



Next-generation even-denominator fractional quantum Hall states of interacting composite fermions

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The discovery of the fractional quantum Hall state (FQHS) in 1982 ushered a new era of research in many-body condensed matter physics. Among the numerous FQHSs, those observed at even-denominator Landau level filling factors are of particular interest as they may host quasiparticles obeying non-Abelian statistics and be of potential use in topological quantum computing. The even-denominator FQHSs, however, are scarce and have been observed predominantly in low-disorder two-dimensional (2D) systems when an excited electron Landau level is half filled. An example is the well-studied FQHS at filling factor $\nu = 5/2$ which is believed to be a Bardeen-Cooper-Schrieffer-type, paired state of flux-particle composite fermions (CFs). Here, we report the observation of even-denominator FQHSs at $\nu = 3/10$, $3/8$, and $3/4$ in the lowest Landau level of an ultrahigh-quality GaAs 2D hole system, evinced by deep minima in longitudinal resistance and developing quantized Hall plateaus. Quite remarkably, these states can be interpreted as even-denominator FQHSs of CFs, emerging from pairing of higher-order CFs when a CF Landau level, rather than an electron or a hole Landau level, is half-filled. Our results affirm enhanced interaction between CFs in a hole system with significant Landau level mixing and, more generally, the pairing of CFs as a valid mechanism for even-denominator FQHSs, and suggest the realization of FQHSs with non-Abelian anyons.

even denominator | fractional quantum Hall effect | lowest Landau level | composite fermion pairing | Landau level mixing

Electrons in flat bands have essentially no kinetic (Fermi) energy and therefore provide a prime platform for exploring many-body physics and emergent interaction phenomena. Nearly perfect flat bands with zero dispersion can be achieved via applying a strong, perpendicular magnetic field (B) to a two-dimensional electron system (2DES), where electrons occupy quantized Landau levels (LLs) (1–4). Over the past decades, electrons (or holes) confined to GaAs heterostructures have been the system of choice to realize this framework because of their extraordinary high purity and transport mobility (1–6). Continuous improvements in the quality of GaAs 2DESs have facilitated the discovery of many exotic interaction phenomena, such as odd- and even-denominator fractional quantum Hall states (FQHSs) (1, 7), Wigner solids, and stripe/nematic and bubble phases (2–4).

Among these phenomena, the FQHS, an incompressible liquid of electrons formed when a rational fraction of a LL is occupied, has been an active topic of interest thanks to its nontrivial topological properties. The vast majority of FQHSs are observed in the lowest orbital LLs at odd-denominator fillings and can be explained by Laughlin's wave function (8) and Jain's composite fermion (CF) model (9). By attaching an even number ($2m$) of magnetic flux quanta to each electron to form a CF, the strongly interacting electrons in B can be equivalently described as weakly interacting CFs in residual magnetic field $B^* = B - B_{1/2m}$, where $B_{1/2m}$ is the magnetic field at filling factor $\nu = 1/2m$. In this picture, CFs occupy effective LLs, termed Lambda levels (ΛLs), formed by B^* , and FQHSs of electrons at $\nu = j/(2mj \pm 1)$ can be understood as the integer QHSs of $2m$ -flux CFs (2^m CFs) at filling $\nu^{2m-CF} = j$, known as the Jain-sequence states (j is an integer) (3, 4, 9).

While in the interpretation of the Jain-sequence states, CFs are viewed as noninteracting quasiparticles, there is evidence that these emergent quasiparticles can indeed interact and exhibit intriguing interaction phenomena of their own. Examples include FQHSs at unconventional fillings (7, 10–13), Bloch ferromagnetism (14), Wigner crystallization (15), and topological bubbles (16). In particular, CF–CF interaction can generate FQHSs at unconventional fillings beyond the Jain-sequence states via different mechanisms.

