free actuation from FES and

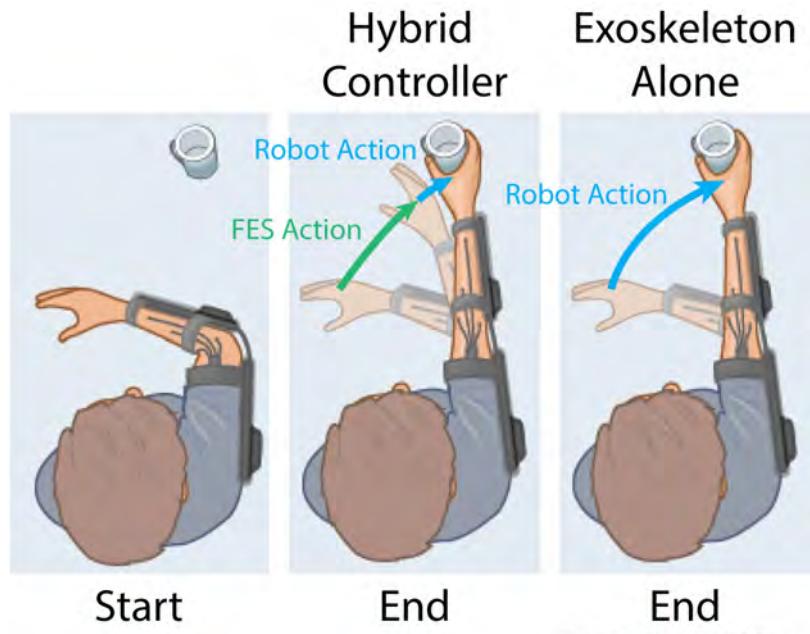


Illustration by Cleveland FES Center

**Figure 1.** An example future application of hybrid systems is shown for a reach and grasp task. The incorporation of both FES and a robot allows for a large portion of the movement to be provided by FES, and the robot can provide small amounts of power to provide minimal movement corrections. With the robot alone, all power for the movement must be provided by the robot.

70 transferring it to the robot. Recently, single-joint hybrid systems that do share actuation on the same joint  
 71 have been explored, but research has been limited, testing only in the elbow flexion extension joint with  
 72 biceps electrodes in a minimum jerk trajectory (Wolf et al., 2017; Burchielli et al., 2022), or in simulation  
 73 (Bardi et al., 2021).

74 In lower limb applications, more advanced hybrid control algorithms have been explored, largely enabled  
 75 by the repetitive nature of gait motions (Ha et al., 2016; Bulea et al., 2014; del Ama et al., 2014). These  
 76 lower limb hybrid systems often use a version of iterative learning control that takes advantage of the  
 77 repetitive movements to fine-tune control over several cycles. Some recent research has begun to use model  
 78 predictive control (MPC) algorithms, which can be more readily adapted to non-cyclic movements in the  
 79 lower limbs (Kirsch et al., 2018; Bao et al., 2021), and which are more similar to the non-cyclic movements  
 80 required of upper-limb movements. Results from these studies using MPC have shown the ability to follow  
 81 a step reference trajectory and hold a position, and the algorithms should generalize to arbitrary trajectories.

82 A truly shared approach for hybrid FES and robotic control of upper limb reaching movements is needed  
 83 to combine these techniques in a manner that achieves generalized upper limb movement assistance in  
 84 an optimal manner. In this paper, we present a model-based control approach to hybrid FES-exoskeleton  
 85 control. Recent works have demonstrated the first steps towards this vision. Wolf et al. demonstrated the  
 86 use of model-based algorithms to power FES in combination with gravity compensation from a robot  
 87 (Wolf and Schearer, 2022). Our group has also demonstrated shared control of elbow flexion and extension  
 88 movements with FES and exoskeleton assistance acting in coordination to follow a desired trajectory  
 89 (Dunkelberger et al., 2022a). In that work, we showed that a model-based controller for our upper limb  
 90 exoskeleton, which has knowledge of the expected contributions of FES, requires significantly less robot  
 91 torque than a standard PD control algorithm, with minimal loss in trajectory following accuracy. Here,  
 92 we expand our initial demonstration along a number of fronts. First, we present an MPC algorithm that

93 removes the integral term used previously and incorporates an additional proportional-integral-derivative  
94 (PID) controller acting in parallel, resulting in improved performance in both trajectory following and  
95 reduction in torque requirements from the exoskeleton compared to our initial controller. We incorporate a  
96 sophisticated model of the user's arm dynamics that accurately captures behavior across the exoskeleton  
97 workspace. We experimentally demonstrate the performance of the hybrid FES-exoskeleton controller in  
98 able-bodied participants completing two trajectories for two degrees-of-freedom of the exoskeleton (elbow  
99 flexion-extension and wrist flexion-extension), and we compare the performance of the hybrid controller to  
100 an exoskeleton-alone case, as illustrated in Figure 1. Finally, we examine longitudinal performance of the  
101 hybrid FES-exoskeleton control for a subset of participants to determine how performance changes one  
102 week after the initial experiment trials.

## 2 MATERIALS AND METHODS

### 103 2.1 Participants

104 Nine able-bodied participants (4 female, avg age 22.9) participated in a single session of the experiment  
105 after providing informed consent. Three of the nine participants, who had experience with FES prior to the  
106 initial experimental session, also completed a second session of testing using the same protocol at least one  
107 week after their first experimental session. The study was approved by the institutional review boards at  
108 Rice University (IRB #FY2017-461) and Cleveland State University (IRB #30213-SCH-HS).

### 109 2.2 Procedure

110 The goal of this study is to develop a new hybrid controller that distributes actuation between an  
111 exoskeleton system and an FES system. The goal of such a controller is that it can reduce the power  
112 requirements in comparison to an exoskeleton alone system, which can lead to more portable devices in  
113 the future that can assist individuals with SCI in completing general activities of daily living. To test the  
114 effectiveness, the developed hybrid controller is used to provide movements on two different degrees of  
115 freedom (DOF), elbow flexion/extension, and wrist flexion/extension. To understand how this compares  
116 to available exoskeleton systems, the resulting torque and position profiles for the hybrid controller are  
117 compared with an exoskeleton-alone controller in following two different trajectories.

### 118 2.3 Materials

119 The hybrid FES-exoskeleton system is comprised of two main subsystems that provide actuation. The  
120 first subsystem, which provides FES, is a transdermic electrical stimulation system (Trier et al., 2001)  
121 which provides 8 output channels of bipolar stimulation. In this study, two channels are used for the elbow  
122 flexion/extension joint, and two channels are used for the wrist flexion/extension joint. To provide varying  
123 levels of output using the FES subsystem, the amplitude and frequencies are kept at a constant value for  
124 each channel, and the pulsedwidth is varied.

125 The second subsystem is the robot, the MAHI Open Exoskeleton (Dunkelberger et al., 2022b). This robot  
126 provides four DOFs of movement support, namely elbow flexion/extension, forearm pronation/supination,  
127 wrist flexion/extension, and wrist radial/ulnar deviation, and each of these joints line up with the equivalent  
128 anatomical degree of freedom of a person using the exoskeleton. These will also be referred to by joint  
129 number throughout this paper, which are joints 1-4 respectively. The exoskeleton has an adjustable  
130 counterweight to account for varying arm masses, an adjustable slider to account for varying forearm  
131 lengths, and an adjustable shoulder abduction angle to keep the participant comfortable. The counterweight

132 and forearm slider parameters are adjusted for each subject at the beginning of the experiment, and locked  
133 for the experiment duration. This shoulder abduction angle was kept at a value of 30° for all participants.

134 **2.4 Methods**

135 The study consists of several model characterization steps related to each of the subsystems, followed  
136 by experimental testing of the hybrid controller which makes use of these characterizations. First, the  
137 electrodes are placed in appropriate locations, and comfortable ranges of stimulation are found. Recruitment  
138 curves are characterized for each set of electrodes to define the relationship between commanded pulse  
139 width and muscle activation level. Gaussian process regression models are created to characterize torque  
140 output for each electrode based on the orientation of the upper-limb. The mass properties of the participant's  
141 arm are then characterized so that a combined dynamic model can be created for the arm-exoskeleton  
142 subsystem. The hybrid controller is created using the characterizations of each of the components. These  
143 characterization steps are more completely described in sections 2.4.1-2.4.5. The hybrid controller is then  
144 compared against an exoskeleton alone controller in a scenario of following two trajectories for each DOF.

145 In this study, the elbow flexion/extension and wrist flexion/extension DOFs are tested independently.  
146 Each of the experimental steps is performed with the elbow flexion/extension joint and corresponding  
147 electrodes, followed by the wrist flexion/extension joint with corresponding electrodes. The explanations  
148 that follow apply to both DOFs.

