Network theory

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Intrinsic simplicity of complex systems

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Predicting the large-scale behaviour of complex systems is challenging because of their underlying nonlinear dynamics. Theoretical evidence now verifies that many complex systems can be simplified and still provide an insightful description of the phenomena of interest.

In our world, we are surrounded by systems of staggering complexity, from societal structures to the neural networks within our brains[1]. One typical strategy to reduce such complexity is to use dimension-reduction techniques, which enable the construction of simplified models to access the behaviour of their intricate counterparts. However, finding the optimal dimension for such reductions remains a challenge. Writing in *Nature Physics*, Vincent Thibeault and colleagues[2] have harnessed the 'low-rank hypothesis' to find an appropriate dimension for a simplified model, ensuring that it retained the essential features of the original high-dimensional network.

One of the first approaches to perform dimensionality reduction of complex networks involved the mapping of systems to effective one-dimensional dynamics that accurately predicted the original macroscopic behaviour[3]. Validated by analyses across a spectrum of different fields, from ecology to biology to power grids, this approach enabled the identification of three structural attributes that are pivotal in determining a system's macroscopic dynamical behaviour: density, heterogeneity and symmetry.

Later research proposed various dimension-reduction approaches through the perspective of spectral analysis[4,5]. For instance, an analytical framework developed for node-specific dynamics allowed the collapse of complex high-dimensional networks into low-dimensional manifolds governed by a few control parameters[6]. This method demonstrated the capability to condense intricate dynamics into effective two- or four-dimensional models[7,8], with particular success in the case of mutualistic networks. Another notable study showed that the dynamics of a networked system could be predicted without full knowledge of the underlying topology[9]. This suggested that dynamics operate in a subspace of astonishingly low dimension compared with the size and heterogeneity of the whole network. Although these previous studies emphasized the potential of dimension-reduction techniques for networks, devising a theory to identify the optimal dimension is still an open problem.

As a network can always be described in the language of matrices and tensors, linear algebra provides vital tools to study network properties. From this perspective, dimension-reduction techniques operate on the implicit assumption that the dynamics of high-dimensional complex systems depend on the behaviour of low-rank matrices. The goal of Thibeault and colleagues was to explore the validity of this low-rank hypothesis and to characterize its impact for many dynamical systems on networks. They first provided theoretical demonstrations that the

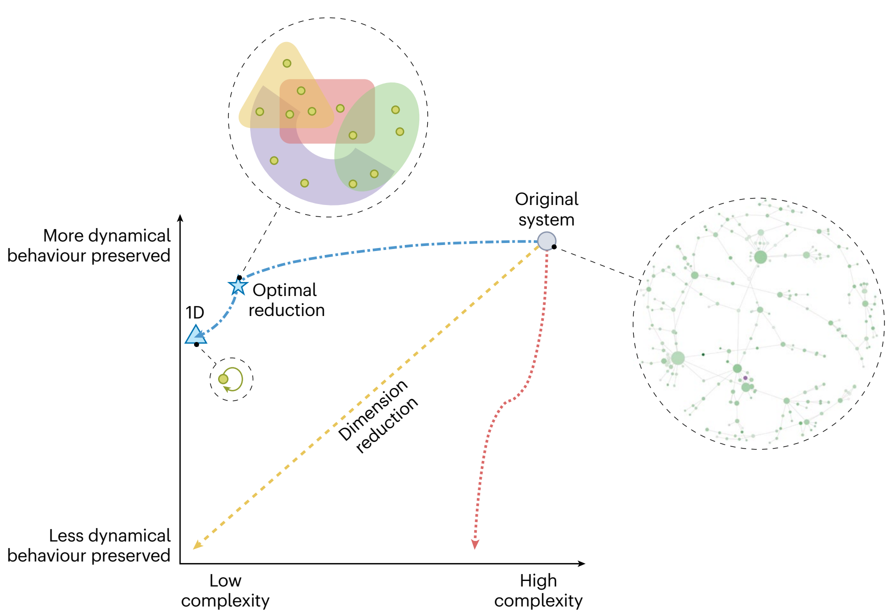

Fig. 1 | Dimension reduction of high-dimensional systems. The original dynamical system is complex, with many dynamical components. The low-rank hypothesis suggests that dimension reduction will lead the system to follow the blue dot-dashed curve instead of the red dotted curve, indicating an optimal dimension reduction (star), yielding higher-order interactions. The ultimate reduction is one-dimensional[3] (triangle).

hypothesis was satisfied in many widely used random networks, finding that the singular values of these networks' weight matrices decreased rapidly, implying low effective ranks. Then the team carried out extensive numerical simulations on many real networks, which were found to exhibit rapidly declining singular values leading to low effective ranks.

Thibeault and co-workers modelled the dimension reduction of a dynamical system as a problem of aligning a low-dimensional vector field with its original one. Minimizing the alignment error yields the optimal dimension-reduction matrix. The optimal low dimension depends on whether it preserves the macroscopic properties of the original system, such as the presence of a phase transition or critical points (Fig. 1). The reduced system resembles low-dimensional dynamics on a smaller network whose dynamical behaviour remains determined, such as a second-order transition in SIS epidemic dynamics, a hysteresis transition in Wilson–Cowan dynamics and a limit cycle in recurrent neural network dynamics. Furthermore, the results provide insights into the emergence of higher-order interactions, as the reduction matrix partially determines the hypergraph's directed, weighted and signed nature. A potential application of the results of Thibeault et al. is to answer the questions of whether the large language models are also low-rank and how to reduce their dimensionality through the low-rank hypothesis.

The main disadvantage of spectral graph theory is its lack of an intuitive interpretation compared with the insights provided by network science more broadly. In contrast, network science, rooted in graph theory, is empirical and focuses on data, offering intuitive and easy-to-design structure properties, such as degree, clustering coefficient and many more. Thus, cross-disciplinary research promotes new ideas and methodologies to understand, predict and control the dynamics and structures of complex systems. The insights offered by the work of Thibeault and colleagues will enrich the toolbox of mathematics and facilitate progress in various scientific and engineering problems, playing a central role in enhancing and controlling the resilience of complex networked systems[10].

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Competing interests

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