Significance

In a large perpendicular magnetic field, electrons confined in a plane occupy quantized, (Landau) energy levels. When a fraction of the lowest-energy Landau level (LLL) is occupied, electron-electron Coulomb interaction dominates over single-electron kinetic energy, and fractional quantum Hall states (FQHSs) manifest themselves as the many-body ground states. Predominantly odd-denominator FQHSs are observed in the LLL and can be understood in a single-particle picture using the theory of noninteracting electron-flux quasiparticles (composite fermions). Here, we report observation of FQHSs at even-denominator fractions ($3/10$, $3/8$, and $3/4$) in the LLL of holes. An understanding of these states necessitates accounting for residual interaction between composite fermions. Our results provide insights into the realization of FQHSs through composite fermion interaction and their pairing.

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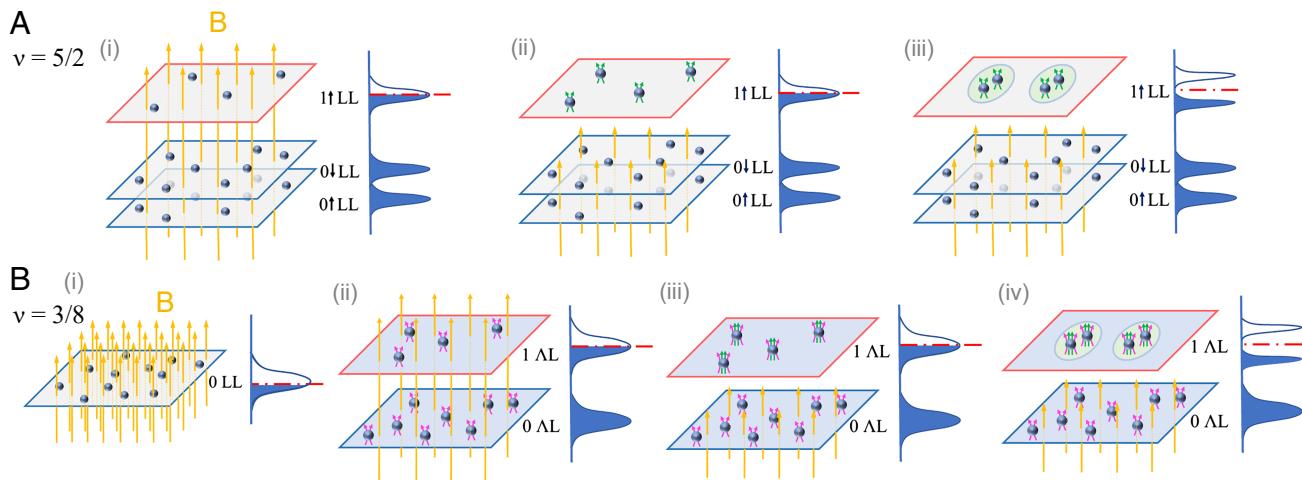


Fig. 1. Schematics of the pairing mechanism for even-denominator FQHs at (A) $v = 5/2$, and (B) $v = 3/8$. The small blue spheres represent electrons (or holes), whereas the gray and blue planes represent electron LLs and CF ALs, respectively. The yellow vertical arrows represent the magnetic field experienced by electrons or CFs. The curved short arrows represent magnetic flux quanta attached to electrons or holes to form CFs (blue spheres with curved arrows). The magenta fluxes are felt by all the CFs, while the green fluxes are only felt by the CFs in the topmost LL or AL. On the Right of each panel, density of states for each LL or AL is shown; the red dash-dotted line is the Fermi energy. At $v = 5/2$ ($3/8$), ^2CFs (^4CFs) in the $1\uparrow$ LL ($1\uparrow$ AL) pair up and open a gap, leading to an incompressible ground state.

An example is a FQHS observed at $v = 4/11$, whose origin could be a FQHS of interacting ^2CFs at $v^{2\text{-CF}} = 1 + 1/3$, namely at a partially occupied CF AL (11, 17). Another, better-known example is the even-denominator FQHS at $v = 5/2$, observed in high-quality GaAs 2DESs (7). In this case, as illustrated in Fig. 1 A, *i*, there are two completely filled, ground-state ($N = 0$) LLs ($0\uparrow$ and $0\downarrow$; arrows represent spins) and a half-filled, excited-state ($N = 1$) LL involved. Electrons in the half-filled $N = 1$ LL capture two flux quanta to turn into ^2CFs [Fig. 1 A, *ii*], which then undergo a pairing instability and condense into a paired FQHS [Fig. 1 A, *iii*]. The $v = 5/2$ FQHS is generally believed to be a non-Abelian, paired state of CFs supporting zero-energy Majorana modes (18, 19), potentially useful in fault-tolerant topological quantum computing (20).