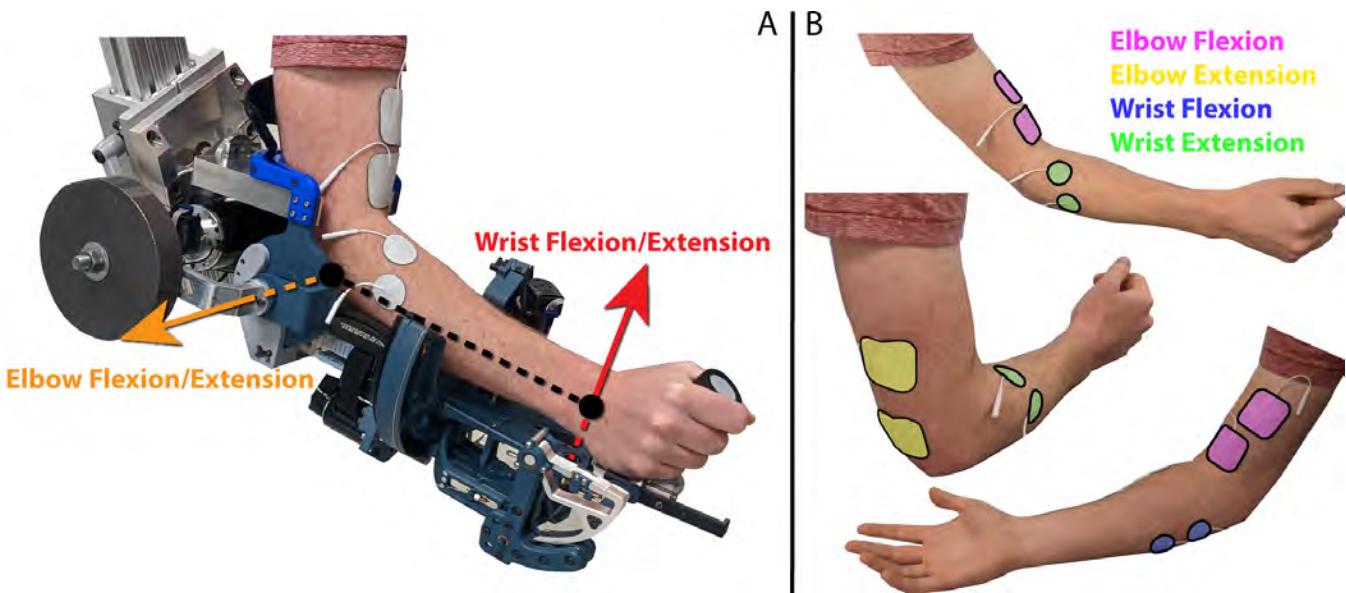
149 **2.4.1 FES Electrode Placement**

150 The experimental protocol began by placing the electrodes on the participants. Each of the electrode pairs  
151 were placed and tested one at a time. A set of electrodes was placed as agonist/antagonist pairs for each  
152 of the active degrees of freedom. This means for the elbow flexion/extension joint, one set of electrodes  
153 was placed to target elbow flexion, and another set was placed to target elbow extension using two inch  
154 square electrodes. For the wrist flexion/extension joint, one set of electrodes was placed to target wrist  
155 flexion, and another set of electrodes was placed to target wrist extension using one inch round electrodes.  
156 Electrode placement locations for each of these movements were chosen based on pilot testing based on  
157 which locations could reliably provide the desired movement. These general chosen locations are shown in  
158 Figure 2.

159 For the elbow flexion electrode placement, a reference electrode was placed, and a Compex motor point  
160 pen was used to find a specific point that generates biceps contraction, and the second electrode was placed  
161 there. For the remaining electrodes, the pair of electrodes were placed in a nominal location, and the pulse  
162 width was increased slowly. The resulting movement with the participant's arm on a table was observed,  
163 and the electrodes were adjusted if the desired movement was not produced. The electrodes were then  
164 wrapped with medical bandage to ensure that the electrodes stayed in the original location.

165 **2.4.2 Threshold Identification**

166 Once the electrodes have been placed, the minimum and maximum pulsedwidth values that will be used  
167 for each participant need to be identified. The robot and arm were moved to a neutral configuration, and  
168 held there using independent PD controllers on all joints. For each electrode placed, the minimum value  
169 that produced a change in torque output in the PD controller is considered the minimum pulse width value,  
170  $pw_{min}$ . The discomfort threshold is then found by increasing the pulsedwidth until the participant verbally  
171 indicates their maximum value which is still comfortable. The maximum pulsedwidth value used throughout  
172 the experiment,  $pw_{max}$ , is taken as a slight reduction from the discomfort threshold. A ramp from the



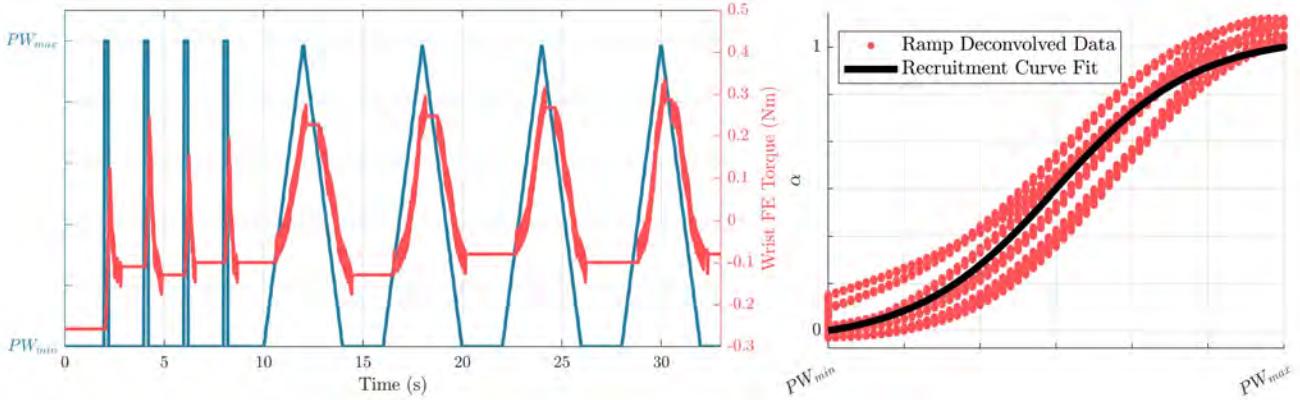
**Figure 2.** (A) A participant with their arm in the robot in the experimental setup, with the axes of rotation for the active joints indicated by orange and red arrows for the elbow flexion/extension and wrist flexion/extension joints respectively. (B) Placement of each of the four sets of electrodes. Electrodes were placed over the biceps for elbow flexion over the triceps for elbow extension. Electrodes were placed on the flexor carpi ulnaris for wrist flexion, and extensor carpi radialis longis and extensor carpi ulnaris muscles for wrist extension.

173  $pw_{min}$  to  $pw_{max}$  is then used to verify that the participant remains comfortable throughout the range, and  
 174 that the  $pw_{min}$  is just below the threshold of providing torque output.

#### 175 2.4.3 Recruitment Curve Characterization

176 With the thresholds defined, a mathematical representation between the pulsedwidth range and muscle  
 177 activation is found, defined as a recruitment curve. Previous research has shown that functional electrical  
 178 stimulation produces a muscle recruitment curve in the form of a sigmoid (Durfee and MacLean, 1989).  
 179 To characterize this recruitment curve, the robot is again moved to a neutral configuration, and held there  
 180 using independent PD controllers on each joint. Each of the electrodes sequentially performs four impulses  
 181 at  $pw_{max}$ , followed by four linear ramps between  $pw_{min}$  and  $pw_{max}$ , as shown in Figure 3.

182 The ramp deconvolution method is used (Durfee and MacLean, 1989) with the input of pulsedwidth values  
 183 and the corresponding torques generated from the stimulation to generate smooth curves to be characterized.  
 184 The sigmoid is then fitted using equation 3 with free parameters of  $c_1$  and  $c_2$ , where  $pw^*$  and  $pw_{max}^*$  are  
 185 defined as pulsedwidths normalized so that a  $pw^*$  value of 0, corresponds a  $pw_{min}$  as defined in equations 1  
 186 and 2.



**Figure 3.** (left) Profiles of commanded pulsewidths, and resulting torque outputs due to stimulation from the wrist extension electrode in the recruitment curve characterization process. (right) Resulting characterized recruitment curve in the form of a sigmoid based on the ramp deconvolved data.

$$pw^* = pw - pw_{min} \quad (1)$$

$$pw_{max}^* = pw_{max} - pw_{min} \quad (2)$$

$$\alpha^* = \frac{c_1}{1 + e^{-c_2(pw^* - \frac{pw_{max}^*}{2})}} - \frac{c_1}{1 + e^{\frac{c_2 pw_{max}}{2}}} \quad (3)$$

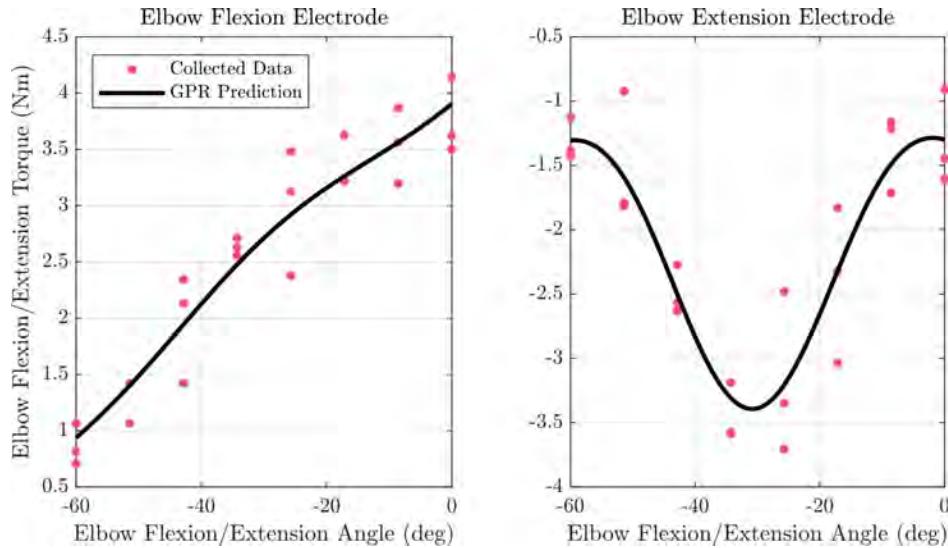
$$\alpha = \frac{\alpha^*}{c_1} \quad (4)$$

187 This equation results in a sigmoid with a minimum value of 0 and a maximum value of  $c_1$ . The term  $c_2$  is  
 188 related to the slope of the function as it crosses the midpoint. To turn this characterization into the standard  
 189 definition of a recruitment curve which varies from 0 to 1,  $\alpha^*$  is divided by  $c_1$  to arrive at an equation for  
 190 activation,  $\alpha$ .

#### 191 2.4.4 Gaussian Process Regression Model Creation

192 The last component needed to mathematically represent the FES subsystem is a representation of the  
 193 torque output based on the arm joint configuration of the participant. A Gaussian Process Regression (GPR)  
 194 model is used to characterize this relationship torque when each of the FES electrode pairs is at a maximum  
 195 activation as a function of the arm configuration. In this case, the black-box representation of the GPR  
 196 models also implicitly capture some of the complex muscle dynamics. For each of the degrees of freedom,  
 197 eight evenly spaced positions are taken between the minimum and maximum values that each joint will see  
 198 throughout the experiment. At each of these positions, PD controllers on each of the individual robot DOFs  
 199 are used to keep the robot at the desired position. The exoskeleton torque required to hold the pose when no  
 200 muscles are stimulated is recorded as  $\tau_{passive}$ . One electrode is increased to its maximum activation, and  
 201 the exoskeleton torque required to hold that pose is recorded as  $\tau_{hold}$ . We consider the difference between  
 202 the two values as the torque produced by the electrode  $\tau_{record}$ .

$$\tau_{record} = \tau_{hold} - \tau_{passive} \quad (5)$$



**Figure 4.** Fitted GPR models are shown along with data points used to fit the model for the elbow flexion/extension joint for the elbow flexion and elbow extension electrode.

