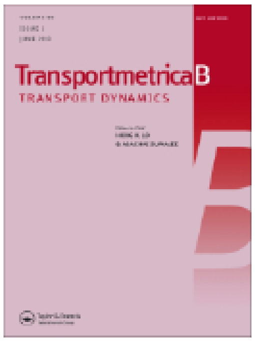

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Managing network congestion with a trip- and area-based tradable credit scheme

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ABSTRACT

This study proposes a trip- and area-based tradable credit scheme (TCS) for congestion management in the context of the morning commute problem using a trip-based Macroscopic Fundamental Diagram model with heterogenous travelers. In our proposed TCS, the regulator distributes credits to all travelers and designs a time-varying and trip-based credit tariff. Credits are traded between travelers and the regulator via a credit market at the price determined by credit demand and supply interactions. The TCS is incorporated into a day-to-day modeling framework to examine travelers' learning process, network state evolution, and credit market properties. The conditions for existence of an equilibrium solution and uniqueness of the equilibrium credit price are established analytically. Simulation results validate the analytical properties, demonstrate that the proposed TCS yields identical social welfare as the congestion pricing while maintaining revenue neutrality, and show the superiority of a trip-based TCS to (trip agnostic) area-based TCS.

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Tradable credits; trip-based macroscopic fundamental diagram; demand management; day-to-day dynamics

1. Introduction

Road traffic externalities are a serious problem that affect urban transportation networks worldwide and their severity continues to increase, imposing significant costs on the traveller, environment, economy, and society. Efforts to alleviate these externalities have been explored from both supply and demand perspectives. Since traditional solutions on the supply side such as building additional infrastructure are known to sometimes be counterproductive (Johnston, Lund, and Craig 1995), demand management solutions, from the widely used price instruments to the emerging, but less explored quantity control instruments have received significant attention.

Since the profound work by Pigou (1920), congestion pricing (CP) has received a great deal of focus over the past century in both theory and practice due to the potential gains in social welfare (Lindsey 2006). Nevertheless, road pricing often receives political and social resistance as it is perceived as a tax (de Palma and Lindsey 2020). For this reason, researchers have been exploring alternative and more appealing demand management solutions such as the tradable credit scheme (TCS) in recent years (Fan and Jiang 2013; Grant-Muller and Xu 2014; Dogterom, Ettema, and Dijst 2017). A typical TCS system has the following features (Fan and Jiang 2013): (1) a total quota of credits available for the area of interest is prespecified; (2) a regulator provides an initial endowment of credits to all potential travellers; (3) the credits can be bought and sold in a market that is monitored by the regulator

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at a price determined by demand and supply interactions; (4) in order to travel, travellers need to spend a certain number of credits (i.e. tariff) to access the urban transportation system. The credits, also termed permits in this study, could vary with the type and attributes of the specific mobility alternative used; (5) enforcement is necessary to ensure the permits are being consumed or traded validly. Consequently, the TCS is revenue-neutral and more equitable than congestion pricing (in the absence of revenue refunding), features that may help address the issue of public acceptance (Verhoef, Nijkamp, and Rietveld 1997; Wu et al. 2012; Palma et al. 2018).

Within the context of congestion pricing, distance-based and travel time-based tariffs are likely to bring improvements in overall efficiency (welfare) relative to cordon- and zonal-based tariffs since they better internalize congestion externalities by charging for the actual distance travelled on the network (or time spent in congestion). Existing literature suggests that these improvements in efficiency can be significant at the network level (Lentzakis et al. 2020). From the standpoint of implementation, the development of ICT and smartphones has provided an effective means of operationalizing a TCS (see Azevedo et al. 2018 for a smartphone-based travel incentive system with all the functionality required by a TCS). Implementing distance- and time-based credit tariffs would arguably not present any additional challenges over link or cordon-based tariff mechanisms within a smartphone-based TCS system.

This paper proposes an area-based TCS with time- and distance-based credit tariffs and incorporates this TCS into a day-to-day modelling framework to investigate the properties of the equilibrium solutions and the performance of the TCS. The day-to-day modelling framework uses a trip-based Macroscopic Fundamental Diagram (MFD) model on the supply side. The framework is then applied to the (departure-time choice) morning commute problem and used to evaluate social welfare, network and traveller-specific performance changes. More specifically, the contributions of this paper are three-fold:

(1) We study the morning commute problem under a TCS with heterogeneous users (in terms of both the valuation of travel time/schedule delay, preferred arrival time and choice sets) using a day-to-day dynamic assignment framework (reflecting day-to-day behaviour), which yields insights into the evolution of market prices and flows. This is contrast with the large body of existing literature (with the exception of Ye and Yang 2013), which solve for an internally consistent system state or equilibrium.
(2) With regard to the design of the TCS, we consider distance and time varying credit tariffs, which have not been considered before. We make use of the trip-based MFD model (which allows for heterogeneous trip-lengths) to effectively model the impacts of the distance-based and travel time-based credit tariffs within the TCS on network efficiency and social welfare. Further, our experiments yield insights into the differences between these schemes relative to traditional zonal schemes and between different TCS tariff profiles.
(3) Analytically, we demonstrate solution existence and the uniqueness of the credit market price under the assumption that the travel time is a continuous function of departure flows, considering heterogeneous travellers.

The rest of this paper is organized as follows. Section 2 reviews relevant literature. Section 3 presents the modelling assumptions, traffic flow model, day-to-day dynamic model, and credit price evolution model. Section 4 discusses the properties of the dynamic system, i.e. the existence of the equilibrium solution and the uniqueness of the credit price at equilibrium state. Section 5 introduces the simulation-based optimization framework to optimize the parameters associated with the tariff profile function. Next, numerical simulation results are discussed in Section 6. Finally, Section 7 concludes this study, discusses the limitations and points out future research directions.

2. Literature review

In this section, we review existing literature in three-related directions: tradable credit schemes, Macroscopic Fundamental diagram (MFD) models, and simulation-based optimization.

Tradable credit schemes have been studied in several contexts including the management of network congestion using static equilibrium models (involving both route and mode choice) and day-to-day dynamic models, and the management of peak-period congestion for the morning commute using bottleneck models (involving departure time choices). A large body of literature exists on the modelling of the TCS considering a static equilibrium in terms of the traffic flow pattern and credit market price. For example, Yang and Wang (2011) established the conditions under which for a given link-specific credit charge scheme (i.e. a credit tariff vector), the user-equilibrium link flow pattern and the credit price are unique. Wang et al. (2012) extended this model to consider heterogeneity. In the context of bottleneck models, Xiao, Qian, and Zhang (2013) showed that the optimal time-varying charge of credits at a bottleneck always exists under the assumption that late arrival is prohibited. Nie (2015) proposed a new type TCS, which does not involve the initial distribution of credits but rewards travellers departing during the designated off-peak time window. The results indicate that a charging to rewarding ratio of 1 provides efficiency gains. Extensions of this scheme were proposed in Nie and Yin (2013) and Xiao, Huang, and Liu (2015), which consider transit as an alternative mode with homogeneous and heterogeneous travelers, respectively. More recently, Bao, Verhoef, and Koster (2019) showed that the equilibrium credit price and departure rate of the bottleneck model with a TCS is not unique although uniqueness is guaranteed using an alternative congestion model developed by Chu (1995).

In addition, the application of TCS to parking management has been investigated. Zhang, Yang, and Huang (2011) proposed parking permit allocation policies and a free trading scheme to improve network performance under limited parking space. Liu et al. (2014) further considered expirable parking permits and showed that the efficiency loss of a non-ideal scheme is bounded. Wang, Wang, and Zhang (2020) utilized the parking permit scheme to eliminate parking competition, which cannot be addressed by a pure parking pricing scheme. Xiao, Liu, and Huang (2021) studied the equilibrium state and system-optimal distributions of parking permits in a many-to-one multi-modal network with parking space constraint. Bao and Ng (2022) empirically tested the parking permit scheme through a lab-in-the-field experiment where participants' willingness to pay, willingness to accept compensation and willingness to give up cars are collected and analyzed by questionnaires. The results suggested that respondents show a strong interest in the tradable parking permit scheme and that such a scheme is effective in encouraging pro-environmental behaviour like reducing car ownership.

Multiperiod TCS schemes and the comparative performance of tradable credits and congestion pricing have also been studied. Miralinaghi and Peeta (2016) developed a multi-period TCS and showed that penalizing credit transfers across periods reduces credit price volatility. de Palma et al. (2018) compared a standard TCS with congestion pricing and showed that the TCS is equivalent to congestion pricing under fully adaptive tolls, but outperforms it under non-adaptive tolls typically when the congestion function is relatively steep compared to the demand function. Seshadri, de Palma, and Ben-Akiva (2021) reached similar conclusions on the comparative performance between TCS and congestion pricing using a more complex within-day dynamic model (departure time context) that explicitly modelled selling behaviour in the market. Further, they found that the performance of the TCS is relatively robust to irrational selling behaviour in the market, although it can result in welfare losses.

