

Evaluating the Performance of Empirical Models of Total Electron Density and Whistler-mode Wave Amplitude in the Earth's Inner Magnetosphere

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11 **Abstract** (**<=350 words**)

12 Empirical models have been previously developed using the large dataset of satellite observations to
13 obtain the global distributions of total electron density and whistler-mode wave power, which are
14 important in modeling radiation belt dynamics. In this paper, we apply the empirical models to
15 construct the total electron density and the wave amplitudes of chorus and hiss, and compare them
16 with the observations along Van Allen Probes orbits to evaluate the model performance. The
17 empirical models are constructed using the Hp30 and SME (or SML) indices, and considering the
18 plasmopause defined using the occurrence rate of high-density regions. The total electron density
19 model provides an overall high correlation coefficient with observations, while large deviations are
20 found in the dynamic regions near the plasmopause or in the plumes. The chorus wave model
21 generally agrees with observations when the plasma trough region is correctly modeled and for
22 modest wave amplitudes of 10 - 100 pT. The model overestimates the wave amplitude when the
23 chorus is not observed or weak, and underestimates the wave amplitude when a large-amplitude
24 chorus is observed. Similarly, the hiss wave model has good performance inside the plasmasphere
25 when modest wave amplitudes are observed. However, when the modeled plasmopause location does
26 not agree with the observation, the model misidentifies the chorus and hiss waves compared to
27 observations, and large modeling errors occur. In addition, strong (>200 pT) hiss waves are observed
28 in the plumes, which are difficult to capture using the empirical model due to their transient nature
29 and relatively poor sampling statistics. We also evaluate four metrics for different empirical models
30 parameterized by different indices. Among the tested models, the empirical model considering a
31 plasmopause and controlled by Hp* (the maximum Hp30 during the previous 24 h) and SME* (the
32 maximum SME during the previous 3 h) or Hp* and SML has the best performance with low errors
33 and high correlation coefficients. Our study indicates that the empirical models are applicable for
34 predicting density and whistler-mode waves with modest power, but large errors could occur,
35 especially near the highly-dynamic plasmopause or in the plumes.

36 1 Introduction

37 The dynamic evolution of Earth's outer radiation belt electron fluxes is strongly affected by whistler-
38 mode waves and the cold electron density through the wave-particle interaction processes (*Thorne et*
39 *al.*, 2021). After the electrons are injected from the nightside plasma sheet, whistler-mode chorus
40 waves scatter the energetic electrons at $\sim 1 - 100$ keV energies, causing their fluxes to decay along the
41 drift trajectory in the magnetosphere and precipitate them into the Earth's upper atmosphere (*Thorne*
42 *et al.*, 2010; *Tao et al.*, 2011; *Ma et al.*, 2012). Following the commencement of geomagnetic storm
43 and the subsequent substorms, chorus waves accelerate relativistic electrons at ~ 100 s keV - 10 MeV
44 energies to build up the Earth's outer radiation belt (*Reeves et al.*, 2013; *Thorne et al.*, 2013; *Li et al.*,
45 2014; *Ma et al.*, 2018). Hiss waves in the plasmasphere and plumes scatter the electrons at ~ 10 keV -
46 1 MeV energies, causing the radiation belt electron flux to decay during the storm recovery phase (*Ni*
47 *et al.*, 2013; *Ma et al.*, 2016a). The energy-dependent slot region forms between the inner and outer
48 radiation belts due to the dominant pitch angle scattering loss by hiss (*Reeves et al.*, 2016; *Ripoll et*
49 *al.*, 2016; *Zhao et al.*, 2019). The total electron density affects the electron resonance energy due to
50 chorus and hiss waves and the efficiencies of pitch angle scattering and acceleration (*Summers et al.*,
51 2007).

52 Whistler-mode chorus waves are commonly observed in the low-density plasma trough over the
53 nightside-dawn-dayside magnetic local time (MLT) sectors (*Li et al.*, 2009; *Meredith et al.*, 2012,
54 2020; *Agapitov et al.*, 2013). Chorus waves on the nightside are strong near the equator, and the
55 waves on dayside have a broad latitudinal coverage with maximum power observed at off-equatorial
56 latitudes (*Agapitov et al.*, 2018). Chorus waves are generated by the unstable anisotropic hot
57 electrons injected from the nightside plasma sheet (*Li et al.*, 2008; *Fu et al.*, 2014), with wave
58 intensities closely related to electron injection events (*Kasahara et al.*, 2009; *Ma, J. et al.*, 2022). The
59 statistical wave power is well correlated with the auroral electrojet index of *AE* or *AL*, which
60 indicates the strength of substorm injections. Chorus waves with high magnetic power are mainly
61 observed to be quasi-parallel propagating. Another group of highly oblique chorus waves have high
62 occurrence rates over the nightside-dawn sector close to the Earth (*Li et al.*, 2016a).

63 Hiss waves are commonly observed in the high-density plasmasphere and plumes in the dayside and
64 afternoon sectors (*Summers et al.*, 2008; *Li et al.*, 2015; *Kim and Shprits*, 2019; *Meredith et al.*, 2018;
65 *Zhang et al.*, 2019). The major sources of hiss include the wave amplification by anisotropic electron
66 distributions in the plumes or near the outer edge of the plasmasphere (*Chen et al.*, 2012a, 2014; *Li et*
67 *al.*, 2013), the chorus waves propagating into the plasmasphere from the plasma trough (*Bortnik et*
68 *al.*, 2008, 2009; *Meredith et al.*, 2021), and the lightning generated whistlers leaking from the
69 ionosphere to the magnetosphere at low *L* shells (*Sonwalkar and Inan*, 1989; *Bortnik et al.*, 2003;
70 *Meredith et al.*, 2006). The statistical wave power is stronger during more disturbed geomagnetic
71 conditions (*Kim et al.*, 2015; *Spasojevic et al.*, 2015). The density structures in the outer
72 plasmasphere or plumes modulate the hiss wave intensity (*Malaspina et al.*, 2016; *Li et al.*, 2019; *Shi*
73 *et al.*, 2019). In addition to the preferred wave amplification regions, the wave propagation could be
74 focused, and enhanced in local high-density regions (*Chen et al.*, 2012b).

75 Using multiple satellite mission data in the magnetosphere, previous statistical studies have revealed
76 the global distribution of wave power and their dependence on geomagnetic activities. The empirical
77 models are widely used to construct the global wave distributions and simulate the radiation belt
78 electron evolution (*Horne et al.*, 2013; *Glauert et al.*, 2014; *Drozdov et al.*, 2015; *Ma et al.*, 2015).
79 The radiation belt simulation using empirical wave models could produce a reasonable estimate of
80 the electron flux decay and acceleration over a period longer than several days. However, event-
81 specific wave distributions from in-situ observations or other techniques are required to simulate the

dynamic electron evolution in a short timescale or during high geomagnetic activities (*Li et al.*, 2016b; *Ma et al.*, 2018).

In this paper, we evaluate the performance of empirical models of total electron density and amplitudes of whistler-mode chorus and hiss waves in the Earth's inner magnetosphere. Section 2 presents two events of total electron density and whistler-mode waves observed by Van Allen Probes. Section 3 presents the empirical model based on the statistics of the Van Allen Probes dataset, and the data distribution comparison between observation and modeling. Section 4 compares the performance of different models using error metrics and the Pearson correlation coefficient. Finally, in Section 5, we summarize and discuss our results.