203 The position tested and  $\tau_{record}$  at that position are saved as training data for the tested electrode. This is  
 204 repeated for the other electrode active for the current DOF, and at each of the other positions, three times in  
 205 a randomized order. The collected training points are then used to generate a GPR model for each electrode  
 206 using Matlab's `fitrgp` function. An example of trained GPR models for elbow flexion/extension torque  
 207 output resulting from the elbow flexion and elbow extension electrodes for a single subject is shown in  
 208 Figure 4. This results in the following equation

$$\tau_{fes} = P(q)\alpha \quad (6)$$

209 where  $P(q) \in \mathbb{R}^{1 \times 2}$  and where column  $i$  is an individual GPR model that provides an estimated output  
 210 torque when electrode set  $i$  is at maximum stimulation, and the robot is at position  $q$ . Recall that this is  
 211 implemented for each joint separately, so there is one  $P(q)$  that corresponds to the elbow flexion/extension  
 212 joint and uses the elbow flexion/extension position as an input, and one  $P(q)$  that corresponds to the wrist  
 213 flexion/extension joint and used the wrist flexion/extension position as an input.

#### 214 2.4.5 Arm Model Characterization

215 An accurate model of the dynamic system is needed for effective MPC implementation. Previous work  
 216 has developed a model of the exoskeleton without an arm (Dunkelberger et al., 2022b). In this study, an  
 217 optimization problem was solved to find an estimate of dynamic properties for the arm to be used with the  
 218 exoskeleton dynamic model, including masses, moments of inertia, and friction components.

219 To add these dynamic properties of the arm to the dynamic model of the exoskeleton, each joint in the  
 220 arm was assumed to be a rigid body rigidly connected to the corresponding joint on the exoskeleton. With  
 221 this assumption, the mass of each arm joint can be added to the mass of the robot joint, and the inertia of  
 222 each arm joint can be combined with the inertia of each robot joint using the parallel axis theorem. While  
 223 this study mainly focuses on the impact on the elbow flexion/extension and wrist flexion/extension joints,  
 224 this arm characterization process utilizes all four joints of the exoskeleton to create a full dynamic model  
 225 as shown in equation 7, which can then be reduced to the single-joint components for the controller.

$$\tau = M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}_f(\dot{\mathbf{q}}) \quad (7)$$

226 In equation 7,  $\tau \in \mathbb{R}^{4 \times 1}$  is a vector consisting of the torques at each joint.  $M \in \mathbb{R}^{4 \times 4}$  is known as the  
 227 mass matrix and consists of different combinations of the mass and inertial terms of each joint.  $\mathbf{V} \in \mathbb{R}^{4 \times 1}$   
 228 is the vector of centrifugal and Coriolis terms.  $\mathbf{G} \in \mathbb{R}^{4 \times 1}$  is the gravity vector and gives the affects of  
 229 gravity on each joint, and  $\mathbf{F}_f \in \mathbb{R}^{4 \times 1}$  gives friction on each joint.  $\mathbf{q}$  is a vector of all joint positions,  $\dot{\mathbf{q}}$  is a  
 230 vector of all joint velocities, and  $\ddot{\mathbf{q}}$  is a vector of all joint accelerations.  $M$ ,  $\mathbf{V}$ ,  $\mathbf{G}$ , and  $\mathbf{F}_f$  were calculated  
 231 using the same methods as previous work (Dunkelberger et al., 2022b), but with the combined arm and  
 232 robot properties serving as lumped parameters in the formulation.

233 Equation 7 can be used to characterize the unknown arm mass properties that appear in the equation, given  
 234 experimentally recorded values for  $\tau$ ,  $\mathbf{q}$ , and  $\dot{\mathbf{q}}$ . To collect these data for characterization, the user's arm was  
 235 placed inside the robot and secured. A chirp signal was used as a position reference for the wrist radial/ulnar  
 236 deviation joint while the other three joints were commanded to remain stationary using independent PD  
 237 controllers. This process was then repeated for each more proximal joint. The torque required to complete  
 238 the motions and the resulting joint positions and velocities were recorded. The recorded velocities were  
 239 filtered, and a finite difference derivative was calculated to approximate the accelerations. With these  
 240 values, the difference between the left side and right side of equation 7, recorded and calculated torques  
 241 respectively, could be found given a guess of mass properties. The difference between these two values at  
 242 every time step is the error in the dynamic model, and this error was used as the optimization criteria to  
 243 estimate the mass properties of the arm when combined with the mass properties of the exoskeleton found  
 244 in previous work (Dunkelberger et al., 2022b).

245 To keep the number of optimization variables small, the problem was solved one joint at a time, starting  
 246 with the most distal joint, wrist radial/ulnar deviation. This joint was the first to be optimized because  
 247 for any given joint, only the more distal joints impact the current mass property analysis. Each more  
 248 proximal joint was then optimized in order, ending with the elbow flexion/extension joint. At each joint,  
 249 the inertia about the axis of rotation and the distances to the center of mass in the other two axes were  
 250 optimization variables. When running the optimization on any joint except wrist radial/ulnar deviation, the  
 251 next distal joint's distance to the center of mass along the distal joint's axis of rotation was also included as  
 252 an optimization variable. This was added because this value does not appear in the calculations for the  
 253 joint moving, but does impact the more proximal joints. Lastly, two optimization variables were added to  
 254 each joint corresponding to the joint kinetic and viscous friction, which were considered to be added to the  
 255 coefficients previously characterized for the exoskeleton by itself. A constant mass was assumed for each  
 256 joint because the mass only appears multiplied by the distance to the center of mass terms. The formulation  
 257 of this optimization problem can be seen in equations 8 and 9.

$$\arg \min_{\mathbf{p}_d} e_d = \sum_{t=1}^M (\tau_{calc\_d\_t} - \tau_{meas\_d\_t})^2 \quad (8)$$

$$\mathbf{p}_d = \begin{cases} [I_{czz\_d}, r_{cx\_d}, r_{cy\_d}, r_{cx\_d+1}, F_{k\_d}, B_d] & \text{if } 1 \leq d \leq 3 \\ [I_{czz\_d}, r_{cx\_d}, r_{cy\_d}, F_{k\_d}, B_d,] & \text{if } d = 4 \end{cases} \quad (9)$$

Active Joint	$q_{hold\_1}$	$q_{hold\_2}$	$q_{hold\_3}$	$q_{hold\_4}$
Elbow F/E	N/A	0°	0°	0°
Wrist F/E	-30°	-30°	N/A	0°

**Table 1.** Holding position of inactive joints throughout testing.

258 In these equations,  $p_d$  represents the vector of parameters for a given joint,  $d$ . A  $d$  of 1 represents the  
 259 elbow flexion/extension joint and  $d = 4$  being the wrist radial ulnar/deviation joint,  $e_d$  refers to the torque  
 260 error between the calculated torque,  $\tau_{calc}$ , and measured torque,  $\tau_{meas}$ ,  $t$  represents a given time step up to  
 261  $M$  total time steps,  $I_{czz}$  is the moment of inertia about the axis of rotation taken about the center of mass,  
 262 and  $r_{cx}$ ,  $r_{cy}$ , and  $r_{cz}$  represent the distance from the axis of rotation to the center of mass in the  $x$ ,  $y$ , and  $z$   
 263 directions respectively.

264 The optimization problem was solved using `fmincon` in Matlab, with initial guesses of zero for all  
 265 optimization variables. The optimal properties found using this method were combined into the lumped  
 266 arm and robot system used in the remainder of this study.

## 267 2.5 Hybrid Controller Design

268 We first present the full four-DOF dynamics for the FES-exoskeleton hybrid system, which we will then  
 269 reduce to the single-DOF dynamics for the control formulation. This is similar to the dynamics of the robot  
 270 and arm system in equation 7, but the inputs to the system arise from both the exoskeleton and the FES  
 271 system, so we separate the torque term into the two components.

$$\tau_{fes} + \tau_{exo} = M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}_f(\dot{\mathbf{q}}) \quad (10)$$

272 In this equation,  $\tau_{exo} \in \mathbb{R}^{4 \times 1}$  and  $\tau_{fes} \in \mathbb{R}^{4 \times 1}$  are torques supplied along each of the robot joints due to  
 273 robot torque outputs, and torques provided by FES respectively.

274 As in the previous sections, the control problem will be described once, but the equations presented  
 275 apply to either the elbow flexion/extension or the wrist flexion/extension DOF. To limit the full dynamics  
 276 in equation 10 to analyze a single DOF with the rest of the joints remaining stationary, all inactive joints  
 277 can be constrained such that  $q_j = q_{hold\_j}$ ,  $\dot{q}_j = 0$ ,  $\ddot{q}_j = 0$  for all joints  $j$  that are inactive. Here,  $q_{hold\_j}$   
 278 is the holding position of joint  $j$  when it is inactive, as shown in Table 1. This results with the following  
 279 equation to describe the dynamics of a single DOF system, either in the elbow flexion/extension or wrist  
 280 flexion/extension case.

$$P(\mathbf{q})\boldsymbol{\alpha} + \tau_{exo\_mpc} = m\ddot{q} + g \sin(q - q_{eq}) + f_f(\dot{q}) \quad (11)$$

281 For the DOF of interest,  $m$  represents the estimated lumped inertia,  $g$  represents the gravitational effects,  
 282  $f_f$  represents the friction effects, and  $q_{eq}$  represents the natural resting position of the combined arm-robot  
 283 system for the DOF of interest. In equation 11, and throughout the remainder of the paper, all variables that  
 284 appear in equations are referring to a single DOF, and the values of these variables are different in the elbow  
 285 flexion/extension DOF and the wrist flexion/extension DOF, but the symbolic expressions apply to both  
 286 DOFs. For example, when this equation is applied to the elbow flexion/extension joint,  $q$ ,  $\dot{q}$ , and  $\ddot{q}$  are the  
 287 position, velocity, and acceleration of the robot elbow flexion/extension joint, and  $\boldsymbol{\alpha}$  is the vector  $[\alpha_1, \alpha_2]^T$ ,  
 288 which are the activation levels of the electrodes placed to induce elbow flexion, and elbow extension.