In contrast, relatively few studies investigate the dynamics of the credit price. A notable exception is Ye and Yang (2013), who employed a day-to-day learning model within a route choice setting to examine the dynamic evolution process of traffic flow and credit price under a TCS. Along similar lines, Guo, Huang, and Yang (2019) proposed a framework containing a period-to-period adjusted credit distribution and charge, and a day-to-day price adjustment and demonstrated analytically that such a TCS ensures convergence to the system optimum.

Towards a practice-ready TCS, Chen et al. (2020) modelled the detailed and joint individual decision making (namely, the buying, selling and departure time choices) together with the regulator's operations in a microscopic time-based simulation framework. Field experiments on TCS are sparse. Brands et al. (2020) empirically tested the market of a TCS through a lab-in-the-field experiment where participants make virtual travel choices and real transactions in a tradable parking permits setting. The results showed that the designed market achieved credit prices within a desired range, and the observed buying and selling was in accordance with a theoretical market equilibrium. Ultimately, and in the absence of empirical evidence, these few existing contributions point to known state-of-the-art day-to-day learning frameworks for capturing demand-supply interactions (see also Cantarella, Velonà, and Watling 2015; Guo et al. 2016; Yildirimoglu and Ramezani 2020). These frameworks are often extremely helpful in ensuring desirable equilibrium properties in disaggregate modelling frameworks of demand-supply interactions, as it is for the TCS schemes at stake in this paper. Yet, on the demand side, both detailed market interactions and day-to-day learning processes are still to be explored by researchers, whereas on the supply side, general and toy networks with link-based credit charging have been used in most studies (e.g. Bao, Gaoa, and Xu 2016; Zhu et al. 2015; Xiao et al. 2019), thus also limiting the design towards practice-ready TCS (Lessan and Fu 2019). In this paper, we examine the TCS for the morning commute problem under a single reservoir network using the Macroscopic Fundamental Diagram (MFD) (Daganzo 2007; Geroliminis and Daganzo 2007) and investigate corresponding properties.

Recent MFD applications, such as Fosgerau and Small (2013), Amirgholy and Gao (2017), quantified potential benefits in terms of congestion reduction under quantity control and pricing schemes by keeping the accumulation no greater than the flow-maximizing value. The individual attributes such as heterogeneous trip lengths were considered in Arnott (2013), Fosgerau (2015), Daganzo and Lehe (2015), Lamotte and Geroliminis (2016), where a reformulation of the computation for trip length is developed, hereafter referred to as the *trip-based MFD*. There are three advantages of the trip-based MFD model: (1) Compared to the traditional MFD model (or *accumulation-based MFD model*, see Leclercq et al. 2015), where the predicted outflow increases instantaneously when there is a sharp increase in the inflow, trip-based MFD model accounts for a reaction time to the sudden change in demand and computes the outflow only considering travellers who have completed their trips, providing more reliable results (Mariotte, Leclercq, and Laval 2017); (2) Trip-based MFD models can accommodate a more realistic heterogeneity of individual travellers in terms of trip length, desired arrival time, and schedule delay penalties (Lamotte and Geroliminis 2018); and (3) Trip-based MFD models allow for testing distance-based TCS schemes, bringing additional degrees of freedom to the scheme's design process towards fairness and efficiency (Daganzo and Lehe 2015).

Moreover, a proper design of the credit charging scheme is required to make the TCS effective for demand management. The system optimal credit scheme is usually derived through a closed-form objective when dealing with a static traffic equilibrium model (e.g.Yang and Wang 2011; Wang et al. 2012; de Palma et al. 2018 for link-specific credit tolls, and Xiao, Qian, and Zhang (2013), Bao, Verhoef, and Koster (2019), Miralinaghi et al. (2019) for time-varying credit tolls). However, it is more complex to obtain the charging scheme that minimizes system cost for simulation-based day-to-day dynamic models. Though several studies have examined the problem of computing the link-specific congestion pricing toll to reach a desired equilibrium (Han et al. 2017; Liu and Geroliminis 2017), or to minimize the system cost (Tan, Yang, and Guo 2015), it remains challenging to determine optimal time-varying charging schemes. To address these challenges, in this paper, we adopt the approach of Liu, Jiang, and Azevedo (2021) who use a Gaussian (mixture) function to parameterize the time-varying road pricing scheme so as to facilitate the use of derivative-free optimization methods, such as evolution algorithms, pattern search, and Bayesian optimization.

Furthermore, we summarize some work studying the TCS for demand management in Table 1 in terms of supply model, demand/behaviour model, credit price, tariff scheme, solution properties, and tariff optimization.

Table 1. Comparisons of present study with existing TCS-related research.

Study	Supply model	Demand/ behaviour model	Credit price	Tariff scheme	Solution properties	Tariff optimization
Yang and Wang (2011)	S; GN	Fixed and elastic; homogeneous traveller; route choice (deterministic)	Constant and unique	Link specific	Flow; credit price	Analytical optimality
Wang et al. (2012)	S; GN	Fixed and elastic; discrete heterogeneous value of time; route choice (deterministic)	Constant and unique	Link specific	E&U: flow, credit price	Analytical optimality
Wu et al. (2012)	S; GN	Elastic; heterogeneous income; mode and route choices (nested-logit model)	Constant	Mode and link specific	Solution of a variational inequality problem	Derivative-free algorithms
Xiao, Qian, and Zhang (2013)	M	Fixed; homogeneous traveller; departure time choice (deterministic)	Constant	Time-varying continuous toll	E: flow, credit price	Analytical optimality
Nie and Yin (2013)	M	Fixed and elastic; homogeneous traveller; departure time and mode choice (deterministic)	Constant	Step tolls with mode-specific rewards	E: flow, credit price	Analytical optimality
Shirmohammadi et al. (2013)	S; GN; fixed and uncertain capacity	Fixed and uncertain; homogeneous traveller; route choice (deterministic)	Under certainty: Constant and unique; Under uncertainty: volatile	Length based; cordon based; link specific	E&U: flow, credit price under uncertainty	Analytical optimality under certainty
Ye and Yang (2013)	S; GN	Fixed; homogeneous traveller; route choice (logit model)	Day-to-day dynamic	Link specific	E&U: flow, credit price	Not considered
Nie (2015)	M	Fixed; homogeneous traveller; departure time choice (deterministic)	Constant	Step tolls with rewards	E: flow, credit price	Analytical optimality
Xiao, Huang, and Liu (2015)	M	Fixed; heterogeneous value of time and schedule delay; departure time and mode choice (deterministic)	Constant	Step tolls with mode-specific rewards	E: flow, credit price	Analytical optimality
Bao, Gaoa, and Xu (2016)	S; GN	Elastic; heterogeneous travellers' framing and labelling of credits; route choice (deterministic)	Constant	Link specific	E&U: flow	Nonlinear optimization
Zhu et al. (2015)	S; GN	Fixed and elastic; continuous heterogeneous value of time; route choice (deterministic)	Constant	Link specific	E&U: flow, credit price	Analytical optimality
Miralinaghi and Peeta (2016)	S; GN	Elastic; homogeneous traveller; route choice (deterministic)	Period-to-period dynamic	Link specific	E&U: flow, credit price	Analytical optimality
Akamatsu and Wada (2017)	Network based on bottleneck model	Fixed and elastic; heterogeneous value of time; departure time and route choice (deterministic and dynamic)	Link specific and time dependent	Link specific and time dependent	Existence conditions	Analytical optimality conditions

(*continued*).

Table 1. Continued.

Study	Supply model	Demand/ behaviour model	Credit price	Tariff scheme	Solution properties	Tariff optimization
Palma et al. (2018)	S; single O-D network	Daily fixed; heterogeneous value of time; mode and route choice (logit model)	Constant and unique	Route and scenario specific	E&U: flow and price for adaptive toll	Nonlinear optimization
Xiao et al. (2019)	S; GN	Fixed; homogeneous traveller; route choice (deterministic)	Constant and unique	Link specific with negative values	E&U: flow, credit price	Analytical optimality
Guo, Huang, and Yang (2019)	S; GN	Elastic; homogeneous traveller; route choice (deterministic)	Day to day dynamic	Link specific and period-to-period adjusted	Convergence to the system optimum	Period-to-period adjusted
Bao, Verhoef, and Koster (2019)	M	Fixed; homogeneous traveller; departure time choice (deterministic)	Constant	Time-varying continuous toll	E: flow, credit price	Analytical optimality
Miralinaghi et al. (2019)	M	Fixed; heterogeneous value of time, schedule delay and preferred arrival time; departure time choice (deterministic)	Constant and unique	Time dependent and value of time specific	E&U: flow, credit price	Analytical optimality
This study	Morning commute problem based trip-based MFD	Fixed; heterogeneous value of time, schedule delay, preferred arrival time and trip length; departure time choice (logit model)	Day to day dynamic	Distance based and time-varying continuous toll	E&U: credit price	Bayesian optimization

Note: S, static congestion with density function; GN, general network; M, morning commute problem based on bottleneck model; E&U, existence and uniqueness.

