

Cross-Scale Modeling of Storm-Time Radiation Belt Variability

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Key Points:

- We developed a novel global test particle model of storm-time radiation belt dynamics with local wave-particle interactions.
- Evolution of the magnetic field and density yields local variations of the magnitude and resonant energy of the wave-particle interactions.
- The new model enables separation of electron acceleration and loss processes driven by both transport and local wave-particle interactions.

15 **Abstract**

16 During geomagnetic storms relativistic outer radiation belt electron flux exhibits large
 17 variations on rapid time scales of minutes to days. Many competing acceleration and loss
 18 processes contribute to the dynamic variability of the radiation belts; however, distin-
 19 guishing the relative contribution of each mechanism remains a major challenge as they
 20 often occur simultaneously and over a wide range of spatiotemporal scales. In this study,
 21 we develop a new comprehensive model for the storm-time radiation belt dynamics by
 22 incorporating electron wave-particle interactions with parallel propagating whistler mode
 23 waves into our global test-particle model of the outer belt. Electron trajectories are evolved
 24 through the electromagnetic fields generated from the Multiscale Atmosphere-Geospace
 25 Environment (MAGE) global geospace model. Pitch angle scattering and energization
 26 of the test particles are derived from analytical expressions for quasi-linear diffusion co-
 27 efficients that depend directly on the magnetic field and density from the magnetosphere
 28 simulation. Using a case study of the 17 March 2013 geomagnetic storm, we demonstrate
 29 that resonance with lower band chorus waves can produce rapid relativistic flux enhance-
 30 ments during the main phase of the storm. While electron loss from the outer radiation
 31 belt is dominated by loss through the magnetopause, wave-particle interactions drive sig-
 32 nificant atmospheric precipitation. We also show that the storm-time magnetic field and
 33 cold plasma density evolution produces strong, local variations of the magnitude and en-
 34 ergy of the wave-particle interactions and is critical to fully capturing the dynamic vari-
 35 ability of the radiation belts caused by wave-particle interactions.

36 **1 Introduction**

37 Relativistic electron intensities in Earth's outer radiation belt are highly dynamic.
 38 During geomagnetic storms, electron intensities in the outer belt can vary over an or-
 39 der of magnitude on rapid time scales of minutes to days and across a wide range of L -
 40 shells (See reviews by W. Li & Hudson, 2019; Ripoll et al., 2020). The system response
 41 of the radiation belts is highly non-linear. Geomagnetic storms can cause a net-enhancement,
 42 depletion, or no relative change in the relativistic electron fluxes in relation to pre-storm
 43 levels (Reeves et al., 2003).

44 Many competing acceleration and loss processes contribute to the dynamic vari-
 45 ability of the radiation belts. Acceleration, transport, and loss occur primarily through
 46 two different processes: radial transport and/or via local resonant wave-particle inter-
 47 actions. Radial transport energizes electrons via conservation of the first and second adi-
 48 abatic invariant (Schulz, 1974) when it acts at time scales longer than the typical bounce
 49 motion of trapped electrons across large spatial scales. Transport processes include con-
 50 vection from the plasma sheet, including mesoscale ($\sim 1 R_E$) injections associated with
 51 fast flows (Gabrielse et al., 2017; Turner et al., 2017), particle injections induced by in-
 52 terplanetary shocks (Foster et al., 2015), and resonant interactions with ultra-low fre-
 53 quency (ULF) waves. Local wave-particle interactions violate the 1st and 2nd adiabatic
 54 invariant through gyroresonance, resulting in pitch angle scattering of the electrons and
 55 energy transfer between the electrons and the waves. Local wave-particle interactions
 56 act on time scales comparable to the electron gyroperiod and cause localized enhance-
 57 ments in phase space density (PSD) at L -shells where waves are present. Many differ-
 58 ent wave modes can resonate with radiation belt electrons (See reviews by Shprits, Sub-
 59 botin, et al. (2008); Thorne (2010); W. Li and Hudson (2019); Ripoll et al. (2020)). Whistler
 60 mode chorus waves are generated outside of the plasmapause (Malaspina et al., 2016)
 61 and have been shown to produce significant flux enhancements in the outer radiation belts
 62 (Horne et al., 2003; Summers et al., 2007b). Both radial transport and wave-particle in-
 63 teractions are substantially enhanced during geomagnetic storms and can lead to per-
 64 manent electron loss from the system. Electron loss occurs either via particle escape through
 65 the magnetopause boundary or by particle scattering into the loss cone and subsequent

66 precipitation into the atmosphere (e.g., Millan & Thorne, 2007; Shprits, Elkington, et
67 al., 2008; Shprits, Subbotin, et al., 2008, and references therein).

68 Mesoscale particle injections and wave-particle interactions often occur simultaneously,
69 and interact in a complex manner. For instance, Jaynes et al. (2015) suggested
70 that substorm injections increase two key populations in the outer radiation belt: the
71 source population (1-10s of keV), which provides free energy for the growth of chorus
72 waves, and the seed population of electrons (10s-100s keV), which are then rapidly ac-
73 celerated by the chorus waves to relativistic energies. Statistical surveys of plasma wave
74 and particle data have found that significant relativistic electron flux enhancements oc-
75 cur outside the plasmapause, in association with prolonged substorm activity, enhanced
76 fluxes of seed electrons, and increased levels of chorus wave activity (Meredith et al., 2003).
77 Furthermore, ultra-relativistic electron flux enhancements are more likely to occur dur-
78 ing intense geomagnetic storms when substorm activity is present (Zhao et al., 2019).

79 Distinguishing relative contributions of the different acceleration and loss processes
80 that govern radiation belt dynamics remains a major challenge. The large spatial, tem-
81 poral, and energy ranges over which radial transport and wave-particle interactions act
82 are difficult for both observations and numerical models to resolve. Several approaches
83 have been used to simulate radiation belt dynamics. One common method is 3D dif-
84 fusion models based upon the quasi-linear approximation. Diffusion models of the radi-
85 ation belts solve the Fokker-Planck equation by reducing electron dynamics to three-dimensional
86 diffusion in pitch angle, energy, and L -shell through gyro-, bounce, and drift averaging
87 of the solution. Diffusion models can account for local pitch-angle scattering, particle
88 energization, and loss due to the combined effect of multiple wave modes interacting with
89 electrons (See review by Ripoll et al., 2020, and references therein). The drift-bounce
90 averaged transport associated with Fokker-Planck diffusion models, however, is only ap-
91 plicable on timescales much longer than the drift period (Ukhorskiy & Sitnov, 2013). Dif-
92 fusion models, therefore, do not encompass the full range of physical processes that gov-
93 ern the storm-time evolution of the outer belt. In particular, diffusion models do not cap-
94 ture the effects of rapid, large-scale reconfiguration of the magnetic field in storm main
95 phase that causes rapid non-adiabatic expansion of electron drift orbits and subsequent
96 magnetopause loss of particles from a broad range of L -shells (Ukhorskiy et al., 2006;
97 Staples et al., 2022). In addition, the inward radial transport is not always slow. Dur-
98 ing storms, earthward convection can occur in the form of mesoscale bursty bulk flows
99 (BBFs) (Baumjohann et al., 1990; Angelopoulos et al., 1994) that swiftly transport source
100 and seed electrons through localized azimuthal electric fields and magnetic gradient trap-
101 ping (Gabrielse et al., 2017; Ukhorskiy et al., 2018; Sorathia et al., 2018; Turner et al.,
102 2021; Kim et al., 2023). Furthermore, boundary layer dynamics on the magnetopause
103 cause additional loss to occur through Kelvin-Helmholtz waves (Sorathia et al., 2017),
104 complicating loss estimated through the last closed drift shell.

105 A more detailed description of electron transport in the outer radiation belt is pro-
106 vided by test-particle simulations. Test-particle simulations evolve relativistic electrons
107 through accurate, time-varying electromagnetic fields provided by global magnetosphere
108 models and can, therefore, describe the full range of three-dimensional effects, includ-
109 ing rapid, non-diffusive radial transport. These effects include key loss and acceleration
110 processes due to: magnetosonic waves driven by coronal mass ejections (CMEs) (Hudson
111 et al., 2015), drift orbit bifurcations (Ukhorskiy et al., 2011), expansion of electron drift
112 orbits in response to large-scale magnetic field perturbations due to the storm-time ring
113 current (Ukhorskiy et al., 2006), injections from magnetotail convection and fast, mesoscale
114 flows (Ukhorskiy et al., 2018; Sorathia et al., 2018; Sorathia et al., 2021), ULF waves (Claudepierre
115 et al., 2016) and their associated radial transport (Kress et al., 2012), and losses asso-
116 ciated with magnetopause boundary dynamics, such as additional loss mediated by Kelvin-
117 Helmholtz waves (Sorathia et al., 2017). Using global magnetosphere and test particle
118 simulations, Sorathia et al. (2018) captured the initial dropout of the radiation belts and

119 subsequent rebuilding through injections in the tail during the 17 March 2013 geomagnetic storm. The test particle simulations obtained good quantitative agreement for radiation belt intensities below 1 MeV throughout all phases of the storm. There were, however, large discrepancies between the modeled intensity and observation at the multi-MeV energies, where the model intensity had a weaker overall enhancement. One possible reason for this discrepancy is that test-particle simulations used in those simulations did not account for local pitch-angle scattering and energization by kinetic plasma waves.

126 Chan et al. (2023) incorporated the effects of cyclotron-resonant wave-particle interactions into their global magnetosphere and test-particle simulation by using a stochastic differential equation (SDE) to solve the Fokker-Planck equation (Tao et al., 2008; Zheng et al., 2014, 2021). Pitch-angle scattering and energy diffusion were calculated using pre-computed, event-specific bounce-averaged diffusion coefficients from Ma et al. (2018). Chan et al. (2023) showed local acceleration can result in rapid changes in PSD and, combined with radial diffusion, and can produce electron PSD enhancements in the outer radiation belts. However, the bounce-averaged diffusion coefficients used in Chan et al. (2023), were computed using a dipolar magnetic field and a static density distribution. The diffusion coefficients, therefore, did not exhibit a realistic variability due to either the storm-time magnetic field or the cold plasma density, which can significantly affect both the estimated wave power (Longley et al., 2022) and the characteristics of the wave-particle interactions themselves (e.g., Kennel & Petschek, 1966).

139 The goal of this study is to incorporate wave-particle interactions with lower band 140 chorus waves into our global test particle simulation in a physically consistent way to 141 directly connect local acceleration and scattering effects to the background plasma and 142 wave fields. This is done within a modular framework to be able to isolate and analyze 143 the relative importance of each loss and acceleration mechanism governing radiation belt 144 dynamics. In this paper, we analyze the main phase of the 17 March 2013 geomagnetic 145 storm and investigate the impact that field-aligned chorus waves have on radiation belt 146 variability. The paper is structured as follows. A description of the numerical models 147 used in this study is presented in Section 2. Section 2.3 describes how the empirical wave 148 model of lower band chorus waves is assimilated into the simulation. Section 2.4 and Ap- 149 pendix B detail the algorithm used to calculate resonant interactions in the test parti- 150 cle model, and Section 3 gives an overview of the 17 March 2013 storm and the appli- 151 cation of the model to simulate the event. Section 4 presents our results. Section 4.1 pro- 152 vides an analysis of how the storm-time magnetic field and density create an magnetic 153 local time (MLT) and energy dependence in the quasi-linear diffusion coefficients. Sec- 154 tion 4.2 describes the resulting enhancement of mulit-MeV electron fluxes at the begin- 155 ning of the recovery phase and Section 4.3 presents quantitative comparisons of the vary- 156 ing storm-time loss processes. Finally, Section 5 discusses the implications of our results 157 on radiation belt dynamics and provides a summary.

158 2 Methodology

159 To fully capture storm-time evolution of the radiation belts, it is necessary to ac- 160 count for the full range of acceleration and transport processes that can be both diffu- 161 sive and non-diffusive. For this purpose, we combine the Fokker-Planck formalism for 162 local resonant wave-particle interactions and our global test-particle radiation belt model 163 that accurately capture storm-time global-to-mesoscale dynamics (Sorathia et al., 2018). 164 The implemented numerical scheme solves the diffusion equation for pitch angle scat- 165 tering and acceleration simultaneously with the integration of the electrons trajectory 166 through fields provided by a global magnetosphere model. We accomplish this through 167 the combination of four separate modules. A schematic diagram summarizing the cou- 168 pling between the modules is shown in Figure 1. The Multiscale Atmosphere-Geospace 169 Environment (MAGE) global geospace model provides accurate evolution of the storm- 170 time electromagnetic fields and plasmaspheric density. The Conservative Hamiltonian

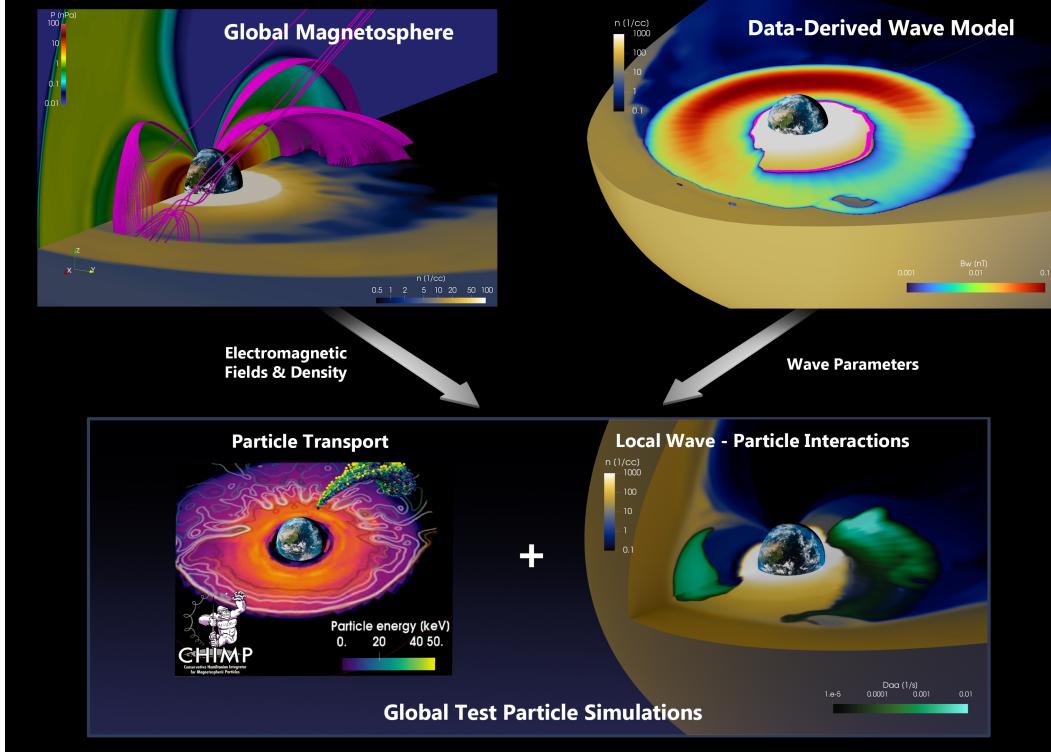


Figure 1. System architecture diagram showing the flow of information between the component modules in the simulation when wave-particle interactions are included.

171 Integrator for Magnetospheric Particles (CHIMP) test-particle model provides an accurate
 172 description of storm-time electron transport and loss. An empirical wave model characterizes
 173 chorus wave power as a function of location, geomagnetic activity, and frequency, within the simulation. Finally, a wave-particle interaction module incorporates local pitch
 174 angle and energy diffusion due to resonance with lower band chorus waves.
 175

2.1 Global Geospace Model

176 The MAGE model combines the Grid Agnostic MHD for Extended Research Application
 177 (GAMERA) global magnetohydrodynamic (MHD) model (Zhang et al., 2019;
 178 Sorathia et al., 2020), and its integrated ionospheric model, REMIX (Merkin & Lyon,
 179 2010), with the Rice Convection Model (RCM) in the inner magnetosphere (Toffoletto
 180 et al., 2003). The MAGE model was designed to capture critical mesoscale features that
 181 regulate storm-time geospace dynamics (Sorathia et al., 2020; Lin et al., 2021; Sorathia
 182 et al., 2021; Pham et al., 2022). For a description of the MAGE model and its compo-
 183 nents, see both Sorathia et al. (2023) and Sciola et al. (2023). The cold plasma density
 184 controls the distribution of different wave populations in the inner magnetosphere (Malaspina
 185 et al., 2016), as well as the resonant interactions with the waves themselves (Summers
 186 et al., 1998; Summers, 2005). In the MAGE model, the plasmaspheric mass is dynam-
 187 ically evolved as a cold, formally zero energy, fluid by the RCM (Lin et al., 2021). The
 188 plasmaspheric density in the inner magnetosphere is initialized with the empirical global
 189 core plasma model (Gallagher et al., 2000) and the observed Kp index at the start of the
 190 event. The plasmasphere is then evolved using the same self-consistent electrostatic po-
 191 tential computed by REMIX. The mass of the cold plasmasphere is ingested back into
 192 the global magnetosphere simulation along with the ring current mass and pressure (Pembroke
 193

194 et al., 2012; Lin et al., 2021). The plasmaspheric density is the field-line averaged value
 195 and is assumed to be constant along the field-line. As the convective electric field erodes
 196 the plasmapause, the plasmaspheric mass is refilled, using an empirical model (Denton
 197 et al., 2012) derived from radio emissions observed by the IMAGE spacecraft. The re-
 198 filling time of the plasmasphere is on the order of several days. Therefore, refilling is more
 199 important during lower activity levels having less impact during shorter time periods as-
 200 sociated with geomagnetic storms, which we are interested in modeling in this work.

201 2.2 Test-Particle Simulation

202 We model the radiation belts with an ensemble of test particles, ranging in ener-
 203 gies and pitch angles that fully capture the radiation belt population. A test particle treat-
 204 ment is valid for radiation belt dynamics, since relativistic electrons make a negligible
 205 contribution to the plasma pressure, and consequently, do not produce feedback on the
 206 fields that drive their motion. CHIMP is a particle integrator, fully integrated to work
 207 with the complex 3D grid geometries used by GAMERA. CHIMP computes the tra-
 208 jectories of particles through the 3D, time-dependent electromagnetic fields generated by
 209 the MAGE model. CHIMP is capable of calculating the trajectories using either the rel-
 210 ativistic Lorentz equations of motion or a relativistic Hamiltonian formulation of the guid-
 211 ing center trajectories (Sorathia et al., 2018; Sorathia et al., 2019). Also available within
 212 CHIMP is a mixed integrator that alternates between a guiding center formulation and
 213 the Lorentz trajectory. The ratio of the particle gyroradius to local magnetic field length
 214 scale is used as the criterion to switch between the two formulations. The test particle
 215 distribution is converted into a PSD by assigning each particle a weight. The particle
 216 weight relates the number of real electrons each test-particle acts as a proxy for and is
 217 calculated to match an initial, specified PSD. The time evolution of PSD is computed
 218 on a discretized phase space grid, using known weights and updated test particle posi-
 219 tions. For more details on the calculation, see appendix A2 of Sorathia et al. (2018).

220 2.3 Data-Derived Wave Module

221 Specification of the properties of all relevant wave modes throughout the inner mag-
 222 netosphere is required to assess the net effect of local wave-particle interactions on elec-
 223 tron intensities. Electromagnetic wave-modes that exhibit cyclotron resonance with en-
 224 ergetic electrons in the radiation belts, such as whistler waves, are driven by kinetic pro-
 225 cesses and, therefore, are not captured by isotropic single fluid MHD models. Most plasma
 226 waves identified as important to sculpting radiation belt population have amplitudes much
 227 smaller than the background fields. Therefore, one can consider the wave mechanisms
 228 independently from other processes. Wave occurrence rates and power depend on the
 229 solar wind and geomagnetic activity. To derive the global wave field specification from
 230 in situ spacecraft measurements, previous studies produced statistically-averaged 2D maps
 231 (L -MLT and/or L -MLAT) of wave amplitudes binned by a geomagnetic activity index
 232 (K_p or AE) of whistler-mode hiss (e.g., W. Li et al., 2015), and chorus (e.g., W. Li et
 233 al., 2016; Agapitov et al., 2018; Wang et al., 2019; Meredith et al., 2020) waves.