289 To develop our control problem, we define the following quantities as the system state,  $x$ , system output,  
 290  $y$ , and control input,  $u$ , where  $C$  is the output matrix describing the variables we can observe.

$$\mathbf{x} = [q, \dot{q}]^T \quad (12)$$

$$C = I_2 \quad (13)$$

$$\mathbf{y} = C\mathbf{x} \quad (14)$$

$$\mathbf{u} = [\tau_{exo\_mpc}, \alpha_1, \alpha_2]^T \quad (15)$$

291 To use standard analysis techniques, we would like to have our dynamics in the form of  $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ ,  
 292 which by definition is the vector  $[\dot{q}, \ddot{q}]^T$ . By solving equation 11 for  $\ddot{q}$  as follows, we can obtain an explicit  
 293 definition for the representation of  $f(\mathbf{x}, \mathbf{u})$ .

$$\ddot{q} = \frac{1}{m}(P(q)\boldsymbol{\alpha} + \tau_{exo\_mpc} - g \sin(q - q_{eq}) - f_f(\dot{q})) \quad (16)$$

294 To implement real time control, it is beneficial to use a linearized form of the dynamics to reduce  
 295 computation time. We can then convert the dynamics to a linearized form by calculating the Jacobian of the  
 296 dynamics about time  $k$  with respect to the input and output. The following gives a estimate for the dynamic  
 297 equations at time  $i$ , linearized at time  $k$ .

$$A_k = \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_k, \mathbf{u}=\mathbf{u}_k} \quad (17)$$

$$B_k = \left. \frac{\partial f}{\partial \mathbf{u}} \right|_{\mathbf{x}=\mathbf{x}_k, \mathbf{u}=\mathbf{u}_k} \quad (18)$$

$$\dot{\mathbf{x}}_i = A_k \mathbf{x}_i + B_k \mathbf{u}_i + \dot{\mathbf{x}}|_{\mathbf{x}=\mathbf{x}_k, \mathbf{u}=\mathbf{u}_k} \quad (19)$$

298 These linearized dynamics are then used in the MPC formulation. The cost function is as follows, where  $i$   
 299 represents a discrete point in time in the standard MPC formulation.

$$J_i = (\mathbf{r}_i - \bar{\mathbf{y}}_i)^T Q (\mathbf{r}_i - \bar{\mathbf{y}}_i) + \Delta \mathbf{u}_i^T R \Delta \mathbf{u}_i + \mathbf{u}_i^T R_m \mathbf{u}_i \quad (20)$$

300 The matrices  $Q \in \mathbb{R}^{2 \times 2}$ ,  $R \in \mathbb{R}^{3 \times 3}$ , and  $R_m \in \mathbb{R}^{3 \times 3}$  are positive diagonal matrices used to weight  
 301 predicted trajectory error, control input rate of change, and control input magnitude respectively. In this  
 302 equation, the control input rate of change at timestep  $i$  is defined as  $\Delta \mathbf{u}_i = \mathbf{u}_i - \mathbf{u}_{i-1}$ . Initial values for  
 303 these gains were chosen based on pilot studies that provided desired behavior as described below.

$$Q = \begin{bmatrix} Q_{pos} & 0 \\ 0 & Q_{vel} \end{bmatrix} \quad (21)$$

$$R = \begin{bmatrix} R_{exo} & 0 & 0 \\ 0 & R_{fes} & 0 \\ 0 & 0 & R_{fes} \end{bmatrix} \quad (22)$$

$$R_m = \begin{bmatrix} R_{m\_exo} & 0 & 0 \\ 0 & R_{m\_fes} & 0 \\ 0 & 0 & R_{m\_fes} \end{bmatrix} \quad (23)$$

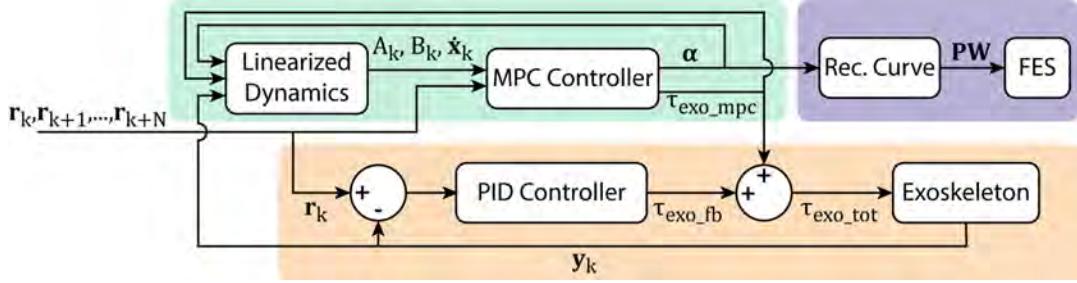
304 The general ideology behind the choice of gains in the hybrid controller is as follows. The gains for  $Q$   
 305 represent the importance for the controller to follow the desired trajectory, with higher gains indicating  
 306 better tracking, but less stable behavior if there are model errors. The gains for  $R_m$  are chosen so that  
 307  $R_{m\_exo} \gg R_{m\_fes}$ , which is the main method by which the hybrid control strategy reduces exoskeleton  
 308 torque compared to a strategy which only uses an exoskeleton. Additionally, these gains are chosen such  
 309 that  $(\bar{\mathbf{y}}_i - \mathbf{r}_i)^T Q (\bar{\mathbf{y}}_i - \mathbf{r}_i) \gg \mathbf{u}_i^T R_m \mathbf{u}_i$ , so that trajectory accuracy is not sacrificed to allow for overall  
 310 torque reduction. The gains for  $R$  are chosen so that  $R_{fes} \gg R_{exo}$  so that the FES system, which has  
 311 significant delay, remains stable by mainly responding with low-frequency changes in torque while the  
 312 exoskeleton does mostly quick corrective actions. This combination of chosen gains for  $R$  and  $R_m$  are  
 313 intended to have the general effect of the FES subsystem providing low frequency, high amplitude torque,  
 314 allowing it to provide a bulk of the power requirement, yet maintain smooth motions despite the time delay.  
 315 The exoskeleton subsystem provides high frequency, low amplitude torque, which provides necessary quick  
 316 corrections without requiring too much power consumption. As a reminder, separate controllers are used  
 317 for the elbow flexion/extension joint and for the wrist flexion/extension joint, and the gains for each of the  
 318 two joints are created independently.

319 Because the  $R_{fes}$  and  $R_{m\_fes}$  gains place costs on activation levels rather than FES torque outputs, in  
 320 some cases, it was necessary to adjust these values for each participant upon initial testing with the hybrid  
 321 controller to account for variations in torque productions for the same activation level. To account for this,  
 322 when the hybrid controller was first tested in the experiment, these gains were increased by a factor of two  
 323 from the original values if there was oscillatory behavior, or decreased by a factor of two if activation levels  
 324 were lower than expected.

325 The final cost function used in the MPC implementation is as follows.

$$\begin{aligned} \arg \min_{u(\cdot)} J_{tot} &= \sum_{i=1}^N J_{k+i} \\ \text{subject to } \bar{\mathbf{y}}_{k+i+1} &= \bar{\mathbf{y}}_{k+i} + \dot{\bar{\mathbf{x}}}_{k+i} T_s, \\ 0 \leq \alpha_e &\leq 1, e = \{1, 2\} \end{aligned} \quad (24)$$

326 In equation 24,  $k$  represents the current point in time, and future discretized timesteps at time  $k + i$  are  $T_s$   
 327 seconds apart, for  $N$  time steps. The dynamics at these future time points are approximated using Euler



**Figure 5.** Hybrid FES-Exoskeleton control block diagram, showing how the different components of the hybrid controller work together to provide torque commands to the robot and pulse width commands to the stimulator given a desired input trajectory.

328 integration as shown by the first constraint on the optimization problem, with the bars representing that  
 329 these are estimated values. The second constraint restricts the activation level,  $\alpha$ , of each electrode,  $e$ , to  
 330 fall between 0 and 1. An additional constraint could be implemented to limit the maximum allowable  
 331 exoskeleton torque; however, in this study, the torque required from the exoskeleton always remained  
 332 below the maximum allowable torque, which meant that this constraint did not need to be implemented.  
 333 The result of the optimization is  $u(\cdot)$  which represents the optimal control inputs over the time prediction  
 334 horizon,  $u_{k+1}, u_{k+2}, \dots, u_{k+N}$ .

335 This MPC formulation is created in C++ using the nonlinear optimization framework CasADi (Andersson  
 336 et al., 2019). The solver for the dynamic problem is compiled into a dll file which can be loaded at  
 337 runtime and interfaced with the Interior Point Solver, IPOPT (Wächter and Biegler, 2006), to solve the  
 338 MPC problem. This MPC problem is solved as fast as possible in a separate thread, and each time a  
 339 solution is found, the solution of the minimization,  $u(\cdot)$ , is sent to the main thread, where those successive  
 340 control solutions are used until the next solution is found. From  $u$ ,  $\tau_{exo\_mpc}$  is used directly, and  $\alpha_1$  and  
 341  $\alpha_2$  are converted to pulsedwidth commands to send to the stimulator using equation 4 which describes the  
 342 recruitment curve.

343 To tune the gains for the MPC algorithm,  $Q$  and  $R$  were first tuned to achieve smooth movements and low  
 344 tracking error, with  $R_m$  values kept at 0. Following this, the  $R_m$  gains were chosen to achieve meaningful  
 345 reduction in the exoskeleton torque, while maintaining similar tracking accuracy. As  $R_m$  gains were tuned,  
 346  $Q$  and  $R$  were further adjusted as necessary.