3. Methodology

We consider a morning commute problem where a fixed demand of N travellers (indexed by i) wish to travel during the morning peak. Assume that the day is discretized into time intervals, $t \in \mathcal{T}$, with a size of Δt. Note that our modelling framework uses both discrete time (on both the demand and supply sides) and continuous time (on the supply side), which will be elaborated on in subsequent sections. Traveller i is assumed to have a trip length L_i, and a desired arrival time T_i^*, which is used to compute an individual-specific set of feasible departure time intervals, or a departure time window, denoted by $TW_i \subset \mathcal{T}$. The collection of disaggregate departure-time decisions of travellers serves as an input to a trip-based MFD supply model, yielding network performance or individual trip-travel times. Thus, in place of the standard bottleneck model, supply is modelled through area-based networks as a reservoir (Daganzo 2007), with the assumption that traffic congestion is spatially uniformly distributed within the network (Daganzo 2007). The key advantage of the trip-based MFD model is that it can accommodate heterogeneity of travellers in terms of trip length.

Travellers are assumed to choose their departure time intervals based on the travel time, schedule delay (the difference between the actual arrival time and desired arrival time), and monetary cost, along the lines of the classic Vickrey model (Vickrey 1969). More specifically, the departure-time choices of travellers are modelled using a logit mixture model, which is described in Section 3.2. Demand-supply interactions are treated using a day-to-day modelling framework (Cantarella and Cascetta 1995), where perceived travel times and schedule delay of travellers evolve from day to day through a learning process. The convergence properties of this day-to-day dynamic model are described in Section 4.

Finally, within the proposed tradable credit scheme, which is used to manage peak-period congestion and achieve peak spreading, the regulator distributes a specified number of credits to each traveller. Travelling in a time interval t will incur a credit charge which depends on a time-interval specific credit tariff denoted by $g(t)$, and either the trip length L_i or travel time T_i, depending on the scheme in place. The TCS is described in detail in section 3.3.

3.1. Supply model

Algorithm 1 Event-based simulation of the trip-based MFD

Step 1. Initialization: Input t_i^{dep}, T_i^*, L_i, speed-MFD function $V(n)$ and number of travellers N; set $n = 0$, event counter $j = 0$, $t_j = 0$; calculate the initially estimated arrival time for all travellers $\forall i, 1 \dots N$ by $L_i/V(0)$.
Step 2. Construct the event list by appending the departure and arrival in the order of time, with a length of $2N$.
Step 3. Calculate the experienced travel time:
While Event list is not empty:
set $j = j + 1$, t_j as the time of the next event
let $L_i = L_i - V(n) \cdot (t_j - t_{j-1}), \forall i$
if the next closest event is a traveller i' departure:
$n = n + 1$, update the credit account balance of traveller i'
else:
$n = n - 1$, compute the experienced travel time of traveller i', $T_{i'}(t_{i'}^{\text{dep}})$
end if
Remove this event from the event list
Update the current average travelling speed $V(n)$
Update the estimated arrival time for travellers currently in the network by Equation (1) considering a constant speed $V(n)$
Sort the event list in the order of time
End while

A single-reservoir network (Daganzo 2007; Geroliminis and Daganzo 2007) is considered in this study, where all trips originate and end within this network. The idea is to describe the aggregate vehicular accumulation, the number of operating vehicles, at the 'neighbourhood' level with a well-defined relationship between the reservoir outflow and the aggregate accumulation. We assume that the travel demand is not excessively large so as to trigger a gridlock. This assumption ensures that the gridlock is never reached and a non-zero flow stable equilibrium can be achieved.

Furthermore, we resort to the *trip-based MFD* as in Arnott (2013), Fosgerau (2015), Daganzo and Lehe (2015), Lamotte and Geroliminis (2018), whose general properties are further investigated in Mariotte, Leclercq, and Laval (2017). The general principle of the *trip-based MFD* is that the trip length of traveller i is computed as the integral of the speed from the entering time t_i^{dep} to the exiting time $t_i^{\text{dep}} + T_i(t_i^{\text{dep}})$, which is written as follows,

$$L_i = \int_{t_i^{\text{dep}}}^{t_i^{\text{dep}} + T_i(t_i^{\text{dep}})} V(n(\tilde{t}))\, \mathrm{d}\tilde{t} \tag{1}$$

Without loss of generality, notation t_i^{dep} is used instead of $t_{i,d}^{\text{dep}}$ in Equation (1), and $\tilde{t}$ represents continuous time. The assumption that $V(n(\tilde{t}))$ is for all travellers in the network and only changes with an event (departure or arrival) facilitates the use of event-based simulation for analysing network properties (Mariotte, Leclercq, and Laval 2017; Yildirimoglu and Ramezani 2020). In this paper we adopt the simulation process described in Algorithm 3.1.

3.2. Behaviour model

A logit mixture model (Ben-Akiva, McFadden, and Train 2019) with a money-metric utility is used to characterize the departure time choice of travellers, wherein the utility for a traveller i departing in a time interval $t \in TW_i$ on day d is given by:

$$U_{i,d}(t) = C_{i,d}(t) + \epsilon_i \tag{2}$$

where ϵ_i is an identically and independently distributed error term; and $C_{i,d}(t)$ is the systematic disutility (defined in equation (4)) for traveller i departing in time interval t on day d. The probability of choosing departure time interval t can be calculated according to the logit model as follows:

$$\Pr_{i,d}(C_{i,d}(t)) = \frac{\exp(\mu \cdot C_{i,d}(t))}{\sum_{s \in TW_i} \exp(\mu \cdot C_{i,d}(s))} \tag{3}$$

where $\mu > 0$ is the scale parameter, reflecting the variance of the unobserved portion of utility, with choices being random for scale parameter equal to zero and deterministic as its value approaches infinity (Ben-Akiva and Lerman 1985). Let $\tilde{c}_{i,d}(t)$ denote the perceived disutility associated with the time components (travel time, schedule delay early, schedule delay late) for traveller i on day d departing in time interval t; the systematic disutility $C_{i,d}(t)$ is defined as:

$$C_{i,d}(t) = \tilde{c}_{i,d}(t) - p_d \cdot g(t) \cdot L_i \cdot w \tag{4}$$

where the credit payment of traveller i is the product of credit price p_d, credit tariff $g(t)$, trip length L_i and w is a factor for scaling down the magnitude of trip length to avoid unrealistically large payments. In principle, one could add the market value of the daily credit endowment to the expression in Equation (4) so that the utility directly captures the net monetary gain or loss from credit transactions. However, in the absence of non-linear income effects, this amounts to adding a constant term to the utility of every alternative, and thus, does not affect the choice and can be omitted. Note also that although in the description of the model and its properties that follows, we will refer solely to the distance-based tariff scheme, in the experiments section, we also examine the travel-time based and area-based schemes.

As noted previously, within the day-to-day modelling framework, travellers update their perception of the time components for day $d+1$, $\tilde{c}_{i,d+1}(t)$, at the end of each day d. We assume that the perception of the time components for day $d+1$, $\tilde{c}_{i,d+1}(t)$, is updated using a combination of the perceived $\tilde{c}_{i,d}(t)$ on day d (historical knowledge) and the experienced (chosen alternatives) and estimated (unchosen alternatives) travel time, schedule delays on day d, $c_{i,d}(t)$, as follows:

$$\tilde{c}_{i,d+1}(t) = \omega \cdot \tilde{c}_{i,d}(t) + (1-\omega) \cdot c_{i,d}(t) \tag{5}$$

where $0 < \omega < 1$ is a learning parameter, which represents the relative weight given to historical experience versus current experience.

The experienced (or estimated) money-metric disutility associated with time components for traveller i on day d departing in time interval t is given by:

$$\begin{aligned} c_{i,d}(t) = & -\theta_i \cdot T_{i,d}(t) - \delta_i \cdot SDE_i \cdot (T_i^* - t - T_{i,d}(t)) \\ & - (1-\delta_i) \cdot SDL_i \cdot (t + T_{i,d}(t) - T_i^*) \end{aligned} \tag{6}$$

where θ_i is the value of time for traveller i, $T_{i,d}(t)$ is the travel time for traveller i on day d departing in time interval t, and δ_i is a binary variable that equals 1 if traveller i arrives early and 0 otherwise. SDE_i and SDL_i are the schedule delay early and schedule delay late parameters for traveller i, respectively, which are defined as the difference between the desired arrival time T_i^* and actual arrival time $t + T_{i,d}(t)$. Note that in expressions that combine a time interval and continuous time, the interval is taken to be its mid point.

Note that using Algorithm 3.1, only the travel time for the chosen departure time can be measured by a particular traveller i. In order to estimate the travel time for all other unchosen departure time intervals in the choice set TW_i, we use the concept of fictional travellers who are assumed to choose these departure time intervals without influencing the accumulation of the network (Lamotte and Geroliminis 2015).