234 In this paper we use a slightly different approach to specify lower band chorus waves
 235 that enables a physically consistent integration of the data-derived wave power distri-
 236 butions into dynamically varying inner magnetosphere as described by the MAGE model
 237 (see Appendix A). The wave model is constructed based on the Van Allen Probes wave
 238 data over the entire mission period and supplies lower band chorus amplitudes as a func-
 239 tion of L , MLT, MLAT, and the SuperMAG SML* index. The SML index is analogous
 240 to the AL index and is indicative of the level of substorm activity. SML* is defined as
 241 the minimum SML index during the preceding three hours (Gjerloev, 2012; Newell & Gjer-
 242 loev, 2012, 2014). An example of the integration of the wave module into MAGE and
 243 CHIMP is depicted in Figure 2 for four different times throughout the simulation. Cho-
 244 rus waves are well organized by distance relative to the plasmapause, dL_{pp} (Malaspina

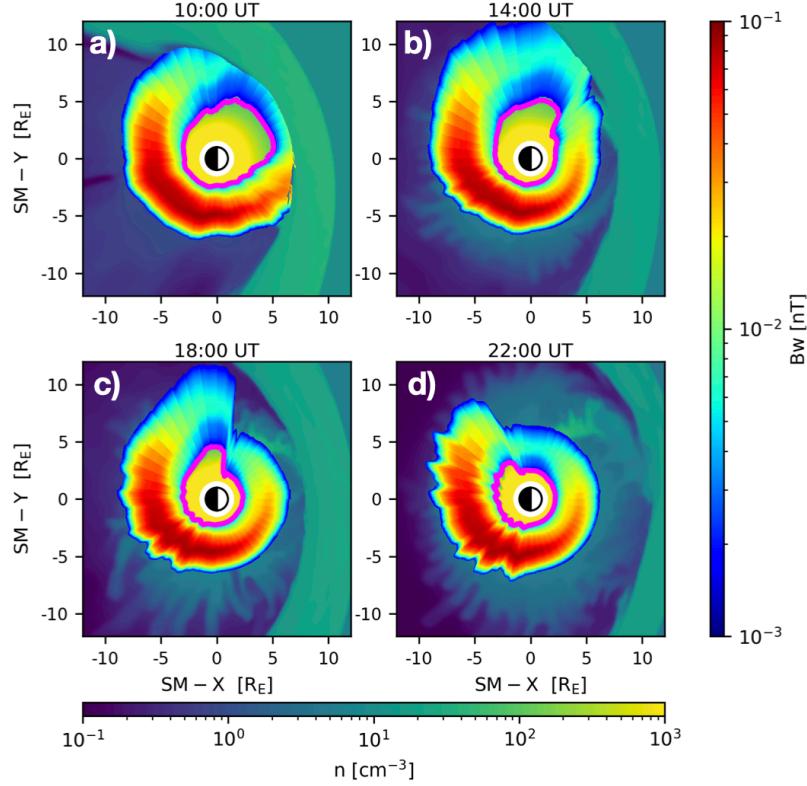


Figure 2. Storm-time evolution of the plasmasphere and resulting chorus wave location throughout the main phase of the 17 March 2013 event. Snapshots in the equatorial plane of the MHD plasma density and ingested chorus wave amplitude at four times, each four hours apart. The magenta line denotes the location of the plasmapause in the model, defined by a plasma density of $n=100 \text{ cm}^{-3}$.

et al., 2016). To maintain physical consistency between the distribution of waves and the plasma environment within the global model, lower band chorus waves from the empirical wave model are ingested into the simulation relative to the modeled plasmapause location. To this end, the wave distribution was re-parameterized according to dL_{pp} . The plasmapause in the empirical model is taken to be the average of the minimum L -shell where chorus waves are present in each MLT bin. The time-dependent plasmapause boundary passed from the RCM module is used to rescale the obtained wave distributions onto the model domain for each step of the global magnetosphere simulation. Here, we define the plasmapause location in the MAGE model to be where the plasma density reaches 100 cm^{-3} , as in Ripoll et al. (2022). We additionally only include waves within the inner magnetosphere, without taking into account waves in the magnetosheath.

2.4 Wave-Particle Interaction Module

Acceleration and scattering of electrons through local wave-particle interactions with lower band chorus waves is computed from the Fokker-Planck diffusion equation via an SDE. The diffusion equation uses the notation of Lyons (1974a, 1974b), defining $D_{\alpha\alpha}/p^2$, $D_{\alpha p}/p^2$, and D_{pp}/p^2 to be in units of s^{-1} . The diffusion rates are solved utilizing analytical expressions for the quasi-linear diffusion coefficients derived by Summers (2005). These coefficients are local, calculated at the particles position, and assume chorus wave prop-

263 agation parallel to the magnetic field. Lower band chorus waves are predominantly quasi
 264 field-aligned, especially during geomagnetically active times (W. Li et al., 2016). The
 265 parallel propagation assumption greatly reduces the complexity of the problem, making
 266 the inclusion into test particle simulations more tractable. Chorus waves typically
 267 have small wave normal angles ($< 20^\circ$), however, large values have also been measured
 268 (e.g., W. Li et al., 2011). Shprits et al. (2006) showed that the bounce-averaged diffusion
 269 coefficients for lower-band chorus interaction with hundreds of keV to a few MeV
 270 electrons were dominated by the resonant harmonic associated with parallel propagating
 271 waves. Restricting our consideration to parallel propagating waves, therefore, should
 272 not significantly affect the analysis of electron acceleration up to the energies of a few
 273 MeV. For electron energies greater than a few MeV, we mimic resonance with obliquely
 274 propagating chorus waves, as described in Appendix A.

275 The diffusion coefficients depend on the background magnetic field and density, see
 276 equation B6. The magnetic field and density are taken directly from the magnetosphere
 277 model at the particle's location. Therefore, local acceleration changes dynamically along
 278 electron trajectories as they propagate through the background plasma with varying den-
 279 sity and ambient magnetic field magnitude.

280 The implementation of the wave-particle interaction module is general and can be
 281 extended to include multiple wave modes. These wave modes can be considered sepa-
 282 rately, or in orchestra, to quantify the effect on the system. A full description of the wave-
 283 particle interaction module included within CHIMP is provided in Appendix B.

284 3 Simulation Setup

285 In this paper we applied our newly developed model to the March 17 2013 storm.
 286 The storm was caused by the interplanetary shock driven by a coronal mass ejection that
 287 struck the Earth at 6 UT, causing an initial rapid drop out of the radiation belt flux likely
 288 attributed to particle loss through the magnetopause (e.g., Baker et al., 2014). Figure
 289 3 provides an overview of the observed SYMH-H and Van Allen Probes (RBSP) mea-
 290 surements throughout the event. A rapid enhancement of electron fluxes for energies be-
 291 low a few MeV followed the dropout, while multi-MeV energies gradually increased in
 292 flux over the next ten hours. Fluxes eventually exceeded the pre-storm values by more
 293 than an order of magnitude (Baker et al., 2014; W. Li et al., 2014; Ukhorskiy et al., 2015).
 294 A minimum storm-time SYM-H index of ≈ -130 nT was reached just prior to 21 UT
 295 under continuous southward B_Z driving. Chorus wave activity was observed directly by
 296 both RBSP A and B, as well as indirectly inferred from POES precipitation measure-
 297 ments. Waves occurred during the entire period of enhancement from 10 UT until mid-
 298 night (W. Li et al., 2014).

299 The global magnetosphere simulation used in this study is described in detail by
 300 Sorathia et al. (2023). In this simulation, GAMERA's modeling domain extended from
 301 $25 R_E$ at the subsolar point to $300 R_E$ down the magnetotail and had a spherical inner
 302 boundary at $1.5 R_E$. GAMERA utilizes a warped spherical grid with $192 \times 192 \times 256$
 303 cells in the radial, polar, and azimuthal directions. This grid is $2 \times$ more refined in each
 304 dimension than the earlier LFM simulation of the same event (Wiltberger et al., 2017;
 305 Sorathia et al., 2018). The REMIX solver for the electrostatic potential used a uniform
 306 grid with 0.5-degree resolution in both latitude and longitude. The low latitude bound-
 307 ary of the REMIX grid was set by the dipole mapping of the MHD inner boundary to
 308 the ionosphere. The REMIX solution was coupled with GAMERA every 5 seconds. The
 309 spatial domain of the RCM grid had a resolution of $0.25^\circ \times 1^\circ$ in latitude and longitude,
 310 respectively. In this work, we used 115 energy channels that consisted of 29 channels for
 311 electrons, 85 channels for protons, and a single zero-energy channel for the cold plasma-
 312 sphere. At geosynchronous orbit, this corresponds to an energy range of ≈ 0.01 –10 keV
 313 for electrons and ≈ 0.1 –100 keV for protons. The updated plasma density and pres-

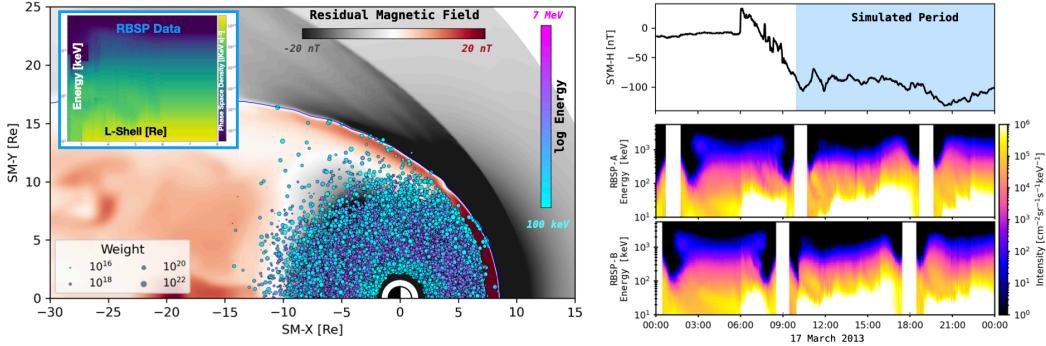


Figure 3. An overview of the global magnetosphere and test particle simulation in the SM equatorial plane at 13:20 UT on 17 March 2013 is shown in the left hand panel. Color contours of the residual magnetic field are plotted in the equatorial plane, with the dipole field removed. Regions where the field is compressed are in red. Test particle locations are projected along magnetic field lines to the equatorial plane. The size of each marker corresponds to the weight of the particle. The initial PSD from the Van Allen Probes used to weight the test particles is provided in the upper left inset. The simulated period relative to the observed SYM-H index is provided in the top right panel. RBSP A and B measurements on March 17, 2013, taken from Boyd et al. (2019), are shown in the bottom right two panels.

sure from RCM were ingested into MHD solution at a frequency set at a cadence of 10 seconds. As shown by Sorathia et al. (2023) through data-model comparison of the SuperMAG indices (Gjerloev, 2012; Newell & Gjerloev, 2012, 2014) with this configuration, the MAGE model captured the large-scale magnetic field perturbations due to the ring current throughout the event.

The MAGE model was driven by solar wind data taken from the OMNI database at 1 minute resolution. Data gaps were linearly interpolated to provide continuous boundary conditions for the simulation. The solar F10.7-cm flux index used for the ionospheric conductance was set to 124.5, the daily flux density during the event, taken from the OMNI data set. The magnetosphere simulation started at 00 UT on 17 March 2013 and was preconditioned with real solar wind for 6 hours before the sudden storm commencement. The simulation was run for a total of 30 hours, encompassing the entire main phase of the storm and several hours into the recovery phase. The three-dimensional electromagnetic fields and plasma solution generated by the simulation were saved at a cadence of 30 seconds.

We utilized the guiding center formalism to compute the test-particle trajectories. Test particles were initiated and integrated on a subdomain of the full MHD grid within a spherical region defined by a radius of $20 R_E$ and centered at the origin. The magnetic field was assumed to be a dipole below the $1.5 R_E$ inner boundary of the MHD grid. Test particle trajectories were solved down to an altitude of $1.05 R_E$. The MHD field is nearly dipolar at $1.5 R_E$, however, there is a small discontinuity as particles cross the MHD inner boundary. We control for the effect by estimating the effective scattering rate to be a thousand times smaller than typical pitch angle diffusion from lower band chorus waves. Particles that exit the outer boundary of the domain or do not bounce before $1.05 R_E$ were considered lost from the system.

To isolate the impact of local chorus acceleration on the radiation belts, we removed the effect of injections (Z. Li et al., 2015; Sorathia et al., 2018) and included only the

341 evolution of the initial radiation belt population. We refer to the modeled population
 342 as the pre-existing belt. The pre-existing particle population was initialized in the SM
 343 equatorial plane, $Z=0$, at 10 UT. The particles were uniformly distributed in MLT and
 344 pitch angle, between $L = 2.5$ and $L = 8$ with energies ranging from 50 keV to 5 MeV.
 345 In total, 2 million particles were evolved for 14 hours throughout the entire acceleration
 346 period from 10 UT until midnight on 18 March 2013. Figure 3 provides an overview of
 347 the test particle simulations. The simulated period of the radiation belts relative to the
 348 observed SYM-H index is also provided in the bottom panel of Figure 3 for additional
 349 context.

350 The pre-event PSD used to weight the test particles was derived from Van Allen
 351 Probes data, using the ECT combined data product provided by Boyd et al. (2019). The
 352 PSD was taken over the outbound orbit between 10 UT and 14 UT on 17 March 2013,
 353 similar to Ma et al. (2018). The initial PSD from RBSP-A and B used to weight the test
 354 particles is given in the inset of the upper panel of Figure 3. Here, the initial PSD was
 355 computed on a grid as a function of energy and L -shell. To ensure all test particles are
 356 weighted, the initial PSD grid spanned energies from 10 keV to 6 MeV and L -shell cov-
 357 erage from $2.5 R_E$ to $8 R_E$, with 55×50 cells in each dimension respectively. The ini-
 358 tial PSD was averaged over the measurements from both RBSP A and B within each
 359 cell. For $L > 6$, the PSD was set to match the value at geosynchronous orbit. This ap-
 360 proximation did not impact the conclusions of this paper; particles initialized at $L >$
 361 6 were quickly lost to the magnetopause due to the compression at the beginning of the
 362 simulation and therefore do not contribute to the final PSD at the end of the main phase.

363 After the test particles were weighted, the time-evolved PSD was computed on a
 364 discretized phase space specified in the SM equatorial plane and parameterized by L , az-
 365 imuthal angle (ϕ), equatorial pitch angle (α_{eq}), and particle kinetic energy (K). L and
 366 ϕ were set for each particle using their equatorial crossing point. The phase space do-
 367 main for the results shown in this work was given by $\Gamma(L, \phi, \alpha_{eq}, K) = [2.5, 10] \times [0, 2\pi] \times$
 368 $[0, \pi] \times [50 \text{ keV}, 7 \text{ MeV}]$. The grid cells were linearly spaced for L , ϕ and α_{eq} and were
 369 logarithmic in energy. The number of cells set in each dimension was set to (30, 24, 9,
 370 30), respectively.

371 To quantify local scattering and acceleration from lower band chorus wave, we per-
 372 formed two test particle simulations. Both simulations evolved initially identical radi-
 373 iation belt populations through the time varying electromagnetic fields form the MAGE
 374 model. The data-derived wave module combined with the wave-particle interaction mod-
 375 ule were incorporated into one simulation and were not included in the other. Through-
 376 out the remainder of manuscript, references to wave-particle interactions will refer only
 377 to resonance with lower band chorus waves.

378 4 Results

379 4.1 The Role of Storm-Time Magnetic Field and Density on Wave-Particle 380 Interactions

381 The storm-time evolution of the magnetic field and cold plasma density can alter
 382 the efficiency of wave-particle interactions by changing the magnitude of scattering and
 383 acceleration as well as the energies electrons resonate at. To assess how the evolution
 384 of the background plasma alters the wave-particle interactions, we performed bounce av-
 385 eraging of the local diffusion coefficients. We note that, while bounce-averaging the dif-
 386 fusion coefficients illustrates the connection to the background plasma conditions, the
 387 local diffusion coefficients were used to solve each individual resonant interaction in Sec-
 388 tion 2.4. In this analysis, we focused on effects longer than a typical bounce period of
 389 electrons. $\langle D_{\alpha\alpha} \rangle$, $\langle D_{\alpha p} \rangle$ and $\langle D_{pp} \rangle$ were computed, as defined in Appendix C, as a func-
 390 tion of energy and α_{eq} . The calculations were performed on a uniform grid, compara-

ble to that used by the wave model, using the magnetic field and density directly from the MAGE model at the specified time. $\langle D_{\alpha\alpha} \rangle$, $\langle D_{\alpha p} \rangle$ and $\langle D_{pp} \rangle$, therefore, provide a snapshot of particle diffusion at a given instance.

Figure 4 presents $\langle D_{\alpha\alpha} \rangle$ for 2 MeV and 100 keV electrons, each with $\alpha_{eq} = 55^\circ$, at the four time instances spread uniformly through the main phase of the storm. The selected times are identical to those used in Figure 2. The panels in Figure 2 are shown again in the Figure 4a-4d as a reference for the storm-time evolution of the modeled plasma-pause and the resultant chorus wave power distribution. $\langle D_{\alpha\alpha} \rangle$ is zero in the blank regions of Figure 4i-4p. These locations denote where either no chorus waves were present or no resonance occurred along the electron bounce. For electrons in resonance with lower band chorus waves at frequencies significantly lower than the plasma frequency, the minimum resonant energy is $E_{min} = E_C (|\Omega_e|/\omega) (1 - \omega/|\Omega_e|)^3$, where Ω_e is the electron gyrofrequency and ω is the resonant wave frequency (e.g., equation 2.19 of Kennel & Petschek, 1966). E_C is the characteristic energy for cyclotron interactions and is defined as $E_C = B^2/8\pi N \propto f_{ce}^2/f_{pe}^2$. Hence, variations in both the magnetic field and the cold plasma density can strongly affect the resonance energy of wave-particle interactions with lower band chorus waves. Figure 4e-4h shows the evolution of the cold plasma parameter, f_{pe}/f_{ce} , in the equatorial plane during the main phase of the storm. While the magnetic field also exhibited a significant non-dipolar structure, much of the variation in f_{pe}/f_{ce} was driven by the cold plasma density. Figure S2 within the supporting information provides a detailed comparison of the influence of the storm-time magnetic field and cold plasma density each have on $\langle D_{\alpha\alpha} \rangle$.

Increased storm-time convection erodes the nightside plasmasphere and forms the plasmaspheric plume (Grebowsky, 1970; Foster & Burke, 2002; Borovsky & Denton, 2006; Darrouzet et al., 2009). This can be seen in the increase in f_{pe}/f_{ce} in Figure 4e at 10 UT, as the cold, dense mass extended to the dayside magnetopause. Several hours into the storm, continued enhanced convection further eroded the plume causing it to narrow in MLT, shown in Figure 4f. In Figure 4g and 4h, the plume ultimately began to rotate as convection decreased. The modeled plume behavior qualitatively matches observations during geomagnetically active periods (Goldstein & Sandel, 2005). This dynamic evolution of the plasmaspheric plume, combined with the reduction of the magnetic field due to the ring current, has major implications for resonance with chorus waves. The extension of the plume to larger L -shells caused an increase in the plasma frequency and lowered the minimum electron energy in resonance with the waves (Kennel & Petschek, 1966). The plume enabled lower energy electrons to be scattered into the loss cone, increasing the precipitation in those regions, as previously reported by observations (W. Li & Hudson, 2019). However, as seen by the absence of $\langle D_{\alpha\alpha} \rangle$ in Figure 4i-4p, the higher ratio of f_{pe}/f_{ce} in the plume shifted the resonance region to lower energies such that ≥ 100 keV electrons do not exhibit resonance with lower band chorus waves.

