Towards an Analytical Quantum Full-Wave Solution of a Transmon Qubit in a 3D Microwave Cavity

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Circuit quantum electrodynamics (cQED) is one of the most promising platforms for building quantum information processors. As quantum technology evolves the hardware is becoming increasingly complex, so there is a need for a general-purpose, high-fidelity numerical modeling method for an accurate analysis. However, there is a lack of analytical solutions that can serve as a validation method to support the development of efficient and accurate numerical models in this new field. In this study, we develop an analytical solution using microwave cavity perturbation and antenna theories within a field-based description of cQED devices (T. E. Roth and W. C. Chew, "Macroscopic circuit quantum electrodynamics: A new look toward developing full-wave numerical models," *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, Vol. 6, 109-124, 2021). The system analyzed corresponds to a transmon inside a rectangular waveguide cavity, where the transmon is formed by connecting a Josephson junction across the terminals of a small dipole antenna. We validate our analytical solution against our same formalism evaluated with numerical eigenmodes and the energy participation ratio quantization approach (Z. K. Minev, Z. Leghtas, S. O. Mundhada, L. Christakis, I. M. Pop, and M. H. Devoret, "Energy-participation quantization of Josephson circuits," *npj Quantum Information*, Vol. 7, No. 1, 1-11, 2021).

To validate the approach, we form a matrix representation of our Hamiltonian operator and calculate the coupled system's eigenvalues and eigenvectors. Using these new eigenvalues and eigenvectors, we compute experimentally-relevant system parameters such as qubit and cavity operating eigenfrequencies, qubit anharmonicity, and AC Stark shifts. These calculations were performed for the transmon positioned at different locations inside the cavity. The average values of the system parameters are shown in Table 1 for the various calculation methodologies. Good agreement is obtained given the typical experimental precision of these quantities.

Table. 1 Average system parameters, where quantities in parentheses correspond to relative percent errors with respect to the numerical eigenmode data.

	Analytical Solution	**Numerical Eigenmode**	**EPR**
Qubit Eigenfrequency (GHz)	6.39 (0.84)	6.44	6.43 (0.21)
Cavity First Eigenfrequency (GHz)	7.55 (1.1e-4)	7.55	7.55 (1.5e-2)
Cavity Second Eigenfrequency (GHz)	9.96 (3.7e-5)	9.96	9.96 (2.6e-2)
Qubit Anharmonicity (MHz)	-371.72 (-1.92)	-379.00	-360.78 (-4.81)
AC stark shift (MHz)	-0.028 (-10.56)	-0.025	-0.026 (-2.28)

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