347 To account for model error in the MPC formulation, a PID controller using only exoskeleton torque is  
 348 implemented in parallel as shown in Figure 5. This has the effect of allowing the MPC portion to control  
 349 most of the action, while still providing a high accuracy on the resulting trajectory tracking. The torque  
 350 provided by the PID controller is defined as  $\tau_{exo\_fb}$ , and the gains for this controller were chosen in pilot  
 351 testing to achieve between 1° and 1.5° RMS tracking error. In the tuning of this controller, the gains were  
 352 slowly increased, and tuned only after fully tuning the MPC system independently, so that the controller  
 353 dynamics achieved from the MPC algorithm were the driving component. This additional controller does  
 354 not change the output applied by the FES subsystem, but the torque applied to the exoskeleton becomes

$$\tau_{exo\_tot} = \tau_{exo\_mpc} + \tau_{exo\_fb} \quad (25)$$

356 To test the effectiveness of the hybrid controller design, it is compared against a purely exoskeleton  
357 controller, defined as the *exoskeleton alone* control case. In this test case, the same general structure is used  
358 with the MPC controller paired with a PID controller, but equation 15 becomes

$$\mathbf{u} = [\tau_{exo}] \quad (26)$$

359 which results in  $R$  and  $R_m$  being single values rather than matrices.

## 360 2.6 Experimental Validation

361 After a participant completed each of the model characterization steps and the MPC problem was  
362 generated, the experimental validation was conducted. Participants were assisted in completing two  
363 different trajectories in two different conditions, using the hybrid controller that combined the FES and  
364 exoskeleton action, and using the exoskeleton alone. The first trajectory is referred to as the *cup* trajectory,  
365 and it is based on a study that tracked healthy individuals' joint-level movements to move a cup to various  
366 target locations with differing grasps (Valevicius et al., 2019). The movement profile for each of the  
367 joints was taken independently and spaced so that it spanned a useful and comfortable trajectory space  
368 for the exoskeleton used in this study which was 30° flexed to 90° flexed from full extension for elbow  
369 flexion/extension and 15° extended to 45° flexed for wrist flexion/extension. The *cup* trajectory is useful to  
370 observe how the hybrid controller behaves when following natural motions that would be expected under  
371 normal use. The second trajectory is referred to as the *sinusoidal* trajectory, and it is an artificially created  
372 trajectory that is the summation of multiple sinusoidal waves at different amplitudes and frequencies.  
373 This trajectory was created to test the controllers' ability to generalize to different movements. The  
374 trajectories are relatively similar in terms of difficulty for the elbow flexion/extension joint, but the wrist  
375 flexion/extension joint movement is significantly easier in the *cup* trajectory than the *sinusoidal* trajectory.  
376 Both trajectories take 42.4 seconds to complete, which is four times the time it took an average able-bodied  
377 individual to complete the *cup* trajectory in (Valevicius et al., 2019). A four times reduction was chosen  
378 because the original trajectory moved through the workspace very quickly, and this reduction empirically  
379 felt an appropriate length to safely perform movements with a human in the robot. Visualizations of these  
380 trajectories are shown in the results in Figures 8 and 9.

381 Each DOF was tested for ten trials on the *cup* trajectory, split evenly between hybrid controller and  
382 exoskeleton alone controller, and 10 trials on the *sinusoidal* trajectory, also split evenly between hybrid  
383 controller and exoskeleton alone controller. While each DOF was being tested, all other DOFs were kept at  
384 their  $q_{hold}$  values as shown in table 1 using independent PD controllers on those joints. Collection of the  
385 experimental data began by running four elbow flexion/extension trials, consisting of one of each possible  
386 combination of trajectory and controller type. This was followed by four wrist flexion/extension trials,  
387 again consisting of each possible combination of trajectory and controller. This sequence was repeated  
388 until all 40 total trials had been collected. Throughout each of the trials, position of the active DOF, total  
389 exoskeleton torque commanded, and activation levels of electrodes were collected at a rate of 1 kHz using  
390 a Quanser Q8-USB data acquisition device.

391 Three of the nine participants repeated the entire protocol (including characterization steps) at least one  
392 week after they completed the first set of data collection. These data were collected to provide insight into  
393 whether results remain similar between sessions within the same participant, rather than only comparing  
394 between participants.

395 **2.7 Data Analysis**

396 The primary objective of these experiments is to understand the extent to which exoskeleton power  
 397 consumption can be reduced in a hybrid system compared to a exoskeleton alone system. We compare  
 398 power consumption by taking the sum of the squared total exoskeleton torque throughout the trajectory for  
 399 each of the conditions tested as shown in equation 27, averaged across each of the five trials with that set  
 400 of conditions. This value is labeled as  $\tau_{ss\_exo}$  for the exoskeleton alone control condition, and  $\tau_{ss\_hybrid}$   
 401 for the hybrid control condition. Because participants have different arm sizes, and require the robot to  
 402 be in different configurations, it is expected that participants will require different amounts of sum of  
 403 squared torque from the system to move through the *cup* and *sinusoidal* trajectories. To normalize the data  
 404 to compare across subjects, the reduction in sum of squared torque in the hybrid control case compared to  
 405 the exoskeleton alone control case is shown by equation 28. This allows us to analyze the varying power  
 406 consumption both between exoskeleton alone and hybrid controllers, as well as how the relative controller  
 407 performance translates between two different trajectories.

$$\tau_{ss} = \sum_{i=1}^N \tau_{exo\_tot}^2 \quad (27)$$

$$\%Imp = 100 \left( 1 - \frac{\tau_{ss\_exo} - \tau_{ss\_hybrid}}{\tau_{ss\_exo}} \right) \quad (28)$$

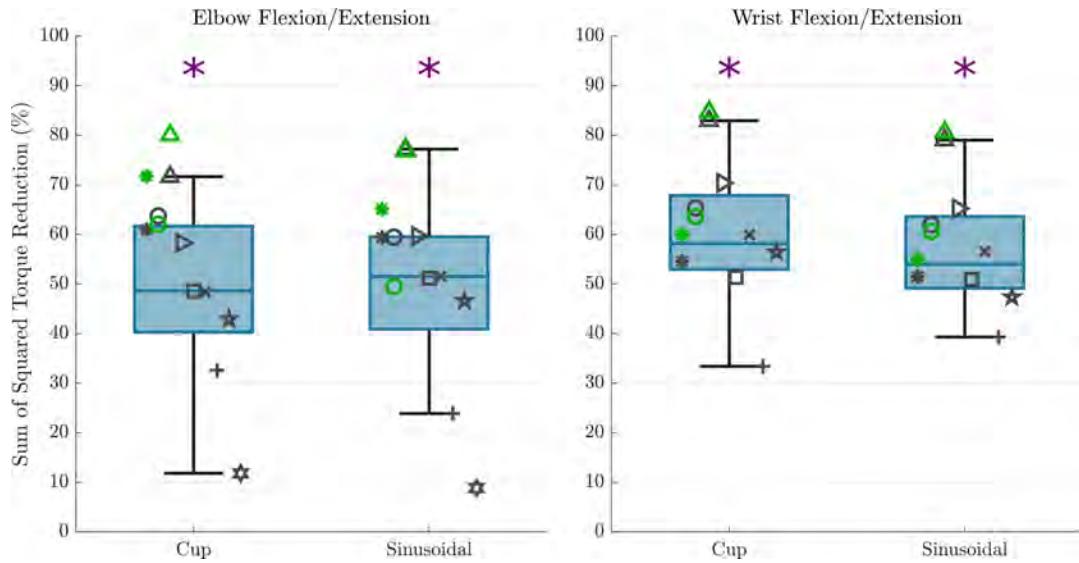
408 In equation 27,  $N$  is the number of data points collected. With this representation, a value of  $\%Imp = 0$   
 409 would represent equal amounts of torque being used in both control cases, which would indicate no  
 410 improvement, a value of  $\%Imp > 0$  would indicate a reduction in power consumption using the hybrid  
 411 controller with a value of  $\%Imp = 100$  indicating no exoskeleton power was consumed, and a value of  
 412  $\%Imp < 0$  would indicate that the hybrid controller required more exoskeleton power than the exoskeleton  
 413 alone case. A paired t-test was performed to understand whether there was a statistically significant  
 414 difference between in the sum of squared torque in the exoskeleton alone control case, and in the hybrid  
 415 control case for each of the trajectories.

416 The secondary objective of these experiments is to understand how the tracking accuracy compares when  
 417 using the two options for controllers. The RMS tracking error is calculated as

$$e_{rms} = \frac{\sum_{i=1}^N \sqrt{(y_i - r_i)^2}}{N} \quad (29)$$

418 A paired t-test was performed to understand whether there was a statistically significant difference between  
 419 the RMS errors in the exoskeleton alone control case, and in the hybrid control case for each of the  
 420 trajectories.

421 One subject was unable to get any detectable torque output from one of the electrodes on the wrist  
 422 flexion/extension DOF, and therefore, did not complete data collection for that DOF. Because of this, there  
 423 are 9 sets of data analyzed for the elbow flexion/extension results, and 8 sets of data analyzed for the wrist  
 424 flexion/extension results.



