# Correspondence

# Wave Propagation Speed and Equivalent Impedance of Dielectric-Coated Conductor Representing Lightning Return-Stroke Channel

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Abstract—In electromagnetic models of lightning return stroke, dielectric coating of channel-representing conductor is used to adjust the current wave propagation speed. In this letter, we examine dependences of both the speed and channel equivalent impedance on the electric permittivity and magnetic permeability of such coating. With increasing the permeability, the propagation speed of current wave decreases and the equivalent impedance increases significantly. In contrast, these parameters are not much influenced by the increase of permittivity. The reason for the difference has been explained in terms of the transmission-line theory.

Index Terms—Channel equivalent impedance, electromagnetic models, finite-difference time-domain (FDTD) method, lightning, propagation speed.

# I. INTRODUCTION

Lightning return-stroke models are needed in studying lightning effects on various objects and systems, and in characterizing the lightning electromagnetic environment. Rakov and Uman [1], based on governing equations, have categorized return-stroke models into four classes: gas dynamic models; electromagnetic models; distributed-circuit models; and "engineering" models.

In the electromagnetic models (see Baba and Rakov and references therein) [2], Maxwell's equations are solved to yield the distribution of current along the lightning channel using numerical techniques, such as the finite-difference time-domain (FDTD) method (Yee) [3]. The resultant distribution of channel current is used to compute electric and magnetic fields radiated by the lightning channel. In these models, it is necessary to reduce the speed of the current wave propagating along the channel-representing vertical conductor to a value lower than the speed of light in air, c. One way to accomplish this is the use of dielectric coating of the conductor (e.g., Kato et al. [4] and Miyazaki and Ishii [5])

The return-stroke speed, along with the current peak, largely determines the radiation field initial peak (e.g., Rakov and Dulzon [6]), while the equivalent impedance of lightning channel influences the magnitude of lightning current and/or the current reflection coefficient at the top of tall structure struck by lightning when a lumped voltage source is employed. It is desirable that model parameters are consistent with the following experimental data.

Manuscript received 16 October 2022; revised 10 December 2022; accepted 3 January 2023. Date of publication 2 February 2023; date of current version 14 April 2023. This work was supported in part by NSF under Grant AGS-2055178. (Corresponding author: Koki Kutsuna.)

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TEMC.2023.3235385.

Digital Object Identifier 10.1109/TEMC.2023.3235385

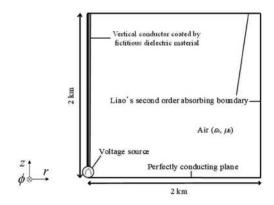


Fig. 1. Configuration of the computational domain in the 2D cylindrical coordinate system. The dielectric has (a) zero conductivity,  $\mu_{\tau}=1$ , and  $\varepsilon_{\tau}>>1$ , (b) zero conductivity,  $\varepsilon_{\tau}=1$ , and  $\mu_{\tau}>>1$ , or (c) zero conductivity,  $\mu_{\tau}>>1$ , and  $\varepsilon_{\tau}>>1$ .

- Typical values of return-stroke wavefront speed are in the range from c/3 to c/2 (Rakov) [7], as observed using optical techniques;
- 2) The equivalent impedance of the lightning return-stroke channel is in the range from 0.6 to 2.5 k $\Omega$  (Gorin and Shkilev) [8], as estimated from measurements of lightning current at different points along the 530-m-high Ostankino Tower in Moscow.

In this letter, we study influences of the permittivity and permeability of the dielectric coating of a vertical conductor, which simulates the lightning return-stroke channel, on the speed of the current wave propagating along the vertical conductor and on its equivalent impedance.

#### II. MODEL DESCRIPTION

Fig. 1 shows the configuration used in the FDTD simulations in the two-dimensional (2-D) cylindrical coordinate system. For FDTD simulations, the 2-D computational domain, whose radial dimension is 2 km and vertical dimension is also 2 km, is divided into rectangular cells of  $\Delta r = 0.5$  m and  $\Delta z = 2$  m. Liao's second-order absorbing boundary condition (Liao et al.) [9] is applied to the upper, bottom, and right-side boundaries to minimize unwanted reflections there. The bottom boundary is set to be a perfectly conducting plane. A vertical perfect conductor of radius 1 m, coated by a 0.5-m-thick fictitious perfect-dielectric material having relative permittivity  $\varepsilon_r$  and relative permeability  $\mu_r$  is located along the left-side boundary (z-axis). The base of the vertical conductor is excited by a lumped voltage source, which generates a Gaussian pulse wave. Influences of the value of the permittivity and permeability of the coating on the speed of current wave propagating along the vertical conductor and on its equivalent impedance are studied. Also, influences of the width of the Gaussian

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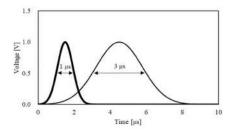


Fig. 2. Waveforms of voltage generated by the voltage source.