Here, we report the observation of exotic even-denominator FQHs in an ultrahigh-mobility GaAs 2D hole system (2DHS). The even-denominator states we observe are particularly remarkable as they are formed in the $N = 0$ (rather than $N = 1$) LL and combine the key features of the two types of unconventional FQHs discussed above: a partially filled CF AL and CF pairing. We interpret these states as even-denominator FQHs of CFs emerging from pairing of CFs in the excited ($N = 1$) CF AL (Fig. 1B), analogous to the $v = 5/2$ FQHS.

Results and Discussion

Fig. 2 highlights our main findings: We observe even-denominator FQHs in the lowest LL at $v = 3/4, 3/8, 3/10$, and the unusual odd-denominator FQHS at $v = 4/11$, evinced by deep R_{xx} minima. A weaker minimum is also seen at $v = 5/12$. The $v = 3/4$ FQHS was also reported very recently in a record-high-mobility 2DHS (21). Numerous high-order Jain-sequence states are also observed near $v = 1/2$, at $v = 1/3, 2/5, \dots$, up to $v = 14/29$, and at $v = 2/3, 3/5, \dots$, up to $15/29$ (Fig. 2, *Left Inset*), comparable to what has been seen in the record-high-quality GaAs 2DESs (5). These features demonstrate the exceptionally high quality of our 2DHS sample. We note that, in contrast to the trace between 7 and 16 T where R_{xx} remains $\lesssim 3$ k Ω , R_{xx} on the flanks of $v = 1/3$ (between $v = 1/3$ and $3/8$, and $1/3$ and $2/7$) reaches very high values, of the order

of $\simeq 100$ k Ω , at $T \simeq 37$ mK. We show traces taken at three different temperatures, revealing an insulating behavior in these regions: As we increase T from 37 to 121 mK, R_{xx} values decrease by more than an order of magnitude. Such reentrant insulating phases appearing between strong FQHs have been reported in GaAs 2DESs near $v = 1/5$ (22, 23) and are generally believed to be Wigner solids pinned by the ubiquitous disorder (2, 24). In GaAs 2DHSs, a similar reentrant behavior was seen at higher fillings, near $v = 1/3$ (25, 26). This is generally attributed to the larger effective mass of 2D holes ($\simeq 0.5 m_0$) compared to GaAs 2D electrons ($0.067 m_0$), and the ensuing severe LL mixing in 2DHSs which favors the Wigner solid states as ground states at higher fillings (27).

The most pronounced R_{xx} minima at $v = 4/11$ and $3/10$ are observed at intermediate $T \simeq 78$ mK. They become shallower at lower $T \simeq 37$ mK. This signals a close competition between the FQHs and Wigner solid states. More specifically, the energies of the FQHS and Wigner solid states are so close that the FQHS wins in a very narrow range of v , but a minuscule density inhomogeneity in the 2DHS can disturb the FQHS. This is reminiscent of what was observed at $v = 1/5$ in GaAs 2DESs where, initially, only an R_{xx} minimum that rose with decreasing temperature was seen but eventually, as the sample quality improved, a vanishing R_{xx} accompanied by a quantized R_{xy} plateau was reported, firmly establishing that the ground state at $v = 1/5$ is a FQHS (22, 23, 28–32). The situation is also similar to what has been reported so far at $v = 1/7$ except that, even in the ultrahigh-mobility 2DESs that have become available very recently, only a deep R_{xx} minimum that rises at the lowest temperatures is observed (33–35). (See *SI Appendix* for more details regarding the historical development of the $v = 1/5$ FQHS in GaAs 2DESs and also additional data for a higher-quality 2DHS which exhibits a decreasing R_{xx} with decreasing temperature at $v = 3/8$.)