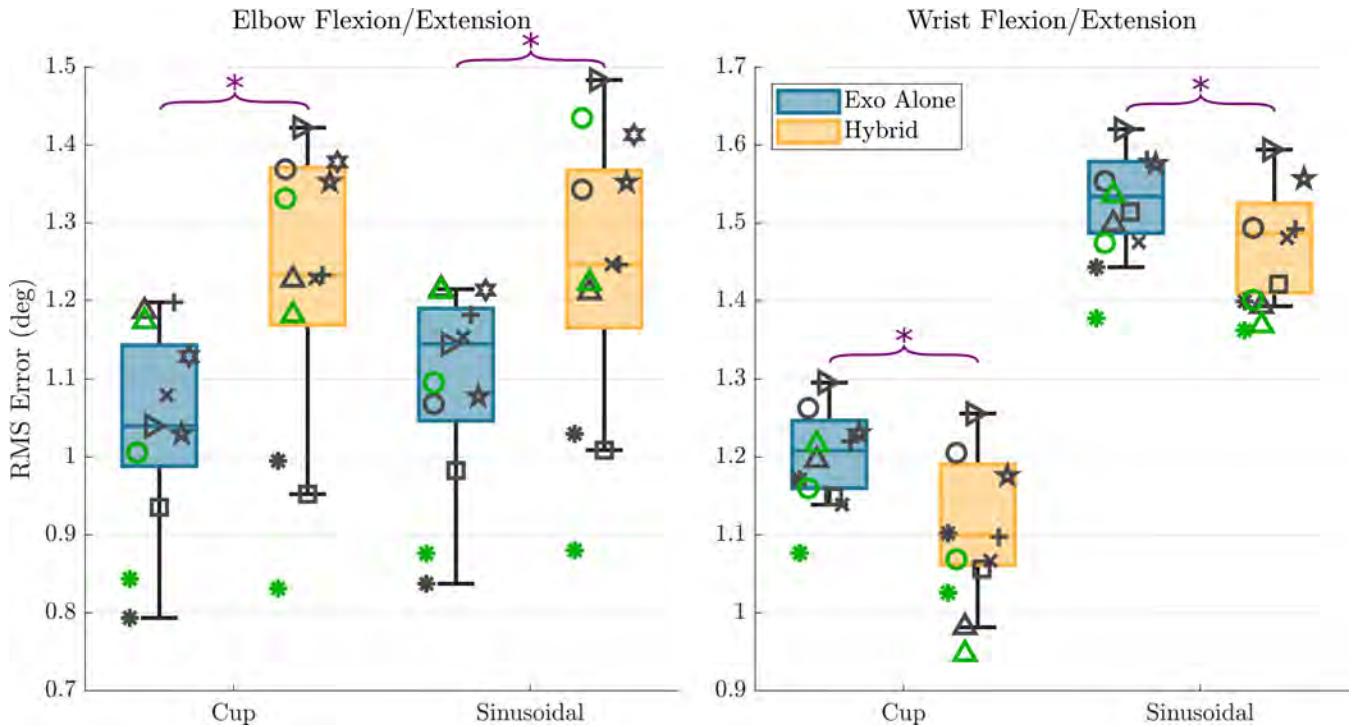
**Figure 6.** Sum of squared torque reduction results are shown for all subjects for each trajectory for the elbow flexion/extension DOF (left) and wrist flexion/extension DOF (right). The overlaid scatterplot shows individual subject results, with the same symbol representing a single subject across figures. Points in green show the repeated data collection for the first three subjects, but repeated data collection does not contribute to boxplot presentation. The purple “\*” above the plots represents a that there was a statistically significant difference in the sum of squared torque between the hybrid and exoskeleton alone control cases.

### 3 RESULTS

425 A summary of the sum of squared torque reduction findings is presented in Figure 6 as boxplots with  
 426 individual subject data overlaid on top. These results show a mean sum of squared torque reduction of 48.8%  
 427 and 48.6% for the *cup* and *sinusoidal* trajectories respectively for the elbow flexion/extension joint when  
 428 comparing the hybrid controller to the exoskeleton alone controller. These values for individual participants  
 429 spanned from 11.8% to 71.6% for the *cup* trajectory, and from 8.8% to 77.2% for the *sinusoidal* trajectory,  
 430 with the lowest data point being an outlier. A mean sum of squared torque reduction of 59.3% and 56.5%  
 431 was shown for the *cup* and *sinusoidal* trajectories respectively for the wrist flexion/extension joint when  
 432 comparing the hybrid controller to the exoskeleton alone controller. These values for individual participants  
 433 spanned from 33.4% to 82.9% for the *cup* trajectory, and from 39.3% to 79.0% for the *sinusoidal* trajectory.  
 434 The statistical tests showed that the sum of squared torques were significantly lower in the hybrid control  
 435 case compared to the exoskeleton alone control case in both DOFs and in both trajectories, with *p* values  
 436 being  $< 0.01$  in both trajectories for the elbow flexion/extension joint, and *p* values being  $< 0.001$  in both  
 437 trajectories for the wrist flexion/extension joint.

438 A summary of the trajectory tracking accuracy findings is presented in Figure 7 as box plots with  
 439 individual subject data overlaid on top. For the elbow flexion/extension joint, mean RMS errors in the *cup*  
 440 trajectory were  $1.04^\circ$  and  $1.24^\circ$  for the exoskeleton alone and hybrid controllers respectively. RMS errors in  
 441 the *sinusoidal* trajectory were  $1.10^\circ$  and  $1.26^\circ$  for the exoskeleton alone and hybrid controllers respectively.  
 442 These results indicate that there is a mean increase of  $0.18^\circ$  in RMS error when using the hybrid controller  
 443 compared to using the exo alone controller in the elbow flexion/extension joint. This difference was shown  
 444 to be statistically significant in the paired t-test, with *p*-values for each of the trajectories  $< 0.01$ .

445 For the wrist flexion/extension joint, RMS errors in the *cup* trajectory were  $1.21^\circ$  and  $1.12^\circ$  for the  
 446 exoskeleton alone and hybrid controllers respectively. RMS errors in the *sinusoidal* trajectory were  $1.53^\circ$

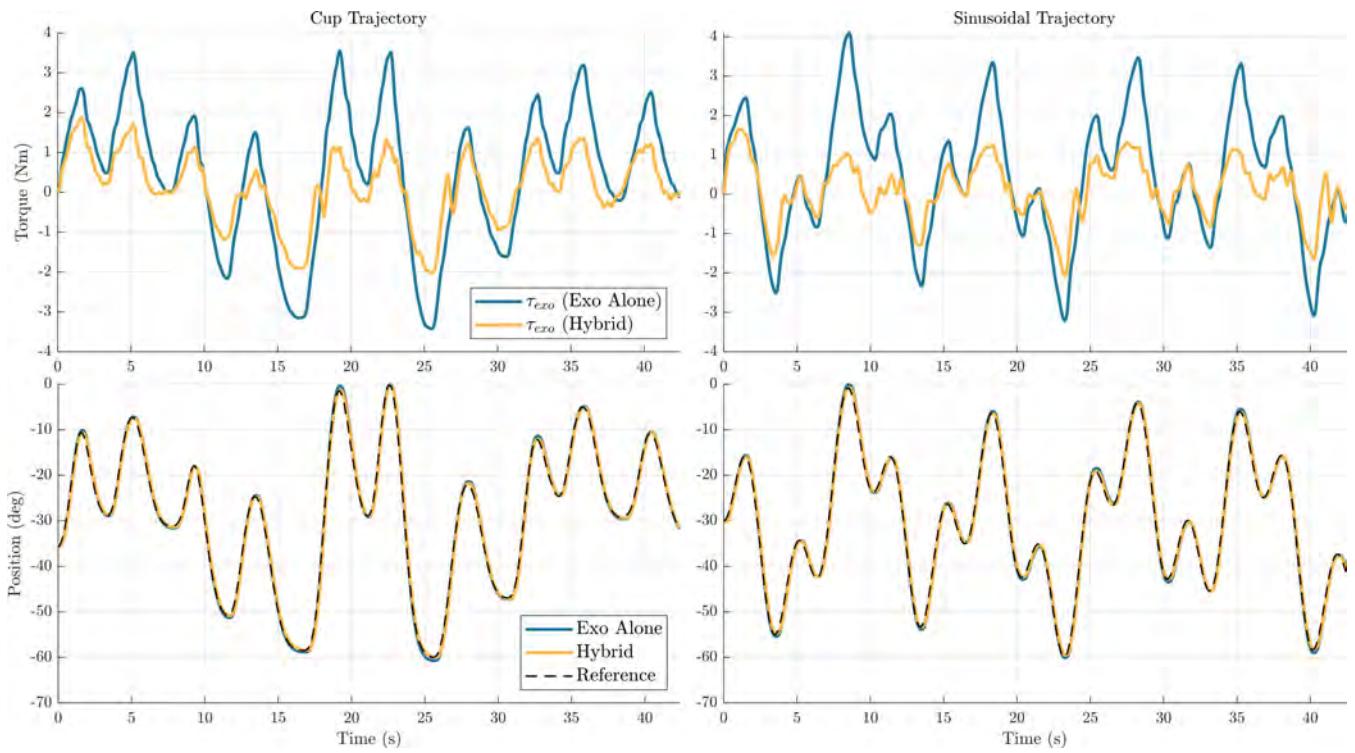


**Figure 7.** RMS error results are shown for all subjects for each trajectory and each controller type for the elbow flexion/extension DOF (left) and wrist flexion/extension DOF (right). The overlaid scatterplot shows individual subject results, with the same symbol representing a single subject across figures. Points in green show the repeated data collection for the first three subjects. The purple “\*” above the plots represents a that there was a statistically significant difference in the RMS errors between the two control types.

447 and  $1.48^\circ$  for the exoskeleton alone and hybrid controllers respectively. These results indicate that there is a  
 448 mean decrease of  $0.07^\circ$  in RMS error when using the hybrid controller compared to using the exoskeleton  
 449 alone controller in the wrist flexion/extension joint. This difference was again shown to be statistically  
 450 significant in the paired t-test, with  $p$ -values for each of the trajectories again remaining  $< 0.01$ .

451 Figures 8 and 9 show time series representations of torque profiles for the best performing subject  
 452 (represented by the  $\triangle$  symbol in Figures 6 and 7) and movement profiles averaged across all subjects. In  
 453 the representative plots of torque profiles, the exoskeleton torque used during the hybrid trials exhibits a  
 454 smaller magnitude than the exoskeleton torque used during exoskeleton alone trials. This result shows that  
 455 the hybrid controller is able to replace a significant amount of the torque requirement from the exoskeleton  
 456 with FES torque. The plots for movement profiles demonstrate how well each of the controllers are able to  
 457 track the trajectory. In all combinations of trajectories and DOFs, the trajectories almost entirely overlap  
 458 each other, showing similar accuracy regardless of controller.

459 The reduction in maximum torque for the torque profile averaged across participants profiles across  
 460 participants is also analyzed, for the hybrid controller compared to the exoskeleton alone controller. For  
 461 this metric, it is interesting to observe both the change in maximum and minimum values, as many  
 462 cable-driven systems would likely require one actuator for each agonist and antagonist pair. In the elbow  
 463 flexion/extension DOF, the maximum torque was reduced by 44.2% and 43.7% in the *cup* and *sinusoidal*  
 464 trajectories respectively, and the minimum torque for the mean profile was reduced by 31% and 27.1% for  
 465 the *cup* and *sinusoidal* trajectories respectively. In the wrist flexion/extension DOF, the maximum torque  
 466 was reduced by 67.1% and 65.3% in the *cup* and *sinusoidal* trajectories respectively, and the minimum



**Figure 8.** Elbow flexion/extension joint exoskeleton torque profile for a single subject (top), and movement profiles averaged across subjects (bottom) are shown for the two different trajectories, *cup* (left) and *sinusoidal* (right). In the plots, the blue line represents data for the exoskeleton alone controller, and the yellow line represents data for the hybrid controller.