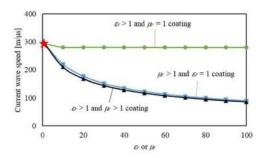


Fig. 3. Dependences of the speed of current wave propagating along the vertical 1-m radius conductor with 0.5-m thick lossless dielectric coating on  $\varepsilon_T$  and/or  $\mu_T$ . The red star corresponds to  $\varepsilon_T=1$  and  $\mu_T=1$  (no coating).

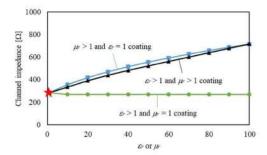


Fig. 4. Dependences of the equivalent impedance of vertical 1-m radius conductor with 0.5-m thick lossless dielectric coating on  $\varepsilon_r$  and/or  $\mu_r$ . The red star corresponds to  $\varepsilon_r = 1$  and  $\mu_r = 1$  (no coating).

pulse, the thickness and conductivity of the coating when it is assumed to be lossy, and the radius of the conductor are examined.

Fig. 2 shows the waveforms of the voltage generated by the voltage source. The peak is 1 V, and the half-peak width is either 1 or 3  $\mu$ s with the risetimes being similar to the return-stroke current risetimes for subsequent and first strokes, respectively.

## III. ANALYSIS AND RESULTS

Fig. 3 shows dependences of the speed of current wave propagating along the vertical conductor on  $\varepsilon_r$  and/or  $\mu_r$ . The current propagation speed is evaluated by measuring the propagation time of the wave peak from the channel base to an altitude of 1 km. Fig. 4 shows dependences of the equivalent impedance of the vertical conductor on  $\varepsilon_r$  and/or  $\mu_r$ . The equivalent impedance is evaluated as the ratio of the applied voltage peak to the peak of the resultant current at the channel base.

It is clear from Figs. 3 and 4 that, with increasing  $\mu_r$ , the propagation speed of current wave decreases and the channel equivalent impedance increases significantly, while they are not much influenced by  $\varepsilon_r$ .

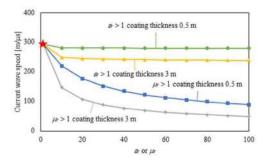


Fig. 5. Dependences of the speed of current wave propagating along the vertical 1-m radius conductor with 0.5-m and 3-m thick lossless dielectric coating on  $\varepsilon_T$  and/or  $\mu_T$ . The red star corresponds to  $\varepsilon_T=1$  and  $\mu_T=1$  (no coating).

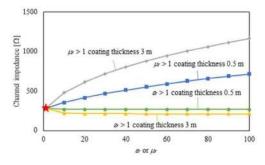


Fig. 6. Dependences of the equivalent impedance of vertical 1-m radius conductor with 0.5-m and 3-m thick lossless dielectric coating on  $\varepsilon_r$  and/or  $\mu_r$ . The red star corresponds to  $\varepsilon_r = 1$  and  $\mu_r = 1$  (no coating).

## IV. DISCUSSION

Using the 3-D FDTD method, Baba and Rakov [2]) have evaluated the speed of a 1- $\mu$ s-risetime ramp current wave propagating along a 0.23-m-radius vertical perfect conductor embedded in a dielectric parallelepiped with a cross section of 8 m  $\times$  8 m and  $\varepsilon_r=9$  to be 200 m/ $\mu$ s (0.67c). This configuration can be viewed as a conductor having about 4-m-thick dielectric coating, considerably thicker than 0.5 m used in this letter. The speed of a Gaussian-pulse current having a half-peak width of 1  $\mu$ s propagating along a 1-m-radius vertical perfect conductor coated by a 4-m-thick dielectric with  $\varepsilon_r=9$ , evaluated using the FDTD simulation in the 2-D cylindrical coordinate system, is 240 m/ $\mu$ s (0.8c), which is not much different from the speed estimated in [2].