The choice set of feasible departure time intervals or departure time window TW_i is individual-specific and defined as $TW_i = \{t_{i,0}^{\text{dep}} - \tau, t_{i,0}^{\text{dep}} - (\tau - 1), \ldots, t_{i,0}^{\text{dep}} + \tau\}$, where τ is a parameter and $t_{i,0}^{\text{dep}}$ represents the initial departure time interval on day 0, which is computed from the preferred arrival time T_i^* and the perceived travel time on day 0. Thus, the departure time window TW_i consists of 2τ time intervals centred around the preferred departure time on day 0, $t_{i,0}^{\text{dep}}$.

Note also that research has shown that travellers' access to information affects perceived costs and convergence of the day to day model. For example, Liu et al. (2017), Liu and Szeto (2020) showed that the predicted travel cost affect system convergence and reduce variations in system performance over days. Along similar lines, Zhu et al. (2019) demonstrated that the combination of historical knowledge and experienced information yields higher levels of convergence.

3.3. Tradable credit scheme

The TCS explored in this paper extends the design proposed in Bao, Verhoef, and Koster (2019), Brands et al. (2020), Chen et al. (2020) to consider trip length-based and travel time-based tariff schemes, and has the following features: (1) The regulator will give out the same amount of credits to every traveller on each day, (2) The credits expire at the end of the day (in other words they are valid only for a single day), (3) All travellers are assumed to trade directly with the regulator so that travellers who are short of credits can buy enough credits to pay the tariff, and travellers who have excess credits can sell them. Note that such a design helps minimize transaction costs associated with information acquisition, negotiation, etc. that are present in markets with peer-to-peer trading and auctions (Brands et al. 2020), (4) The tariff profile (i.e. the time-varying tariff in credits per trip length or travel time unit) is designed by the regulator in advance and is invariant across the entire day-to-day process.

It should be pointed out that the proposed TCS scheme is not identical to a congestion pricing system in which toll revenues are equally redistributed to all users (often termed revenue refunding or revenue recycling schemes). The key difference between the two is that the TCS scheme involves a market which allows for trading of credits resulting in an endogenous credit price. This market mechanism lends the TCS scheme additional flexibility that has been shown to yield both efficiency and equity gains (see Seshadri, de Palma, and Ben-Akiva 2021; Palma et al. 2018) in the presence of demand and supply uncertainty, even when credit allocation rates are fixed (across days or time periods).

The buying and selling behaviour of individuals in the market is not explicitly modelled in this study, in contrast with Chen et al. (2020). Here, for each day, after selecting a departure time interval, a traveller will have to pay a credit tariff according to the tariff profile function $g(t)$ and her/his trip-length or travel time. If the traveller is short of credits, she/he can only buy the credits needed for the payment directly from the regulator; otherwise, she/he will sell extra credits to the regulator at the market price. We assume a credit balance of 0 at the end of the day (note that since credits expire at the end of the day, no user will leave credits unused) and we do not make an explicit assumption on the number of trades. This assumption is commonly adopted in the literature (Yang and Wang 2011; Wu et al. 2012; Brands et al. 2020). It is expected that complex and strategic buying and selling behaviours will affect credit price evolution (Dogterom, Ettema, and Dijst 2017; Chen et al. 2020) through both the credit value perception and effective demand and supply of credits. A common way to avoid the speculation is to set a specific validity period; then no one can benefit from credit stocking and banking behaviours (de Palma and Lindsey 2020). In this study, the validity period is set as one day. Note that while the end-of-day 'traveller-to-regulator' trading is beneficial in reducing transaction costs (associated with finding a buyer/seller in a market, etc.), its main disadvantage is that the budget neutrality of credits is not guaranteed in the short-term (Brands et al. 2020). A possible solution could be setting a cap for the supply of credits from the regulator. Nevertheless, as we demonstrate through numerical experiments, the proposed price adjustment mechanism in Section 3.3.1 ensures market clearance of credits at equilibrium, and hence, budget neutrality will be guaranteed eventually. Our design of the system is motivated by the need to minimize transaction costs, which can affect efficiency of the TCS (see Brands et al. 2020; Nie 2012 for a detailed discussion).

3.3.1. *Credit price evolution*

As credits are bought and sold in a market, the price is determined endogenously by credit demand-supply interactions (Yang and Wang 2011; Ye and Yang 2013). Specifically, the supply of credits is predetermined by the regulator while the demand or consumption of credits is governed by the credit tariff profile, credit price and the traffic flow generated from the executed travel plans (departure times choices) of all the agents. The difference between the supply and demand determines the credit price, i.e. the market price should increase when demand exceeds supply, and reduce when supply exceeds demand, which in turn influences the departure time choices of agents.

We assume that the credit price on day $d+1$ is based on the previous day's credit price p_d and the expected excess credit consumption Z_d, defined as the difference between the expected credit consumption and the total endowment of credits in a given day. The endowment $I_{i,d}$ is assumed to be the same for all travellers and constant across days, hence, we denote it by I hereafter, dropping the subscripts i and d. Thus,

$$Z_d = \sum_i \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_d)) \cdot g(t) \cdot L_i \cdot w - I \cdot N \tag{7}$$

$$p_{d+1} = p_d + Q(p_d, Z_d) \tag{8}$$

where the change of price is represented by function Q.

Function $Q(p, Z)$ needs to satisfy the following assumptions to guarantee a non-negative price p: (1) $\forall p > 0$, $Q(p, Z)$ is strictly increasing with $Z \in \mathbb{R}$; (2) If $p = 0$, $Q(0, Z)$ is strictly increasing with $Z \geq 0$; (3) $Q(p, 0) = 0,\ \forall p \geq 0$; $Q(0, Z) = 0,\ \forall Z \leq 0$. Then, we have

$$Q(p, Z) = 0 \Longleftrightarrow p \cdot Z = 0, \quad p \geq 0,\ Z \leq 0 \tag{9}$$

4. Solution analysis

This section builds on the work of Ye and Yang (2013), who showed that the equilibrium of market price and flow dynamics under a TCS in a route choice context is unique and stable. Note that due to the lack of a closed-form expression for the trip-based MFD model, it is not straightforward to prove that the travel cost vector $\bar{c}_d$ is monotone with respect to the departure flows, especially with heterogeneous travellers. Without this property, establishing uniqueness of the equilibrium analytically is difficult. For the same reason, we do not have an analytical form of the corresponding Jacobian matrix of the travel cost $\bar{c}_d$, which is required for proving convergence. Hence, we only discuss existence of the equilibrium and uniqueness of the credit price. Nevertheless, our numerical experiments indicate that the day to day dynamical system converges to the same equilibrium flow pattern for different initial conditions.

4.1. *Solutions to the day-to-day model*

As defined in Cantarella and Cascetta (1995), Watling (1996), a dynamic day-to-day model could be either a deterministic process (DP) or a stochastic process (SP) of evolution towards, respectively, a stochastic equilibrium and stationary probability distribution. In a DP, the fraction of users departing in a particular interval is assumed to be equal to its expected value, i.e. the choice probability, and hence, departure flows are treated as deterministic. Thus, in a DP, knowledge of the entire history of the system completely describes the current system state. This is in contrast with an SP, where flows are treated as random variables (we refer the reader to Cantarella and Cascetta 1995; Watling 1996 for a more detailed discussion). Thus, implicitly, in the DP, demand is modelled as a continuous variable (also adopted in Mariotte, Leclercq, and Laval 2017; Lamotte and Geroliminis 2018). We adopt the DP assumption, and thus, the number of travellers departing in time interval t on day d is assumed to be equal to the sum of choice probabilities of interval t across individuals.

When the proposed day-to-day model reaches equilibrium, the total number of consumed credits should not exceed the total number of endowed credits, which is given as:

$$\lim_{d\to D}\sum_{i}\sum_{t\in TW_i}\Pr_i(C_i(t))\cdot g(t)\cdot L_i\cdot w\le I\cdot N \tag{10}$$

As the credit tariff profile within the peak hour period is predetermined and constant across days, when the departure time windows for all travellers are fixed, it allows for the calculation of the theoretical minimum consumption of credits for all travellers. In this case, the credit consumption in the dynamic system will adapt to the credit endowment, as long as the feasibility condition that the credit endowment is not smaller than the minimum demand is satisfied.

4.2. Existence of the equilibrium and uniqueness of the price

For the proposed day-to-day dynamic system, at an equilibrium $\tilde{c}_{i,*}(t)$, $\forall i\in\{1,2,\ldots,N\}$, $t\in TW_i$, the vector of perceived travel costs of all travellers $\tilde{\mathbf{c}}_d$ is equal to the vector of the experienced travel cost of all travellers $\mathbf{c}_d$ (Cantarella and Cascetta 1995). This implies that when the system evolves to $c_{i,*}(t)$, $\forall i\in\{1,2,\ldots,N\}$, $t\in TW_i$, it will remain at this state for all subsequent days. By definition, for all travellers $i\in\{1,2,\ldots,N\}$, the equilibrium or fixed-point condition for this system is:

$$\begin{cases}\tilde{c}_{i,*}(t)=\omega\cdot\tilde{c}_{i,*}(t)+(1-\omega)\cdot c_{i,*}(t), & \forall t\in TW_i\\ p_*=p_*+Q(p_*,Z_*)\end{cases} \tag{11}$$

which is equivalent to

$$\begin{cases}\tilde{c}_{i,*}(t)=c_{i,*}(t), & \forall t\in TW_i\\ Q(p_*,Z_*)=0\end{cases} \tag{12}$$

We now introduce the following two theorems for the existence of an equilibrium and the uniqueness of the price.