A quantitative measure of the robustness of a FQHS is the size of its energy gap. However, as stated above, R_{xx} minima at $v = 3/8, 4/11$, and $3/10$ in our 2DHS are strongly affected by the insulating background on their flanks. Alternatively, we perform a pseudogap analysis and estimate gaps of the order of a few hundred mK for these FQHs (*SI Appendix*, Figs. S1 and S2).

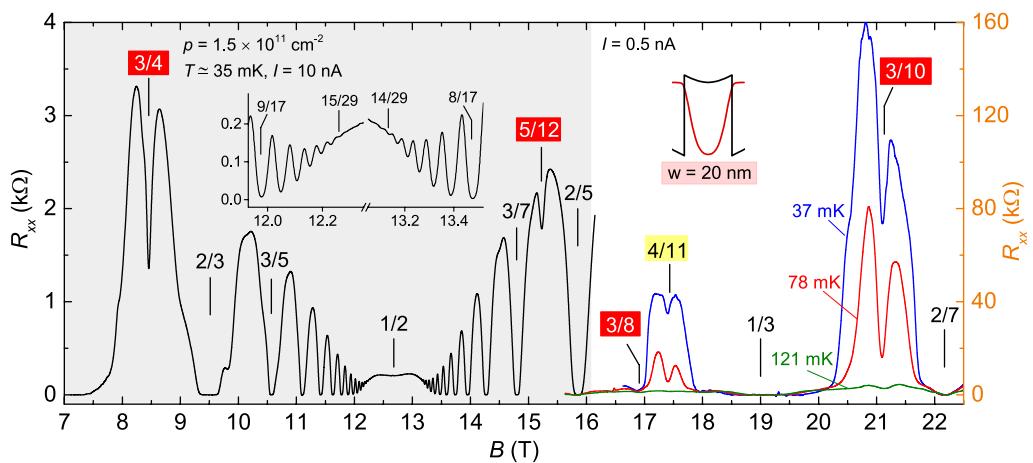


Fig. 2. Longitudinal resistance R_{xx} vs. perpendicular magnetic field B . Data are shown between $v = 1$ and $2/5$ at $T \simeq 35$ mK (black), and for $B > 16$ T at three different temperatures and with a different scale as indicated on the Right. Field positions of different fillings are marked with vertical lines. Left Inset: A magnified version of R_{xx} vs. B near $v = 1/2$. Right Inset: The self-consistently calculated hole charge distribution (red) and potential (black) for our 2DHS.

These gaps are somewhat larger than the energy gap deduced for the $v = 3/4$ FQHS in a 2DHS sample with slightly smaller density ($\simeq 20$ mK) (21), but we emphasize that there is a large uncertainty in all these values.

In order to further demonstrate that the ground state at $v = 3/8$, $4/11$, and $3/10$ is a FQHS, in Fig. 3, we present Hall traces near these fillings. At $v = 1/3$, $2/5$, and $2/7$, R_{xy} exhibits flat, quantized plateaus at the expected values, and $R_{xx} \simeq 0$. Between these strong Jain-sequence states, we observe several developing Hall plateaus at $v = 3/8$, $4/11$, and $3/10$, consistent with the well-defined but nonzero R_{xx} minima at these fillings. We also see a weak R_{xx} minimum at $v = 5/13$, but no R_{xy} plateau. Note that the traces are taken at the optimal temperatures of $\simeq 93$ and $\simeq 134$ mK, respectively. At very low temperatures, R_{xx} near these fillings becomes extremely large because of the dominant insulating background. This induces severe R_{xx} mixing, preventing us from measuring an accurate R_{xy} . When the temperature is too high, the FQHSs become very weak and R_{xy} features disappear. We emphasize that the developing R_{xy} plateaus are observed at the expected magnetic fields to within 0.4% and centered at $R_{xy} = h/e^2 v$ to within 0.3%.