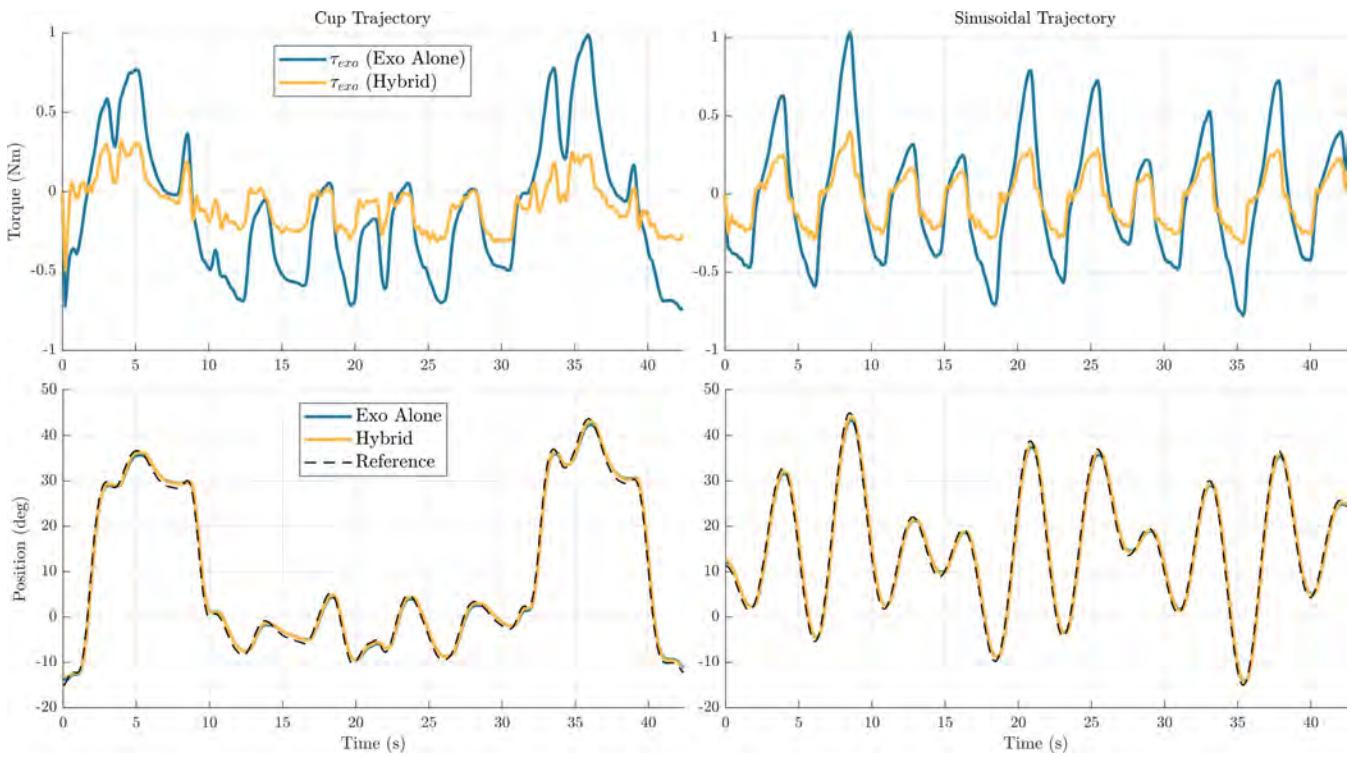
467 torque for the mean profile was reduced by 36.9% and 36.6% for the *cup* and *sinusoidal* trajectories  
468 respectively.

## 4 DISCUSSION

469 There is a need for devices to provide assistance in completing activities of daily living for individuals with  
470 SCI. For this population, return of upper-limb function is among their top priorities (Anderson, 2004). Both  
471 FES and exoskeletons provide some framework to assist with movement, but each of these technologies  
472 has fundamental limitations preventing meaningful assistance for the upper-limbs in activities of daily  
473 living. FES is unable to provide accurate and repeatable movements by itself, and using feedback control  
474 causes instability due to the inherent time delays in muscle response to stimulation. Exoskeletons are able  
475 to provide accurate and repeatable movements, but require bulky systems and large amounts of power to  
476 support upper-limb movements against gravity. In this paper, we have proposed a hybrid FES-exoskeleton  
477 controller that combines the two technologies, with the goal of reducing power consumption compared to  
478 a robot alone, and providing accurate movement, similar to that of an exoskeleton alone. This controller  
479 uses the model predictive control cost function to leverage the strengths of each of the subsystems, while  
480 minimizing the weaknesses of each.

### 481 4.1 Torque Reduction

482 An average reduction of 48.7% and 57.9% of sum of squared torque was found on the elbow  
483 flexion/extension and wrist flexion/extension DOFs respectively with the use of the hybrid controller  
484 compared to the exoskeleton alone controller. These results in the EFE joint are an improvement over



**Figure 9.** Wrist flexion/extension joint exoskeleton torque profile from a single subject (top), and movement profiles averaged across subjects (bottom) are shown for the two different trajectories, *cup* (left) and *sinusoidal* (right). In the plots, the blue line represents data for the exoskeleton alone controller, and the yellow line represents data for the hybrid controller.

485 the 32.1% reduction found in our previous implementation using only the *cup* trajectory (Dunkelberger  
 486 et al., 2022a). This improvement shows that the inclusion of the feedback controller instead of using an  
 487 integral term, and the incorporation of a more sophisticated arm model, resulted in greater benefits in this  
 488 hybrid control scheme, while even extending to more generalized trajectory cases. This shows promise for  
 489 meaningful power consumption reduction for a hybrid system when comparing to an exoskeleton alone  
 490 controller. Practically, this could mean that a portable hybrid system could be powered for roughly twice as  
 491 long as an equivalent exoskeleton alone system, given the same battery capacity. In the future, this could  
 492 lead to more portability and longevity in hybrid assistive devices for impaired populations.

493 It is worth noting that while the participants are able-bodied and can move their arm through the desired  
 494 trajectories without assistance, we should not expect to see a torque reduction of 100%. With FES we  
 495 often cannot achieve the full capabilities of the user's muscles, and in this study, many of the participants  
 496 were not able to produce the maximum required torque solely through FES, even at maximum activation.  
 497 Additionally, FES is known to not provide accurate or repeatable movements by itself, so at a minimum,  
 498 the exoskeleton needs to provide corrective torques to account for these inaccuracies.

499 The average reduction in minimum and maximum torques shows potential for actuator sizes to be  
 500 reduced while still achieving the same resultant motion, which would result in less bulky assistive robotic  
 501 systems. In the future, this could be more directly tested by artificially limiting the maximum torque of the  
 502 exoskeleton joints to observe how the FES can make up for the lack of torque.

503 **4.2 Accuracy**

504 FES systems by themselves do not provide reliable repeatability when trying to perform generalized  
505 movements. The goal of hybrid FES and exoskeleton systems is to achieve trajectory-following accuracies  
506 significantly better than FES systems by themselves, ideally approaching accuracies that are achievable  
507 using exoskeleton-alone systems. In the elbow flexion/extension joint, the hybrid algorithm had on average  
508  $0.20^\circ$  and  $0.16^\circ$  more RMS tracking error on the *cup* and *sinusoidal* trajectories, respectively, when  
509 comparing the hybrid controller to the exoskeleton alone controller. While this was a decrease in accuracy,  
510 this still resulted in a very similar motion over the trajectory, as shown in Figure 8. To put this in perspective,  
511 for a forearm length of 30 cm, the RMS error in positioning the wrist, given the error in angular tracking, is  
512 approximately 1 mm. For the wrist flexion/extension joint, the hybrid controller had on average  $0.09^\circ$  and  
513  $0.05^\circ$  less RMS tracking error on the *cup* and *sinusoidal* trajectories, respectively. Again, while there is a  
514 small decrease in accuracy, the resultant trajectories are very similar, as shown in Figure 9. These results  
515 demonstrate that the hybrid controller is able to achieve similar tracking accuracies to the exoskeleton  
516 alone controller in both of the individual DOFs.

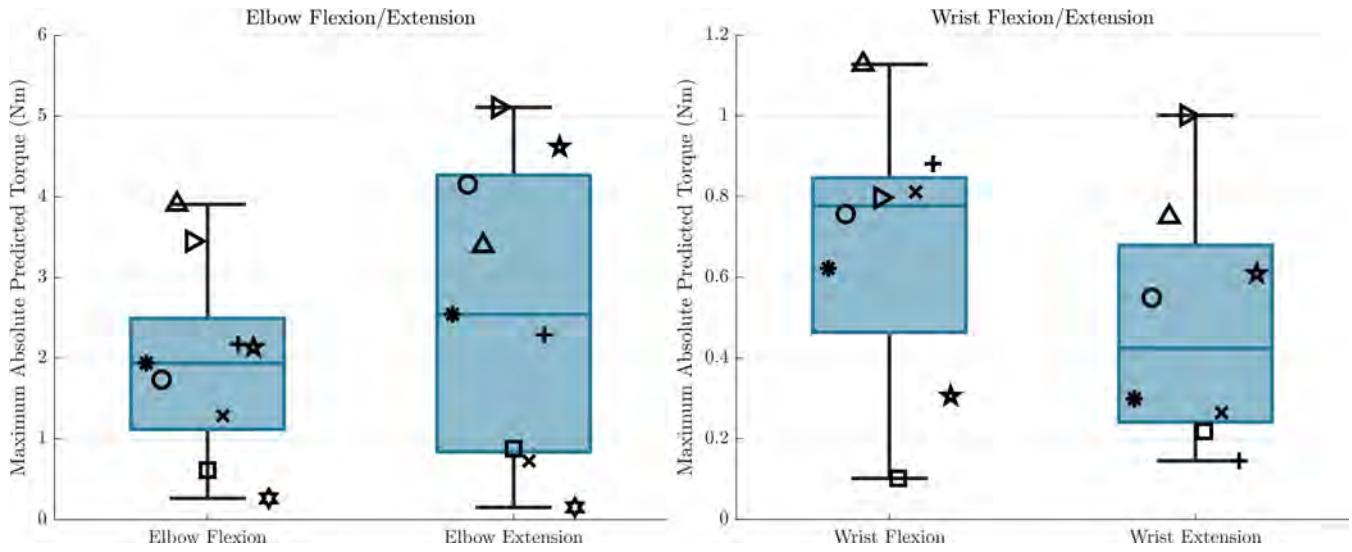
517 It is worth noting the difference in tracking accuracy between the *cup* and *sinusoidal* trajectories on the  
518 wrist flexion/extension joint. Recall that the *cup* trajectory requires significantly less movement, with an  
519 average velocity of 7.3 deg/sec compared to the sinusoidal trajectory with an average velocity of 14.3  
520 deg/sec. The difference in difficulty between the trajectories is likely the cause for more tracking error in  
521 the *sinusoidal* trajectory. Still, we see that the general relationship of the hybrid controller having a 0.06  
522 degree RMSE improvement is similar to the 0.09 degree RMSE improvement on the *cup* trajectory.