Theorem 4.1 (Existence of the equilibrium): *If the travel demand is not excessively large so as to trigger a gridlock Lamotte and Geroliminis (2016), Miralinaghi et al. (2019), and if*

$$I\cdot N>I_{\min}\cdot N\triangleq\lim_{p\to\infty}\sum_{i}\sum_{t\in TW_i}\Pr_i(C_i(t|p))\cdot g(t)\cdot L_i\cdot w,$$

then there exists an equilibrium solution $(\tilde{c}_{i,*},p_*)$, $\forall i\in\{1,2,\ldots,N\}$, $t\in TW_i$ of the proposed dynamic system, if the cost function $c_i(t)$ is a continuous function of departure flows.

Proof: The proof is detailed in A. It is directly inspired by a proof of Ye and Yang (2013). ■

Theorem 4.2 (Uniqueness of the price): *Assume the minimum credit endowment condition is satisfied, if the total credit demand, $\sum_i\sum_{t\in TW_i}\Pr_i(C_i(t\,|\,p_1))\cdot g(t)\cdot L_i\cdot w$, is strictly decreasing with credit price, i.e.*

$$(p_1-p_2)\left(\sum_{i}\sum_{t\in TW_i}\Pr_i(C_i(t\,|\,p_1))\cdot g(t)\cdot L_i\cdot w-\sum_{i}\sum_{t\in TW_i}\Pr_i(C_i(t\,|\,p_2))\cdot g(t)\cdot L_i\cdot w\right)<0 \tag{13}$$

then the equilibrium price is unique.

Proof: The proof is detailed in A. ■

Remark 4.1: Notes on the conditions for inequality (13) to hold:

(1) There are at least two departure times with different credit charges. This condition holds as the credit tariff is time-varying.
(2) According to equation (3), for each traveller i, the probability of choosing departure time t, $\Pr_i(C_i(t))$, takes the logit form, thus the following conditions are satisfied for all $i \in \{1, 2, \ldots, N\}$:
  (i) (i)$\frac{\partial \Pr_i(C_i(t))}{\partial C_i(t)} > 0 \quad \forall t \in TW_i$
  (ii) (ii)$\frac{\partial \Pr_i(C_i(t_1))}{\partial C_i(t_2)} < 0 \quad \forall t_1 \neq t_2 \in TW_i$
  (iii) (iii)$\frac{\partial \Pr_i(C_i(t_1))}{\partial C_j(t_2)} = 0 \quad \forall t_1 \in TW_i,\ t_2 \in TW_j,\ i \neq j$
  (iv) (iv)$\frac{\partial \Pr_i(C_i(t_1))}{\partial C_i(t_2)} = \frac{\partial \Pr_i(C_i(t_2))}{\partial C_i(t_1)} \quad \forall t_1, t_2 \in TW_i$
  (v) (v)$\sum_{t' \in TW_i} \Pr_i(C_i(t')) = 1$

Condition (i) states that the partial derivative of $\Pr_i(C_i(t))$ with regard to $C_i(t)$ is positive, which means that the probability of choosing departure time interval t becomes higher if the $C_i(t)$ increases (perceived cost decreases). Condition (ii) means that for traveller i the probability of choosing a departure time interval t_1 becomes lower if for another time interval t_2, $C_i(t_2)$ increases (perceived cost of t_2 decreases). Condition (iii) states that the decision making of traveller i depends only on his/her own perceived utility. In addition, it is trivial to derive that $\frac{\partial \Pr_i(C_i(t_1))}{\partial C_i(t_2)} = \mu \cdot \Pr_i(C_i(t_1)) \cdot \Pr_i(C_i(t_2)),\ t_1 \neq t_2$, then condition iv) holds. Finally, the probabilities of all alternatives should sum up to 1, as shown in condition v).

(3) Let $\boldsymbol{J}_{\mathbf{Pr},\boldsymbol{l}}$ be the Jacobian matrix of $\mathbf{Pr}_i(\cdot)$ with respect to $\boldsymbol{C}_i(\boldsymbol{t}) = (C_i(t'),\ t' \in TW_i)$. It can be shown that $\boldsymbol{J}_{\mathbf{Pr},\boldsymbol{l}}$ is positive semidefinite as condition (2) is satisfied and the systematic utility is a non-increasing function of the corresponding perceived cost (see details in Cantarella and Cascetta 1995; Ye and Yang 2013). Accordingly, we have $\boldsymbol{g} \cdot \boldsymbol{J}_{\mathbf{Pr},\boldsymbol{l}} \cdot \boldsymbol{g}^T \geq 0$, where $\boldsymbol{g} = (L_i \cdot w \cdot g(t'),\ t' \in TW_i)$. The equality holds if and only if condition (1) is not satisfied. Thus, the following inequality holds:

$$\begin{aligned}
&(p_1 - p_2)(\mathbf{Pr}_i(\boldsymbol{C}_i(\boldsymbol{t} \mid p_1))\boldsymbol{g}^T - \mathbf{Pr}_i(\boldsymbol{C}_i(\boldsymbol{t} \mid p_2))\boldsymbol{g}^T) \\
&\quad = (p_1 - p_2)\left(\sum_{t \in TW_i} \Pr_i(C_i(t \mid p_1)) \cdot g(t) \cdot L_i \cdot w - \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_2)) \cdot g(t) \cdot L_i \cdot w\right) \\
&\quad < 0
\end{aligned} \tag{14}$$

Then, we can derive inequality (13) by summing over the population.

Inequality (13) can also be interpreted in the following manner. Under the logit form choice probability function, when the credit price goes up, the departure time interval that is associated with a relatively higher tariff is less likely to be chosen and the credit consumption will decrease.

Then, it is reasonable to propose the following hypothesis:

Hypothesis 4.1: p_* is decreasing with l in interval $(l_{\min}, l_{UE}]$, where $l_{\min}$ is defined in Section 4.1, and l_{UE} is the average credit consumption for equilibrium pattern without TCS. Besides, $p_* = 0$ when $l \geq l_{UE}$.

A similar hypothesis is analytically proved in Ye and Yang (2013), where the TCS is applied to a route choice problem using a link-based network congestion model. We will instead validate this hypothesis by simulations in Section 6.

5. Simulation-based optimization framework

Practically, the DP is solved using simulation. This is necessary because of the heterogeneity in trip lengths, which makes it difficult to derive travel times as a function of aggregate flows (Watling 1996;

Lamotte and Geroliminis 2018). In the simulations, we assume that the error terms for a given individual are perfectly correlated across days (this implies that the error term in its entirety captures a persistent agent effect). In this case, for a given draw of the error terms (which are identical across days) for the population of travellers, one can think of an equilibrium system state since once travel times converge, the choices of a given individual remain stable across days. Average flows and performance measures across multiple simulation replications (draws of the error terms for the population of travellers) are used to approximate the solution of the DP. The average flows (and performance measures) are thus associated with an error of estimation, which can be made arbitrarily low with a suitably large number of replications (Watling 1996).

The social welfare per capita W at the equilibrium state is used to measure the performance of scenarios with and without TCS. First, in the no tariff case (or NTE, the scenario without TCS), the social welfare per capita W_{NTE} is the average consumer surplus (CS) per traveller, i.e.the average of observed travel utilities $U_{i,d}(t_{i,d}^{\mathrm{dep}})$, including travel time cost, schedule delay penalty and the random component, which is written as

$$\begin{aligned} W_{\mathrm{NTE}} &= \frac{\mathrm{CS}}{N} = \frac{1}{N}\sum_{i=1}^{N} U_{i,d}(t_{i,d}^{\mathrm{dep}}) \\ &= \frac{1}{N}\sum_{i=1}^{N}[\tilde{c}_{i,d}(t_{i,d}^{\mathrm{dep}}) + \epsilon_i(t_{i,d}^{\mathrm{dep}})] \end{aligned} \tag{15}$$

where d is the day when the system reaches an equilibrium.