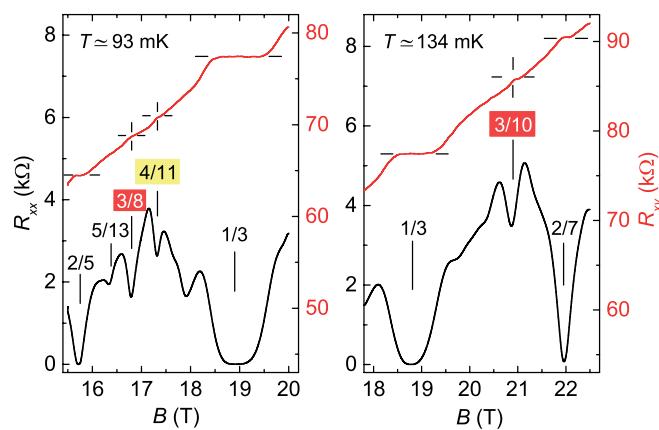


Fig. 3. Longitudinal (R_{xx} , in black) and Hall (R_{xy} , in red) resistances near $v = 3/8$ and $4/11$ (Left), and near $v = 3/10$ (Right). The vertical and horizontal lines mark the expected values for magnetic field positions and R_{xy} plateaus, respectively.

Phenomenologically, the 2-flux CFs (2 CFs) near $v = 1/2$ experience an effective magnetic field $B^* = B - B_{1/2}$ and form their own LLs (namely, ALs). A FQHS at fractional filling factor v can then be viewed as an integer QHS of 2 CFs at 2 CFs filling factor v^{2-CF} , with $v = v^{2-CF}/(2v^{2-CF} \pm 1)$. In order to compare the behavior of bare holes and 2 CFs, in Fig. 4A, we present R_{xx} vs. B traces in different field ranges and shift the x-axis of the *Upper* panel by $B_{1/2}$ to match the zero B of the *Lower* panel (4). The correspondence of R_{xx} minima at fractional fillings $v = 1/3, 2/5, 3/7, \dots$ ($v^{2-CF} = 1, 2, 3, \dots$) in the *Upper* panel of Fig. 4A with the R_{xx} minima at $v = 1, 2, 3, \dots$ in the *Lower* panel is clear; the relevant relation is $v = v^{2-CF}/(2v^{2-CF} + 1)$. Similarly, as seen in Fig. 4A, *Lower* panel, FQHSs on the lower-field side of $v = 1/2$ at $v = 2/3, 3/5, 4/7, \dots$ map to $v^{2-CF} = 2, 3, 4, \dots$, obeying the simple relation $v = v^{2-CF}/(2v^{2-CF} - 1)$. Moreover, the Jain-sequence FQHSs between $v = 1/3$ and $1/4$ (*Upper* panel) resemble the FQHSs between $v = 1$ and $1/2$ (*Lower* panel), and map to integer QHSs of 4-flux CFs with integer fillings v^{4-CF} ; e.g., FQHSs at $2/7, 3/11, 4/15, \dots$ map to $v^{4-CF} = 2, 3, 4, \dots$, following the relation $v = v^{4-CF}/(2v^{4-CF} - 1)$.

In very high-quality GaAs 2DESSs, however, besides the Jain-sequence states, several FQHSs that do not map into integer QHSs of CFs are observed (10–13). For example, between $v = 1/3$ and $2/5$, unconventional odd-denominator FQHSs have been reported at $v = 4/11$ and $5/13$. These can be viewed as QHSs of 2 CFs at fractional 2 CF fillings, $v^{2-CF} = 4/3$ and $5/3$.

Now, besides the $4/11$ and $5/13$ states, in our 2DHS sample, we observe numerous even-denominator FQHSs at $v = 3/4$, $3/8$, and $3/10$. Note the striking resemblance of the R_{xx} vs. B traces near $v = 3/10$ and $3/4$. Similar to the $3/4$ state which is the only FQHS observed between $v = 1$ and $2/3$, the $3/10$ state is also the only FQHS seen between the two nearby strong FQHSs at $v = 1/3$ and $2/7$. In fact, the $v = 3/4$ and $3/8$ states we observe can be mapped to even-denominator FQHSs of 2 CFs at $v^{2-CF} = 3/2$, and the $3/10$ state can be mapped to even-denominator FQHSs of 4 CFs at $v^{4-CF} = 3/2$. In Fig. 4B–D, we show schematics of CF AL occupations at $v = 3/8, 3/4$, and $3/10$ with 2 CFs and 4 CFs. They all have a completely filled lowest AL (0AL) and a half-filled excited AL (1AL), reminiscent of the half-filled $N = 1$ electron LL where pairing of 2 CFs presumably leads to an even-denominator FQHS at $v = 5/2$.