2 Van Allen Probes observation of total electron density and whistler-mode waves

We use Van Allen Probes (RBSP) measurements (*Mauk et al.*, 2013) to obtain the total electron density (N_e) and whistler-mode wave amplitudes (B_w) at $L < 6.5$ in the Earth's inner magnetosphere. The Electric and Magnetic Field Instrument Suite and Integrated Science (EMFISIS) measures the DC magnetic fields and the AC signals of wave electric and magnetic fields over a broad frequency range (*Kletzing et al.*, 2013). The background magnetic fields in three orthogonal directions are measured by the fluxgate magnetometer. The wave electric and magnetic power spectral densities at 10 Hz - 12 kHz frequencies are measured by the Waveform Receiver (WFR), which also provides the wave polarization properties, including the wave normal angle, ellipticity, and planarity, calculated using the Singular Value Decomposition (SVD) method (*Santolik et al.*, 2003). The High Frequency Receiver (HFR) measures the wave electric power spectral density from 10 kHz to 400 kHz, capturing the upper hybrid resonance band waves. The total electron density is calculated using the measured upper hybrid resonance frequency (*Kurth et al.*, 2015). We also use the total electron density inferred using the spacecraft potential measured by the Electric Field and Waves (EFW) instrument (*Wygant et al.*, 2013) when the upper hybrid resonance frequency is unavailable from HFR measurements. The density measurements from EFW have been calibrated against the more accurate EMFISIS density measurements (*Breneman et al.*, 2022).

In this section, we describe the total electron density and whistler-mode wave measurements during two events when Van Allen Probes apogees were at different MLTs. Then, we will compare modeling and observation during the two events in Section 3.3.

2.1 Observations on 01 March 2023

Figure 1 shows the total electron density and whistler-mode waves measured over a full day by Van Allen Probe A (panels C-F) and Probe B (panels G-J) on 01 March 2013. Figure 1A shows the geomagnetic H_p30 index (black) and H_p^* values (blue). The H_p30 index is analogous to the K_p index but has a 30-min resolution (*Matzka et al.*, 2022), and we define H_p^* as the maximum H_p30 index during the previous 24 h. The 24 h timescale is chosen following the plasmopause models by *Carpenter and Anderson* (1992) and *O'Brien and Moldwin* (2003). Figure 1B shows the SuperMAG (*Gjerloev*, 2012) Auroral Electrojet index SML and the negative of SME^* , where SME^* is defined as the maximum SME index during the previous 3 h. A modest geomagnetic storm occurred on 01 March 2013, as analyzed by *Ma et al.* (2016b) and *Bortnik et al.* (2018). The maximum H_p30 index was close to 6, and the minimum SML index was about -1400 nT, indicating a disturbed geomagnetic condition and particle injection activities.

The Van Allen Probes apogees were on the nightside (Figures 1C and 1G), suitable for measuring the whistler-mode chorus waves during this event. The measured total electron densities (Figures 1D and

1H) show a clear plasmapause structure with a large density gradient, the separation of the dense plasmasphere (blue) and tenuous plasma trough (black). Using the method described by *Ma et al.* (2021), we identified the high-density region (blue), including the plasmasphere and plumes, and the low-density region (black) of the plasma trough. The density data were averaged into a 1-min time cadence which is used in the statistical analysis in the following sections.

The wave magnetic power spectrograms (Figures 1E and 1I) show intense chorus waves at frequencies above (upper-band) and below (lower-band) $0.5 f_{ce}$, where f_{ce} is the equatorial electron gyrofrequency. The chorus waves were observed over a wide range of MLTs on the nightside. The hiss waves at frequencies from ~ 50 Hz to 1 kHz were observed mainly at $L < 3.5$ during this event, with weaker intensities compared to the chorus. We selected the whistler-mode wave intensities from the wave power spectrograms at frequencies from 20 Hz (or the equatorial proton gyrofrequency f_{cp} when $f_{cp} > 20$ Hz) to f_{ce} (or 10 kHz when $f_{ce} > 10$ kHz). In addition, we excluded the highly oblique magnetosonic waves by requiring that the wave ellipticity is greater than 0.5, the wave normal angle is below 80° , and the wave planarity is above 0.2. The chorus and hiss wave amplitudes were calculated in both the low-density and high-density regions, respectively, as shown by the black and blue lines in Figures 1F and 1J. During this event, the peak of chorus wave amplitude reached about 500 pT, and the hiss waves had weak amplitudes of tens of pT. We calculated the root-mean-square (RMS) wave amplitudes during each 1-min time cadence for statistical purposes.

2.2 Observations on 02 September 2013

Figure 2 shows the density and waves measurements made over a full day by Van Allen Probe A (panels C-F) and Probe B (panels G-J) on 02 September 2013. The maximum Hp30 index (Figure 2A) was 4, and the minimum *SML* index (Figure 2B) was about -900 nT, indicating a modestly disturbed geomagnetic condition.

The Van Allen Probes apogees were located on the dusk side (Figures 2C and 2G), which is suitable for measuring the plasmaspheric plumes and hiss waves during this event, as analyzed by *Li et al.* (2019). The measured total electron densities (Figures 2D and 2H) show evident density perturbations during each orbit of Van Allen Probes except for the Probe B observation after 16:00 UT. For example, during 13-21 UT, Figure 2D shows that Van Allen Probe A first traveled from the plasmasphere (blue) to the plasma trough (black), encountered plumes (blue) during 16:00 - 18:30 UT, and then traveled from the plasma trough (black) back to the plasmasphere (blue). The density measurements on different orbits suggest a highly dynamic variation of the plume on the dusk side.

Figures 2E and 2I show hiss wave activities with extended coverage in the high-density plasmasphere and plumes. The hiss waves at frequencies of ~ 50 Hz - 1 kHz are correlated with the high-density region, both during the extended plume period of 16:00 - 18:30 UT measured by Van Allen Probe A (Figures 2D-E) and for the short periods of density variations during 01 - 04 UT measured by Van Allen Probe B (Figures 2I-J). The magnetosonic waves were also observed below 50 Hz during 6 - 7 UT by Probe A (Figure 2E) and during 7 - 9 UT by Probe B (Figure 2I), but they were excluded from our wave data using the spectral criteria described above. The chorus waves were observed in the plasma trough during 19:00 - 22:30 UT by Probe B (Figure 2I). Figures 2F and 2J show that the peaks of hiss wave amplitudes (blue) are about 100 pT both in the plasmasphere and plumes.

3 Empirical model

3.1 Development of the empirical model

In previous studies, the surveys of whistler-mode chorus and hiss waves were usually modeled separately, and the full L -MLT distributions of both chorus and hiss waves were parameterized for different solar wind or geomagnetic conditions. However, the chorus and hiss waves are usually separated in space, with the chorus observed in the low-density plasma trough and the hiss observed in the high-density plasmasphere or plumes [e.g., Meredith et al., 2018; 2020]. Therefore, an additional model of the plasmopause location or plasma density is required to construct the global distributions of chorus and hiss using the previous models.

We performed a survey using a unified dataset to analyze the total electron density, chorus wave amplitudes, and hiss wave amplitudes. This approach allowed us to construct consistent statistical distributions among them. Van Allen Probes measurements from September 2012 to October 2019 were used. To obtain the global distributions, we selected data when the magnetic latitude was within 10° from the magnetic equator. The survey of total electron density was performed at $2 < L < 6.5$, considering that the density at $L < 2$ may not be reliable when the upper hybrid resonance frequency is higher than 500 kHz (Hartley et al., 2023). Surveys of chorus and hiss were performed at $1 < L < 6.5$. The whistler-mode waves at $L < 2$ were identified as hiss throughout our survey. The data of average density and root-mean-square wave amplitudes were binned in every 1 h MLT and $0.5 L$ shell.

We used the combination of Hp^* and SME^* (or SML) indices to categorize the statistical distributions of the total electron density, chorus wave amplitude, and hiss wave amplitude. The plasmopause location in L -MLT was previously fitted as a function of the maximum Kp index in the previous 24 - 36 h by O'Brien et al. (2003). The large amplitude chorus waves were generally found to be correlated with electron injections. The hiss waves are also related to the AE index, and were previously categorized by AE^* by Li et al. (2015). Using the combination of Hp^* and SME^* or Hp^* and SML , the model may better capture the plasmopause location and the wave activity. We set 7 levels of Hp^* ($Hp^* < 1$, $1 < Hp^* < 2$, $2 < Hp^* < 3$, ... $Hp^* > 6$), and up to 4 levels of SME^* (or SML) within each range of Hp^* . The ranges of SME^* (or SML) were set so that the data sample number is sufficient in each combination of Hp^* and SME^* (or SML), and the possible variation of SME^* (or SML) is captured.