523 A benefit of the proposed control architecture is that the feedback controller portion can be adjusted  
524 independently of the model predictive control portion. This means that if a specific movement needs  
525 high-precision, the gains of the feedback controller can be modified in a straightforward manner to increase  
526 accuracy, although it would result in an increase in exoskeleton torque usage. Additionally, while this  
527 study focused on the challenging task of tracking time-varying trajectories, it would also be an interesting  
528 translation to modify the implementation to achieve desired setpoint positions, where FES could be used  
529 for a majority of the movement generation when it is far from the target, and the exoskeleton could be used  
530 to fine-tune the position when it is close to the desired setpoint.

531 **4.3 Generalization Across Tasks**

532 Many of the previous applications using FES for assistance provide the stimulation using a pre-  
533 programmed profile for a specific movement. An important feature of the proposed hybrid controller  
534 is that it does not rely on knowing the desired trajectory before use, and works with any given input  
535 trajectory. By testing two different trajectories, we were able to observe how the different outcome metrics  
536 varied in different movements. Tracking performance across several tasks has been reported by a few  
537 studies that use both FES and exoskeletons (Memberg et al., 2014; Ajiboye et al., 2017; Rohm et al., 2013),  
538 but none of these studies use a controller to distribute actuation between the two systems on the same joint.

539 The sum of squared torque reduction was similar between the two trajectories for both the elbow  
540 flexion/extension DOF and the wrist flexion/extension DOF. Along with the means and ranges being the  
541 same, the general spacing of the participants within the range of results remained the same between the  
542 two trajectories. This means that the benefits in power reduction did generalize well to these different  
543 trajectories, and that users could expect similar results on trajectories that require similar motions. It is  
544 especially interesting that a similar level of sum of squared torque reduction was found on the two different



**Figure 10.** The maximum absolute values of the GPR predictions throughout the workspace for all participants are shown for each electrode, and for each DOF. This represents how different participants are able to achieve different levels of torque from FES when the participant is receiving maximum stimulation. The symbols here correspond to the same symbols from Figures 6 and 7.

545 trajectories for the wrist flexion/extension joint, especially because one of the trajectories was significantly  
546 more challenging than the other.

547 While the elbow flexion/extension DOF saw similar tracking accuracies in the two different trajectories  
548 when comparing the two controllers, the wrist flexion/extension DOF did see a difference in trajectory  
549 tracking accuracy on the two different trajectories. Despite this, the relationship between the exoskeleton-  
550 alone tracking accuracy and the hybrid tracking accuracy remained similar in all cases, with the elbow  
551 flexion/extension DOF showing average increase of 19.2%, and 14.5% in RMS error on the *cup* and  
552 *sinusoidal* trajectories respectively, and the wrist flexion/extension DOF showing average reduction of  
553 7.4% and 3.3% on the *cup* and *sinusoidal* trajectories respectively.

554 While not implemented in this paper, another benefit of this proposed controller is the ability to intuitively  
555 adjust controller behavior to generalize to different objectives of movement. If a specific task requires high  
556 precision in a movement, the gains of the  $Q$  matrix or feedback could be increased to favor more accurate  
557 movement at a cost of more torque. If there is an onset of fatigue, the weights of the  $R_m$  matrix can be  
558 adjusted to prefer more exoskeleton torque, and allow the muscles to recover.

#### 559 4.4 Consistency Across Participants

560 While the results between trajectories were consistent within participants, there is a significant distribution  
561 of results between participants, especially for the sum of squared torque reduction observed for the hybrid  
562 controller compared to the exoskeleton alone controller. Even though all results showed improvement,  
563 except for the single participant who could not achieve an FES response in one of the wrist flexion/extension  
564 electrodes, some participants had significantly better results than others. There are many factors that can  
565 impact the effectiveness of FES, including electrode placement, size of muscles, body fat levels, and  
566 fatigue, many of which are not modifiable. These variations in ability to produce torques due to FES can  
567 be visualized across participants in Figure 10, where the maximum absolute value that the GPR model  
568 predicts that each participant can produce throughout the workspace is shown. We can see that there are  
569 wide variations in the predicted amount of FES torque production. As an example, one participant cannot

570 produce more than about 0.25 Nm of torque throughout the entire workspace with either the elbow flexion  
571 or elbow extension electrodes, but two other participants can produce more than 3 Nm in both of these  
572 cases. With these differences in mind, it is clear that some participants would never be able to achieve  
573 high reductions in power consumption with this hybrid control approach. To increase consistency between  
574 participants, it would be interesting to test with implanted FES systems, which are more reliable and  
575 targeted, and to model fatigue which can help modify the controller in real-time to account for it.

576 When observing the results of the three participants who performed the same protocol twice separated by  
577 at least a week, we see that the results remained similar between the two time points. The difference between  
578 sessions in sum of squared torque reduction when comparing the hybrid controller to the exoskeleton alone  
579 controller remained within 17% across participants for the elbow flexion/extension DOF, and below 10%  
580 for the wrist flexion/extension DOF. The difference between sessions in RMS tracking error for the hybrid  
581 controller compared to the exoskeleton alone controller remained below 7% across participants for the  
582 elbow flexion/extension DOF, and below 12% on the wrist flexion/extension DOF. It is encouraging that  
583 even though it is difficult to generalize across participants, these preliminary repeatability results seem to  
584 indicate that results hold steady within users if the same implementation procedure is followed during each  
585 use. It is important to note here that the participants repeated the entire protocol, and it is expected that the  
586 model that the FES production will change from day to day (especially when using surface electrodes),  
587 meaning that the model will necessarily have to be tuned for each use, even for the same participant.

588 One factor of this controller implementation that does not generalize across participants is that it relies on  
589 the relative weighting between exoskeleton torque inputs and FES activation levels. While the exoskeleton  
590 torque outputs are relatively consistent across participants, the activation levels do not map directly to  
591 torque outputs, because each participant produces a different amount of torque, given an activation level.  
592 In this case, the  $R_{fes}$  and  $R_{m-fes}$  parameters as defined in equations 22 and 23 must be scaled for each  
593 participant, based on the torque outputs expected from the GPR models. However, once the parameters are  
594 scaled once they should only need to be modified if electrodes need to be moved, or if fatigue occurs.

595 One participant had a particularly weak response to the FES, with a very low response from the elbow  
596 flexion/extension electrodes, and no response from the wrist flexion/extension electrodes. This difference  
597 compared to the remainder of participants shows the importance in characterizing each individual's FES  
598 behavior to understand the potential effectiveness of using the proposed hybrid controller.

## 599 4.5 Future Work

600 An area of interest in observing the behavior of hybrid systems would be to identify how maximum  
601 torque allowed by the exoskeleton changes the resulting behavior in terms of torque output and tracking  
602 error. We observed the maximum torque used by the exoskeleton in this study, but it was not limited in any  
603 particular way to influence controller behavior. We should expect the controllers to behave differently if the  
604 maximum torques are limited at the start, as the future-looking MPC controller is able to predict a torque  
605 limit onset and proactively compensate for it.

606 Modeling of fatigue is another area of interest when using FES, and has received much attention in  
607 the FES research community. While this study aimed to keep the stimulation time to a minimum to  
608 reduce the effects of fatigue, there were likely at least some effects of fatigue present in results. Modeling  
609 and compensating for fatigue would be a meaningful addition to the hybrid controller to see improved  
610 performance.

611 The overall results from this study show promise for power reduction while maintaining high accuracy  
612 when performing movements with a single-DOF through the implementation of the hybrid FES-exoskeleton  
613 controller. Importantly, these algorithms should translate to a multi-DOF use case with only small  
614 modifications. To realize truly shared control for generalized upper-limb movements, these algorithms  
615 should be tested in multi-DOF circumstances to understand potential benefits and complications in this  
616 scenario.

## 5 CONCLUSION

617 In this paper, we presented a model-based control approach to hybrid FES-exoskeleton control. We  
618 experimentally demonstrated the benefits of using this model-based controller to distribute robot and FES  
619 contributions to control elbow and wrist movements with a hybrid FES-exoskeleton system. This control  
620 strategy reduced exoskeleton torque for the hybrid system with similar tracking accuracy compared to  
621 using the exoskeleton alone. To realize practical implementation of hybrid FES-exoskeleton systems, the  
622 control strategy requires translation to multi-DoF movements, achieving more consistent improvement  
623 across participants, and balancing control to more fully leverage the muscles' capabilities.

## CONFLICT OF INTEREST STATEMENT

624 The authors declare that the research was conducted in the absence of any commercial or financial  
625 relationships that could be construed as a potential conflict of interest.

## AUTHOR CONTRIBUTIONS

626 ND, JB, ES, and MO contributed to conception and design of the study. ND conducted the experiments  
627 and performed the statistical analysis. ND wrote the first draft of the manuscript. ND, JB, ES, and MO  
628 wrote sections of the manuscript. All authors contributed to manuscript revision, read, and approved the  
629 submitted version.

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## DATA AVAILABILITY STATEMENT

635 The datasets created for this study can be found on github  
636 (<https://github.com/mahilab/SingleDofHybridControlDataFrontiers>).

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