The density also significantly impacts the magnitude of the scattering and energization resulting from the interaction (Summers et al., 1998; Summers, 2005). Continuing storm-time convection significantly eroded the dayside plasmasphere. The erosion formed a low-density trough. This reduced the ratio of f_{pe}/f_{ce} just outside the plasma-pause, seen in Figure 4f-4h. The trough initially formed on the dawnside at pre-noon before extending to the dayside. The density depletion, in combination with the reduced magnetic field strength from the ring current, reduced this f_{pe}/f_{ce} to ~ 0.1 at very low L -shells from 14 UT to 22 UT. The low f_{pe}/f_{ce} shifted the resonance zone to higher energies to include multi-MeV electrons. Therefore, despite having lower wave amplitudes in this region (see Figure 2), the scattering and acceleration for 2 MeV electrons was enhanced. This is exhibited by the increase in $\langle D_{\alpha\alpha} \rangle$ in the pre-noon sector just outside the plasmapause in Figure 4j-4l. From Figure S2 within the supporting information, we see that the enhancement in scattering for 2 MeV electrons results primarily from the erosion of the plasmasphere despite the reduction in the magnetic field due to the ring

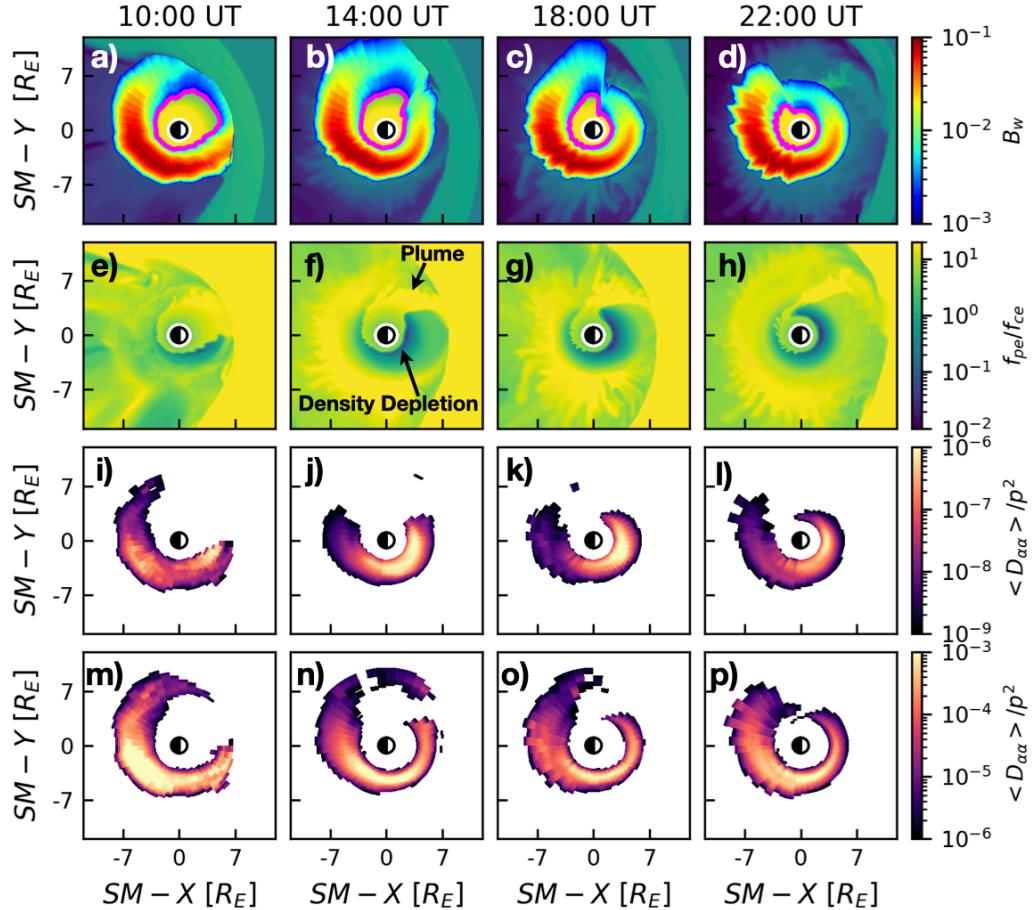


Figure 4. Overview of the influence of the background plasma conditions on quasi-linear diffusion coefficients. The times selected match those shown in Figure 2, which are plotted again in the top row. The cold plasma parameter, f_{pe}/f_{ce} , in the equatorial plane is shown in the second row. The middle and bottom rows show $\langle D_{\alpha\alpha} \rangle$ due to chorus waves, in units of s^{-1} , for 2 MeV and 100 keV electrons, respectively, both with $\alpha_{eq} = 55^\circ$. $\langle D_{\alpha\alpha} \rangle$ is calculated using the instantaneous magnetic field, density, wave amplitudes and locations from the magnetosphere and empirical wave models.

444 current. In contrast, $\langle D_{\alpha\alpha} \rangle$ for 100 keV electrons was only weakly affected by the den-
 445 sity erosion on the dayside. Instead, the diffusion rates remained the largest in the post-
 446 midnight to dawn sectors where the wave amplitudes were the largest, shown in Figure
 447 4m-4p.

4.2 Electron Flux Enhancements

449 Figure 5 presents the evolution of the radiation belt intensity, $j = p^2 f_{PSD}$. The
 450 intensity is calculated by averaging the PSD over pitch angle, gyrophase, and MLT. Fig-
 451 ure 5 compares the initial distribution of j as a function of energy and radius to the average
 452 intensity at the end of both simulations, when an enhancement of multi-MeV elec-
 453 trons was observed by the Van Allen Probes. Shown in Figure 5e, the combination of
 454 radial transport and electron loss through the magnetopause reduced the the intensity
 455 of the radiation belts by almost an order of magnitude for $R > 5 R_E$ and energies be-

456 low 500 keV. Inward radial transport increased 300-900 keV electron intensities by an
 457 order of magnitude for $R < 4 R_E$.

458 By the end of the main phase of the storm, the Van Allen Probes observed enhancement
 459 of relativistic radiation belt fluxes by an order of magnitude relative to pre-storm
 460 levels (Baker et al., 2014; W. Li et al., 2014; Ukhorskiy et al., 2015), shown in Figure 3.
 461 Figure 5e, shows that acceleration via transport increased MeV intensities by a factor
 462 of two to three for $R < 4 R_E$ but did not produce the significant enhancement across
 463 all L -shells of the outer radiation belt, agreeing with the previous results of Sorathia et
 464 al. (2018). Furthermore, in Figure 5b, there is no distinguishable increase in average in-
 465 tensity for ≥ 1 MeV electrons below $R < 4 R_E$ relative to the initial distribution, while,
 466 at larger distances, a reduction in average intensity occurred. Figure 5c and 5d show,
 467 however, that local acceleration from lower band chorus waves did accelerate electrons
 468 to MeV energies, as shown by previous studies (W. Li et al., 2014; Ma et al., 2018; Chan
 469 et al., 2023). Additional acceleration from wave-particle interactions increased the av-
 470 erage intensity by up to three orders of magnitudes for multi-MeV electrons at $R > 2 R_E$.
 471 Using global MHD and test particle simulations, Sorathia et al. (2018) show that injec-
 472 tions during the 17 March 2013 storm do not produce significant enhancements in > 1
 473 MeV electrons. Electron resonance with chorus waves is a likely candidate for the ad-
 474 dditional source needed to capture the multi-MeV enhancement at the end of the storm
 475 main phase, as also shown by Ma et al. (2018).

476 The enhancement arose due to the acceleration of radiation belt electrons with ini-
 477 tial energies below 1 MeV to multi-MeV energies. The simulation was restricted to the
 478 evolution of the pre-existing belt and did not contain electron injections from the cen-
 479 tral plasma sheet that supply the continuous source of electrons between 50-100s, as shown
 480 by Sorathia et al. (2018). Consequently, there was a depletion in electrons below ~ 500
 481 keV, as shown in Figure 5d.

482 Figure 6 presents the equatorial pitch angle and energy distribution of MLT-averaged
 483 PSD at the peak of the outer belt average intensity at $L = 3.7$. We compare the ini-
 484 tial PSD distribution to the PSD from the end of both simulations with and without wave-
 485 particle interactions. Shown in Figure 6d, there is a reduction in the ratio of the PSD
 486 for the lowest equatorial pitch angles between the simulation including chorus wave res-
 487 onance and the simulation with only radial transport effects. This difference is attributed
 488 to diffusion into the loss cone caused by chorus wave resonance with subsequent parti-
 489 cle loss into the atmosphere. Wave-particle interactions also caused the pitch angle dis-
 490 tribution to become more anisotropic, which is indicative of electron acceleration by their
 491 resonant interaction with the waves. Due to the absence of a parallel electric field, res-
 492 onance with parallel propagating waves conserves the particle kinetic energy in the frame
 493 moving with the phase velocity wave. Particles are therefore constrained to diffuse along
 494 resonant phase-space curves that uniquely determine the change in momentum from a
 495 given change in pitch angle (Summers et al., 1998). For a given resonant diffusion curve,
 496 particles gain energy as they are scattered to larger equatorial pitch angles (Kennel &
 497 Petschek, 1966). This leads to a pitch angle distribution at higher energies that is ini-
 498 tially more equatorial, forming a pancake distribution at multi-MeV energies, similar to
 499 what we see in Figure 6c.

500 In both simulations with and without wave-particle interactions, there was a peak
 501 in the PSD at $\alpha_{eq} \sim 90$ deg for energies below ~ 400 keV, seen in Figure 6b and 6c.
 502 This feature is attributed to the lack of resonance of these electrons with lower band cho-
 503 rus waves. Therefore, the electrons have had few opportunities to be scattered out of these
 504 PSD cells from wave-particle interactions. If present, the higher frequency of upper band
 505 chorus waves would extend the resonance region to include the electrons near $\alpha_{eq} = 90$ deg
 506 below ~ 400 keV and would enable scattering of this population of electrons.

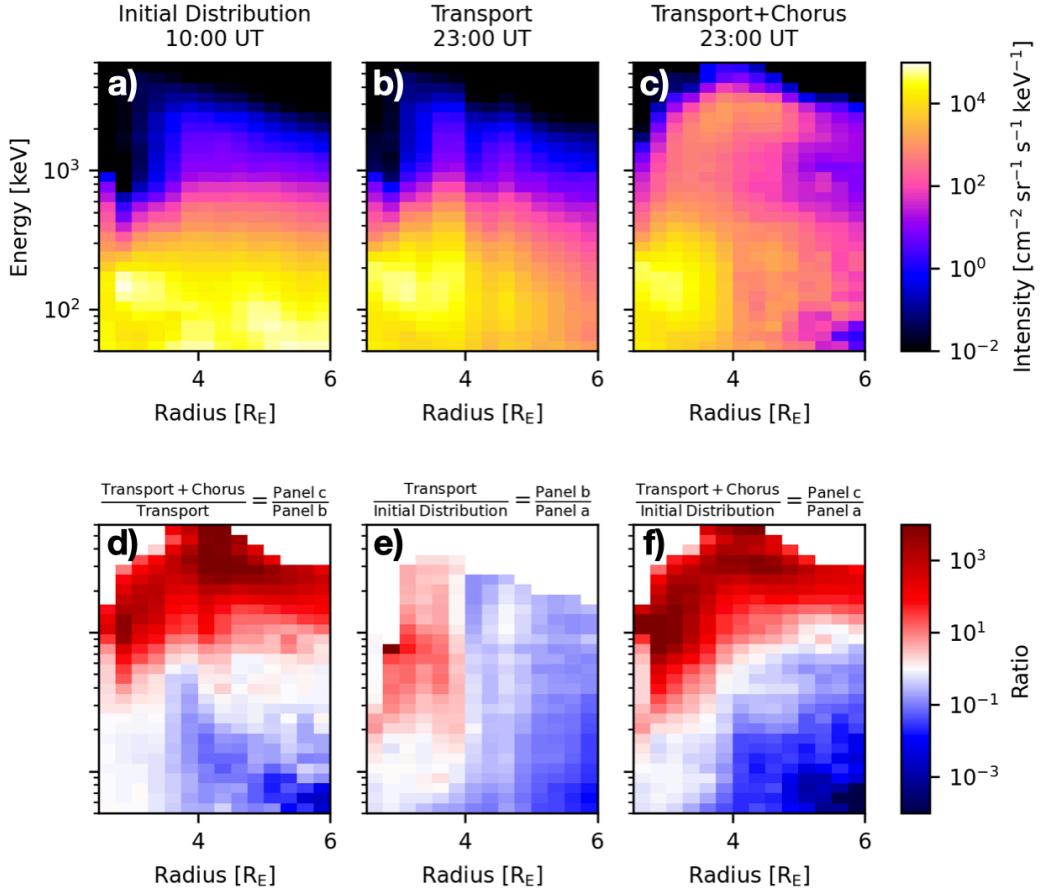


Figure 5. MLT-averaged intensity as a function of energy and radial distance. The initial distribution is shown in panel a. The average intensity at 23 UT for the cases with transport only and when wave-particle interactions with lower band chorus waves were included are presented in panels b and c, respectively, with their ratio provided in panel d. The ratio of the final intensity relative to the initial intensity for both runs are provided in panels e and f.

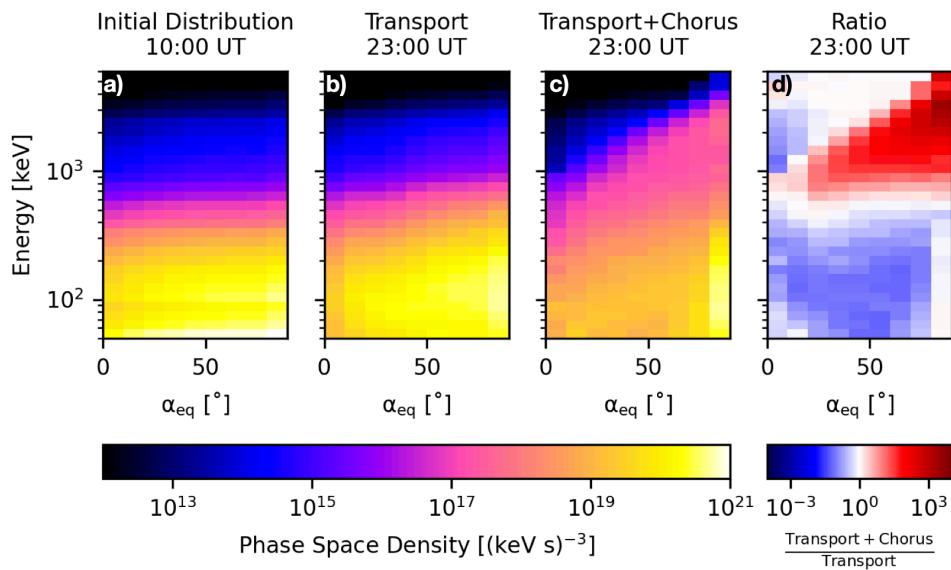


Figure 6. Similar to Figure 5, showing the MLT-averaged phase space density at $L = 3.7$ as a function of energy and equatorial pitch angle.

The purpose of this investigation is to compare the impact of wave-particle interactions in the model to the stand-alone radial transport simulation. Due to this and the exclusion of injections, we do not directly compare the simulated average intensity to RBSP-A measurements. The full data-model comparison is left for a future paper that will include the full range of diffusive and non-diffusive processes. Figure 7 presents a quantitative comparison of the intensities between the simulations when wave-particle interactions were included, shown in green, and when they were not, shown in orange. It shows "synthetic measurements" at RBSP-A, i.e., the average electron intensity extracted from test particle simulations computed along the spacecraft orbit from the time-evolution of the simulated PSD. The modeled average intensity in Figure 5 incorporates contributions from all particles along a given field line that maps from the spacecraft location to the corresponding PSD cell in the equatorial plane. To account for the fraction of electrons that mirror before reaching the spacecraft location, we performed field line tracking through the time-dependent global magnetosphere model to extract the equatorial projection of the spacecraft position along with the local and equatorial magnetic field strength. We further assumed the pitch angle distribution has a $\sin^n(\alpha)$ dependence to determine the attenuation factor, J_{scl} , calculated as

$$J_{scl} = \frac{\int_0^{\alpha_M} \sin^n(\alpha) d\alpha}{\int_0^{\pi} \sin^n(\alpha) d\alpha} \quad (1)$$

507 where α_M is the largest equatorial pitch angle that can be measured at the spacecraft
 508 location. In this work, we set $n = 2$, as done in Sorathia et al. (2018). The average
 509 intensities plotted in Figure 7 are energy weighted averages across the selected MagEIS
 510 MeV energy channels (Blake et al., 2013).

511 The comparison in Figure 7 between the simulations with and without wave-particle
 512 interactions highlights the importance of local electron acceleration by lower band chro-
 513 rus waves. The inclusion of local acceleration resulted in an order of magnitude or more
 514 enhancement of the radiation belt intensity relative to the simulation when only trans-
 515 port effects were included. Local wave-particle interactions led to a rapid increase in the
 516 radiation belt intensity within the first few hours, while RBSP-A was within the core
 517 of the outer belt, shown by the green curve in Figure 7. The increase was primarily due
 518 to electrons with large initial equatorial pitch angles ($\alpha_{eq} > 60$ deg). The bounce mo-
 519 tion of these electrons is entirely contained within the magnetic latitudes where waves
 520 were present. This enabled continuous resonance and acceleration to higher energies. The
 521 apogee of the RBSP-A orbit was on the nightside, near midnight, during this event. Seen
 522 in Figure 4b, the cold plasma parameter, f_{pe}/f_{ce} , increased as RBSP-A approached apogee
 523 around 15:00 UT. The acceleration at the highest energies was therefore reduced despite
 524 wave amplitudes being large at those L -shells. This decrease in $\langle D_{\alpha\alpha} \rangle$ on the nightside
 525 for MeV electrons is shown in Figure 4f. By the time of the second outbound orbit, start-
 526 ing at 20 UT, continuous resonance with chorus waves caused a several decade enhance-
 527 ment, relative to the previous apogee at 15 UT, of the \geq MeV average intensity across
 528 the entire outer belt up to geosynchronous orbit. In the simulation with wave-particle in-
 529 teractions, green curve in Figure 7, the enhancement of the electron intensity between
 530 the first and second apogees was not as significant at 1 MeV as it was at higher ener-
 531 gies. This can be attributed to a combined effect of 1 MeV electrons being accelerated
 532 to multi-MeV energies and the absence of electron injections to replenish to 1 MeV pop-
 533 ulation. Through comparison to the simulation when only transport effects were included,
 534 Figure 7 shows that the inclusion of local acceleration due to resonance with lower band
 535 chorus waves resulted in an order of magnitude or more enhancement of the radiation
 536 belt intensity at the end of the main phase.

537 It should be noted that chorus waves in the data-derived wave model, described
 538 in Section 2.3 and Appendix A, were distributed within 45° magnetic latitude through-
 539 out all MLT sectors in order to mimic pitch-angle scattering by oblique chorus waves.
 540 Statistically, lower band chorus waves are observed below 15° in the evening to dawn sec-

541 tor, below 30° in the dawn to afternoon sector, and up to 45° on the dayside (e.g., Agapitov et al., 2018; Meredith et al., 2012). While the wave amplitude used in the model decreased with magnetic latitude (equation A1), the assumption that lower band chorus waves were contained up to 45° in all MLT sectors may cause the model to overestimate the amount of local scattering and acceleration of relativistic electrons that occurred when compared to a more realistic MLT distribution of the latitudinal extent of the waves, if the chorus activity is dominated by parallel propagating waves. Calibration of the wave amplitude as a function of magnetic latitude and MLT is left for the future paper containing the full data-model comparison with RBSP observations.

550 4.3 Storm-Time Loss Processes

551 Figure 8 presents an overview of the losses throughout the simulation that includes
 552 chorus wave-particle interactions. There are two possible mechanisms responsible for ra-
 553 diation belt losses. Electrons either escape through the magnetopause due to rapid changes
 554 in configuration or via precipitation into the atmosphere. In our model simulations, elec-
 555 trons were considered to precipitate into the atmosphere if they did not bounce before
 556 reaching the spherical inner boundary of CHIMP, set at $1.05 R_E$. Evaluating the loss
 557 cone at $1.05 R_E$, rather than 100 km, results in precipitation loss to be over estimated
 558 by $\sim 10\%$. A more physically accurate representation of precipitation would require in-
 559 cluding energy and altitude dependent deposition into the atmosphere which is outside
 560 the scope of this work.

561 The test particles were considered to be lost through the magnetopause if they crossed
 562 from closed to open field lines, referred to here as the open-closed boundary. To iden-
 563 tify magnetopause crossings, field line tracing was performed for every test particle at
 564 each simulation step. A field line was defined to be magnetospheric, open, or interplan-
 565 etary magnetic field (IMF), if both, one, or none of its foot points map to the ionosphere,
 566 respectively. Particles are considered lost through the magnetopause if they cross the open-
 567 closed boundary and exit the CHIMP simulation domain at $20 R_E$ while on an open field
 568 line. We do not make a distinction between transitions to open or to IMF field lines when
 569 determining magnetopause losses, as was previously done in Sorathia et al. (2017).

570 Electrons did not precipitate and were lost only through the magnetopause when
 571 only radial transport effects were taken into account in the test particle simulation. Even
 572 in the presence of wave-particle interactions with lower-band chorus, the majority of elec-
 573 trons were lost through the magnetopause. Magnetopause loss contributed to $\sim 60\%$
 574 of total losses, or $\sim 40\%$ of the initial radiation belt, shown by the blue curve in Fig-
 575 ure 8. The largest increases in magnetopause losses occurred in rapid increments as the
 576 dynamic pressure of the solar wind quickly compressed the magnetosphere. The simu-
 577 lation, without scattering from wave-particle interactions, had a similar magnetopause
 578 loss profile to the blue curve in Figure 8, however, $\sim 46\%$ of the initial belt was lost.
 579 The difference between the simulations potentially can be attributed to electrons lost to
 580 precipitation when resonant interactions were included that might otherwise have been
 581 lost through the magnetopause later in the storm.

582 When compared to the model where only transport was included, wave-driven pre-
 583 precipitation increased the total loss by $\sim 14\%$ during the 14 hour simulated period. The
 584 precipitation accounted for $\sim 1/3$ of the total loss when both transport and pitch-angle
 585 scattering were present, denoted by the red curve in Figure 8. The precipitation loss rate
 586 also decreased over time. This is caused by the reduction in diffusive scattering rates as
 587 electrons are accelerated to higher energies. Therefore, fewer electrons were scattered into
 588 the loss cone as a function of time.