Using the Van Allen Probes dataset, we first obtained the occurrence rates of high-density (plasmasphere or plume) and low-density (plasma trough) flags in each L -MLT bin under each category of Hp^* and SME^* . Then, to construct a deterministic empirical model, we set the regions with a high-density occurrence rate higher than 0.5 to be the modeled plasmasphere or plume, and the regions with a low-density occurrence rate higher than 0.5 to be the modeled plasma trough.

The total electron densities with a high-density flag (blue) or a low-density flag (black) in Figures 1D, 1H, 2D, and 2H were averaged in each L -MLT bin under each combination of Hp^* and SME^* . To construct the global density distribution, we used the average densities in the high-density region as the densities in the modeled plasmasphere or plume, and the average densities in the low-density region as the densities in the modeled plasma trough.

Similar to the density, we obtained the root-mean-square amplitudes of chorus and hiss waves in each L -MLT bin under each geomagnetic condition. To construct the global distributions for a certain geomagnetic condition, we assigned the chorus wave amplitudes in the modeled plasma trough, and hiss wave amplitudes in the modeled plasmasphere or plume. The modeled chorus and hiss waves are well separated in the space. Note that the chorus and hiss waves could be modeled using different parameters after the low- and high-density regions were modeled. For example, we used Hp^* and

SML for chorus and Hp^* and SME^* for hiss. The model performances for different parameters are discussed in Section 4. The performance of the chorus wave amplitude model using Hp^* and *SML* is found to be close to the performance of the model using Hp^* and SME^* .

3.2 Statistical distributions

Figure 3 shows the statistical distributions of the total electron density (panels A-D), chorus wave amplitude (panels E-H), and hiss wave amplitude (panels I-L) for the selected Hp^* , SME^* , and *SML* conditions. The sample numbers are shown in Figures 3M-T. The chorus waves are shown in the modeled plasma trough region, and hiss waves are shown in the modeled plasmasphere or plume. During quiet times (Figures 3A, E, and I), the modeled plasmopause is mainly located at $L > 6.5$, and thus the densities mainly represent the plasmaspheric density, and the hiss waves are widely distributed at $L < 6.5$, consistent with an extended plasmasphere region. As geomagnetic conditions become more disturbed, the total electron densities are eroded over the nightside-dawn-dayside sectors, showing an MLT-dependent plasmopause. The high density in the dusk sector during the disturbed time (Figure 3D) includes the data samples of plumes or extended plasmasphere compared to other MLTs. The chorus wave power is enhanced over the nightside-dawn-dayside sectors as the geomagnetic activity becomes more disturbed (Figures 3F-H). Figures 3I-L show that the hiss wave powers are enhanced on the dayside and the dusk side at high L shells when Hp^* and SME^* increase, although the overall spatial coverage becomes more limited due to the erosion of the plasmopause. The statistical distributions of total electron density and whistler-mode waves are consistent with the previous survey results (e.g., *Sheeley et al.*, 2001; *Li et al.*, 2009, 2015; *Meredith et al.*, 2018, 2020). Although the distributions under 4 conditions of Hp^* and SME^* (or *SML*) are shown in Figure 3, the empirical models cover all geomagnetic conditions and the full models are provided in the data repository.

3.3 Comparison between observation and modeling during the events

For a given value of Hp^* and SME^* (or *SML*) at a specific time, the empirical model provides the distribution of density and whistler-mode waves on a global scale by selecting data from the corresponding geomagnetic categories. The total electron density and whistler-mode wave amplitudes were modeled along the L shell and MLT of Van Allen Probes from September 2012 to October 2019. The modeled results were produced at $2 < L < 6.5$ for the density and $1 < L < 6.5$ for the wave amplitudes at a 1-min time cadence to compare with the observation. It is worth noting that the empirical model was developed using the data samples within 10° from the equator, while the Van Allen Probes measurements had additional sampling at latitudes up to 20° .

Figures 1D, 1F, 1H, and 1J show the comparison between observation and modeling on 01 March 2013, when the Van Allen Probes apogees were on the nightside. The model (red) well captures the location of plasmopause and the density values in the plasmasphere and plasma trough (Figures 1D and 1H). The modeling was not performed during 16:30 - 19:00 UT on Van Allen Probe B (Figure 1H) because the L shell was larger than 6.5. The wave mode of chorus (orange) or hiss (red) is also correctly identified by the model (Figures 1F and 1J). Overall, the observations of chorus (black) and hiss (blue) show larger wave amplitude fluctuations than the modeling. The modeled chorus and hiss waves are persistently present in the low- and high-density regions, respectively, and may not reproduce the strong bursts or rapid disappearance of the observed waves. The large discrepancies are found at the peak of observed wave amplitude or when the whistler-mode waves were absent, i.e., at the extreme amplitudes. The modeling significantly overestimates the observed chorus waves during 15:00 - 16:30 UT observed by Probe B (Figure 1H), but the satellite was at high latitudes before

255 traveling towards $L \sim 7$, and the chorus waves were possibly damped in the nightside high-latitude
256 region.

257 Figures 2D, 2F, 2H, and 2J show the comparison on 02 September 2013 when the satellite apogees
258 were on the dusk side. Plasmaspheric plumes were observed by Van Allen Probes (blue), which were
259 not captured by the modeling. In Figure 2D, the modeling (red) shows the density structures of the
260 plasmasphere and plasma trough during 04 - 12 UT, and only the plasmasphere during 13 - 21 UT.
261 The modeling agrees with the observation when the high- and low-density regions are identified
262 correctly.

263 Because the high- and low-density regions are potentially mis-identified by the model, the model
264 predicts hiss waves when chorus waves are observed, and vice versa. The discrepancies in the wave
265 amplitude modeling are larger than those in Figure 1 due to the mis-identification of the wave modes.
266 For example, during 7 - 9 UT in Figure 2F, the observation shows that hiss $B_w > 0$ and chorus $B_w = 0$,
267 and the modeling suggests that hiss $B_w = 0$ and chorus $B_w > 0$. This causes an additional major source
268 of modeling error, especially for the waves inside and near the plumes, in addition to the error
269 sources found in Figure 1.

270 3.4 Comparison of Data Distributions from Observation and modeling

271 We compare the data distribution from observations and modeling using the ~ 7 -year dataset at 1-min
272 time cadence. In this comparison and the evaluation of modeling performance discussed in Section 4,
273 we only used the data when the Van Allen Probes were located within 10° from the equator.

274 Figures 4A, 4D, and 4G show the average total electron density, RMS amplitude of chorus, and RMS
275 amplitude of hiss wave as a function of L . The observation generally agrees with the modeling
276 results. The high- and low-density regions were modeled using a criterion of occurrence rate at 0.5
277 (see Section 3.1), causing the slight overestimate (underestimate) of chorus (hiss) B_w at $L > 4.5$, and
278 slight underestimate (overestimate) of chorus (hiss) B_w at $L < 4.5$. If the average density and RMS B_w
279 were weighted by the occurrence rates of high- and low-density regions in the models, the modeling
280 would perfectly match the observation. However, such a model would not be very useful because it
281 mixes the plasmasphere and plasma trough densities, and the modeled chorus and hiss waves appear
282 simultaneously.