When the half-peak width of the Gaussian pulse is increased from 1 to 3  $\mu$ s, the speed of current wave propagating along the dielectric-coated vertical conductor remains essentially the same. On the other hand, the channel equivalent impedance increases by about 20% regardless of the value of  $\varepsilon_r$  or  $\mu_r$ , probably due to the dependence of the characteristic impedance of a vertical conductor above ground on height (Baba and Rakov) [10].

When the thickness of dielectric coating was increased from 0.5 to 3 m, for relative permeability  $\mu_r=30$ , 60 or 90 and  $\varepsilon_r=1$ , the current wave propagation speed reduced by about 40, 45% or 45%, respectively, as shown in Fig. 5, and the channel equivalent impedance increased by about 50, 60% or 60%, respectively, as shown in Fig. 6. On the other hand, for relative permittivity  $\varepsilon_r=30$ , 60 or 90 and  $\mu_r=1$ , the current wave propagation speed reduced by about 15%, as shown in Fig. 5, and the channel equivalent impedance reduced by about 20%, as shown in Fig. 6, for the same change in the coating thickness from 0.5 to 3 m.

When the radius of conductor was increased from 1 to 3 m, for relative permeability  $\mu_r = 30$ , 60 or 90 and  $\varepsilon_r = 1$ , the current wave propagation speed reduced by about 25, 30 or 35%, respectively, as

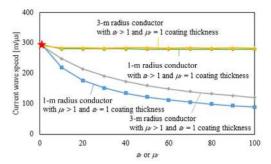


Fig. 7. Dependences of the speed of current wave propagating along the vertical 1-m and 3-m radius conductors with 0.5-m thick dielectric coating on  $\varepsilon_T$  and/or  $\mu_T$ . The red star corresponds to  $\varepsilon_T=1$  and  $\mu_T=1$  (no coating).

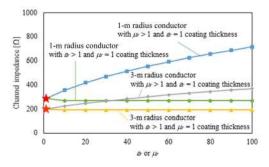


Fig. 8. Dependences of the equivalent impedance of vertical 1-m and 3-m radius conductors with 0.5-m thick dielectric coating on  $\varepsilon_T$  and/or  $\mu_T$ . The red star corresponds to  $\varepsilon_T=1$  and  $\mu_T=1$  (no coating).

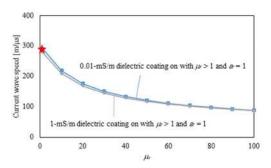


Fig. 9. Dependences of the speed of current wave propagating along the vertical 1-m radius conductor with 0.5-m thick 0.01 or 1-mS/m lossy dielectric coating on  $\mu_r$ . The red star corresponds to  $\varepsilon_r=1$  and  $\mu_r=1$  (no coating).

shown in Fig. 7, and the channel equivalent impedance decreased by about 45, 45% or 50%, respectively, as shown in Fig. 8. On the other hand, for relative permittivity  $\varepsilon_r = 30$ , 60, or 90 and  $\mu_r = 1$ , the current wave propagation speed increased only by about 1%, as shown in Fig. 7, and the channel equivalent impedance decreased by about 30%, as shown in Fig. 8, for the same change in the conductor radius from 1 to 3 m.

We now consider the case of lossy dielectric. When the conductivity of dielectric coating was increased from 0 to 0.01 mS/m, for relative permeability  $\mu_r = 30$ , 60 or 90 and  $\varepsilon_r = 1$ , the current wave propagation speed decreased by only about 1%, as shown in Fig. 9, and the channel equivalent impedance decreased by about 20%, as shown in Fig. 10. When the conductivity of dielectric coating was increased from 0 to 1 mS/m, for relative permeability  $\mu_r = 30$ , 60 or 90 and  $\varepsilon_r = 1$ , the current wave propagation speed decreased by only about 4, 3% or 2%, respectively, as shown in Fig. 9, and the channel equivalent impedance decreased by about 25%, as shown in Fig. 10.

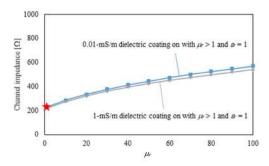


Fig. 10. Dependences of the equivalent impedance of vertical 1-m radius conductor with 0.5-m thick 0.01 or 1-mS/m lossy dielectric coating on  $\mu_{\tau}$ . The red star corresponds to  $\mu_{\tau}=1$  (no coating).