589 While the latitudinal distribution of the waves used within the wave model may
 590 result in an overestimation of the precipitation, the magnetopause loss in our simulations
 591 did not include the initial dropout event at 6 UT due to the shock arrival. In addition,

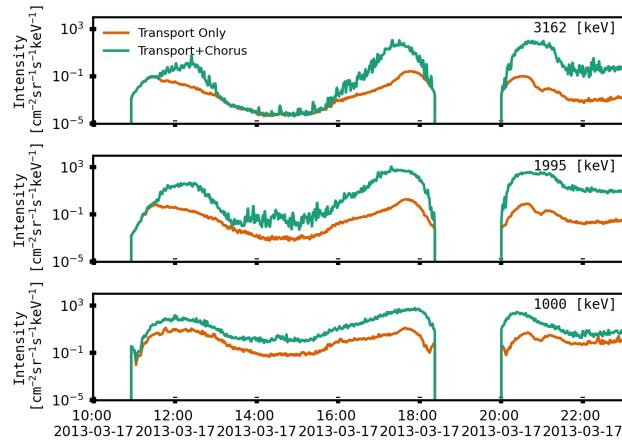


Figure 7. Comparison of model-predicted average intensity at RBSP-A when local wave-particle interactions were included (green) and when they were not (orange) at selected MeV energy channels.

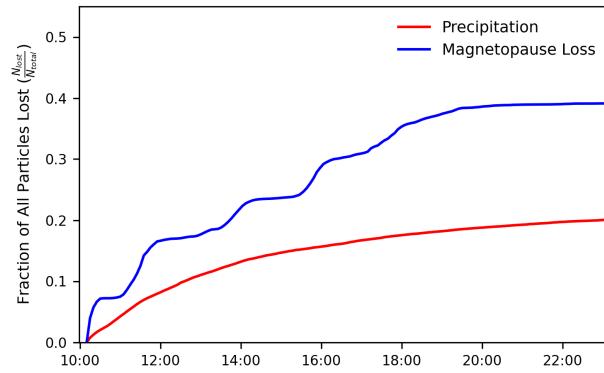


Figure 8. Overview of the evolution radiation belt losses throughout the simulation. The blue and red curves denote the fraction of the electrons, relative to the total number of electrons initially present, that were lost either through the magnetopause or precipitated into the atmosphere, respectively. Particles lost to precipitation was determined if they did not bounce before reaching our inner boundary of $1.05 R_E$.

592 injections would have provided an additional source that could either be scattered into
 593 the loss cone or immediately lost to the magnetopause if they are not on trapped orbits.
 594 Even without an additional particle source, we found that that precipitation can still be
 595 a significant component of radiation belt loss when wave activity is high.

596 Figure 9 presents the number flux, mean energy, and energy flux of precipitating
 597 electrons in the ionosphere. The precipitation shows the combined contribution from both
 598 hemispheres and was accumulated over the entire test particle simulation. The major-
 599 ity of precipitation, both in terms of number flux and energy flux, occurred in the post-
 600 midnight to dawn sector where wave amplitudes were the largest, as shown in Figure 2.
 601 The precipitation in this region comprised mostly of electrons with energies ≤ 200 keV,
 602 shown in the middle panel of Figure 9. This is consistent with the corresponding $\langle D_{\alpha\alpha} \rangle$
 603 for 100 keV electrons in the bottom row of Figure 4. The shift in resonant scattering of
 604 higher energy electrons to lower L -shells in the pre-noon sectors due to erosion of the
 605 plasmasphere is also apparent in the mean energy of the precipitating flux. Shown in the
 606 middle panel of Figure 9, the mean energy of precipitation increased with increasing co-
 607 latitude in both the pre-noon sector and towards the dayside. This highlights the strong
 608 dependence of wave-particle interactions on the evolution of the background plasma con-
 609 ditions.

610 5 Discussion and Conclusions

611 Understanding the outer electron radiation belt is a system level problem of geospace.
 612 This is especially true during geomagnetic storms where rapid variability of the radia-
 613 tion belt fluxes is prevalent. Realistic representation of storm-time magnetospheric elec-
 614 trodynamics is critical for capturing the dynamic response of the outer radiation belt in-
 615 tensity. A physically consistent solution of the outer radiation belt within the global mag-
 616 netosphere is important to capturing key acceleration and loss processes, such as injec-
 617 tions from the plasma sheet, magnetopause losses, and wave-particle interactions. A flexi-
 618 ble, modular framework enables global radiation belt models to isolate the effects of each
 619 mechanism, including the balance between diffusive and non-diffusive processes that pro-
 620 duce similar features, such as localized peaks in electron PSD (Kim et al., 2023) .

621 In the present paper we presented a new model of the radiation belts that incor-
 622 porates quasi-linear diffusion from resonant wave-particle interactions with whistler-mode
 623 chorus waves into our global magnetosphere and test particle model. The radiation belt
 624 model was driven solely by the upstream solar wind conditions, the solar F10.7-cm flux,
 625 and a data-derived specification of the chorus wave power. Pitch-angle scattering and
 626 acceleration from chorus wave-particle interactions was computed using a time-forward
 627 SDE to solve the Fokker-Planck diffusion equation. The diffusion equation was solved
 628 simultaneously with the integration of electron trajectories through electromagnetic fields
 629 provided by the magnetosphere model and was based on analytical expressions for the
 630 *local* quasi-linear diffusion coefficients (Summers, 2005). The local diffusion coefficients
 631 were derived for cyclotron resonance with field-aligned electromagnetic waves and are
 632 calculated for each instance of resonant chorus wave-particle interactions are computed.
 633 An empirical wave model was used to set the chorus wave amplitude as a function of lo-
 634 cation and geomagnetic activity. The chorus waves were dynamically ingested into the
 635 model based on the dynamically changing plasmapause location in the simulation. We
 636 used analytical expressions for the local diffusion coefficients that depend on the back-
 637 ground magnetic field and cold plasma density. The background plasma conditions were
 638 taken directly from the magnetosphere model at the particle's location. Therefore, the
 639 diffusion coefficients varied throughout the storm as a function of time and location con-
 640 sistently with the plasma and the wave model.

641 While previous studies have applied SDEs to solve the Fokker-Planck equation (Tao
 642 et al., 2008; Zheng et al., 2014, 2021; Chan et al., 2023), this is the first study where pitch-

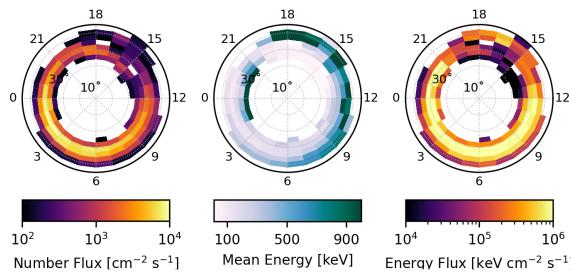


Figure 9. Number Flux (left), mean energy (middle), and energy flux (right) of precipitating electrons accrued over both hemispheres between 10 UT and 24 UT, shown as a function of colatitude and MLT. Regions with a number flux below $100 \text{ cm}^{-2} \text{ s}^{-1}$ have been masked for clarity.

angle scattering and acceleration have been included within a global magnetosphere and test-particle simulation using local quasi-linear diffusion rates that were physically consistent with the varying background plasma conditions. During geomagnetic storms, the enhanced convection electric field strongly impacts the evolution of the plasmasphere (e.g., Delzanno et al., 2021, and references therein). The plasmaspheric structure and dynamics govern the generation and propagation of whistler mode waves prevalent in the inner magnetosphere, creating a strong MLT dependence in the pitch angle diffusion and resulting precipitation patterns during storm time. The cold plasma density not only influences the global distribution of the waves, but also plays a major role in mediating the resulting wave-particle interactions.

Additionally, we have shown that low values of the cold plasma density enable stronger scattering with multi-MeV electrons. Through scaling the density in the Sheeley et al. (2001) trough model, previous studies have established that very low values of the density accelerate electrons to ultra-relativistic energies (Allison et al., 2021). Using a physics-based model, we have demonstrated that lower density regions formed due to the plasmasphere erosion and efficiently accelerated electrons to multi-MeV energies. Furthermore, we have demonstrated that determining both the plasma trough density and plasma-pause location was important to regulating the presence of chorus waves, the energies of resonant electrons, and the magnitude of the scattering and energization. For that reason, consistently incorporating the storm-time evolution of the cold plasma density and chorus wave power is critical to capturing the dynamical effect of wave-particle interactions during all phases of geomagnetic storms.

We performed a case study of the enhancement of the outer radiation belt during the 17 March 2013 storm. RBSP PSD data was used as the initial condition to set the electron fluxes. To isolate the impact of the chorus waves, we simplified the simulation and neglected electron injections from the tail (Sorathia et al., 2018). We evaluated identical radiation belt distributions through two simulations, one included diffusion due to wave-particle interactions with lower band chorus waves and the other did not. We showed that:

- Radial transport through accurate storm-time electromagnetic fields alone did not produce a significant flux enhancement at multi-MeV energies.
- Resonance with parallel propagating lower band chorus waves increased the modeled radiation belt intensity of > 1 MeV electrons by more than an order of magnitude at all L -shells within geosynchronous orbit.
- Evolution of the storm-time magnetic field and cold plasma density strongly affected the resonant energies of electrons and the magnitude of the resulting wave-particle interactions with lower band chorus waves. The dynamic variation of the cold plasma density and magnetic field resulted in a strong energy and MLT dependence in pitch angle diffusion, atmospheric precipitation, and energy diffusion throughout the event.
- Wave-particle interactions with lower band chorus waves produced an anisotropic pitch angle distribution as the electrons were accelerated to multi-MeV energies.
- In the presence of waves, precipitation accounted for a third of the total loss of electrons from the outer belt.
- Precipitation consisted predominantly of 100s keV electrons, scattered into the loss cone on the dawnside where the chorus wave amplitudes were the largest.
- The mean energy of the precipitation increased to above 600 keV at low L -shells on the dayside due to a deep depletion of the cold plasma density caused by the erosion of the plasmasphere.

While the present simulation accounted for local wave-particle interactions only with lower band chorus waves, our approach for modeling wave-particle interactions is gen-

694 The technique can easily be extended to incorporate other wave modes that play
 695 an important role in scattering and energizing radiation belt electrons. Each additional
 696 wave mode can be modeled individually, or in concert with the others, to study their rel-
 697 ative contribution. In addition, the algorithm can be modified to account for the oblique
 698 propagation of certain wave modes.

699 Our test-particle approach enables us to distinguish how electrons are accelerated,
 700 transported, and lost from the system. Global magnetosphere and test particle simu-
 701 lations now enable us to directly quantify the relative role local wave-particle interactions,
 702 mesoscale particle injections, and large-scale radial transport via ULF resonance in gov-
 703 erning radiation belt dynamics. Due to the physically consistent modeling of the mag-
 704 netosphere and its impact on the radiation belts, we are now able to connect each mech-
 705 anism directly back to the magnetic field and plasma conditions through which they are
 706 evolved on the time scales of seconds to hours that drive variability during geomagnetic
 707 storms.

708 6 Open Research

709 All RBSP-ECT data (Spence et al., 2013; Boyd et al., 2019) are publicly available
 710 at the Web site <http://www.RBSP-ect.lanl.gov/>. The OMNI data set are available
 711 at <https://cdaweb.gsfc.nasa.gov/index.html> by selecting OMNI from the avail-
 712 able spacecraft and then the 1-minute resolution data. Simulation data used to create
 713 the figures are archived on Zenodo (Michael et al., 2024) and available online via <https://doi.org/10.5281/zenodo.10831861>. All figures were made using Matplotlib <https://matplotlib.org/>. The format of the files and their contents are described in a docu-
 714 ment available on Zenodo, including an example Python script.

717 Appendix A Specification of the Empirical Wave Model

718 The empirical wave model contains statistical properties of the root-mean-square
 719 wave amplitude of lower band chorus waves. The dataset is derived from Van Allen Probes
 720 A and B measurements between January 2013 and July 2019. As detailed in Shen et al.
 721 (2019), whistler activity was selected outside the plasmapause using the electron cyclotron
 722 harmonic (ECH) wave power. The wave amplitude is parameterized by L -shell, MLT,
 723 magnetic latitude ($|\lambda_{MLAT}|$), and SML*. We include only waves observed near the equa-
 724 tor with $|\lambda_{MLAT}| < 10^\circ$. The grid of the empirical wave model extends in L -shell from
 725 $L = 2$ to 6 with $0.2 L$ resolution. The model has 1 hour resolution in MLT and six ge-
 726 omagnetic activity bins. The geomagnetic activity bins are defined as $SML^* > -100$
 727 nT, -500 nT $< SML^* < -100$ nT, -750 nT $< SML^* < -500$ nT, -1000 nT $< SML^* <$
 728 -750 nT, $SML^* < -1000$ nT. L -shells were determined using the TS05 magnetic field
 729 model (Shen et al., 2019). Bins with sampling times less than three minutes, correspond-
 730 ing to 180 samples, were not included. This was done to ensure the statistical significance
 731 of the model. The wave spectrum is assumed to be Gaussian, with a peak at $0.3 f_{ce}$ and
 732 a lower and upper cutoff between $\omega_{LC} = 0.05 f_{ce}$ and $\omega_{UC} = 0.55 f_{ce}$, respectively.

733 Before ingesting the wave model into the test particle simulation, the wave ampli-
 734 tudes are interpolated onto a new grid with double the resolution in each dimension in
 735 L and MLT. Waves are routinely observed at L -shells larger than the apogee of the Van
 736 Allen Probes (Meredith et al., 2020). To avoid the large discontinuity in wave amplitude
 737 at geosynchronous orbit due to lack of data, we attenuate the wave amplitude in each MLT
 738 bin out to $L = 8$. This is done using a Gaussian profile that peaks at $L = 6$. The em-
 739 pirical model is smoothed with a 2D Gaussian filter to remove any artifacts. Wave am-
 740 plitudes were excluded outside of $L=8$ as the injected populations from the plasma sheet
 741 are not included in this study and magnetopause losses quickly remove the electrons at
 742 larger L -shells.

743 Higher order harmonics become dominant for ≥ 1 MeV electrons as they resonate
 744 with obliquely propagating waves (Shprits et al., 2006; Shprits & Ni, 2009). For a given
 745 energy above ~ 1 MeV and a fixed f_{pe}/f_{ce} ratio, resonance with oblique waves extends
 746 scattering and energization rates to lower equatorial pitch angle. This enables more MeV
 747 electrons to resonate at a single location and, consequently, causes a single electron to
 748 be in resonance at a larger range of magnetic latitudes. Broadening the magnetic lat-
 749 itudes of the wave region, while retaining the parallel propagating wave assumption, has
 750 a qualitatively similar affect. The increased magnetic field at higher magnetic latitudes
 751 shifts the resonating electrons to higher energies. In addition, the increased latitudinal
 752 range permits MeV particles with lower equatorial pitch angles to resonate near their
 753 bounce point.

As described in Section 2.4, we assume the chorus waves propagate parallel to the magnetic field. The Van Allen Probes is able to measure the wave environment between $|\lambda_{MLAT}| = 20^\circ$. As a proxy for resonance with obliquely propagating waves, we increase the latitudinal range of the waves to 45° . We define the latitudinal profile as a flat-top Gaussian distribution

$$f(\lambda_{MLAT}) = B_{W,eq} \exp \left[\left(\frac{\lambda_{MLAT}^2}{2\sigma} \right)^P \right] \quad (A1)$$

754 where $B_{W,eq}$ is the equatorial wave amplitude, $\sigma = 0.4$, and $P = 2.5$. The latitudinal
 755 profile in equation A1 was applied in all MLT sectors. A simple comparison of the bounce
 756 averaged diffusion coefficients calculated using this method to the full calculation with
 757 oblique waves (J. M. Albert, 2005, 2007, 2008; J. Albert, 2018) within the 20° is provided
 758 in the supplemental material. Both the wave amplitude profile in magnetic latitude and
 759 the distribution of the latitudinal extent of the waves with respect to MLT can be eas-
 760 ily adjusted to provide better agreement with observations or statistical distributions.

761 Appendix B Stochastic Modeling of Wave-Particle Interactions

Electrons resonate with whistler mode waves when the Doppler shifted wave frequency is a harmonic of the relativistic gyrofrequency. This is determined by the resonance condition:

$$\omega - kv_{\parallel} = \frac{n\Omega_e}{\gamma}, \quad n = 0, \pm 1, \pm 2, \dots \quad (B1)$$

where ω is the wave frequency, k is the wave number in the direction of propagation, v_{\parallel} is the parallel velocity of the particle, γ is the Lorentz factor, and n is an integer denoting the cyclotron harmonic. Ω_e is the nonrelativistic electron gyrofrequency defined by $|\Omega_e| = e|\mathbf{B}|/(m_e c)$, where \mathbf{B} is the magnetic field vector, m_e is the electron mass, and c is the speed of light. The resonant frequencies are obtained by simultaneously solving the resonance condition with the dispersion relation of the wave. For R mode whistler waves this is given by

$$\left(\frac{ck}{\omega} \right)^2 = 1 - \left(\frac{\omega_{pe}}{\Omega_e} \right)^2 \frac{1 + \epsilon}{(\omega/|\Omega_e| - 1)(\omega/|\Omega_e| + \epsilon)} \quad (B2)$$

762 where $\omega_{pe} = \sqrt{4\pi N_0 e^2/m_e}$ is the plasma frequency, N_0 is the plasma density, and e
 763 is the electron charge. Finally, $\epsilon = m_e/m_p$ is the electron to proton mass ratio.

764 Many cyclotron harmonics contribute to electron resonance and scattering when
 765 waves propagate obliquely to the magnetic field (J. M. Albert & Young, 2005). Numer-
 766 ical codes that calculate diffusion rates for oblique waves often include resonance with
 767 ± 5 cyclotron harmonics (Horne et al., 2003; J. M. Albert & Young, 2005). While these
 768 codes are efficient, computation of diffusion rates including an angular spread in wave
 769 normal angle can still be computationally expensive. This is especially true when try-
 770 ing to integrate the calculation within a test particle approach. Test particles in our sim-

771 ulation undergo on the order of $\sim 10^8$ resonant interactions over the course of the storm
 772 main phase. Several million test particles are also needed to fully resolve phase space.

773 Lower band chorus waves tend to be quasi field-aligned, with their wave normal
 774 distribution peaking at 0° and are often approximated using an angular distribution of
 775 30° (Horne et al., 2003). In addition, for 100 keV to a few MeV electrons, the bounce-
 776 averaged diffusion coefficients are dominated by the resonant harmonic associated with
 777 parallel propagating waves (Shprits et al., 2006). To reduce the complexity of the com-
 778 putation, we assume waves propagate parallel to the magnetic field, as electrons, there-
 779 fore, resonate with the first-order harmonic only (Summers et al., 1998). This enables
 780 us to quickly and efficiently determine the resonant wave frequency for each resonant wave-
 781 particle interaction. To better capture the scattering of \geq a few MeV electrons, we mimic
 782 resonance with obliquely propagating chorus waves, as described in Appendix A.

783 As discussed in Summers (2005), solution of equations (B1) and (B2) for parallel
 784 propagating waves yields up to potentially three resonant roots. We further assume that
 785 lower band chorus waves only propagate away from the equator. This assumption results
 786 in a single resonant frequency for each wave-particle interaction.