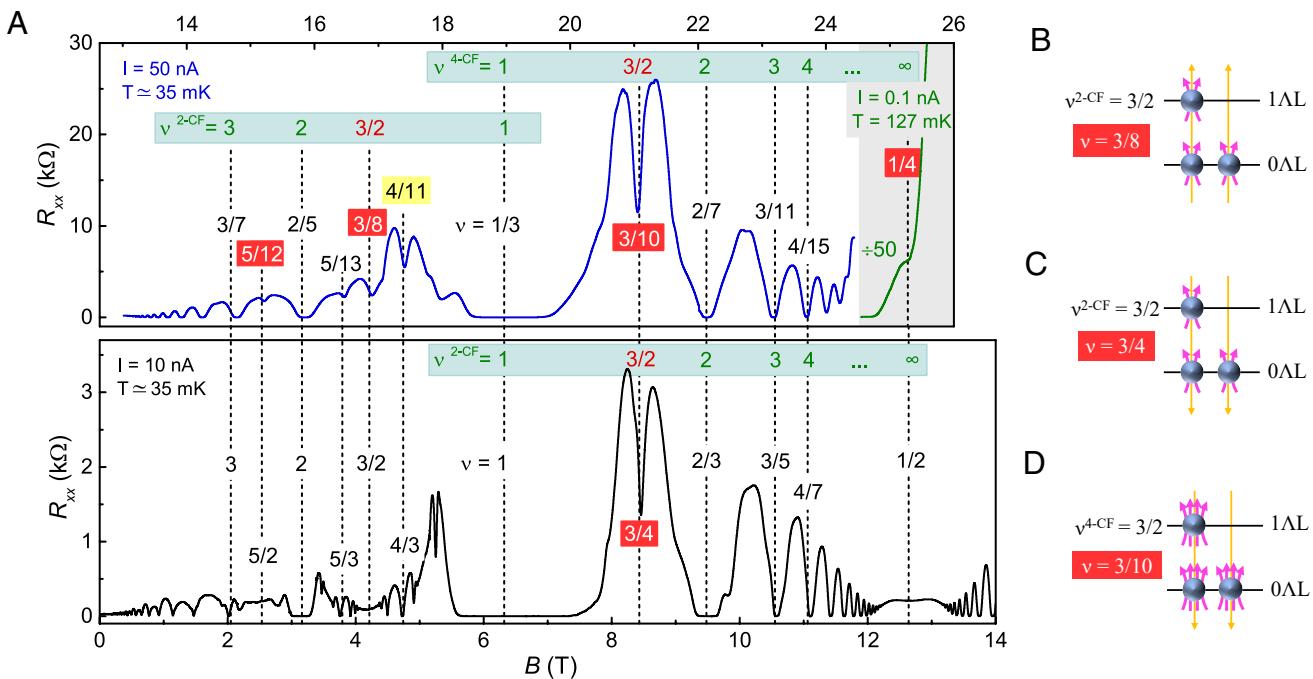


Fig. 4. FQHSs in CF picture. (A) R_{xx} vs. B traces. The x-axis of the *Upper* panel is shifted by $B_{1/2}$ to match the $B = 0$ of the *Lower* panel. The blue trace in the *Upper* panel was measured with an excitation current of 50 nA. As a result, the R_{xx} values of the insulating phases are suppressed because of nonlinear effect, and the hole temperature near the insulating phases may be higher due to the heating caused by the current. (B–D) Schematics of CF AL occupations at $v = 3/8$, $3/4$, and $3/10$. Holes capture two or four flux quanta (curved magenta arrows) to turn into CFs. The long yellow arrows indicate the residual magnetic field experienced by the CFs. Note that in Fig. 1, we emphasize the mechanism of CF pairing when a LL of electrons (Fig. 1A) or a AL of CFs (Fig. 1B) is half occupied. This requires additional flux attachment in the topmost, half-filled LL or AL; for clarity, in Fig. 4 B–D, we are not showing this flux attachment.