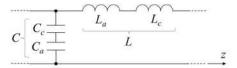


Fig. 11. Conceptual per-unit-length equivalent circuit of an above-ground conductor coated by fictitious dielectric material with high values of  $\varepsilon_r$  and  $\mu_r$ . For a lossless transmission line, the phase velocity  $v=(LC)^{-1/2}$  and the characteristic impedance  $Z_0=(L/C)^{1/2}$ .

In the following, we will interpret the results presented above in terms of the transmission-line (TL) theory. Fig. 11 shows the conceptual equivalent circuit of a vertical conductor coated by a fictitious dielectric material of high  $\varepsilon_r$  and/or  $\mu_r$ , where  $C_c$  is the per-unit-length shunt capacitance of high-permittivity-coating layer,  $C_a$  is the per-unit-length shunt capacitance between the coating outer surface and the ground,  $L_c$  is the per-unit-length inductance due to the magnetic flux in high-permeability-coating layer, and  $L_a$  is the per-unit-length series inductance due to the magnetic flux between the coating outer surface and the ground. Applicability of the classical TL theory to a vertical lightning channel above ground, which is not a uniform TL, was discussed and justified by Rakov [11], Baba and Rakov [2], and de Conti et al. [12] among others.

The series inductance L per unit length of a horizontal dielectric coated conductor, which is located at height h from perfectly conducting ground and has radius a and dielectric thickness (b-a) is given as follows:

$$\begin{cases} L = L_a + L_c \\ L_a = \frac{\mu_0}{2\pi} \ln \frac{2h}{b}, \quad L_c = \frac{\mu_T \mu_0}{2\pi} \ln \frac{b}{a} \quad [\text{H/m}] \quad \text{(for } h >> b > a) \end{cases}$$
(1)

Note that it is common practice to apply the horizontal conductor formula to a vertical conductor (e.g., Baba and Rakov [2]); for a vertical conductor, L (and C) is a function of h, but the dependence is relatively weak (logarithmic). The inductance  $L_0$  per unit length of a bare horizontal conductor with radius a at height h is given by

$$L_0 = \frac{\mu_0}{2\pi} \ln \frac{2h}{a}$$
 [H/m] (for  $h >> a$ ). (2)

Thus, the ratio of the inductances  $L/L_0$  is as follows:

$$\frac{L}{L_0} = \frac{L_a + L_c}{L_0} = \frac{\ln \frac{2h}{b} + \mu_r \ln \frac{b}{a}}{\ln \frac{2h}{a}}.$$
 (3)

Similarly, the ratio of the capacitance C per unit length of a horizontal dielectric-coated conductor to the capacitance  $C_0$  of the corresponding

bare conductor is given by

$$\frac{C}{C_0} = \frac{1}{\frac{1}{C_a} + \frac{1}{C_c}} \frac{1}{C_0} = \frac{\ln \frac{2h}{a}}{\ln \frac{2h}{b} + \frac{1}{\varepsilon_r} \ln \frac{b}{a}}.$$
 (4)

For h = 100 m, a = 1 m, b = 1.5 m,  $\varepsilon_r = 20$  and  $\mu_r = 20$ , (3) and (4) yield  $L/L_0 = 2.5$  and  $C/C_0 = 1.1$ . It follows from (3) and (4) that the higher relative permeability of the coating contributes to the increase of series inductance more significantly than does the higher relative permittivity to the increase of shunt capacitance. This is the reason why the current propagation speed and the channel equivalent impedance are sensitive to variation in  $\mu_r$ , but not much influenced by variation in  $\varepsilon_r$  (see Figs. 3 and 4).

#### V. SUMMARY

Influences of the permittivity and permeability of the lossless dielectric coating of a vertical conductor on the current-wave propagation speed and the equivalent impedance of the return-stroke channel have been studied. With increasing the permeability, the propagation speed of current wave decreases and the channel equivalent impedance increases to values expected for lightning. On the other hand, they are not much influenced by the increase of permittivity. The reason for the difference has been explained in terms of the TL theory. Influences of the thickness of dielectric coating with high relative permeability and the radius of the coated conductor on both the current propagation speed and the equivalent impedance are significant. The conductivity of lossy dielectric coating, 0.01 mS/m or 1 mS/m, has little influence on the current propagation speed, while the equivalent impedance decreases by about 25%, probably because it serves to effectively increase the conductor radius.

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