We solve the Fokker-Plank diffusion equation using a time-forward stochastic differential equation (SDE) to compute pitch angle scattering and momentum diffusion from resonant wave-particle interactions. Derived by Tao et al. (2008), the SDE is formulated using the Itô method and is defined as:

$$\begin{aligned}\Delta\alpha &= a_\alpha\Delta t + b_{\alpha\alpha}\sqrt{2\Delta t}\eta_\alpha \\ \Delta p &= a_p\Delta t + b_{p\alpha}\sqrt{2\Delta t}\eta_\alpha + b_{pp}\sqrt{2\Delta t}\eta_p\end{aligned}\quad (\text{B3})$$

where

$$\begin{aligned}a_\alpha &= \frac{1}{Gp}\partial_\alpha\left(\frac{GD_{\alpha\alpha}}{p}\right) + \frac{1}{G}\partial_p\left(\frac{GD_{\alpha p}}{p}\right) \\ a_p &= \frac{1}{Gp}\partial_\alpha(GD_{\alpha p}) + \frac{1}{G}\partial_p(GD_{pp})\end{aligned}\quad (\text{B4})$$

and $G = p^2 \sin \alpha$. The second term, and third for Δp , on the right-hand side of equations (B4) describe a Wiener process. η_α, η_p are separate random numbers generated from a normal distribution with a mean of zero and a variance of 1 and Δt represents the time the particle is in resonance with the wave. For real $D_{\alpha p} = D_{p\alpha}$, the diffusion matrix, \mathbf{b} , is overdetermined. Following Tao et al. (2008), we set $b_{\alpha p} = 0$. The diffusion matrix reduces to:

$$\begin{aligned}b_{\alpha\alpha} &= \sqrt{D_{\alpha\alpha}/p}, & b_{\alpha p} &= 0 \\ b_{p\alpha} &= D_{p\alpha}/\sqrt{D_{\alpha\alpha}}, & b_{pp} &= \sqrt{D_{pp} - D_{p\alpha}^2/D_{\alpha\alpha}}.\end{aligned}\quad (\text{B5})$$

The local quasi-linear diffusion rates used here are defined such that $D_{\alpha\alpha}/p^2, D_{\alpha p}/p^2$, and D_{pp}/p^2 are in units of s^{-1} . This corresponds to the notation of Lyons (1974a, 1974b). Summers (2005) derived analytical expressions for the local $D_{\alpha\alpha}, D_{\alpha p}$ and D_{pp} . The derivation uses the exact cold plasma dispersion relationship and the resonance condition for field-aligned electromagnetic waves in a hydrogen plasma. Following Summers (2005), $D_{\alpha\alpha}, D_{\alpha p}$ and D_{pp} are defined as:

$$\begin{aligned}\frac{D_{\alpha\alpha}}{p^2} &= \frac{\pi}{4}\frac{\Omega_e^2}{W_0}\frac{1}{\gamma^2}\left(1 - \frac{\omega_j \cos \alpha}{k_j v}\right)^2 \frac{W(k_j)}{|v \cos \alpha - d\omega_j/dk_j|} \\ \frac{D_{\alpha p}}{p^2} &= -\frac{\pi}{4}\frac{\Omega_e^2}{W_0}\frac{\sin \alpha}{\beta \gamma^2}\left(1 - \frac{\omega_j \cos \alpha}{k_j v}\right)\left(\frac{\omega_j}{k_j}\right) \frac{W(k_j)}{|v \cos \alpha - d\omega_j/dk_j|} \\ \frac{D_{pp}}{p^2} &= -\frac{\pi}{4}\frac{\Omega_e^2}{W_0}\frac{\sin^2 \alpha}{\beta^2 \gamma^2}\left(\frac{\omega_j}{k_j}\right)^2 \frac{W(k_j)}{|v \cos \alpha - d\omega_j/dk_j|}\end{aligned}\quad (\text{B6})$$

787 where α is the pitch angle, $W_0 = B_0^2/8\pi$ is the magnetic energy density of the back-
 788 ground magnetic, ω_j and k_j are the resonant frequency and wave number that satisfy
 789 equation (B1) and (B2), and $\beta = v/c$, $d\omega_j/dk_j$ is obtained from equation (B1), and $W(k_j)$
 790 is the wave spectral density. The diffusion coefficients differ from Summers (2005) by a
 791 factor of 2, as noted in J. M. Albert (2007). In addition to being faster to evaluate, the
 792 closed form expressions enable the model to solve for diffusion coefficients for each res-
 793 onant interaction. The background magnetic field, density, and wave amplitude at each
 794 electron's location are taken directly from either the global magnetosphere or empirical
 795 wave model. This removes the need to pre-compute or bounce-average the diffusion co-
 796 efficients based on empirical density or magnetic field reconstructions. The analytical
 797 expressions also permits the local diffusion coefficients to vary throughout the storm as
 798 a function of time and 3D location. The advantage to this approach is the temporal vari-
 799 ations of $D_{\alpha\alpha}$, $D_{\alpha p}$ and D_{pp} are physically consistent with the plasma conditions through
 800 which the test particle trajectories are being integrated.

801 The SDE is evaluated concurrently with the test particle integrator. The average
 802 state of the test particle over the time step is used in the wave-particle interaction to re-
 803 duce any error introduced in operator splitting. The particle only experiences a resonant
 804 interaction if lower band chorus waves are present at the particle's location and if the
 805 resonant frequency resides between the lower and upper cutoff set for the spectrum. The
 806 location of the last equatorial crossing is used to determine the particle's L -shell. This
 807 assumes that the bounce timescale of the particle is much less than the drift motion and
 808 avoids the need to perform field line tracing for each individual interaction. To deter-
 809 mine the corresponding wave amplitude from the empirical wave model, we compute dL_{pp}
 810 of the particle using the plasmapause location in the global magnetosphere model. The
 811 plasmapause location is calculated in the equatorial plane as a function of MLT, discretized
 812 into 5° bins. For each MLT bin, we define the plasmapause location where the plasma
 813 density first decreases to 100 cm^{-3} , as in Ripoll et al. (2022).

814 If present, the wave amplitude in the empirical wave model at the particle's dL_{pp}
 815 and MLT is retrieved. The wave amplitude is scaled according to the particle's magnetic
 816 latitude using equation A1. The magnetic field and density at the particle's location is
 817 obtained from the MAGE model. ω_j and k_j are calculated from the resonance condition
 818 and dispersion relation using the energy and pitch angle of the particle along with the
 819 background plasma conditions. $D_{\alpha\alpha}$, $D_{\alpha p}$, D_{pp} and their derivatives, are then computed
 820 and used within the SDE to determine the pitch angle scattering, $\Delta\alpha$, and momentum
 821 diffusion, Δp , of the particle. $\Delta\alpha$ and Δp are used to update the electron's momentum
 822 and energy according to $p_{\text{new}} = p_{\text{initial}} + \Delta p$, $\alpha_{\text{new}} = \alpha_{\text{initial}} + \Delta\alpha$, respectively. Δt in
 823 equation (B4) is initially set to the time step of the particle. To reduce error in the cal-
 824 culation, each resonant interaction is limited to not exceed $\Delta\alpha = 0.5^\circ$ or $\Delta p = 0.05p_{\text{initial}}$.
 825 Δt is updated to ensure this criteria is met. The SDE is advances iteratively until the
 826 cumulative resonance time matches the original time step. The SDE is advanced every
 827 time step that the particle remains in resonance with waves, updating the particle en-
 828 ergy, pitch angle, wave amplitude, and background magnetic field and density accord-
 829 ingly.

830 Appendix C Calculation of Bounce-Averaged Diffusion Coefficients

Bounce averaging is performed following a similar approach to Lyons et al. (1972);
 Shprits et al. (2006); Summers et al. (2007a). The local diffusion coefficients, $D_{\alpha\alpha}$ and
 $D_{\alpha p}$ are multiplied by $\left(\frac{\partial\alpha_{eq}}{\partial\alpha}\right)^2$, $\frac{\partial\alpha_{eq}}{\partial\alpha}$, respectively, to convert them to the equivalent equa-
 torial coefficients, where

$$\frac{\partial\alpha_{eq}}{\partial\alpha} = \frac{\tan\alpha_{eq}}{\tan\alpha}. \quad (\text{C1})$$

The bounce-averaged values of the diffusion coefficients (B6) therefore become

$$\begin{aligned}\langle D_{\alpha\alpha} \rangle &= \frac{1}{\tau_b} \int_0^{\tau_b} D_{\alpha\alpha}(K, \alpha, B, \rho, B_w) \left(\frac{\partial \alpha_{eq}}{\partial \alpha} \right)^2 dt \\ \langle D_{\alpha p} \rangle &= \frac{1}{\tau_b} \int_0^{\tau_b} D_{\alpha p}(K, \alpha, B, \rho, B_w) \left(\frac{\partial \alpha_{eq}}{\partial \alpha} \right) dt \\ \langle D_{pp} \rangle &= \frac{1}{\tau_b} \int_0^{\tau_b} D_{pp}(K, \alpha, B, \rho, B_w) dt\end{aligned}\quad (C2)$$

where τ_B is the bounce-period of the particle and is defined as

$$\tau_b = 2 \int_{-S_{m1}}^{S_{m2}} \frac{ds}{v_{\parallel}} = \frac{2}{v} \int_{-S_{m1}}^{S_{m2}} \frac{1}{\sqrt{1 - \frac{B}{B_{eq}} \sin^2 \alpha_{eq}}} ds \quad (C3)$$

831 This converts the integral of the bounce period along the magnetic field line from one
 832 mirror point in the southern hemisphere, $-S_{m1}$, to the other mirror point in the north-
 833 ern hemisphere, S_{m2}

Similarly, we recast the integrals of the bounce averaged diffusion coefficients to be along the magnetic field line, as done in equation C3. We do not assume that the magnetic field is a dipole, rather we perform field line tracing in the MAGE model to extract the storm-time magnetic fields and density profile. Combining equations C1-C3, we obtain the new expressions

$$\begin{aligned}\langle D_{\alpha\alpha} \rangle &= \frac{1}{T(\alpha_{eq})} \int_{-S_{m1}}^{S_{m2}} D_{\alpha\alpha} \frac{\tan^2 \alpha_{eq} \cos \alpha(s)}{\sin^2 \alpha(s)} ds \\ \langle D_{\alpha p} \rangle &= \frac{1}{T(\alpha_{eq})} \int_{-S_{m1}}^{S_{m2}} D_{\alpha p} \frac{\tan \alpha_{eq}}{\sin \alpha(s)} ds \\ \langle D_{pp} \rangle &= \frac{1}{T(\alpha_{eq})} \int_{-S_{m1}}^{S_{m2}} D_{pp} \frac{1}{\cos \alpha(s)} ds\end{aligned}\quad (C4)$$

834 where $\alpha(s)$ is a function of α_{eq} and the magnetic field strength at a particular location
 835 along the field line, $B(s)$. $T(\alpha_{eq})$ is defined as $T(\alpha_{eq}) = \int_{-S_{m1}}^{S_{m2}} \frac{1}{\sqrt{1 - \frac{B}{B_{eq}} \sin^2 \alpha_{eq}}} ds$. To
 836 perform calculation of the bounce averaged diffusion coefficients for a given energy and
 837 α_{eq} , the resonant frequency and resulting local diffusion coefficient must be determined
 838 at each location along the discretized field line.

Acknowledgments

840 This work was supported by the NASA Early Career Investigator Program grant 80NSSC21K0464,
 841 Living With a Star grant 80NSSC17K0679, NASA O2R grant 80NSSC19K0241, the Van
 842 Allen Probes mission NNN06AA01C, as well as by the NASA DRIVE Science Center
 843 for Geospace Storms (CGS) under award 80NSSC20K0601. KS was also supported by
 844 80NSSC20K1833. WL and XS acknowledge the NASA grants 80NSSC20K0698 and 80NSSC24K0239,
 845 and NSF grant AGS-1847818. We would like to acknowledge high-performance comput-
 846 ing support from Cheyenne (doi:10.5065/D6RX99HX) provided by NCAR's Compu-
 847 tational and Information Systems Laboratory, sponsored by the National Science Foun-
 848 dation.

849 **References**

- 850 Agapitov, O. V., Mourenas, D., Artemyev, A. V., Mozer, F. S., Hospodarsky, G.,
 851 Bonnell, J., & Krasnoselskikh, V. (2018). Synthetic empirical chorus wave
 852 model from combined van allen probes and cluster statistics. *Journal of Geo-*
 853 *physical Research: Space Physics*, 123(1), 297-314. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2017JA024843> doi:
 854 <https://doi.org/10.1002/2017JA024843>
- 855 Albert, J. (2018). Diagonalization of diffusion equations in two and three dimen-
 856 sions. *Journal of Atmospheric and Solar-Terrestrial Physics*, 177, 202-207.
 857 Retrieved from <https://www.sciencedirect.com/science/article/pii/S1364682617302663> (Dynamics of the Sun-Earth System: Recent Observa-
 858 tions and Predictions) doi: <https://doi.org/10.1016/j.jastp.2017.08.008>
- 859 Albert, J. M. (2005). Evaluation of quasi-linear diffusion coefficients for whistler
 860 mode waves in a plasma with arbitrary density ratio. *Journal of Geo-*
 861 *physical Research: Space Physics*, 110(A3). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2004JA010844> doi:
 862 <https://doi.org/10.1029/2004JA010844>
- 863 Albert, J. M. (2007). Simple approximations of quasi-linear diffusion coefficients.
 864 *Journal of Geophysical Research: Space Physics*, 112(A12). Retrieved
 865 from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2007JA012551> doi: <https://doi.org/10.1029/2007JA012551>
- 866 Albert, J. M. (2008). Efficient approximations of quasi-linear diffusion coefficients in
 867 the radiation belts. *Journal of Geophysical Research: Space Physics*, 113(A6).
 868 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2007JA012936> doi: <https://doi.org/10.1029/2007JA012936>
- 869 Albert, J. M., & Young, S. L. (2005). Multidimensional quasi-linear diffusion of
 870 radiation belt electrons. *Geophysical Research Letters*, 32(14). Retrieved
 871 from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2005GL023191> doi: <https://doi.org/10.1029/2005GL023191>
- 872 Allison, H. J., Shprits, Y. Y., Zhelavskaya, I. S., Wang, D., & Smirnov, A. G. (2021).
 873 Gyroresonant wave-particle interactions with chorus waves during extreme
 874 depletions of plasma density in the van allen radiation belts. *Science Ad-*
 875 *vances*, 7(5), eabc0380. Retrieved from <https://www.science.org/doi/abs/10.1126/sciadv.abc0380> doi: 10.1126/sciadv.abc0380
- 876 Angelopoulos, V., Kennel, C. F., Coroniti, F. V., Pellat, R., Kivelson, M. G.,
 877 Walker, R. J., ... Gosling, J. T. (1994). Statistical characteristics of bursty
 878 bulk flow events. *Journal of Geophysical Research: Space Physics*, 99(A11),
 879 21257-21280. doi: 10.1029/94ja01263
- 880 Baker, D. N., Jaynes, A. N., Li, X., Henderson, M. G., Kanekal, S. G., Reeves,
 881 G. D., ... Shprits, Y. Y. (2014, March). Gradual diffusion and punctuated
 882 phase space density enhancements of highly relativistic electrons: Van Allen
 883 Probes observations. *Geophysical Research Letters*, 41(5), 1351-1358. doi:
 884 [10.1002/2013GL058942](https://doi.org/10.1002/2013GL058942)
- 885 Baumjohann, W., Paschmann, G., & Lühr, H. (1990). Characteristics of high-speed
 886 ion flows in the plasma sheet. *Journal of Geophysical Research: Space Physics*
 887 (1978-2012), 95(A4), 3801-3809. doi: 10.1029/ja095ia04p03801
- 888 Blake, J. B., Carranza, P. A., Claudepierre, S. G., Clemmons, J. H., Crain, W. R.,
 889 Dotan, Y., ... Zakrzewski, M. P. (2013, November). The Magnetic Electron
 890 Ion Spectrometer (MagEIS) Instruments Aboard the Radiation Belt Storm
 891 Probes (RBSP) Spacecraft. *Space Science Reviews*, 179(1-4), 383-421. doi:
 892 [10.1007/s11214-013-9991-8](https://doi.org/10.1007/s11214-013-9991-8)
- 893 Borovsky, J. E., & Denton, M. H. (2006). Effect of plasmaspheric drainage plumes
 894 on solar-wind/magnetosphere coupling. *Geophysical Research Letters*, 33(20).
 895 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2006GL026519> doi: <https://doi.org/10.1029/2006GL026519>

- 904 Boyd, A. J., Reeves, G. D., Spence, H. E., Funsten, H. O., Larsen, B. A., Skoug,
 905 R. M., ... Jaynes, A. N. (2019). Rbsp-ect combined spin-averaged elec-
 906 tron flux data product. *Journal of Geophysical Research: Space Physics*,
 907 124(11), 9124-9136. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JA026733> doi: <https://doi.org/10.1029/2019JA026733>
- 910 Chan, A. A., Elkington, S. R., Longley, W. J., Aldhurais, S. A., Alam, S. S., Al-
 911 bert, J. M., ... Li, W. (2023). Simulation of radiation belt wave-particle
 912 interactions in an mhd-particle framework. *Frontiers in Astronomy and Space
 913 Sciences*, 10. Retrieved from <https://www.frontiersin.org/articles/10.3389/fspas.2023.1239160> doi: 10.3389/fspas.2023.1239160
- 914 Claudepierre, S. G., Toffoletto, F. R., & Wiltberger, M. (2016). Global mhd
 915 modeling of resonant ulf waves: Simulations with and without a plasmas-
 916 phere. *Journal of Geophysical Research: Space Physics*, 121(1), 227–244. doi:
 917 [10.1002/2015ja022048](https://doi.org/10.1002/2015ja022048)
- 918 Darrouzet, F., Gallagher, D. L., André, N., Carpenter, D. L., Dandouras, I.,
 919 Décréau, P. M. E., ... et al. (2009). Plasmaspheric density structures and
 920 dynamics: Properties observed by the cluster and image missions. *Space Sci-
 921 ence Reviews*, 145(1–2), 55–106. doi: [10.1007/s11214-008-9438-9](https://doi.org/10.1007/s11214-008-9438-9)
- 922 Delzanno, G. L., Borovsky, J. E., Henderson, M. G., Resendiz Lira, P. A., Royter-
 923 shteyn, V., & Welling, D. T. (2021). The impact of cold electrons and cold
 924 ions in magnetospheric physics. *Journal of Atmospheric and Solar-Terrestrial
 925 Physics*, 220, 105599. Retrieved from <https://www.sciencedirect.com/science/article/pii/S1364682621000596> doi: <https://doi.org/10.1016/j.jastp.2021.105599>
- 926 Denton, R. E., Wang, Y., Webb, P. A., Tengdin, P. M., Goldstein, J., Redfern,
 927 J. A., & Reinisch, B. W. (2012, March). Magnetospheric electron density long-
 928 term (\gtrsim 1 day) refilling rates inferred from passive radio emissions measured
 929 by IMAGE RPI during geomagnetically quiet times. *Journal of Geophysical
 930 Research (Space Physics)*, 117(A3), A03221. doi: [10.1029/2011JA017274](https://doi.org/10.1029/2011JA017274)
- 931 Foster, J. C., & Burke, W. J. (2002). Saps: A new categorization for sub-auroral
 932 electric fields. *Eos, Transactions American Geophysical Union*, 83(36), 393-
 933 394. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2002EO000289> doi: <https://doi.org/10.1029/2002EO000289>
- 934 Foster, J. C., Wygant, J. R., Hudson, M. K., Boyd, A. J., Baker, D. N., Erick-
 935 son, P. J., & Spence, H. E. (2015). Shock-induced prompt relativistic
 936 electron acceleration in the inner magnetosphere. *Journal of Geophysi-
 937 cal Research: Space Physics*, 120(3), 1661-1674. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2014JA020642> doi:
 938 <https://doi.org/10.1002/2014JA020642>
- 939 Gabrielse, C., Angelopoulos, V., Harris, C., Artemyev, A., Kepko, L., & Runov, A.
 940 (2017). Extensive electron transport and energization via multiple, localized
 941 dipolarizing flux bundles. *Journal of Geophysical Research: Space Physics*,
 942 122(5), 5059-5076. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2017JA023981> doi: [10.1002/2017JA023981](https://doi.org/10.1002/2017JA023981)
- 943 Gallagher, D. L., Craven, P. D., & Comfort, R. H. (2000). Global core plasma
 944 model. *Journal of Geophysical Research: Space Physics*, 105(A8), 18819-
 945 18833. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/1999JA000241> doi: [10.1029/1999JA000241](https://doi.org/10.1029/1999JA000241)
- 946 Gjerloev, J. W. (2012). The SuperMAG data processing technique. *Journal
 947 of Geophysical Research: Space Physics*, 117(A9). Retrieved 2022-08-30,
 948 from <https://onlinelibrary.wiley.com/doi/abs/10.1029/2012JA017683> (eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1029/2012JA017683>) doi:
 949 [10.1029/2012JA017683](https://doi.org/10.1029/2012JA017683)
- 950 Goldstein, J., & Sandel, B. R. (2005). The global pattern of evolution of plasmas-
 951

- 959 pheic drainage plumes. In *Inner magnetosphere interactions: New perspectives*
 960 *from imaging* (p. 1-22). American Geophysical Union (AGU). Retrieved from
 961 <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/159GM02>
 962 doi: <https://doi.org/10.1029/159GM02>

963 Grebowsky, J. M. (1970). Model study of plasmapause motion. *Journal of Geo-*
 964 *physical Research (1896-1977)*, 75(22), 4329-4333. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JA075i022p04329>
 965 doi: <https://doi.org/10.1029/JA075i022p04329>

966 Horne, R. B., Glauert, S. A., & Thorne, R. M. (2003). Resonant diffusion of radi-
 967 ation belt electrons by whistler-mode chorus. *Geophysical Research Letters*,
 968 30(9). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2003GL016963>
 969 doi: <https://doi.org/10.1029/2003GL016963>