283 Figure 4B shows the probability density function (PDF) between the modeled and observed total
284 electron densities. Most of the data are distributed around the diagonal line, suggesting a good model
285 performance. The overall Pearson correlation coefficient (R) is 0.89. To examine the modeled data
286 distribution for a fixed observation, Figure 4C shows the PDF divided by the sum of probability
287 density in each range of observed N_e , denoted as the normalized PDF. The total normalized PDF for
288 each range of observation is 1, so the good model will show a normalized PDF of $\sim 0.5 - 1$ along the
289 diagonal line. Figure 4c indicates good model performance when the observed density is higher than
290 100 cm^{-3} . The modeled data shows a wide spread for the observed densities from 10 cm^{-3} to 100 cm^{-3} ,
291 suggesting a large deviation at and near the plasmopause or plumes. For the observed densities below
292 10 cm^{-3} , the modeled density is mainly at $3 - 10 \text{ cm}^{-3}$, suggesting that the model correctly identifies
293 the low-density region of the plasma trough, but the modeled density is overall higher than the
294 observation when the observed density is below 3 cm^{-3} .

295 Figure 4E shows the probability density function for chorus waves. The modeled and observed
296 chorus waves with 0 pT amplitude, including the high-density region data (chorus $B_w = 0$) in the
297 modeling and observation, are included at $B_w < 2$ pT bins, and considered when evaluating the model

performance. The high PDF at observed $B_w < 2$ pT and modeled $B_w < 2$ pT suggests that the majority of the plasmasphere regions are correctly modeled. A second group of data are found at observed $B_w > 2$ pT and modeled $B_w < 2$ pT, representing the times when the satellite was outside the plasmopause and observed chorus wave activity while the model suggests a high-density region. Similarly, the group of data at observed $B_w < 2$ pT and modeled $B_w > 2$ pT represents the times when the satellite was inside the plasmopause or chorus was not observed outside the plasmopause, while the model suggests a low-density region with chorus wave activity. Figure 4F shows the normalized PDF for each range of observed chorus B_w . The model underestimates the strong chorus wave amplitudes for observed $B_w > 100$ pT; specifically, the model most likely predicts a wave amplitude of 100 - 200 pT for the observed wave amplitude of 500 - 1000 pT. The model provides a good estimate of the chorus wave amplitude for the observed chorus B_w from 10 pT to 100 pT. For the weak chorus waves with $B_w < 10$ pT, the model most likely overestimates the observation with modeled B_w of 10 - 20 pT. The overall Pearson correlation coefficient for chorus B_w is 0.67, which is lower than that of the density model. The correlation coefficient is affected by the groups of data at observed $B_w > 2$ pT and modeled $B_w < 2$ pT, and at observed $B_w < 2$ pT and modeled $B_w > 2$ pT; i.e., the R is strongly affected by the mis-identification of the plasmasphere or plasma trough.

Figure 4H shows the probability density function for hiss. The data distribution and scattering are similar to those of the chorus waves, while the amplitudes of observed and modeled hiss waves are overall lower than those of chorus. The group of data at observed $B_w > 2$ pT and modeled $B_w < 2$ pT represents the times when the spacecraft was in the high-density region and the model suggests a low-density region. The data group at observed $B_w < 2$ pT and modeled $B_w > 2$ pT represents the times when the spacecraft was in the low-density region or hiss was not observed, while the model suggests hiss wave activity in a high-density region. The normalized PDF distribution (Figure 4I) suggests that the modeling agrees with observation for observed B_w from 10 pT to 50 pT, underestimates for observed $B_w > 50$ pT, and overestimates for observed $B_w < 10$ pT. In addition, another evident group of data is found for observed $B_w > 200$ pT and modeled $B_w < 2$ pT in Figure 4I. The very large amplitude hiss waves, despite their overall low occurrence, are usually observed with large density variations or in the plumes (*Shi et al.*, 2019). The empirical model may not fully capture the density variations, plume structures, or their evolution. Instead, these regions are likely mis-identified as the low-density plasma trough by the model, since the L shell of the perturbed density is usually high, and the geomagnetic condition is usually disturbed (*Shi et al.*, 2019). As a result, the empirical model cannot capture the very large amplitude hiss waves. The Pearson correlation coefficient for hiss is 0.53, which is lower than that of chorus.

4 Performance of different models

4.1 Metrics for evaluating the model performance

We further evaluate the model performances using various error metrics and correlation coefficients for different L shells. In each L shell bin, we consider the data in the range $\Delta L = \pm 0.25$. Following *Morley et al.* (2018), we consider Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Median Symmetric Accuracy (MSA), and Pearson correlation coefficient. Below we define x as the observed N_e or B_w and y as the modeled N_e or B_w .

We calculate Mean Absolute Error normalized by the average of N_e or RMS of B_w as

$$MAE/Mean = \overline{|y - x|}/\bar{x} \quad \text{for } N_e, \quad (1)$$

$$MAE/RMS = \overline{|y - x|}/\sqrt{\overline{x^2}} \quad \text{for } B_w. \quad (2)$$

341 The Root Mean Square Error normalized by the average of N_e or RMS of B_w is calculated as

$$342 \quad RMSE/Mean = \sqrt{(y-x)^2/\bar{x}} \quad \text{for } N_e, \quad (3)$$

$$343 \quad RMSE/RMS = \sqrt{(y-x)^2/\sqrt{x^2}} \quad \text{for } B_w. \quad (4)$$

344 The Median Symmetric Accuracy is calculated as

$$345 \quad MSA = 100(e^{Median(|\ln \frac{y}{x}|)} - 1) \quad \text{for } N_e \text{ and } B_w. \quad (5)$$

346 The Pearson correlation coefficient is calculated as

$$347 \quad R = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \Sigma(y-\bar{y})^2}} \quad \text{for } N_e \text{ and } B_w. \quad (6)$$

348 All data, including the observed or modeled $B_w = 0$, are considered in Equations (1 - 4) and (6). To
 349 calculate MSA in Equation (5), the logarithm function is used, so we consider all density data, and
 350 only the wave amplitude data with observed and modeled $B_w > 10$ pT. The good model is evaluated
 351 as having low MAE, low RMSE, low MSA, and high R.

352 Although the MSA cannot be calculated for $B_w = 0$, it provides a symmetric error for underestimate
 353 and overestimate in terms of the multiplication factor, while MAE and RMSE provide symmetric
 354 errors in terms of the percentage relative to the observation. For example, if $y = 0.5x$, then $MAE =$
 355 0.5 , $RMSE = 0.5$, $MSA = 100$; if $y = 2x$, then $MAE = 1$, $RMSE = 1$, $MSA = 1$ (same as for $y =$
 356 $0.5x$); if $y = 1.5x$, then $MAE = 0.5$ (same as for $y = 0.5x$), $RMSE = 0.5$ (same as for $y =$
 357 $0.5x$), $MSA = 50$.

358 4.2 Comparing different models

359 We evaluate the performances of 6 models categorized by different combinations of $Hp30$ or Hp^* ,
 360 and SML or SME^* indices, and the incorporation of the plasmopause. For each model, the surveys of
 361 total electron density, chorus wave amplitude, and hiss wave amplitude are performed using the same
 362 combination of geomagnetic indices. The statistical methods are the same as those described in
 363 Section 3 except for those described below. We perform the surveys of density and wave amplitudes
 364 using

- 365 • Empirical model 1: use $Hp30$ and SML indices, and consider the low- and high-density
- 366 categorizations in the models (same as the density categorization method described in Section
- 367 3.1, and denoted as 'PP');
- 368 • Empirical model 2: use $Hp30$ index, and consider the low- and high-density categorizations;
- 369 • Empirical model 3: we perform the surveys of density and wave amplitudes using SML index,
- 370 and consider the low- and high-density categorizations;
- 371 • Empirical model 4: use Hp^* and SML indices, and consider the low- and high-density
- 372 categorizations;
- 373 • Empirical model 5: use Hp^* and SME^* indices, and consider the low- and high-density
- 374 categorizations;

- Empirical model 6: use Hp^* and SME^* indices; the low- and high-density categorization is not considered, but the model adopts the averages over both low- and high-density conditions weighted by their occurrence rates (denoted as 'No PP').