[Fig. 1A]. It is then tempting to interpret the $v = 3/8$, $3/4$ and $3/10$ states as even-denominator FQHSs of CFs emerging from paring of CFs in the excited AL. The mechanism is schematically illustrated in Fig. 1B for $v = 3/8$. In the lowest LL, the holes capture two flux quanta to turn into 2 CFs, which fill CF ALs formed by the residual magnetic field. The 2 CFs in the half-filled 1AL capture two more flux quanta to turn into higher-order (4-flux) 4 CFs which undergo a pairing instability and condense into a paired FQHS. Similarly, the $3/4$ ($3/10$) state can be interpreted as the consequence of pairing of 4 CF (6 CF) in 1AL. Note that at $v = 3/4$ and $3/10$ CFs experience negative effective field, that is, the magnetic fluxes captured by holes to form CFs are opposite to the direction of the residual magnetic field felt by the CFs (Fig. 4 C and D).

Pairing of CFs has been explored theoretically as a mechanism for even-denominator FQHSs (18, 19, 36–43). Although CF pairing was initially proposed to explain the $v = 5/2$ FQHS, it has been theoretically extended to the even-denominator fillings in the lowest LL when a CF AL is half filled. Scarola et al. (38) investigated the possible CF pairing at $v = 3/8$ in the spin-reversed AL with a two-body inter-CF model, which supports an incompressible, paired FQHS. The same problem was later revisited using a more accurate quantitative theory employing the method of CF diagonalization (40, 41). It was found that a paired state of CFs with anti-Pfaffian wave function is favored at $v = 3/8$ in both fully spin-polarized and partially spin-polarized systems. A CF diagonalization study of the fully polarized $v = 3/10$ state also suggests an incompressible ground state arising from an anti-Pfaffian type pairing of CFs in 1AL (42). Our observation of FQHSs at $v = 3/10$ and $3/8$ indeed supports that CF pairing applies to half-filled ALs. The predicted energy gaps of the $3/8$ and $3/10$ FQHSs are very small (e.g., $\simeq 0.002e^2/\epsilon l$

for $v = 3/8$) (40), suggesting that these states are very fragile, and thus, their observation requires ultrahigh-quality samples.

Our observation of even-denominator FQHSs at $v = 3/10$, $3/8$, and $3/4$ raises a natural question. Why are these very fragile states seen so clearly in GaAs 2DHSs rather than in 2DESs which in general exhibit much higher transport mobility (5, 6)? In GaAs 2DESs, only very weak dips in R_{xx} are observed at $v = 3/10$ and $3/8$ at very low temperatures with no discernible R_{xy} plateaus, and there is no R_{xx} minimum or R_{xy} plateau at $3/4$ (5, 11–13). In contrast, in our 2DHS, the R_{xx} minima at these fillings are robust, persisting to $T \simeq 134$ mK, and are accompanied by reasonably well-developed R_{xy} plateaus (Figs. 2 and 3). Similarly, there is a relatively strong $v = 3/4$ FQHS (Fig. 2 and also ref. 21). The fact that these even-denominator FQHSs are favored in 2DHSs strongly suggests that the larger effective mass of holes and the ensuing LL mixing play a significant role in inducing interaction between CFs and CF pairing in half-filled ALs.

Our conjecture regarding the importance of LL mixing and CF interaction in our 2DHS is supported by three additional observations. First, in our sample, we also see an odd-denominator FQHS at $v = 4/11$ [Fig. 3, *Left*]. This state, which is also believed to be stabilized by CF interaction (17, 39), is observed in GaAs 2DESs too but appears to be more robust in our 2DHS as it is seen at higher temperatures ($\simeq 93$ mK) and has a pseudogap energy of $\simeq 100$ mK (*SI Appendix*, Fig. S2), much higher than the energy gap ($\simeq 10$ mK) reported for 2DESs (12, 13). Second, in our 2DHS, we observe insulating phases (IPs) surrounding the $v = 3/10$, $4/11$, and $3/8$ FQHSs (Fig. 1). Note that these IPs are re-entrant around the $v = 1/3$ FQHS. This is in contrast to 2DESs which exhibit qualitatively similar reentrant IPs around a FQHS at much smaller filling, $v = 1/5$ (22–24). These reentrant IPs are generally believed to manifest pinned Wigner crystal