970 Hudson, M. K., Paral, J., Kress, B. T., Wiltberger, M., Baker, D. N., Foster,
 971 J. C., ... Wygant, J. R. (2015). Modeling cme-shock-driven storms in
 972 2012-2013: Mhd test particle simulations. *Journal of Geophysical Re-*
 973 *search: Space Physics*, 120(2), 1168-1181. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2014JA020833>
 974 doi: <https://doi.org/10.1002/2014JA020833>

975 Jaynes, A. N., Baker, D. N., Singer, H. J., Rodriguez, J. V., Loto'aniu, T. M., Ali,
 976 A. F., ... Reeves, G. D. (2015). Source and seed populations for relativis-
 977 tic electrons: Their roles in radiation belt changes. *Journal of Geophys-
 978 ical Research: Space Physics*, 120(9), 7240-7254. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2015JA021234>
 979 doi: <https://doi.org/10.1002/2015JA021234>

980 Kennel, C. F., & Petschek, H. E. (1966). Limit on stably trapped particle fluxes.
 981 *Journal of Geophysical Research*, 71(1), 1-28. doi: 10.1029/jz071i001p00001

982 Kim, H.-J., Kim, K.-C., Noh, S.-J., Lyons, L., Lee, D.-Y., & Choe, W. (2023).
 983 New perspective on phase space density analysis for outer radiation belt
 984 enhancements: The influence of mev electron injections. *Geophysical
 985 Research Letters*, 50(14), e2023GL104614. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2023GL104614>
 986 (e2023GL104614 2023GL104614) doi: <https://doi.org/10.1029/2023GL104614>

987 Kress, B. T., Hudson, M. K., Ukhorskiy, A. Y., & Mueller, H.-R. (2012). Non-
 988 linear radial transport in the earth's radiation belts. In *Dynamics of the
 989 earth's radiation belts and inner magnetosphere* (p. 151-160). American Geo-
 990 physical Union (AGU). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2012GM001333>
 991 doi: <https://doi.org/10.1029/2012GM001333>

992 Li, W., Bortnik, J., Thorne, R. M., & Angelopoulos, V. (2011). Global distribu-
 993 tion of wave amplitudes and wave normal angles of chorus waves using themis
 994 wave observations. *Journal of Geophysical Research: Space Physics*, 116(A12).
 995 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2011JA017035>
 996 doi: <https://doi.org/10.1029/2011JA017035>

1000 Li, W., & Hudson, M. (2019). Earth's Van Allen Radiation Belts: From Discovery
 1001 to the Van Allen Probes Era. *Journal of Geophysical Research: Space Physics*,
 1002 124(11), 8319-8351. doi: 10.1029/2018ja025940

1003 Li, W., Ma, Q., Thorne, R. M., Bortnik, J., Kletzing, C. A., Kurth, W. S., ...
 1004 Nishimura, Y. (2015). Statistical properties of plasmaspheric hiss derived
 1005 from van allen probes data and their effects on radiation belt electron dy-
 1006 namics. *Journal of Geophysical Research: Space Physics*, 120(5), 3393-3405.
 1007 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2015JA021048>
 1008 doi: <https://doi.org/10.1002/2015JA021048>

1009 Li, W., Santolik, O., Bortnik, J., Thorne, R. M., Kletzing, C. A., Kurth, W. S.,
 1010 & Hospodarsky, G. B. (2016). New chorus wave properties near the equa-
 1011 tor from van allen probes wave observations. *Geophysical Research Let-*

- ters, 43(10), 4725-4735. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2016GL068780> doi: <https://doi.org/10.1002/2016GL068780>
- Li, W., Thorne, R. M., Ma, Q., Ni, B., Bortnik, J., Baker, D. N., ... Claude-pierre, S. G. (2014). Radiation belt electron acceleration by chorus waves during the 17 march 2013 storm. *Journal of Geophysical Research: Space Physics*, 119(6), 4681-4693. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2014JA019945> doi: <https://doi.org/10.1002/2014JA019945>
- Li, Z., Hudson, M., Kress, B., & Paral, J. (2015). Three-dimensional test particle simulation of the 17–18 march 2013 cme shock-driven storm. *Geophysical Research Letters*, 42(14), 5679-5685. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2015GL064627> doi: <https://doi.org/10.1002/2015GL064627>
- Lin, D., Sorathia, K., Wang, W., Merkin, V., Bao, S., Pham, K., ... et al. (2021). The role of diffuse electron precipitation in the formation of sub-auroral polarization streams. *Earth and Space Science Open Archive*, 20. Retrieved from <https://doi.org/10.1002/essoar.10508315.1> doi: [10.1002/essoar.10508315.1](https://doi.org/10.1002/essoar.10508315.1)
- Longley, W. J., Chan, A. A., Jaynes, A. N., Elkington, S. R., Pettit, J. M., Ross, J. P. J., ... Horne, R. B. (2022). Using meped observations to infer plasma density and chorus intensity in the radiation belts. *Frontiers in Astronomy and Space Sciences*, 9. Retrieved from <https://www.frontiersin.org/articles/10.3389/fspas.2022.1063329> doi: [10.3389/fspas.2022.1063329](https://doi.org/10.3389/fspas.2022.1063329)
- Lyons, L. R. (1974a). General relations for resonant particle diffusion in pitch angle and energy. *Journal of Plasma Physics*, 12(1), 45–49. doi: [10.1017/S0022377800024910](https://doi.org/10.1017/S0022377800024910)
- Lyons, L. R. (1974b). Pitch angle and energy diffusion coefficients from resonant interactions with ion–cyclotron and whistler waves. *Journal of Plasma Physics*, 12(3), 417–432. doi: [10.1017/S002237780002537X](https://doi.org/10.1017/S002237780002537X)
- Lyons, L. R., Thorne, R. M., & Kennel, C. F. (1972, January). Pitch-angle diffusion of radiation belt electrons within the plasmasphere. *Journal of Geophysical Research*, 77(19), 3455. doi: [10.1029/JA077i019p03455](https://doi.org/10.1029/JA077i019p03455)
- Ma, Q., Li, W., Bortnik, J., Thorne, R. M., Chu, X., Ozeke, L. G., ... Claude-pierre, S. G. (2018). Quantitative evaluation of radial diffusion and local acceleration processes during gem challenge events. *Journal of Geophysical Research: Space Physics*, 123(3), 1938-1952. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2017JA025114> doi: <https://doi.org/10.1002/2017JA025114>
- Malaspina, D. M., Jaynes, A. N., Boulé, C., Bortnik, J., Thaller, S. A., Ergun, R. E., ... Wygant, J. R. (2016). The distribution of plasmaspheric hiss wave power with respect to plasmapause location. *Geophysical Research Letters*, 43(15), 7878–7886. doi: [10.1002/2016gl069982](https://doi.org/10.1002/2016gl069982)
- Meredith, N. P., Cain, M., Horne, R. B., Thorne, R. M., Summers, D., & Anderson, R. R. (2003). Evidence for chorus-driven electron acceleration to relativistic energies from a survey of geomagnetically disturbed periods. *Journal of Geophysical Research: Space Physics*, 108(A6). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2002JA009764> doi: <https://doi.org/10.1029/2002JA009764>
- Meredith, N. P., Horne, R. B., Shen, X.-C., Li, W., & Bortnik, J. (2020). Global model of whistler mode chorus in the near-equatorial region ($|\lambda_m| < 18^\circ$). *Geophysical Research Letters*, 47(11), e2020GL087311. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2020GL087311> (e2020GL087311 10.1029/2020GL087311) doi: <https://doi.org/10.1029/2020GL087311>

- 1069 Meredith, N. P., Horne, R. B., Sicard-Piet, A., Boscher, D., Yearby, K. H., Li,
 1070 W., & Thorne, R. M. (2012). Global model of lower band and up-
 1071 per band chorus from multiple satellite observations. *Journal of Geo-
 1072 physical Research: Space Physics*, 117(A10). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2012JA017978> doi:
 1073 <https://doi.org/10.1029/2012JA017978>
- 1075 Merkin, V. G., & Lyon, J. G. (2010, Oct). Effects of the low-latitude ionospheric
 1076 boundary condition on the global magnetosphere. *Journal of Geophysical Re-
 1077 search (Space Physics)*, 115(A10), A10202. doi: 10.1029/2010JA015461
- 1078 Michael, A. T., Sorathia, K. A., Ukhorskiy, A. Y., Albert, J., Shen, X., Li, W., &
 1079 Merkin, V. G. (2024). Cross-scale modeling of storm-time radiation belt vari-
 1080 ability. *Zenodo*. Retrieved from <https://doi.org/10.5281/zenodo.10831861>
 1081 doi: 10.5281/zenodo.10831861
- 1082 Millan, R., & Thorne, R. (2007). Review of radiation belt relativistic electron
 1083 losses. *Journal of Atmospheric and Solar-Terrestrial Physics*, 69(3), 362-377.
 1084 Retrieved from <https://www.sciencedirect.com/science/article/pii/S1364682606002768> (Global Aspects of Magnetosphere-Ionosphere Coupling)
 1085 doi: <https://doi.org/10.1016/j.jastp.2006.06.019>
- 1086 Newell, P. T., & Gjerloev, J. W. (2012). SuperMAG-based par-
 1087 tial ring current indices. *Journal of Geophysical Research: Space
 1088 Physics*, 117(A5). Retrieved 2022-08-30, from <https://onlinelibrary.wiley.com/doi/abs/10.1029/2012JA017586> (*eprint*:
 1089 <https://onlinelibrary.wiley.com/doi/pdf/10.1029/2012JA017586>) doi:
 1090 <https://doi.org/10.1029/2012JA017586>
- 1091 Newell, P. T., & Gjerloev, J. W. (2014). Local geomagnetic indices and the
 1092 prediction of auroral power. *Journal of Geophysical Research: Space
 1093 Physics*, 119(12), 9790–9803. Retrieved 2022-08-30, from <https://onlinelibrary.wiley.com/doi/abs/10.1002/2014JA020524> (*eprint*:
 1094 <https://onlinelibrary.wiley.com/doi/pdf/10.1002/2014JA020524>) doi:
 1095 <https://doi.org/10.1002/2014JA020524>
- 1096 Pembroke, A., Toffoletto, F., Sazykin, S., Wiltberger, M., Lyon, J., Merkin, V., &
 1097 Schmitt, P. (2012). Initial results from a dynamic coupled magnetosphere-
 1098 ionosphere-ring current model. *Journal of Geophysical Research: Space
 1099 Physics*, 117(A2). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2011JA016979> doi: 10.1029/2011JA016979
- 1100 Pham, K. H., Zhang, B., Sorathia, K., Dang, T., Wang, W., Merkin, V., ... Lyon,
 1101 J. (2022, February). Thermospheric Density Perturbations Produced by
 1102 Traveling Atmospheric Disturbances During August 2005 Storm. *Jour-
 1103 nal of Geophysical Research (Space Physics)*, 127(2), e2021JA030071. doi:
 1104 <https://doi.org/10.1029/2021JA030071>
- 1105 Reeves, G. D., McAdams, K. L., Friedel, R. H. W., & O'Brien, T. P. (2003).
 1106 Acceleration and loss of relativistic electrons during geomagnetic storms.
 1107 *Geophysical Research Letters*, 30(10). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2002GL016513> doi:
 1108 <https://doi.org/10.1029/2002GL016513>
- 1109 Ripoll, J.-F., Claudepierre, S. G., Ukhorskiy, A. Y., Colpitts, C., Li, X., Fennell,
 1110 J. F., & Crabtree, C. (2020). Particle dynamics in the earth's radiation belts:
 1111 Review of current research and open questions. *Journal of Geophysical Re-
 1112 search: Space Physics*, 125(5), e2019JA026735. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JA026735>
 1113 (e2019JA026735 2019JA026735) doi: <https://doi.org/10.1029/2019JA026735>
- 1114 Ripoll, J.-F., Thaller, S. A., Hartley, D. P., Cunningham, G. S., Pierrard, V., Kurth,
 1115 W. S., ... Wygant, J. R. (2022). Statistics and empirical models of the plas-
 1116 masphere boundaries from the van allen probes for radiation belt physics. *Geo-
 1117 physical Research Letters*, 49(21), e2022GL101402. Retrieved from <https://doi.org/10.1029/2022GL101402>

- 1124 agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2022GL101402
1125 (e2022GL101402 2022GL101402) doi: <https://doi.org/10.1029/2022GL101402>

1126 Schulz, M. (1974). Particle lifetimes in strong diffusion. *Astrophysics and Space Sci-
1127 ence*, 31(1), 37–42. doi: 10.1007/bf00642599

1128 Sciola, A., Merkin, V. G., Sorathia, K., Gkioulidou, M., Bao, S., Toffoletto, F., ...
1129 Ukhorskiy, A. (2023). The contribution of plasma sheet bubbles to storm-
1130 time ring current buildup and evolution of its energy composition. *Journal of
1131 Geophysical Research: Space Physics*, 128(11), e2023JA031693. Retrieved
1132 from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2023JA031693> (e2023JA031693 2023JA031693) doi: <https://doi.org/10.1029/2023JA031693>

1133 Sheeley, B. W., Moldwin, M. B., Rassoul, H. K., & Anderson, R. R. (2001). An
1134 empirical plasmasphere and trough density model: Crres observations. *Journal
1135 of Geophysical Research: Space Physics*, 106(A11), 25631-25641. Retrieved
1136 from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2000JA000286> doi: <https://doi.org/10.1029/2000JA000286>

1137 Shen, X.-C., Li, W., Ma, Q., Agapitov, O., & Nishimura, Y. (2019). Statistical
1138 analysis of transverse size of lower band chorus waves using simultaneous multi-
1139 satellite observations. *Geophysical Research Letters*, 46(11), 5725-5734.
1140 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019GL083118> doi: <https://doi.org/10.1029/2019GL083118>

1141 Shprits, Y. Y., Elkington, S. R., Meredith, N. P., & Subbotin, D. A. (2008). Review
1142 of modeling of losses and sources of relativistic electrons in the outer radiation
1143 belt i: Radial transport. *Journal of Atmospheric and Solar-Terrestrial Physics*,
1144 70(14), 1679-1693. Retrieved from <https://www.sciencedirect.com/science/article/pii/S1364682608001648> (Dynamic Variability of Earth's
1145 Radiation Belts) doi: <https://doi.org/10.1016/j.jastp.2008.06.008>

1146 Shprits, Y. Y., & Ni, B. (2009). Dependence of the quasi-linear scattering
1147 rates on the wave normal distribution of chorus waves. *Journal of Geo-
1148 physical Research: Space Physics*, 114(A11). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2009JA014223> doi:
1149 <https://doi.org/10.1029/2009JA014223>

1150 Shprits, Y. Y., Subbotin, D. A., Meredith, N. P., & Elkington, S. R. (2008). Re-
1151 view of modeling of losses and sources of relativistic electrons in the outer
1152 radiation belt ii: Local acceleration and loss. *Journal of Atmospheric and
1153 Solar-Terrestrial Physics*, 70(14), 1694-1713. Retrieved from <https://www.sciencedirect.com/science/article/pii/S1364682608001673> (Dy-
1154 namic Variability of Earth's Radiation Belts) doi: <https://doi.org/10.1016/j.jastp.2008.06.014>

1155 Shprits, Y. Y., Thorne, R. M., Horne, R. B., & Summers, D. (2006). Bounce-
1156 averaged diffusion coefficients for field-aligned chorus waves. *Journal of
1157 Geophysical Research: Space Physics*, 111(A10). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2006JA011725> doi:
1158 <https://doi.org/10.1029/2006JA011725>

1159 Sorathia, K. A., Merkin, V. G., Panov, E. V., Zhang, B., Lyon, J. G., Gar-
1160 retson, J., ... Wiltberger, M. (2020). Ballooning-interchange insta-
1161 bility in the near-earth plasma sheet and auroral beads: Global magne-
1162 tospheric modeling at the limit of the mhd approximation. *Geophys-
1163 ical Research Letters*, 47(14), e2020GL088227. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2020GL088227> (e2020GL088227 10.1029/2020GL088227) doi: 10.1029/2020GL088227

1164 Sorathia, K. A., Merkin, V. G., Ukhorskiy, A. Y., Allen, R. C., Nykyri, K., & Wing,
1165 S. (2019). Solar wind ion entry into the magnetosphere during northward
1166 imf. *Journal of Geophysical Research: Space Physics*, 124(7), 5461-5481.
1167 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JA025502> (e2019JA025502 10.1029/2019JA025502) doi: 10.1029/2019JA025502

1168 Sorathia, K. A., Merkin, V. G., Ukhorskiy, A. Y., Allen, R. C., Nykyri, K., & Wing,
1169 S. (2019). Solar wind ion entry into the magnetosphere during northward
1170 imf. *Journal of Geophysical Research: Space Physics*, 124(7), 5461-5481.
1171 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JA025502> (e2019JA025502 10.1029/2019JA025502) doi: 10.1029/2019JA025502

1172 Sorathia, K. A., Merkin, V. G., Ukhorskiy, A. Y., Allen, R. C., Nykyri, K., & Wing,
1173 S. (2019). Solar wind ion entry into the magnetosphere during northward
1174 imf. *Journal of Geophysical Research: Space Physics*, 124(7), 5461-5481.
1175 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JA025502> (e2019JA025502 10.1029/2019JA025502) doi: 10.1029/2019JA025502

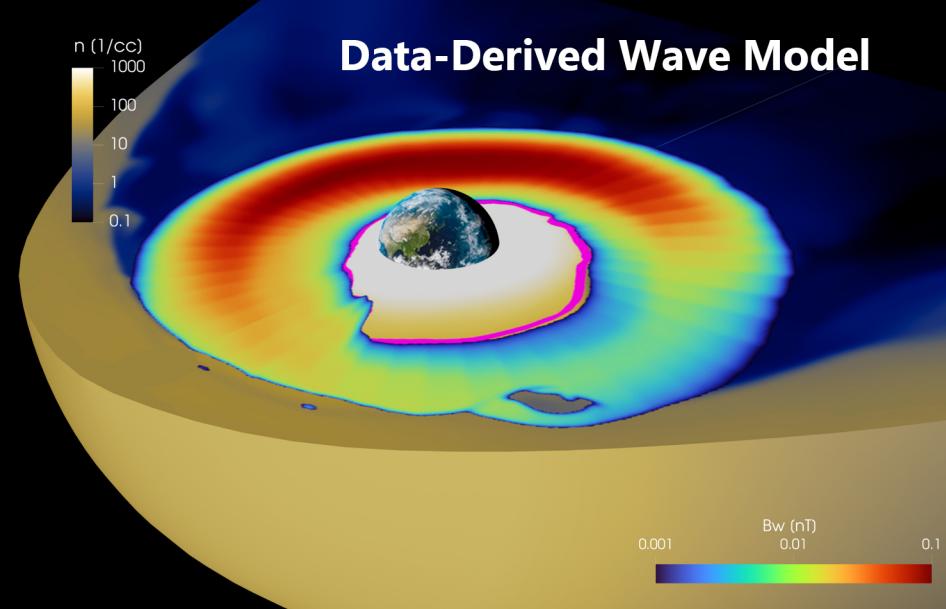
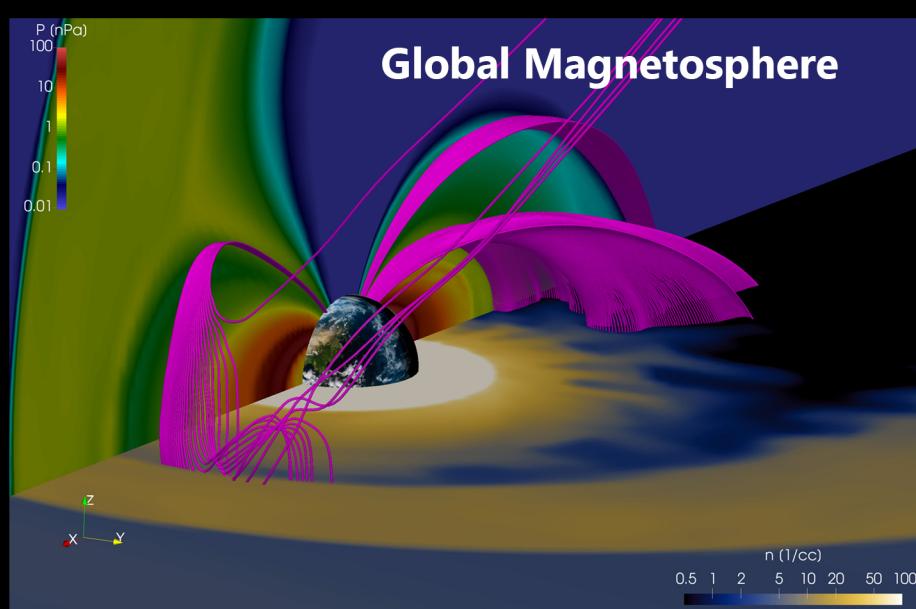
1176 Sorathia, K. A., Merkin, V. G., Ukhorskiy, A. Y., Allen, R. C., Nykyri, K., & Wing,
1177 S. (2019). Solar wind ion entry into the magnetosphere during northward
1178 imf. *Journal of Geophysical Research: Space Physics*, 124(7), 5461-5481.
1179 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JA025502> (e2019JA025502 10.1029/2019JA025502) doi: 10.1029/2019JA025502