For the TCS scenario, the social welfare is computed as the average of the CS, the travellers' revenue (TR) from unused credits, the regulator revenue (RR) from credit tariff collection, the regulator cost (RC) from buying unused credits, and the value of the used credits from travel endowment (TE). Let $\phi_{i,d}(t_{i,d}^{\mathrm{dep}})$ represent the number of credits sold by traveller i on day d, $\psi_{i,d}(t_{i,d}^{\mathrm{dep}})$ denote the number of credits bought by traveller i on day d, and $g_{i,d}^{e}(t_{i,d}^{\mathrm{dep}})$ denote the number of credits used to travel by traveller i on day d from the endowment. Note that $\phi_{i,d}(t_{i,d}^{\mathrm{dep}}) > 0$ when traveller i uses fewer credits than the endowment, i.e. $g_{i,d}^{e}(t_{i,d}^{\mathrm{dep}}) < l$, and $\phi_{i,d}(t_{i,d}^{\mathrm{dep}}) = 0$ when $g_{i,d}^{e}(t_{i,d}^{\mathrm{dep}}) \geq l$, while $\psi_{i,d}(t_{i,d}^{\mathrm{dep}})$ takes a contrary relationship. Thus, we can compute the above social welfare components: $\mathrm{TR} = \sum_{i=1}^{N} \phi_{i,d}(t_{i,d}^{\mathrm{dep}}) \cdot p_d$, $\mathrm{RR} = \sum_{i=1}^{N} \psi_{i,d}(t_{i,d}^{\mathrm{dep}}) \cdot p_d$, $\mathrm{RC} = \sum_{i=1}^{N} \phi_{i,d}(t_{i,d}^{\mathrm{dep}}) \cdot p_d$, $\mathrm{TE} = \sum_{i=1}^{N} g_{i,d}^{e}(t_{i,d}^{\mathrm{dep}}) \cdot p_d$. In addition, the paid credits by traveller i can be considered as the sum of two components, the credits from endowment and credits bought from the regulator, i.e. $g(t) \cdot L_i \cdot w = \psi_{i,d}(t_{i,d}^{\mathrm{dep}}) + g_{i,d}^{e}(t_{i,d}^{\mathrm{dep}})$. Based on the considerations above, the social welfare per capita W_{TCS} is calculated as follows,

$$\begin{aligned} W_{\mathrm{TCS}} &= \frac{1}{N}[\mathrm{CS} + \mathrm{TR} + \mathrm{RR} - \mathrm{RC} + \mathrm{TE}] \\ &= \frac{1}{N}\sum_{i=1}^{N}[\tilde{c}_{i,d}(t_{i,d}^{\mathrm{dep}}) - p_d \cdot g(t_{i,d}^{\mathrm{dep}}) \cdot L_i \cdot w + \epsilon_i(t_{i,d}^{\mathrm{dep}})] \\ &\quad + \frac{1}{N}\left[\sum_{i=1}^{N} \phi_{i,d}(t_{i,d}^{\mathrm{dep}}) \cdot p_d + \sum_{i=1}^{N} \psi_{i,d}(t_{i,d}^{\mathrm{dep}}) \cdot p_d \right. \\ &\quad \left. - \sum_{i=1}^{N} \phi_{i,d}(t_{i,d}^{\mathrm{dep}}) \cdot p_d + \sum_{i=1}^{N} g_{i,d}^{e}(t_{i,d}^{\mathrm{dep}}) \cdot p_d \right] \\ &= \frac{1}{N}\sum_{i=1}^{N}[\tilde{c}_{i,d}(t_{i,d}^{\mathrm{dep}}) + \epsilon_i(t_{i,d}^{\mathrm{dep}})] \end{aligned} \tag{16}$$

Note that the TCS welfare measure is equivalent to the NTE case and equals the combination of travel time cost and schedule delay penalty plus the random utility component.

Our simulation captures the system model presented in Section 3 including detailed traveller and regulator states in the single-reservoir network under the designed market conditions.[1] A simulation-based optimization problem is then formulated to determine the optimal credit tariff scheme which leads to the maximum social welfare W_{TCS}. Note that gradient-based algorithms will be computationally expensive for our optimization problem because they involve numerical computation of derivatives for an objective that has no closed form and is the outcome of a complex simulation. Furthermore, the simulation framework can be a time-consuming process when the number of scenarios is large.

Based on the considerations above, the Bayesian optimization (BO) approach is adopted, which can approximate well the mapping from the input credit tariff profile to the output simulation-based objective function using few evaluations. In our context, the input consists of parameters of the credit tariff profile, which is parameterized by a Gaussian function, containing three parameters: mean, variance and amplitude. A pair of the input and output is defined as a sample. A BO framework essentially consists of two main steps (Frazier 2018): (1) update a Bayesian statistical model that approximates a complex map from the input (i.e. the credit tariff profile parameters) to the output (i.e. the social welfare W_{TCS}); (2) determine the next input by optimizing an acquisition function. The first step uses a Gaussian Process (GP) to approximate the joint distribution of social welfare and tariff profile parameters and updates the GP with new samples generated by the second step. BO is shown to be effective in optimizing time-varying road pricing mechanisms (parameterized by Gaussian mixture functions) in Liu, Jiang, and Azevedo (2021). Further, as the results in Section 6 show, it performs well for the problem at hand.

6. Numerical experiments

This section begins by introducing the simulation settings. Next, we present the results of (1) the day-to-day model properties and convergence (Section 6.2) under a given credit tariff profile; (2) the comparison between the optimized TCS against the NTE (Section 6.3); (3) the comparison between the optimized TCS and time-of-day pricing (or CP) (Section 6.4); and finally (4) the comparison among alternative types of credit tariff profiles and credit schemes (Section 6.5).

6.1. Experiment settings

The settings are presented in Table 2.

The experiments consider a single-reservoir network with a capacity of 4500 travellers, with the speed function adopted from Lamotte and Geroliminis (2018) and other parameters (trip length, time window parameters and time interval) used in Yildirimoglu and Ramezani (2020). The MFD is also characterized by the critical value of the accumulation, denoted by n_{cr}, that can be computed according to the adopted speed function and network capacity n_{jam} as $n_{cr} = n_{jam}/3$. In the experiments, we assume $n_{jam} = 4500$, and hence, $n_{cr} = 1500$ travellers. Two demand scenarios, moderate congestion ($N_1 = 3700$ travellers) and high congestion ($N_2 = 4500$ travellers), are considered, where N_1 makes the accumulation at no tariff equilibrium just exceed the critical value of the accumulation n_{cr}, and N_2 is the largest possible value that will not trigger gridlock. The profiles of the accumulation in the two scenarios are shown in Section 6.2. The initial departure time $t_{i,0}^{dep}$ is generated from a truncated Gaussian distribution (note that hereafter $t_{i,0}^{dep}$ refers to a specific departure time rather than an interval). The desired arrival time T_i^* is then computed as $t_{i,0}^{dep} + L_i/v_f$ for all travellers, which is also normally distributed. Additionally, heterogeneous travellers are captured by drawing their trip lengths from a truncated Gaussian distribution and schedule delay penalties from a lognormal distribution,

Table 2. Numerical settings.

Parameters (unit)	Specification
Credit endowment (credit)	$I = 5$
Demand (traveller)	$N_1 = 3700, N_2 = 4500$
Initial departure time (min)	$t_{i,0}^{\text{dep}} = \mathcal{N}(80, 18), t_{i,0}^{\text{dep}} \in (20, 150]$
Trip length (m)	$L_i = 4600 + \mathcal{N}(0, (0.2 \times 4600)^2), L_i > 0$
Trip length scale factor	$w = 2 \times 10^{-4}$
Schedule delay penalty (DKK/min)	$SDE_i \sim \text{Lognormal}(-1.9, 0.2^2) \times 4$
	$SDL_i = SDE_i \times e^1$
Value of time (DKK/min)	$\theta_i = SDE_i \times e^{0.5}$
Time window parameter	$\tau = 30$
Time interval (min)	$\Delta t = 1$
Network capacity (vehicle)	$n_{\text{jam}} = 4500$
Free flow speed (m/s)	$v_f = 9.78$
Speed function (m/s)	$V(n) = v_f(1 - \frac{n}{n_{\text{jam}}})^2$
Learning parameter	$\omega = 0.9$
Function $Q(p, Z)$	$Q(p, Z) = kZ$, if $p > 0$; $Q(p, Z) = \max\{0, kZ\}$, if $p = 0$, where $k = 2 \times 10^{-5}$
Tariff profile function	$g(t \mid A, \xi, \sigma) = A \times e^{\frac{-(t-\xi)^2}{2\sigma^2}}$

respectively. The mean value of the generated value of time is 1 (DKK/min) (Fosgerau, Hjorth, and Lyk-Jensen 2007) and the standard deviation is 0.2 (DKK/min). The schedule delay penalties are assumed to vary proportionally to the value of time and satisfy the widely used trip timing preferences relationship, i.e. $SDE_i < \theta_i < SDL_i$ (Vickrey 1969). In both demand scenarios, the same distributions are used while all other parameters are constant (see Table 2). To set up a time-varying credit charging scheme, the tariff function is assumed to take the form of a (positive) Gaussian function, which is controlled by three parameters, mean ξ, variance σ and amplitude A. Without loss of generality, the method described below can be extended to a Gaussian mixture function to allow for asymmetric and more flexible tariff profiles (Liu, Jiang, and Azevedo 2021). Moreover, alternative step tariff and triangular tariff profiles are also tested under the proposed framework in Section 6.5.1. Finally, several additional points are noteworthy:

- in the simulations, the excess credit consumption Z_d is computed using the observed credit consumption on day d: $Z_d = \sum_i g(t_{i,d}^{\text{dep}}) \cdot L_i \cdot w - I \cdot N$; the expected credit consumption expression in Equation (7) allows for a theoretical analysis of model properties;
- for realistic behaviour we show the results under a feasible time window $TW_{i,d}$ that changes from day-to-day based on the updated perceived travel times. $TW_{i,d}$ is still centred around the preferred arrival time, but lagged by the traveller's perceived travel time for day d, i.e.: $TW_{i,d} = \{T_i^* - T_{i,d-1}(t_{i,d-1}^{\text{dep}}) - \tau \cdot \Delta t, T_i^* - T_{i,d-1}(t_{i,d-1}^{\text{dep}}) - (\tau - 1) \cdot \Delta t, \ldots, T_i^* - T_{i,d-1}(t_{i,d-1}^{\text{dep}}) + \tau \cdot \Delta t\}$. The simulations under a fixed time window TW_i also confirmed convergence properties of the day-to-day dynamic model and uniqueness of the credit price.
- the properties demonstrated in Section 6.2 are for a single draw of the choice model error terms. Thus, an error of estimation can be computed for each performance measure from multiple simulations with different draws of the error terms for the population. Given the size of the population, our results indicated that this error of estimation is small for all performance measures of interest, and hence, for computational reasons we report results of a single draw.

6.2. Day-to-day evolution process

6.2.1. Day-to-day process without TCS

In this subsection, we first focus on examining the equilibrium properties of the day-to-day dynamic model without the TCS in both the moderate congestion and high congestion scenarios. Theoretically, when the day-to-day evolution reaches an equilibrium, the vector of the perceived travel cost of all travellers, $\tilde{\mathbf{c}}_d$, should be equal to the vector of the experienced travel cost of all travellers, $\mathbf{c}_d$. Thus, the inconsistency between $\tilde{\boldsymbol{c}}_d$ and $\boldsymbol{c}_d$ is used as a measure of convergence towards the equilibrium. Specifically, the L1 norm of the difference between them divided by the number of travellers N, i.e. $\| \tilde{\boldsymbol{c}}_d - \boldsymbol{c}_d \|_1 /N$, is computed to represent the inconsistency. Further, a normalized version of this gap is given by $\| (\tilde{\boldsymbol{c}}_d - \boldsymbol{c}_d/\tilde{\boldsymbol{c}}_d \|_1 \times 100\%$.

Figure 1(a) presents the convergence of the perceived travel cost of all travellers $\tilde{\boldsymbol{c}}_d$. It is found that the inconsistency becomes stable and close to 0 after 80 days, with a gap around 0.05%, implying that the day-to-day evolution reaches an equilibrium state. Note that, as travellers do not have perceived cost and use a predetermined departure time on day 0, the inconsistency in the perceived travel times is computed only from day 1.[2] Figure 1(b) illustrates the evolution process of the average consumer surplus (CS) per capita across days, and Figure 1(c) shows the evolution process of the social welfare per capita. These two plots are identical since the social welfare equals the consumer surplus in the no tariff case. Figure 1(d) plots the evolution process of the average travel time cost across days. Figure 1(e) demonstrates the departure rates for every 5-minute interval on different days, and Figure 1(f) depicts the states of accumulation on different days, where the accumulation on day 85 overlaps with that of day 99. This also indicates that an equilibrium state is reached. In these two plots, the curves of day 0 represent the initial generated state specified in Section 6.1.

We also investigate convergence properties in the moderate congestion scenario although the plots are omitted here. The results demonstrate that with the same learning parameter $\omega = 0.9$, the perceived generalized cost reaches a stable state. In comparison, we observe a higher consumer surplus, higher social welfare and lower travel time cost at the equilibrium state due to less congestion. In addition, there is a lower peak of departure rates compared to the high congestion scenario, and the peak accumulation at the equilibrium state is also lower than the one in Figure 1(f) but higher than the critical value n_{cr}.

6.2.2. Day-to-day evolution with TCS

In this subsection, we present the convergence and equilibrium properties mentioned in Section 4.2 of the day-to-day dynamic model with the TCS for the moderate and high congestion scenarios. It is worth noting that the equilibrium states of the base cases are used as the starting states (i.e. day 0) of the TCS cases for both demand scenarios. The parameters of the tariff profile function are arbitrarily set to satisfy the condition in Theorem 4.1: $A = 11$, $\xi = 18$ and $\sigma = 80$.

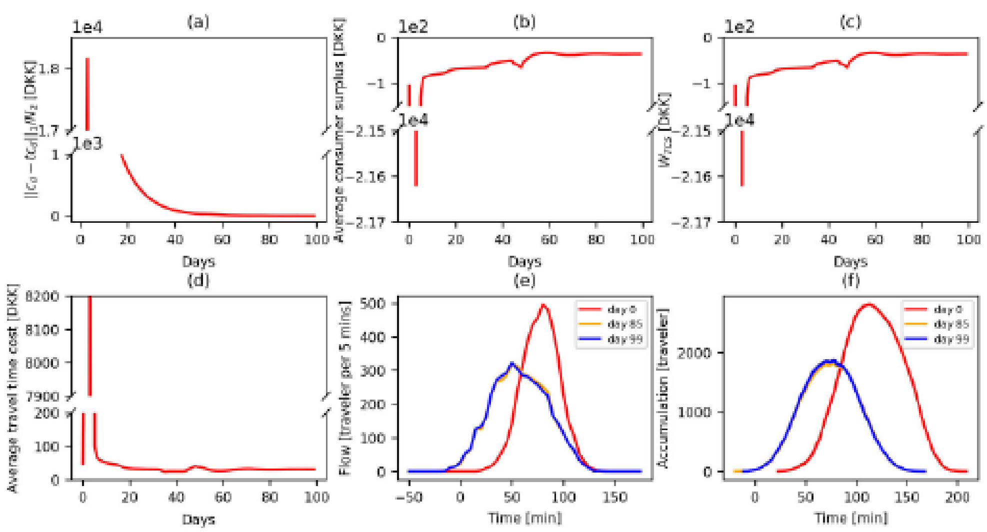

Figure 1. The evolution process in the high congestion scenario without TCS.

According to Figure 2(a–d), it can be seen that the day-to-day evolution under a given TCS converges to an acceptable degree, where the system reaches an equilibrium after 60 days, with a gap of 0.04%. Note that the social welfare W_{TCS} in Figure 2(c) is smaller than that in the no tariff case. This is because the credit tariff scheme is not optimal but arbitrarily specified, shown as the gray dashed line in Figure 2(f). The results in Figure 2(e,f) also support this observation that the peak departure rate and accumulation are not reduced compared to day 0. In addition, Figure 2(g) displays the evolution of the credit price, which goes up from 0 (DKK) to the equilibrium price 1.1 (DKK). This process is consistent with the evolution of the credit transactions shown in Figure 2(h). Initially, the number of bought credits (i.e. the credit demand) is much higher than the number of sold credits (i.e. the credit supply), implying that the market is short of credits and travellers need to buy extra credits from the regulator. Thus, the credit price increases. After perceiving a high travel cost due to the relatively expensive credit payment, travellers adjust their departure time to avoid being charged excessively, leading to a smaller credit demand and consequently slower credit price increment. Finally, at the equilibrium state, the credit supply nearly is equal to the credit demand and the corresponding credit price becomes stable. Figure 2(i) illustrates the evolution process of the average credit payment, which is the value of the used credits (or the tariff payment). Note that this tariff payment is exactly the difference between the consumer surplus and the social welfare, as derived in Equation (16).

When applying the same tariff profile to the moderate congestion scenario, a similar pattern is observed, the detailed plots are once again omitted here.

Next, we examine the uniqueness of the credit price by setting different initial prices and price adjustment parameter k for the high congestion scenario. Similar patterns, not presented here, are observed in these tests for the moderate congestion scenario. Let $k \in \{1 \times 10^{-5}, 2 \times 10^{-5}, 4 \times 10^{-5}\}$; we then examine the influence of the price adjustment parameter on the price evolution. The results are shown in Figure 3(a). It can be observed that when k becomes larger, the credit market shows a greater reaction to the difference between the credit demand and supply, leading to a higher peak value and rapider change in price.