Models 1 - 3 only use the instantaneous geomagnetic conditions, and models 4 - 6 also consider the most disturbed Hp condition in the past 24 h. Model 6 is equivalent to the method of directly using the models of density, chorus, and hiss waves without imposing a plasmapause. The density and wave models are not coupled, the plasmaspheric density could be mixed with the plasma trough density, and the chorus and hiss wave could appear simultaneously in the same location.

Figure 5 shows the metrics for different models as a function of the L shell. For the total electron density (Figures 5A-D), models 4 and 5 using Hp^* have lower MAE, lower RMSE, lower MSA and higher R , than models 1, 2 and 3 using the $Hp30$ index. Although model 6 has a slightly lower RMSE and higher R than models 4 and 5, the MSA of model 6 is much higher than that of models 4 and 5. The best density model is model 5 using Hp^* , SME^* , and PP, with slightly lower errors and a higher correlation coefficient than model 4. Note that the R for the density data in each L shell bin is much lower than the overall R for all L shells (0.89 in Figure 4B). This is because the total electron density has a persistent L -dependence with high densities generally at low L shells, contributing to the high correlation coefficient when the data at different L shells are included in the R calculation.

For the chorus B_w (Figures 5E-H), models 4 and 5 have slightly lower MAE and RMSE and slightly higher R at $L < 4$ than models 1, 2, and 3, and the improvement evaluated using MSA is significant. Although model 6 shows the lowest RMSE and MSA at $L > 3.5$, it has the lowest R overall and significantly large MAE at $L < 4$. The best chorus wave models are 4 and 5 considering Hp^* , SML (or SME^*), and PP, and the R of model 4 is slightly higher than model 5.

For the hiss B_w (Figures 5I-L), models 4 and 5 have slightly lower RMSE and MSA than models 1, 2, and 3, and noticeable improvement evaluated using MAE and R . Compared to models 4 and 5, model 6 shows large MAE at $L > 4$ and low R at $4 < L < 6$, and the performances at other regions or for other metrics are similar. The best model for hiss is model 5 considering Hp^* , SME^* , and PP, with higher R than model 4.

The overall metrics for different models of density, chorus B_w and hiss B_w are also tabulated in Table 1, considering the data at all different L shells. Similar to the discussions above, the overall best model is model 4 for the density and hiss and model 5 for chorus, while their performance difference is very small. Significant modeling improvement is obtained using the Hp^* index compared to the $Hp30$ index, and considering the low- and high-density categorization in the model.

5 Conclusions and discussions

We evaluated the performance of a number of empirical models describing the total electron density and whistler-mode wave amplitudes in the Earth's inner magnetosphere. The empirical models of density, chorus, and hiss waves were developed using the ~ 7 years of Van Allen Probes data, categorized using $Hp30$, SME , SML indices, and their past maximum values, with a classification of high- or low-density regions (i.e., plasmasphere, plume or plasma trough). The models were used to reproduce the density and wave amplitudes along the Van Allen Probes trajectories, and the data distribution was compared between the observation and modeling results. We further used 4 metrics to compare the performances of 6 different models, categorized using different geomagnetic indices or excluding the density region classification. Our model performance evaluation indicates that:

- Incorporating the plasmapause (i.e., classifying the high- and low-density regions) significantly improves the modeling of total electron density as well as the amplitudes of chorus and hiss waves.
- Using the maximum values of geomagnetic indices during the past 3 for SME and 24 h for Hp30 improves the modeling results compared to using only the instantaneous indices.
- The total electron density is well-modeled with high Pearson correlation coefficients using geomagnetic indices. The model agrees with the observation when the observed $N_e > 3 \text{ cm}^{-3}$ and overestimates for smaller density observations. The additional errors are near the plasmapause or in the plumes, causing the large data spread in the probability density function distribution.
- The amplitudes of whistler-mode chorus and hiss waves are well-modeled when the observed wave amplitudes are moderate, with amplitudes between 10-100 pT.
- For the observed amplitude $B_w < 10 \text{ pT}$ or in the absence of whistler-mode waves, the chorus and hiss models tend to provide the average wave amplitudes, which overestimate the observation.
- The models underestimate the whistler-mode wave amplitudes when the observed amplitude is intense, with values larger than 100 pT.
- The mis-identification of the plume region or the errors in identifying the plasmapause boundary causes large errors in modeling chorus and hiss wave amplitudes, because the chorus and hiss waves are mis-labeled by the model.
- The empirical model cannot capture the very large amplitude ($> 200 \text{ pT}$) hiss waves, probably because these hiss waves are present in the plume region at high L shells during disturbed conditions, which is identified as the plasma trough by the model.
- To investigate the model performance properly, it is necessary to evaluate multiple error metrics and correlation coefficients. Using a single metric may provide a biased judgment for the model comparison.

Although we evaluated the performances of 6 different models, the chosen 'best' model is not yet optimized. For example, our model comparison mainly focused on the *Hp30* and *SME* (or *SML*) indices and their derivatives, while the impacts of other geomagnetic indices and solar wind parameters have not been investigated. The solar wind dynamic pressure may significantly impact the whistler-mode waves at $L > 6$ due to the compression of the magnetosphere (Yue *et al.*, 2017; Zhou *et al.*, 2015). Following previous studies (O'Brien *et al.*, 2003; Li *et al.*, 2015), we incorporated the history of *Hp30* and *SME* indices by simply using their maximum values in the past 24 h and 3 h, respectively. The history lengths of the indices are not tested, and the alternative method of using mean values of the indices is not investigated. The model optimization requires significant work efforts for an empirical model. However, the machine learning models inherently optimize the dependences of the model target on the parameters (Bortnik *et al.*, 2016; Chu *et al.*, 2017, 2021; Huang *et al.*, 2022; Ma, D., *et al.*, 2022). The test of different empirical models could directly suggest the importance of each parameter, while the machine learning technique is more efficient in providing the best model fit for many parameters.

Our empirical models are developed using the Van Allen Probes data within 10° from the magnetic equator at $L < 6.5$. Additional data from the other spacecraft missions (THEMIS, Cluster, MMS, and Arase) provide the waves and density measurements at higher L shells or higher latitudes. In this paper, the comparison between Van Allen Probes data and the model results is limited to latitudes within $\pm 10^\circ$. The chorus waves are confined close to the equator at the nightside, while the high powers of the dayside chorus are found at higher latitudes (Agapitov *et al.*, 2018). A more comprehensive wave model in L shell, MLT, and magnetic latitude is required to properly capture the

high-latitude wave power. The evaluation of the model performance in other regions is left as a future work.

Although the accuracy of the empirical models may be lower than the accuracy of machine learning models, the empirical model inherently provides the average density and wave power under a certain condition, which is stable and generally matches the data averaged over a sufficiently long period. The empirical model is robust if the data sampling time is sufficiently high in each category. The empirical models of total electron density and whistler-mode wave power are applicable to radiation belt modeling on a timescale longer than several days (*Horne et al.*, 2013; *Glauert et al.*, 2014; *Drozdov et al.*, 2015), or under quiet to modestly disturbed geomagnetic conditions (*Ma et al.*, 2015, 2017). During a short and disturbed period, the chorus and hiss wave amplitudes may be underestimated, or the plume regions may be misidentified. Therefore, the wave model based on observation (*Li et al.*, 2016b; *Ma et al.*, 2018) or from machine learning prediction (*Bortnik et al.*, 2018) may provide better radiation belt modeling results. Our study of the empirical model performance provides a reference for the future development of machine learning models, by investigating the different error metrics and revealing the key factors affecting the model performance.

6 Conflict of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

7 Author Contributions

Q.M. performed the surveys using satellite data, constructed the empirical models, made model-data comparisons, evaluated the model performance, and wrote the manuscript. X.C., D.M. and S.H. performed machine learning modeling, which helped inspire the evaluation methods for the performance of the empirical models, discussed the strengths and weaknesses of the empirical model, and revised the manuscript. W.L. and J.B. discussed the results of this project, suggested improvements to the models, revised the manuscript, and provided support for this project. X.S. helped with the data preparation and global surveys, verified the data distribution, and revised the manuscript.