- 10.1029/2019JA026728 doi: <https://doi.org/10.1029/2019JA026728>
- 1179 Sorathia, K. A., Merkin, V. G., Ukhorskiy, A. Y., Mauk, B. H., & Sibeck, D. G.
 1180 (2017). Energetic particle loss through the magnetopause: A combined global
 1181 mhd and test-particle study. *Journal of Geophysical Research: Space Physics*,
 1182 122(9), 9329–9343. doi: 10.1002/2017ja024268
- 1183 Sorathia, K. A., Michael, A., Merkin, V., Ukhorskiy, A., Turner, D. L., Lyon, J., ...
 1184 Toffoletto, F. (2021). The role of mesoscale plasma sheet dynamics in ring cur-
 1185 rent formation. *Frontiers in Astronomy and Space Sciences*, 8. Retrieved from
 1186 <https://www.frontiersin.org/articles/10.3389/fspas.2021.761875>
 1187 doi: 10.3389/fspas.2021.761875
- 1188 Sorathia, K. A., Michael, A., Merkin, V. G., Ohtani, S., Keesee, A. M., Sciola, A.,
 1189 ... Pulkkinen, A. (2023). Multiscale magnetosphere-ionosphere coupling
 1190 during stormtime: A case study of the dawnside current wedge. *Journal of*
 1191 *Geophysical Research: Space Physics*, 128(11), e2023JA031594. Retrieved
 1192 from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2023JA031594> (e2023JA031594 2023JA031594) doi: <https://doi.org/10.1029/2023JA031594>
- 1193 Sorathia, K. A., Ukhorskiy, A. Y., Merkin, V. G., Fennell, J. F., & Claudepierre,
 1194 S. G. (2018, Jul). Modeling the Depletion and Recovery of the Outer Radi-
 1195 ation Belt During a Geomagnetic Storm: Combined MHD and Test Particle
 1196 Simulations. *Journal of Geophysical Research (Space Physics)*, 123(7), 5590-
 1197 5609. doi: 10.1029/2018JA025506
- 1198 Spence, H. E., Reeves, G. D., Baker, D. N., Blake, J. B., Bolton, M., Bourdarie, S.,
 1199 ... Thorne, R. M. (2013, November). Science Goals and Overview of the
 1200 Radiation Belt Storm Probes (RBSP) Energetic Particle, Composition, and
 1201 Thermal Plasma (ECT) Suite on NASA's Van Allen Probes Mission. *Space*
 1202 *Science Reviews*, 179(1-4), 311-336. doi: 10.1007/s11214-013-0007-5
- 1203 Staples, F. A., Kellerman, A., Murphy, K. R., Rae, I. J., Sandhu, J. K., &
 1204 Forsyth, C. (2022). Resolving magnetopause shadowing using multimis-
 1205 sion measurements of phase space density. *Journal of Geophysical Re-*
 1206 *search: Space Physics*, 127(2), e2021JA029298. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2021JA029298>
 1207 (e2021JA029298 2021JA029298) doi: <https://doi.org/10.1029/2021JA029298>
- 1208 Summers, D. (2005, Aug). Quasi-linear diffusion coefficients for field-aligned electro-
 1209 magnetic waves with applications to the magnetosphere. *Journal of Geophysi-
 1210 cal Research (Space Physics)*, 110(A8), A08213. doi: 10.1029/2005JA011159
- 1211 Summers, D., Ni, B., & Meredith, N. P. (2007a). Timescales for radiation belt
 1212 electron acceleration and loss due to resonant wave-particle interactions: 1.
 1213 theory. *Journal of Geophysical Research: Space Physics (1978–2012)*, 112(A4),
 1214 n/a-n/a. doi: 10.1029/2006ja011801
- 1215 Summers, D., Ni, B., & Meredith, N. P. (2007b). Timescales for radiation belt elec-
 1216 tron acceleration and loss due to resonant wave-particle interactions: 2. evalua-
 1217 tion for vlf chorus, elf hiss, and electromagnetic ion cyclotron waves. *Journal*
 1218 *of Geophysical Research: Space Physics*, 112(A4). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2006JA011993> doi:
 1219 <https://doi.org/10.1029/2006JA011993>
- 1220 Summers, D., Thorne, R. M., & Xiao, F. (1998). Relativistic theory of wave-
 1221 particle resonant diffusion with application to electron acceleration in the
 1222 magnetosphere. *Journal of Geophysical Research: Space Physics*, 103(A9),
 1223 20487-20500. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/98JA01740> doi: 10.1029/98JA01740
- 1224 Tao, X., Chan, A. A., Albert, J. M., & Miller, J. A. (2008). Stochastic model-
 1225 ing of multidimensional diffusion in the radiation belts. *Journal of Geo-*
 1226 *physical Research: Space Physics*, 113(A7). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2007JA012985> doi:
 1227 10.1029/2007JA012985
- 1228
- 1229
- 1230
- 1231
- 1232
- 1233

- 1234 <https://doi.org/10.1029/2007JA012985>
- 1235 Thorne, R. M. (2010). Radiation belt dynamics: The importance of wave-particle
1236 interactions. *Geophysical Research Letters*, 37(22). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2010GL044990> doi:
1237 <https://doi.org/10.1029/2010GL044990>
- 1238 Toffoletto, F., Sazykin, S., Spiro, R., & Wolf, R. (2003). Inner magnetospheric modeling with the Rice Convection Model. *Space Science Reviews*, 107(1-2), 175–196. doi: 10.1023/a:1025532008047
- 1239 Turner, D. L., Cohen, I. J., Michael, A., Sorathia, K., Merkin, S., Mauk, B. H.,
1240 ... Reeves, G. D. (2021). Can earth's magnetotail plasma sheet produce a source of relativistic electrons for the radiation belts? *Geophysical Research Letters*, 48(21), e2021GL095495. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2021GL095495>
1241 (e2021GL095495 2021GL095495) doi: <https://doi.org/10.1029/2021GL095495>
- 1242 Turner, D. L., Fennell, J. F., Blake, J. B., Claudepierre, S. G., Clemmons, J. H.,
1243 Jaynes, A. N., ... Reeves, G. D. (2017). Multipoint observations of energetic
1244 particle injections and substorm activity during a conjunction between magnetospheric multiscale (mms) and van allen probes. *Journal of Geophysical Research: Space Physics*, 122(11), 11,481-11,504. doi: 10.1002/2017ja024554
- 1245 Ukhorskiy, A. Y., Anderson, B. J., Brandt, P. C., & Tsyganenko, N. A. (2006).
1246 Storm time evolution of the outer radiation belt: Transport and losses. *Journal of Geophysical Research: Space Physics*, 111(A11). Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2006JA011690> doi:
1247 <https://doi.org/10.1029/2006JA011690>
- 1248 Ukhorskiy, A. Y., & Sitnov, M. I. (2013, November). Dynamics of Radiation Belt
1249 Particles. *Space Science Reviews*, 179(1-4), 545-578. doi: 10.1007/s11214-012-
1250 -9938-5
- 1251 Ukhorskiy, A. Y., Sitnov, M. I., Millan, R. M., & Kress, B. T. (2011). The role
1252 of drift orbit bifurcations in energization and loss of electrons in the outer
1253 radiation belt. *Journal of Geophysical Research: Space Physics*, 116(A9).
1254 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2011JA016623> doi: <https://doi.org/10.1029/2011JA016623>
- 1255 Ukhorskiy, A. Y., Sitnov, M. I., Millan, R. M., Kress, B. T., Fennell, J. F., Claude-
1256 pierre, S. G., & Barnes, R. J. (2015). Global storm time depletion of the outer
1257 electron belt. *Journal of Geophysical Research: Space Physics*, 120(4), 2543-
1258 2556. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2014JA020645> doi: <https://doi.org/10.1002/2014JA020645>
- 1259 Ukhorskiy, A. Y., Sorathia, K. A., Merkin, V. G., Sitnov, M. I., Mitchell, D. G.,
1260 & Gkioulidou, M. (2018). Ion trapping and acceleration at dipolarization
1261 fronts: High-resolution mhd and test-particle simulations. *Journal of Geophysical Research: Space Physics*, 123(7), 5580-5589. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JA025370> doi:
1262 <https://doi.org/10.1029/2018JA025370>
- 1263 Wang, D., Shprits, Y. Y., Zhelavskaya, I. S., Agapitov, O. V., Drozdov, A. Y.,
1264 & Aseev, N. A. (2019). Analytical chorus wave model derived from van
1265 allen probe observations. *Journal of Geophysical Research: Space Physics*,
1266 124(2), 1063-1084. Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JA026183> doi: <https://doi.org/10.1029/2018JA026183>
- 1267 Wiltberger, M., Merkin, V., Zhang, B., Toffoletto, F., Oppenheim, M., Wang,
1268 W., ... Stephens, G. K. (2017). Effects of electrojet turbulence on a
1269 magnetosphere-ionosphere simulation of a geomagnetic storm. *Journal of Geophysical Research: Space Physics*, 122(5), 5008-5027. doi:
1270 <https://doi.org/10.1002/2016ja023700>
- 1271 Zhang, B., Sorathia, K. A., Lyon, J. G., Merkin, V. G., Garretson, J. S., & Wilt-
1272

- 1289 berger, M. (2019, sep). GAMERA: A three-dimensional finite-volume
1290 MHD solver for non-orthogonal curvilinear geometries. *The Astrophysical
1291 Journal Supplement Series*, 244(1), 20. Retrieved from <https://doi.org/10.3847%2F1538-4365%2Fab3a4c> doi: 10.3847/1538-4365/ab3a4c
- 1292 Zhao, H., Baker, D. N., Li, X., Jaynes, A. N., & Kanekal, S. G. (2019). The
1293 effects of geomagnetic storms and solar wind conditions on the ultrarelativistic
1294 electron flux enhancements. *Journal of Geophysical Research: Space Physics*, 124(3), 1948-1965.
1295 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JA026257> doi:
1296 <https://doi.org/10.1029/2018JA026257>
- 1297 Zheng, L., Chan, A. A., Albert, J. M., Elkington, S. R., Koller, J., Horne, R. B.,
1298 ... Meredith, N. P. (2014). Three-dimensional stochastic modeling of
1299 radiation belts in adiabatic invariant coordinates. *Journal of Geophysical Research: Space Physics*, 119(9), 7615-7635.
1300 Retrieved from <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2014JA020127> doi:
1301 <https://doi.org/10.1002/2014JA020127>
- 1302 Zheng, L., Chen, L., Chan, A. A., Wang, P., Xia, Z., & Liu, X. (2021). Uber v1.0:
1303 a universal kinetic equation solver for radiation belts. *Geoscientific Model Development*, 14(9), 5825-5842. Retrieved from <https://gmd.copernicus.org/articles/14/5825/2021/> doi: 10.5194/gmd-14-5825-2021
- 1304
- 1305
- 1306
- 1307
- 1308

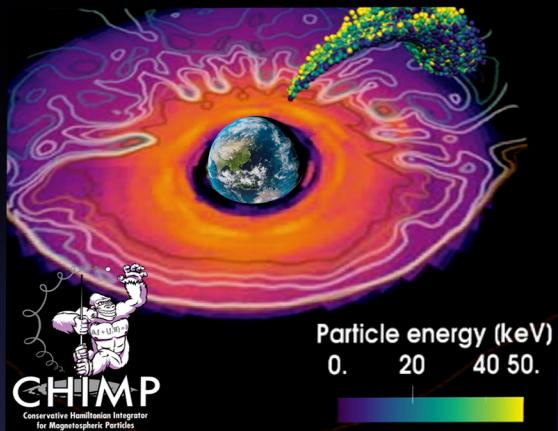
Figure 1.



**Electromagnetic
Fields & Density**

Wave Parameters

Particle Transport



+

Local Wave - Particle Interactions

D_{aa} (1/s) 1.e-5, 0.0001, 0.001, 0.01

Global Test Particle Simulations

Figure 2.

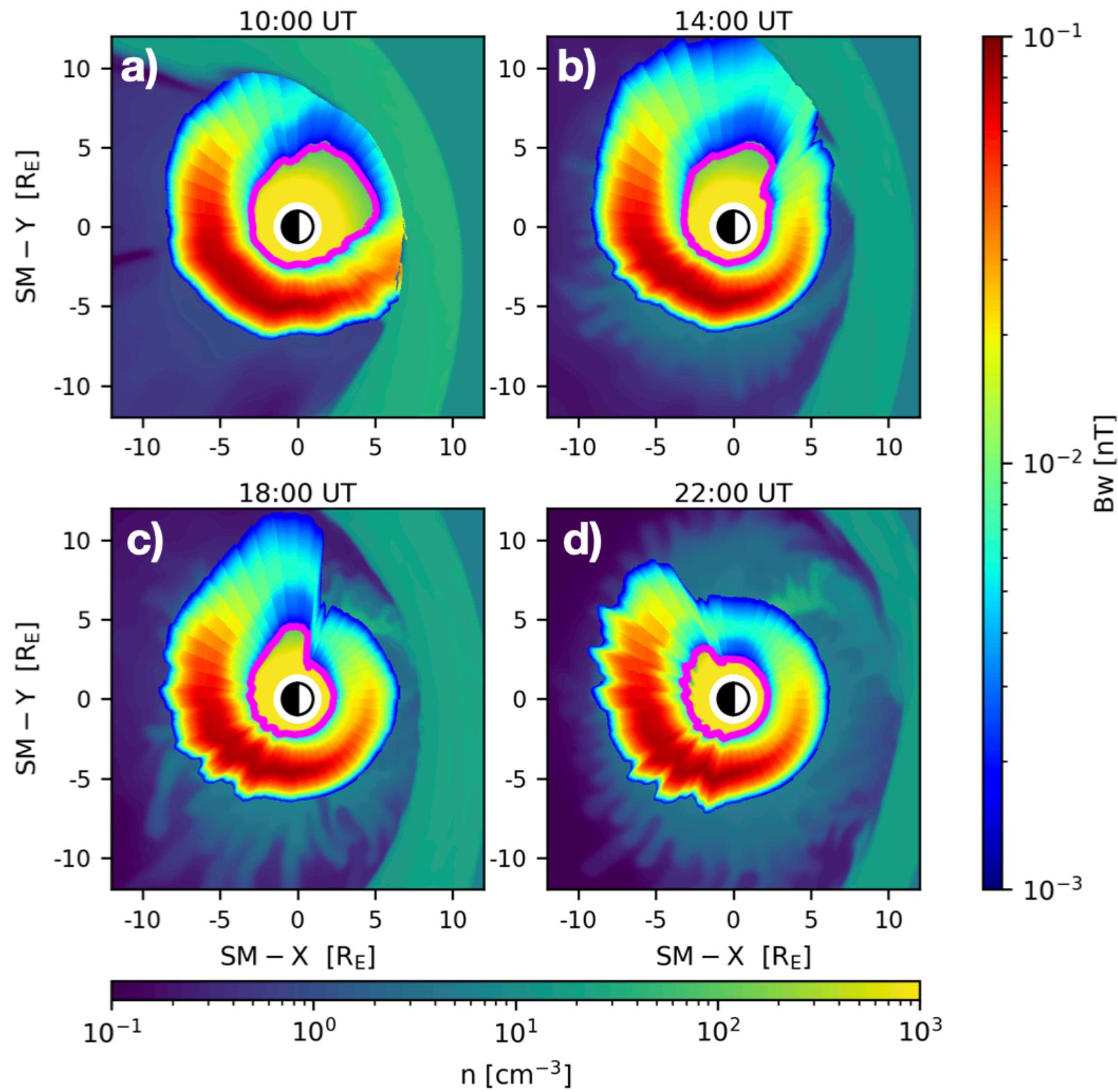


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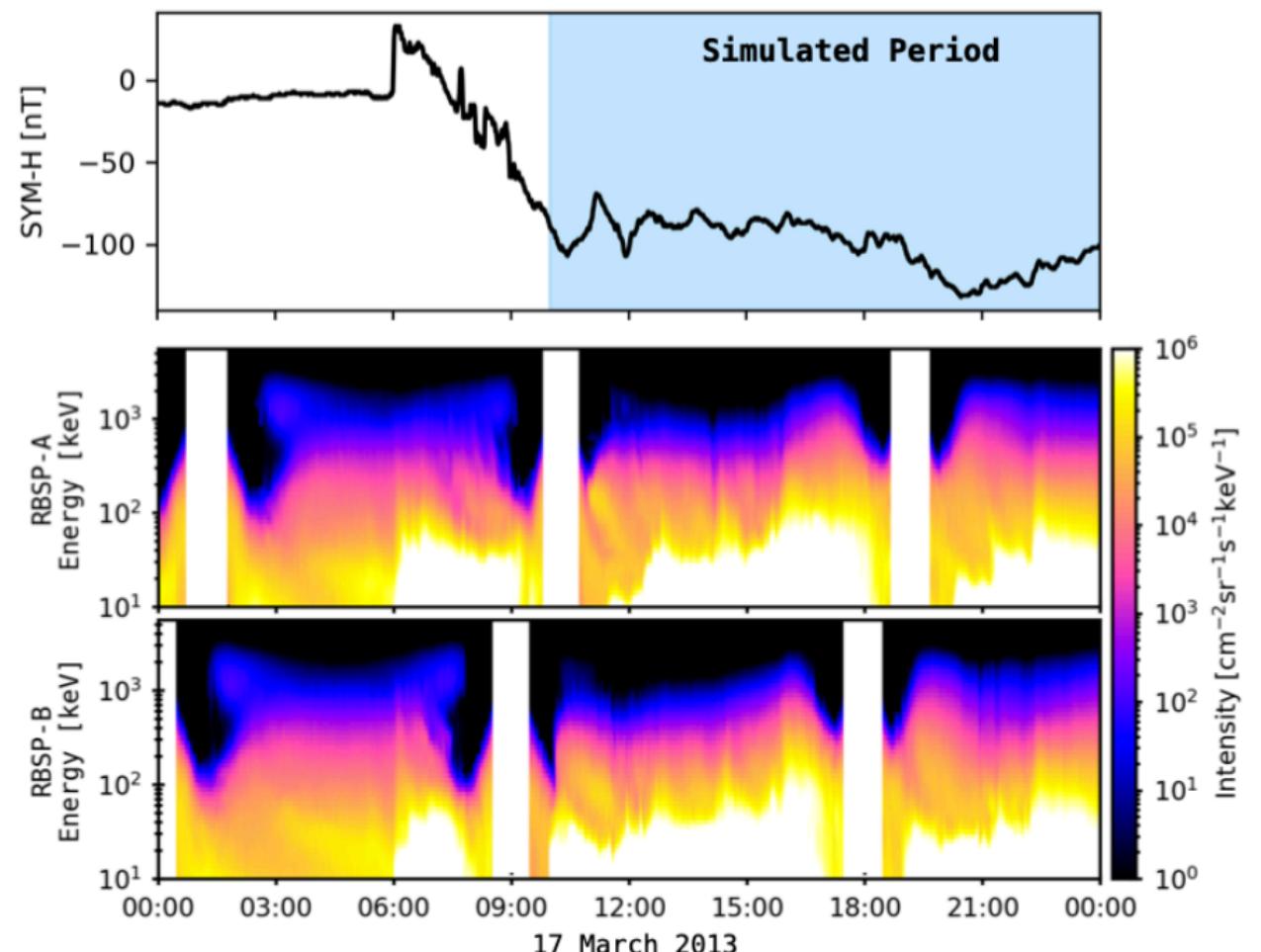
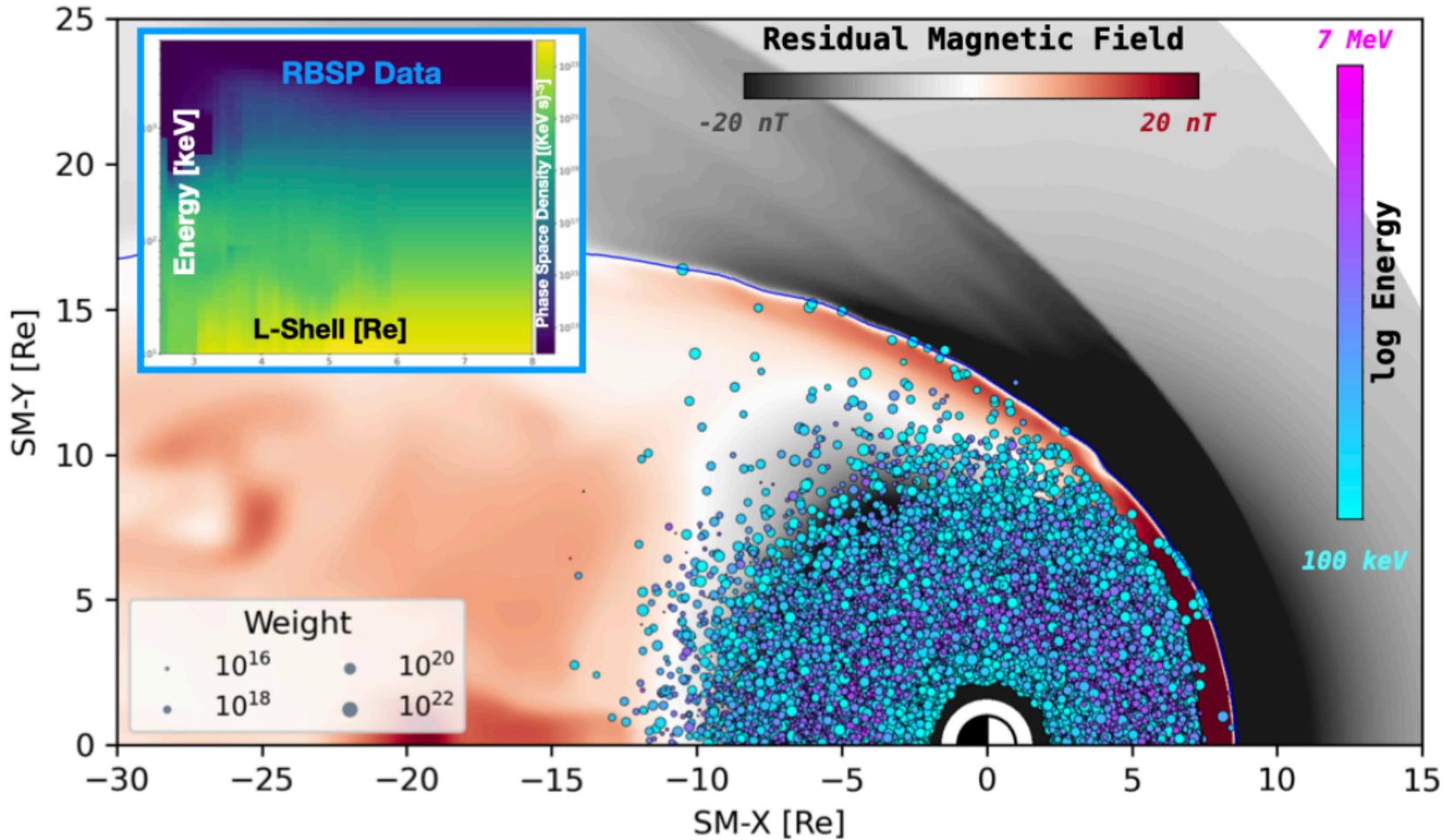