states, and the much higher filling of $1/3$ in 2DHSs compared to $1/5$ in 2DESs has been attributed to the more significant LL mixing in 2DHSs (25–27). Third, a very recent theoretical study, stimulated by the observation of the $v = 3/4$ FQHS in 2DHSs, points to the role of LL mixing in enhancing the CF interaction and leading to the emergence of even-denominator FQHSs in the lowest LL, e.g., at $v = 1/4$, in 2DHSs (43). (As detailed elsewhere, in our 2DHSs we do indeed observe a developing FQHS at $v = 1/4$ (44).

It is also worth emphasizing that the even-denominator FQHSs we report here are observed in the lowest LL, in contrast to the well-known $v = 5/2$ FQHS which is seen in the excited LL (7). Even-denominator FQHSs have been observed in the lowest LL at $v = 1/2$ and $1/4$ but only in GaAs 2DESs confined to wide or double quantum wells (QWs) with bilayer-like charge distributions (45–48). In double QWs (46), there is negligible interlayer tunneling, and the $1/2$ FQHS is widely believed to be a two-component (Halperin-Laughlin Ψ_{331}) state which is Abelian. The $1/2$ and $1/4$ FQHSs in wide QWs, however, have been enigmatic, although the latest experiments (49) favor a one-component state, consistent with the calculations which support a non-Abelian (Pfaffian) state (50, 51). Evidently, the thick and bilayer-like electron charge distribution in a wide QW softens the short-range part of the Coulomb repulsion, thereby destabilizing the CF-Fermi sea and resulting in a paired state of CFs at $v = 1/2$ and $1/4$. In our 2DHS, which is confined to a narrow QW and has a single-layer charge distribution (Fig. 2, *Right Inset*), it is the severe LL mixing that results in a similar softening (43).

The $v = 3/10$ and $3/8$ FQHSs we observe in our ultrahigh-quality 2DHS, together with the $v = 3/4$ FQHS (21), expand the family of even-denominator FQHSs. They suggest that CF pairing occurs not only at half-filled LLs but also at half-filled ALs, leading to the next-generation even-denominator FQHSs. Our results should stimulate future work. Theorists are bound to explore and calculate more rigorously the role of LL mixing and CF–CF interaction. On the experimental front, our report will affect the burgeoning area of research on many-body phenomena in monolayer 2D materials, such as transition-metal dichalcogenides (e.g., WSe₂). These materials also host carriers with a rather large effective mass, comparable to GaAs 2D holes (52, 53) and, if their quality could be further improved, should

provide a fertile ground for the observation of exotic many-body states such as those reported here.

Materials and Methods

The high-quality 2DHS studied here is confined to a 20-nm-wide GaAs quantum well grown on a GaAs (001) substrate by molecular beam epitaxy. The 2DHS has a hole density of $1.5 \times 10^{11} \text{ cm}^{-2}$ and a low-temperature (0.3 K) mobility of $3.2 \times 10^6 \text{ cm}^2/\text{Vs}$. We performed our experiments on a $4 \times 4 \text{ mm}^2$ van der Pauw geometry sample, with alloyed In:Zn contacts at the four corners and side midpoints. The sample was cooled in a dilution refrigerator with a base temperature of $\leq 35 \text{ mK}$. We measured the longitudinal and Hall resistances, R_{xx} and R_{xy} , using the conventional lock-in amplifier technique with low frequencies ($\leq 13 \text{ Hz}$) and small excitation currents ($\leq 50 \text{ nA}$).

Data, Materials, and Software Availability. All study data are included in the article and/or supporting information, and the original data can be found at online data repository (<https://doi.org/10.5281/zenodo.10279058>) (54).

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