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10 Data Availability Statement

The Van Allen Probes EMFISIS data are retrieved from <http://emfisis.physics.uiowa.edu/Flight/>, and EFW data are retrieved from <http://rbsp.space.umn.edu/data/rbsp/>. The SuperMAG magnetic indices

are retrieved from <https://supermag.jhuapl.edu/mag/>. The *Hp30* index is retrieved from <https://kp.gfz-potsdam.de/en/hp30-hp60/data/>. The empirical model and results of this paper are available in the data repository <https://doi.org/10.6084/m9.figshare.22762247>.

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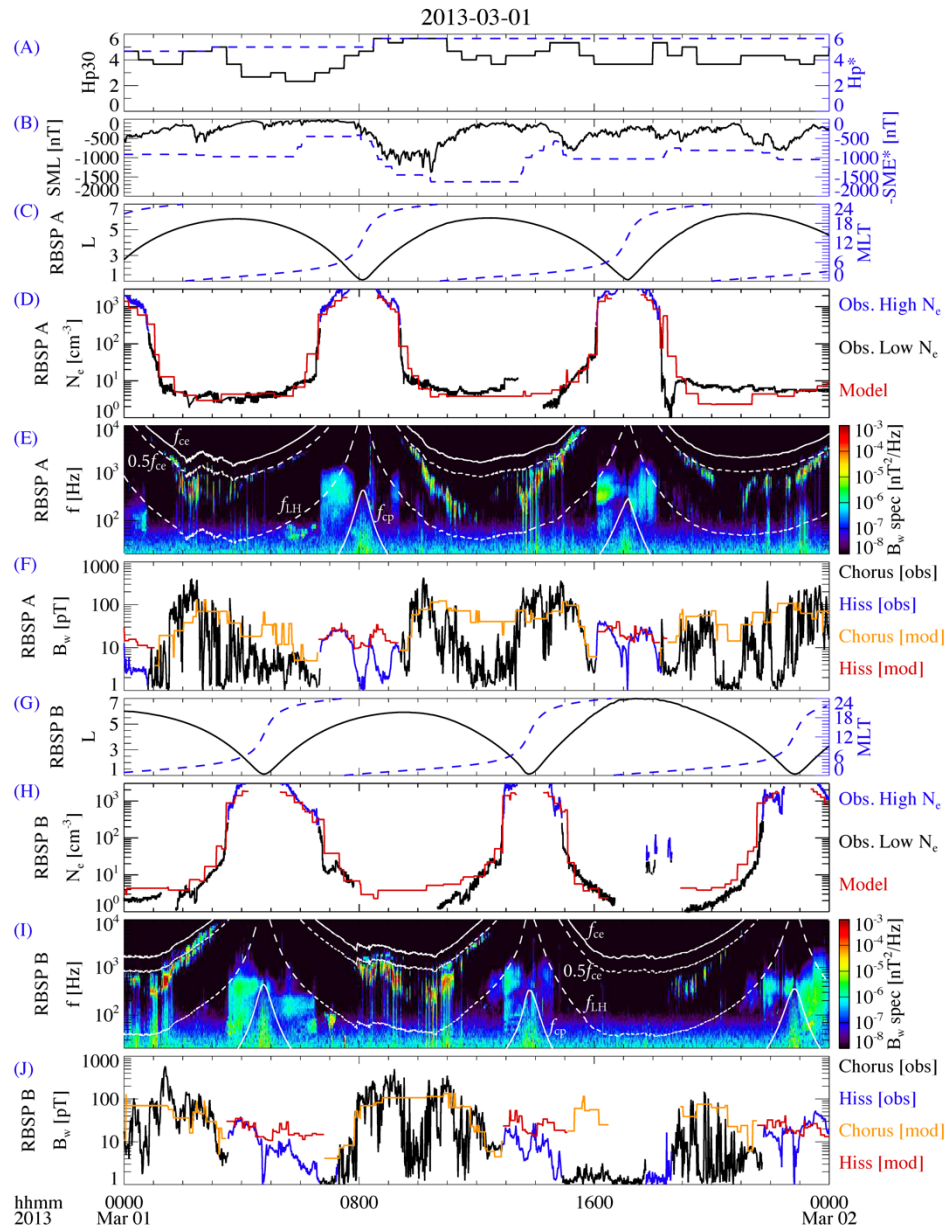
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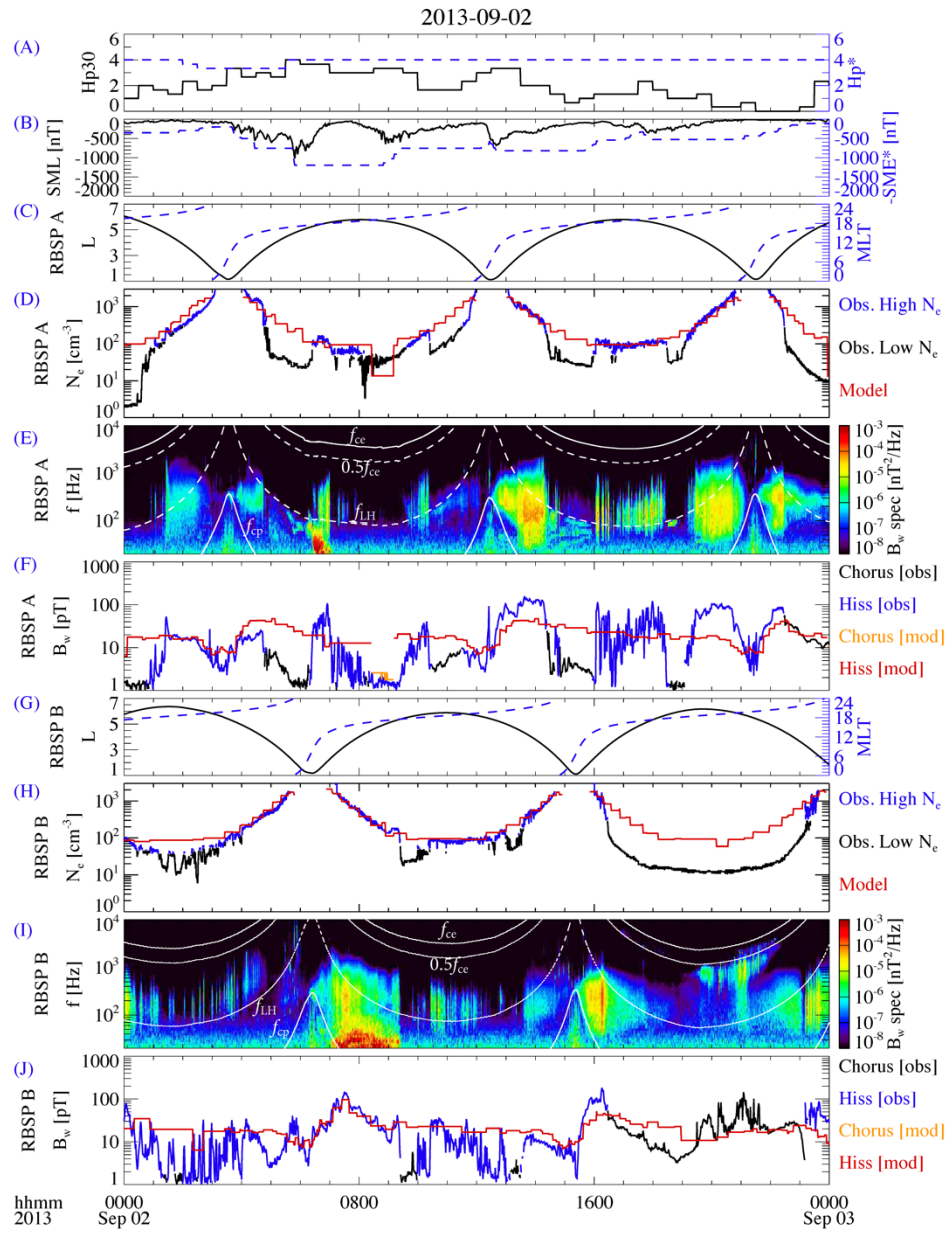
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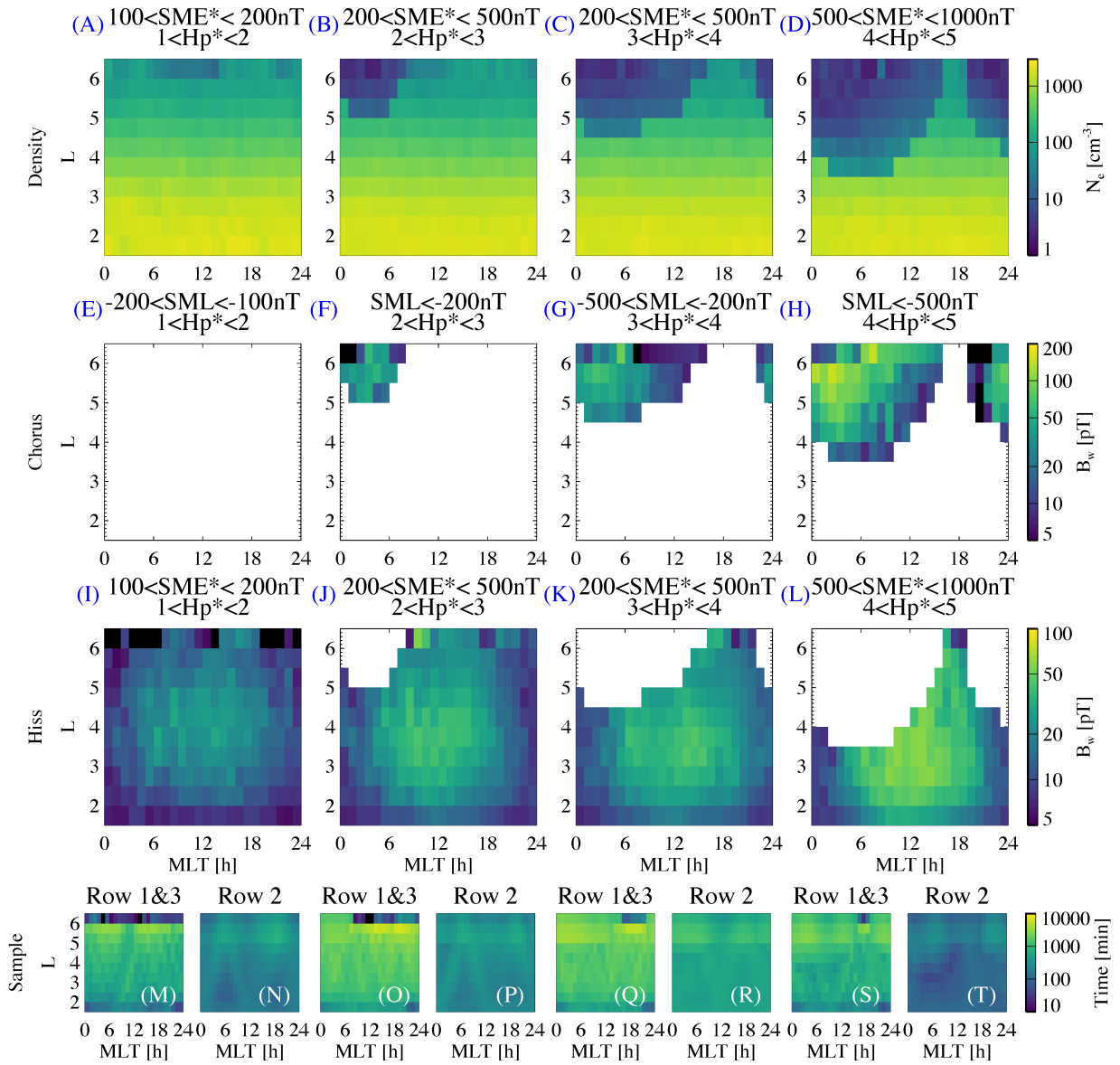
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Figure 1. Van Allen Probes observation and empirical modeling of the total electron density and whistler-mode chorus and hiss waves on 01 March 2013. (A) Geomagnetic $Hp30$ index and Hp^* , which is the maximum $Hp30$ index during the previous 24 h; (B) geomagnetic SML index and SME^* , which is the maximum SME index in the previous 3 h; (C) L shell and MLT of Van Allen Probe A; (D) total electron densities observed by Van Allen Probe A in the high-density plasmasphere or plume (blue) and in the low-density plasma trough (black), and produced by the empirical model (red); (E) magnetic power spectrogram at 20 Hz - 10 kHz frequencies observed by Van Allen Probe A, where the four white lines are equatorial electron gyrofrequency (f_{ce}), $0.5 f_{ce}$, lower hybrid resonance frequency (f_{LH}), and proton gyrofrequency (f_{cp}); (F) chorus (black) and hiss (blue) wave amplitudes observed by Van Allen Probe A, and chorus (orange) and hiss (red) wave amplitudes produced from the empirical model. (G-J) Same as (C-F) except for the density and waves along the trajectory of Van Allen Probe B.