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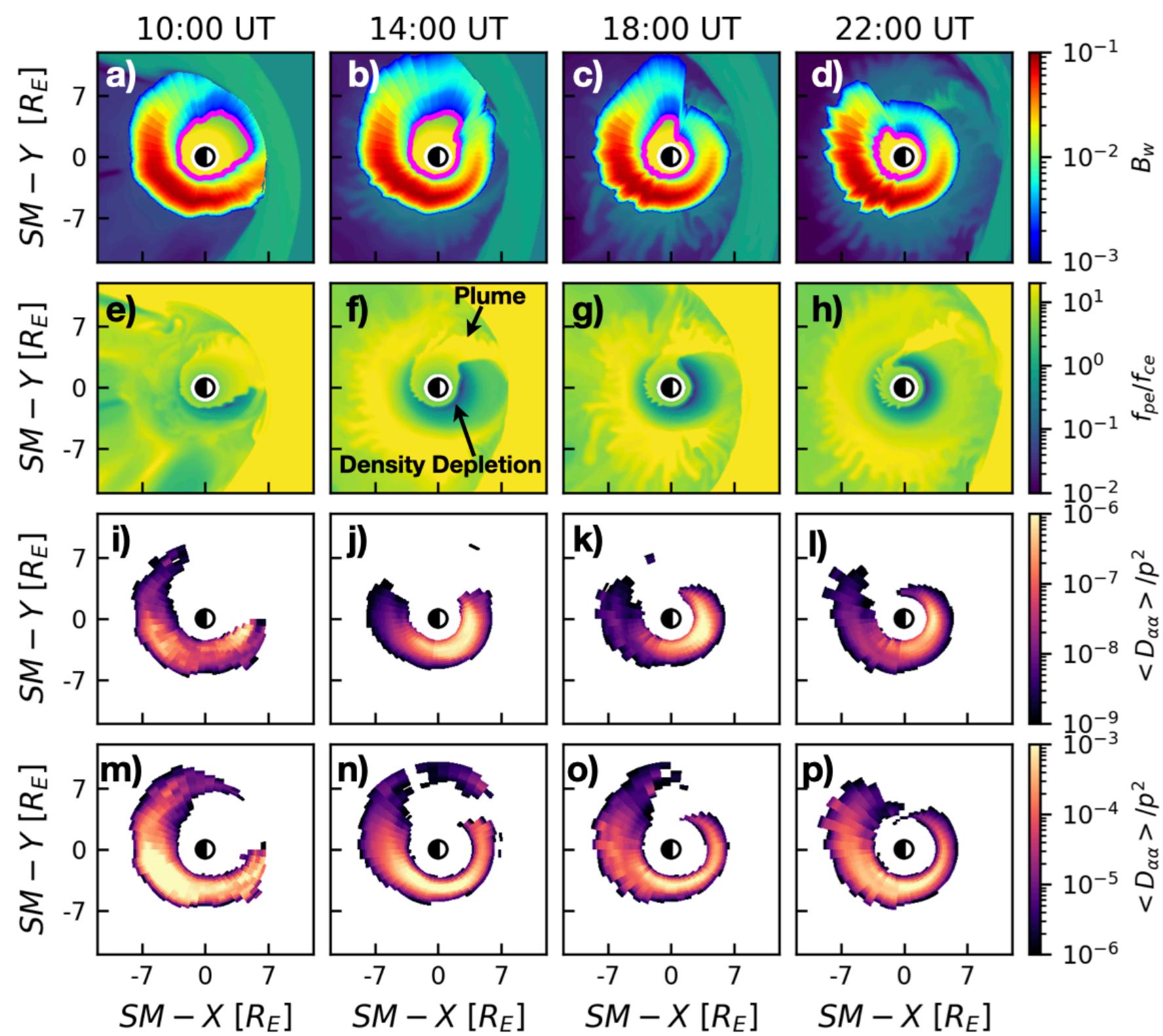


Figure 5.

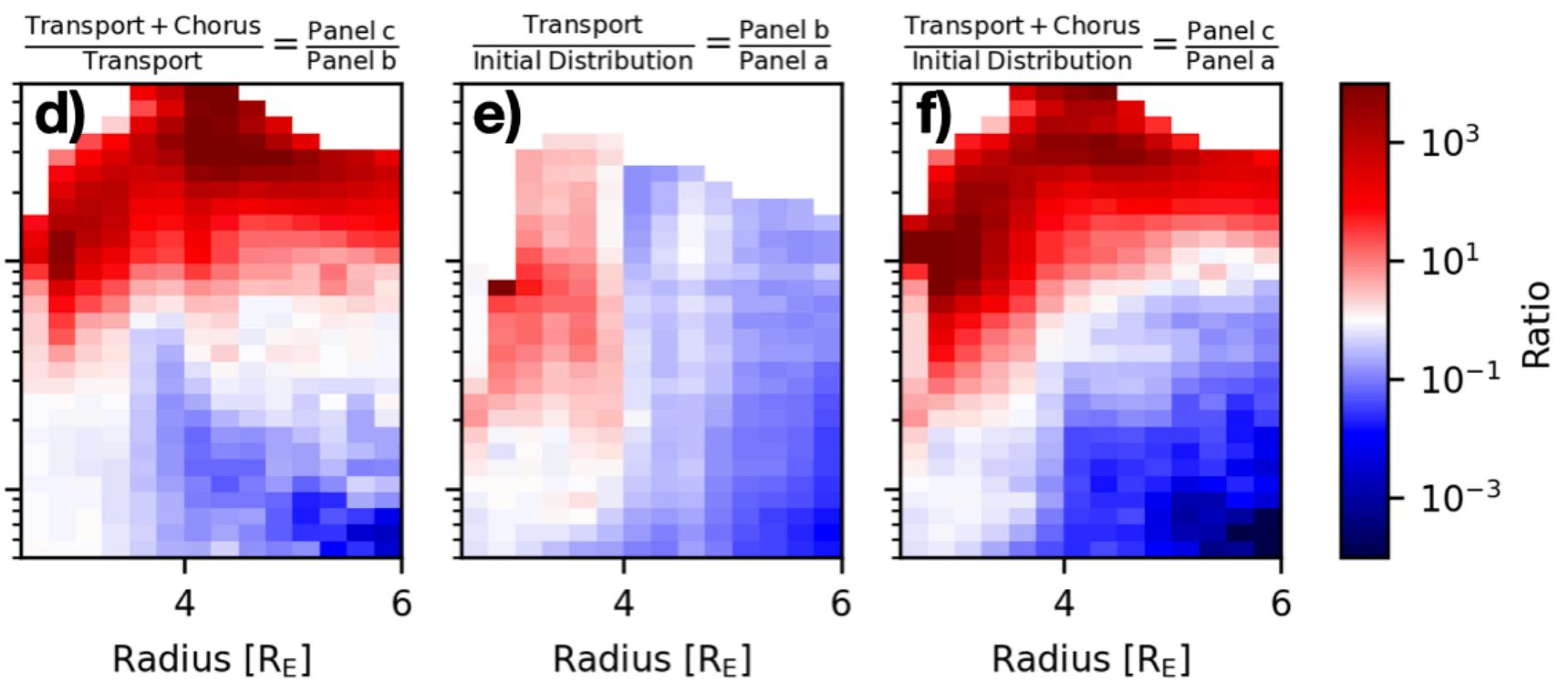
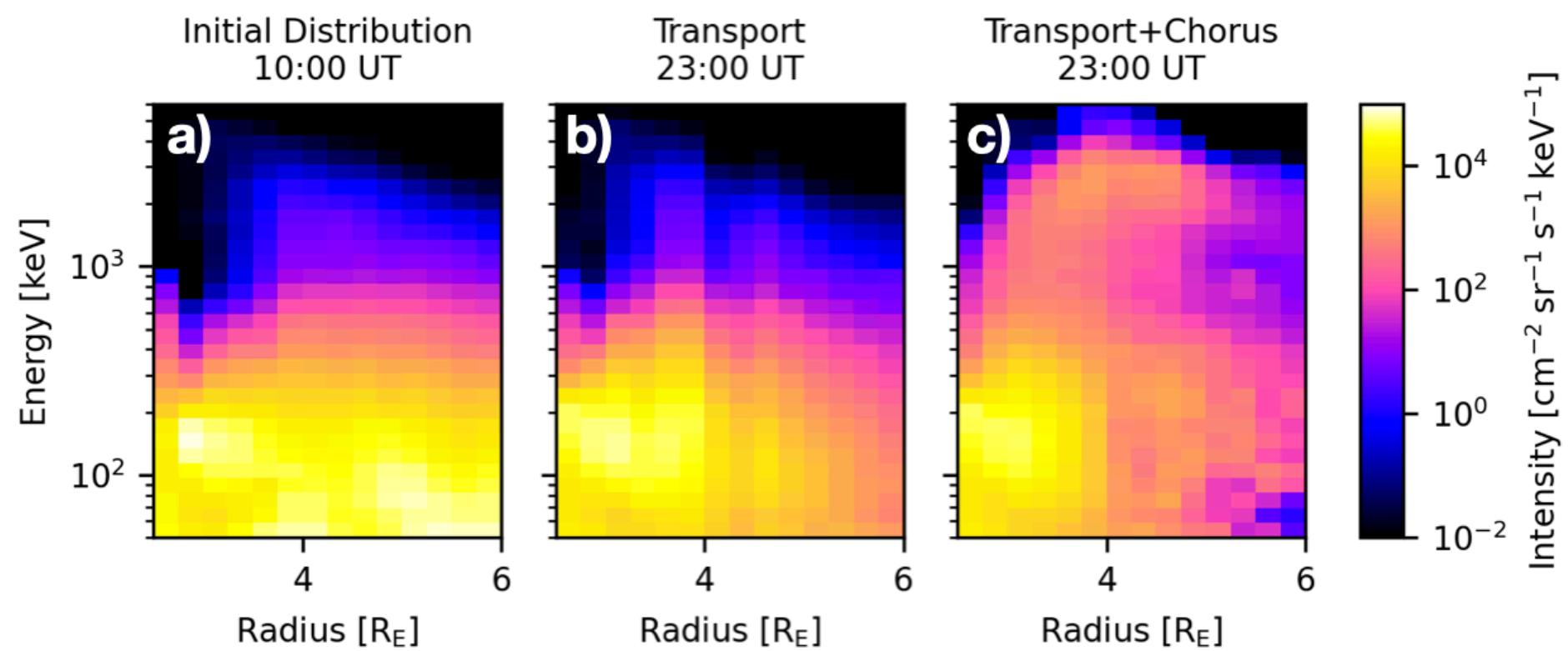


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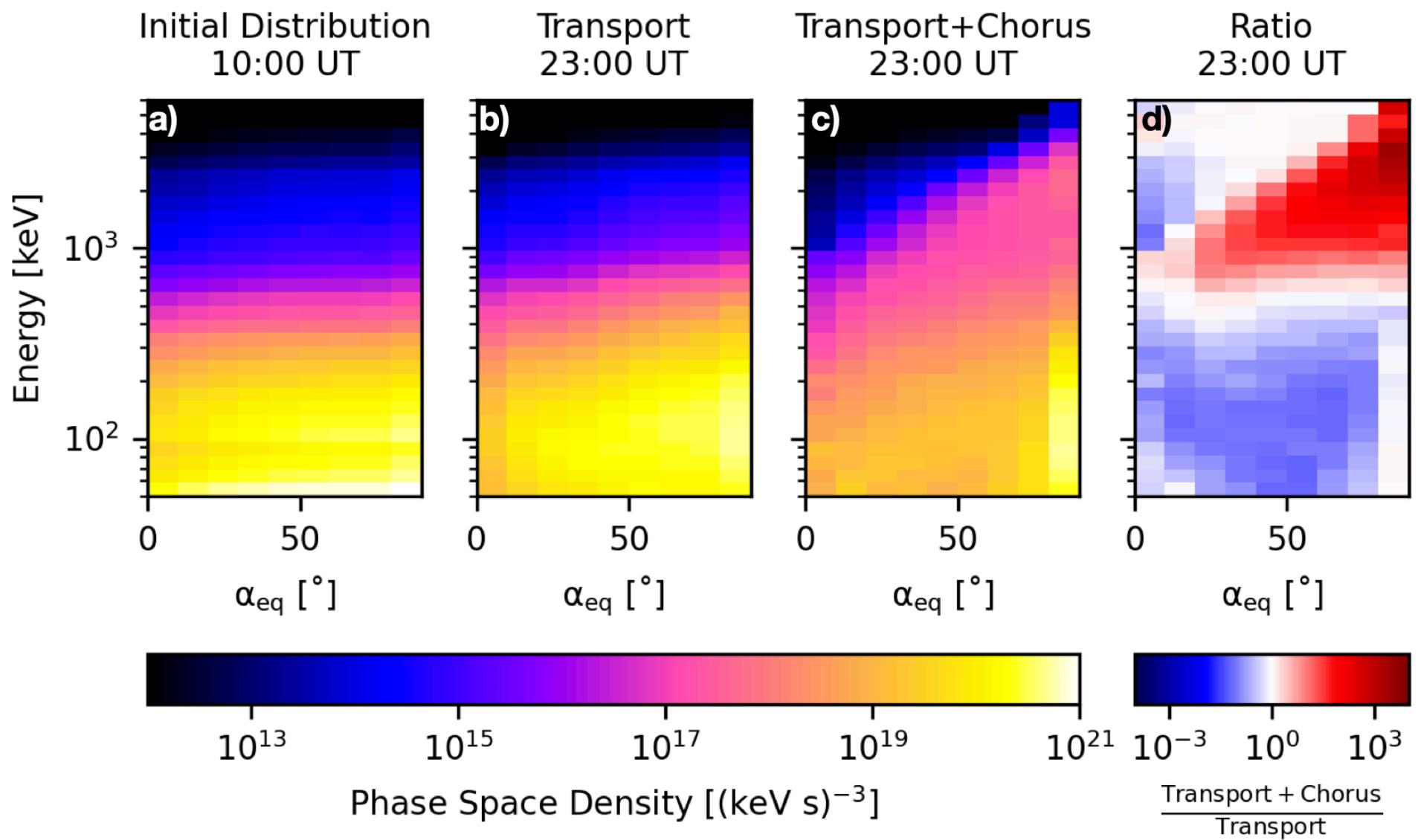


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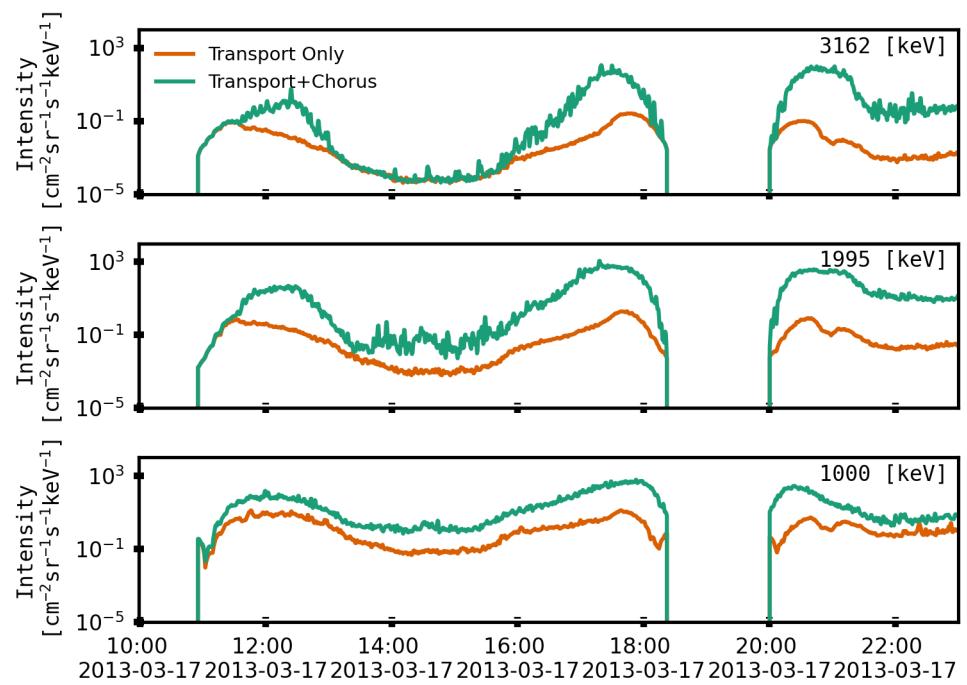


Figure 8.

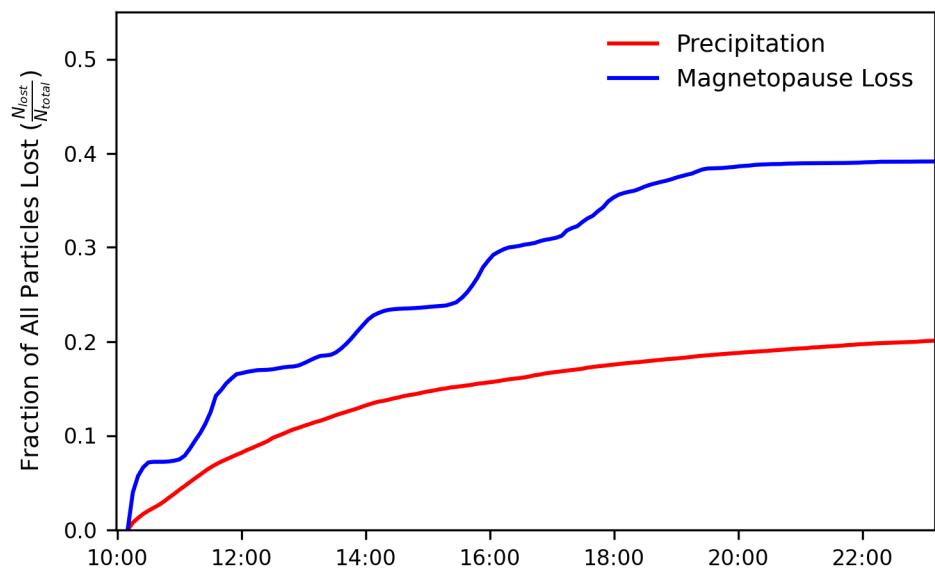


Figure 9.