749

750 **Figure 2.** Same as Figure 2 except for the event on 02 September 2013.



751

752 **Figure 3.** Statistical surveys of total electron density (A-D), and root-mean-square amplitude of
 753 chorus (E-H) and hiss (I-L) as a function of MLT and L shell, categorized by different Hp^* and
 754 SME^* or Hp^* and SML indices. (M) Total sample time under the geomagnetic condition for total
 755 electron density (Row 1, Panel A) or hiss (Row 3, Panel E), and (N) total sample time under the
 756 geomagnetic condition for chorus (Row 2, Panel E). (O-T) Same as (M-N) except for different
 757 geomagnetic conditions. Panels (E-H) only show the data in the region where the low-density
 758 occurrence rate is higher than the high-density occurrence rate; Panels (I-L) only show the data in the
 759 region where the high-density occurrence rate is higher.

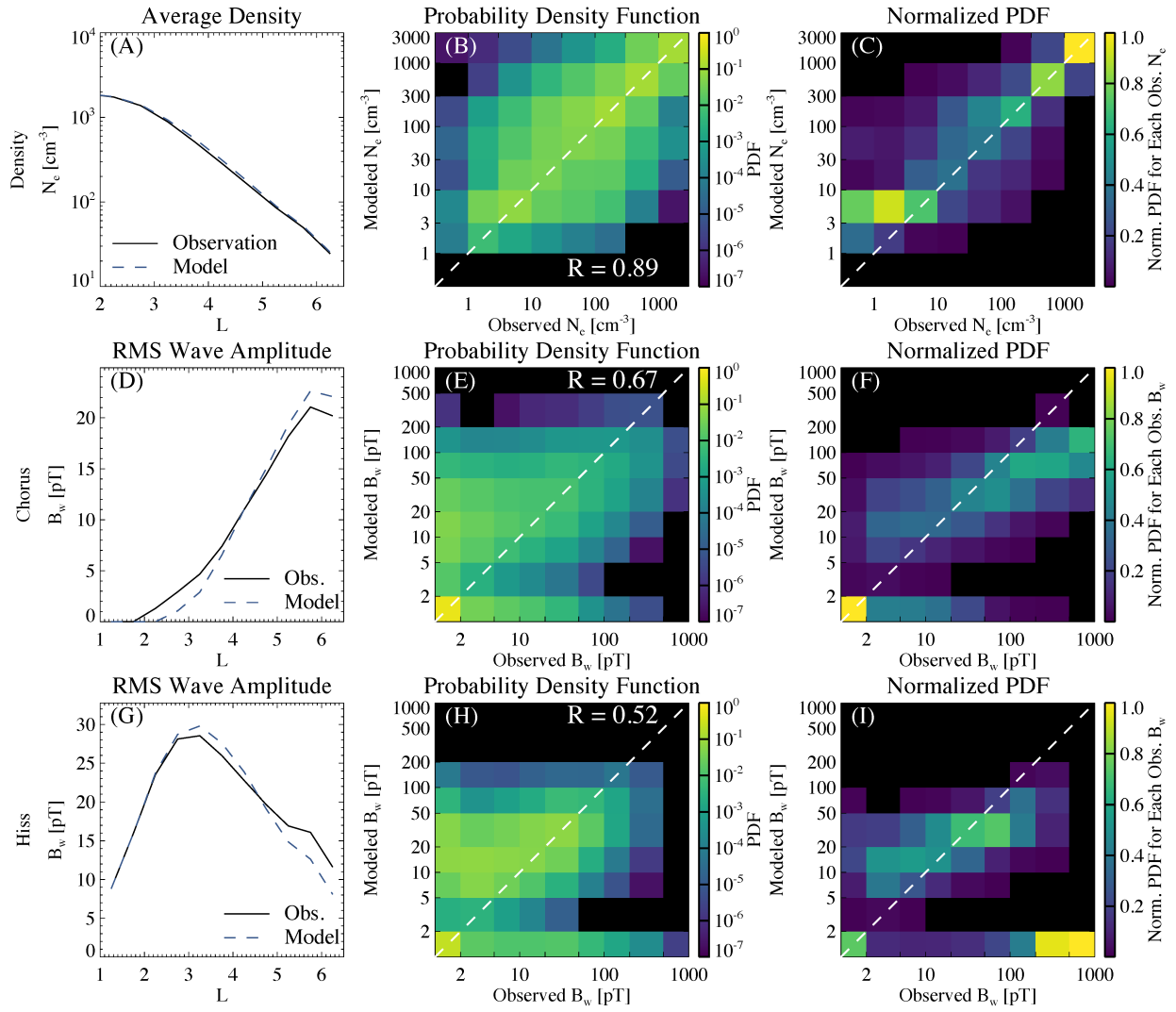


Figure 4. The comparison between observation and the empirical model, considering the Hp^* , SME^* index and modeled plasmopause for density and hiss, and considering the Hp^* , SML index and modeled plasmopause for chorus. (A) The average density as a function of L , where the solid and dashed lines are observation and modeling results, respectively; (B) probability density distribution as a function of modeled and observed densities; (C) probability densities divided by the total probability density within each bin of observed density. (D-F) Same as (A-C) except for the chorus wave amplitude. (G-I) Same as (A-C) except for the hiss wave amplitude.

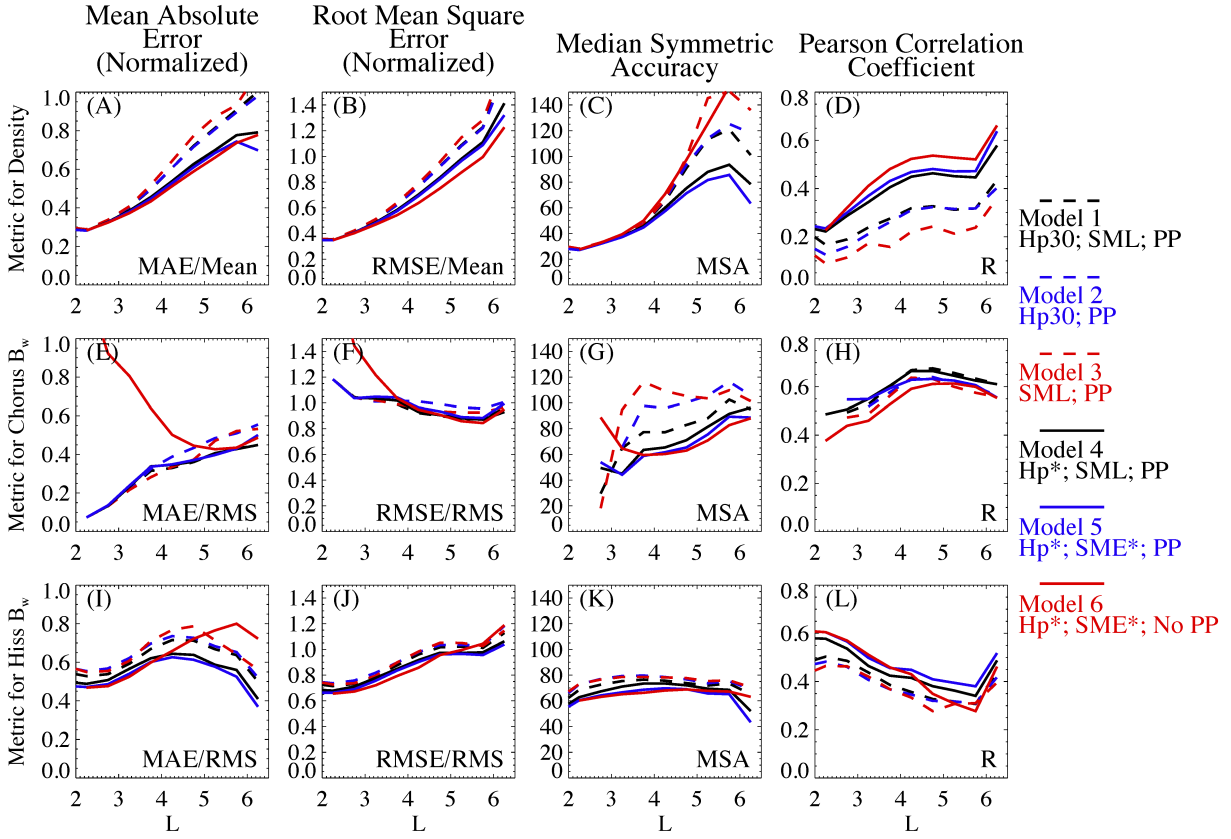


Figure 5. The performance of different empirical models evaluated using four metrics for different L shells. (A-D) Total electron density; (E-H) chorus wave amplitude; (I-L) hiss wave amplitude. The metrics include: Mean Absolute Error divided by the average of N_e or by the RMS B_w (chorus and hiss); Root Mean Square Error divided by the average of N_e or by the RMS B_w ; Median Symmetric Accuracy; Pearson Correlation Coefficient R . The different empirical models are illustrated by different line styles or colors.

775 **Table 1.** The overall performance of different empirical models evaluated using four metrics. The
776 best performance evaluated by each metric is highlighted in bold blue font; the second best is
777 highlighted in blue font; the worst performance is highlighted in bold black font.

Model	Mean Absolute Error			Root Mean Square Error			Median Symmetric Accuracy			Pearson Correlation Coefficient		
	MAE/Mean for N_e MAE/RMS for B_w			RMSE/Mean for N_e RMSE/RMS for B_w			MSA			R		
	Density N_e	Chorus B_w	Hiss B_w	Density N_e	Chorus B_w	Hiss B_w	Density N_e	Chorus B_w	Hiss B_w	Density N_e	Chorus B_w	Hiss B_w
<i>Hp30</i> , <i>SML</i> , PP	0.427	0.366	0.647	0.72	0.927	0.946	69.5	163	58.8	0.881	0.679	0.45
<i>Hp30</i> , PP	0.428	0.433	0.668	0.724	1.01	0.966	71.1	162	60.2	0.88	0.649	0.441
<i>SML</i> , PP	0.441	0.421	0.68	0.732	0.976	0.965	77.6	243	61	0.876	0.634	0.432
<i>HP*</i> , <i>SML</i> , PP	0.398	0.368	0.585	0.697	0.929	0.911	60.8	102	59.4	0.888	0.672	0.491
<i>HP*</i> , <i>SME*</i> , PP	0.393	0.369	0.562	0.691	0.945	0.894	57.8	104	57.1	0.89	0.653	0.517
<i>HP*</i> , <i>SME*</i> , No PP	0.401	0.451	0.682	0.702	0.926	0.908	84.3	89.3	51.8	0.888	0.637	0.475

778