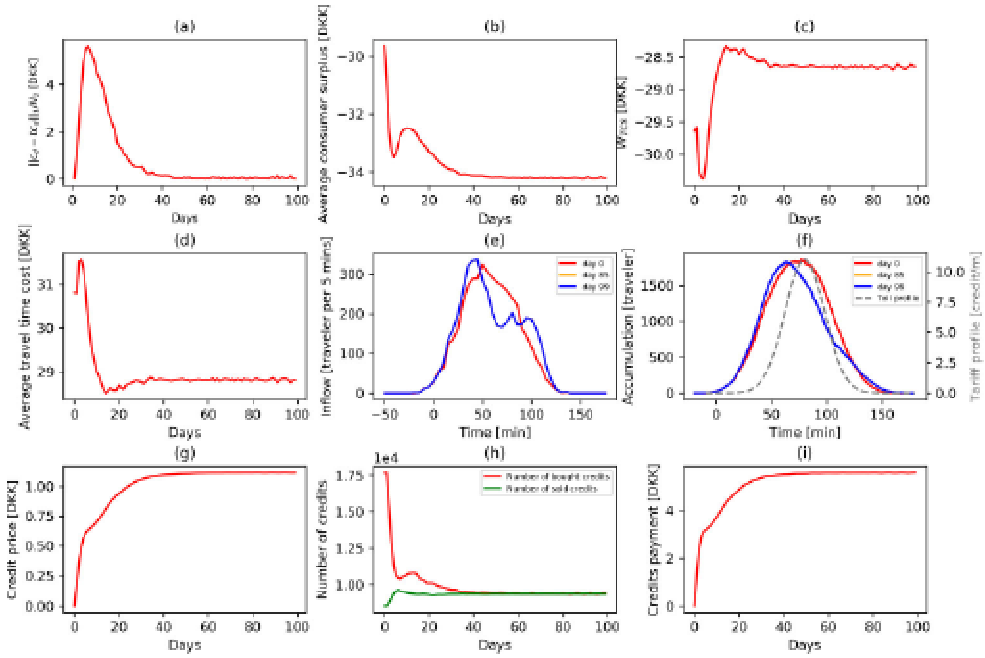

Figure 2. The evolution process in the high congestion scenario with TCS.

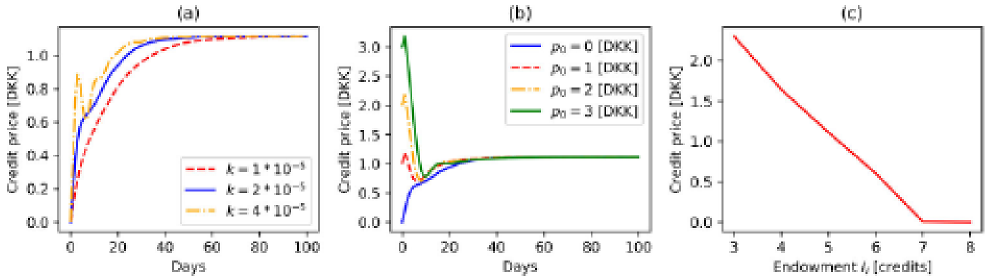

Figure 3. The evolution of credit price with (a) different price adjustment parameters and (b) different initial prices; (c) The equilibrium credit price with different credit endowment.

In addition, Figure 3(b) presents the credit price evolution with different initial prices $p_0 = 0, 1, 2, 3$ (DKK). It appears that though the evolution processes are different and start from different initial values, the credit price eventually converges to the same level.

Moreover, we validate Hypothesis 4.1 by varying the credit endowment.[3] Under the given credit tariff profile, the minimum possible endowment $l_{\min} = 1.61$ and $l_{UE} = 7.01$. Note the credit endowment is identical among all the travellers and keeps constant across days. Let $l = \{3, 4, \ldots, 7, 8\}$, the results are demonstrated in Figure 3(c). It is clear that the credit price monotonically decreases with l and reaches 0 when l exceeds $l_{UE} = 7.01$.

6.3. *Bayesian optimization results*

In this subsection, we present the performance of the TCS under an optimized tariff profile obtained from the developed BO method for both demand scenarios.

The domains of the tariff profile function parameters are set as $A \in [2, 20]$ (unit: credit/meter), $\xi \in [30, 90]$ and $\sigma \in [10, 70]$. The initial samples are generated via the Latin Hypercube Sampling (LHS) (McKay, Beckman, and Conover 2000) consisting of 30 points. For each sample input point, we run the day-to-day simulation and compute the travel time cost and schedule delay cost using the average value of the last 10 days after the equilibrium. The social welfare per capita can therefore be calculated using Equation (16). Figure 4 shows the evolution process for the high congestion scenario with an optimized tariff profile, the parameters of which are $A = 4.8$, $\xi = 67.3$ and $\sigma = 33.5$. It can be seen that the system becomes stable after 80 days, and converges to a credit price of 3.05 (DKK). Compared to the no tariff case, the departure rate curve is flattened, and the peak accumulation is reduced from 1867 to 1053 (traveller], which overall, leads to an improvement in the social welfare, raising from −29.8 (DKK) per capita to −23.8 (DKK) by 6.0 (DKK). We observe from Figure 4(e ,f) that more travellers are departing later under the optimized tariff profile compared to the no tariff case, in order to avoid high credit tariffs. However, due to the highly reduced travel time, there are 70.8% travellers arriving at their destinations earlier than their desired arrival time under the optimized tariff profile, while 67.7% in the no tariff case.

Similar patterns are also observed in the moderate congestion scenario, although the plots are omitted here. The daily average values of measurement variables for the last 10 days after the equilibrium for the no tariff case, moderate congestion and high congestion scenario are listed in Table 3, where the second to eighth columns are the daily average monetary travel time cost, schedule delay cost, random utility, consumer surplus, social welfare per capita, tariff payment and credit price, respectively. The 'mean' and 'std.dev' rows in Table 3 represent the mean values and standard deviation across the last 10 days after the equilibrium for each scenario, respectively. It can be seen that, in the moderate congestion scenario, the travel time cost per capita is reduced from 26.7 (DKK) to 20.9 (DKK) by 21.8%, while the schedule delay cost is increased from 3.3 (DKK) to 6.2 (DKK) by 90.5%, and overall social welfare per capita is improved by 3.2 (DKK). In the high congestion scenario, the average travel

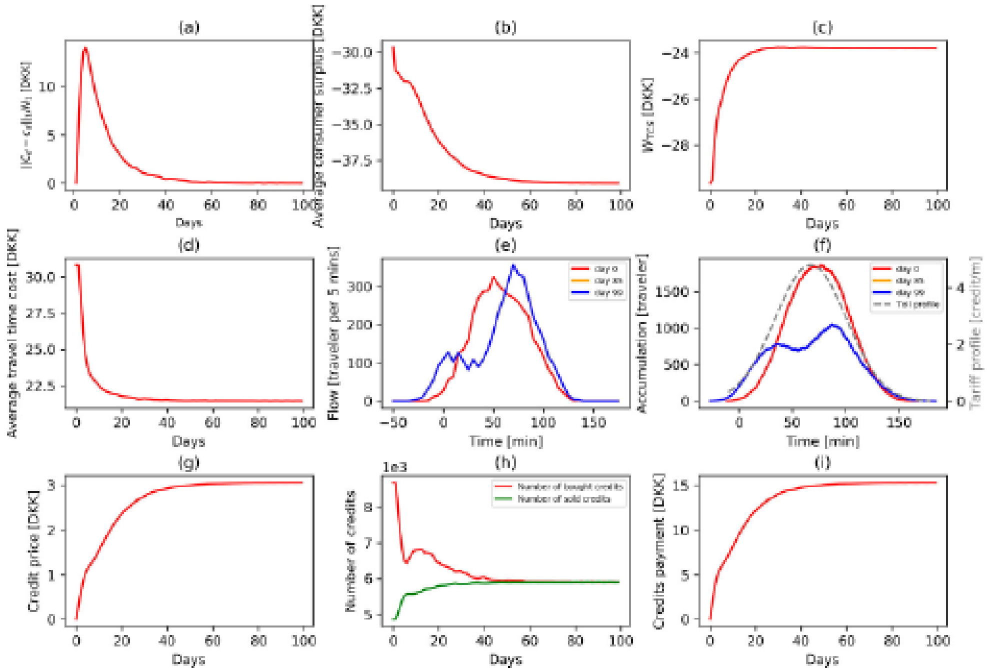

Figure 4. The evolution process in the high congestion scenario with optimized TCS.

Table 3. Comparisons among no tariff case, optimized TCS and CP.

Unit (DKK/cap)	Travel time cost	Schedule delay	Random utility	Consumer surplus	Social welfare	Tariff payment	Credit price (DKK)
			No tariff case (*N*1)				
Mean	−26.7	−3.3	5.5	−24.5	−24.5	–	–
Std. dev.	0.020	0.004	0.004	0.013	0.013	–	–
			Optimized TCS (*N*1)				
Mean	−20.9	−6.2	5.8	−34.0	−21.3	12.8	2.6
Std. dev.	0.005	0.005	0.001	0.002	0.001	0.003	0.001
			Optimized CP (*N*1)				
Mean	−20.8	−6.3	5.8	−34.2	−21.3	12.9	–
Std. dev.	0.001	0.001	0.001	0.002	0.002	0.002	–
			No tariff case (*N*2)				
Mean	−31.0	−4.7	5.9	−29.8	−29.8	–	–
Std. dev.	0.080	0.050	0.010	0.120	0.120	–	–
			Optimized TCS (*N*2)				
Mean	−21.5	−8.3	6.0	−39.1	−23.8	15.3	3.1
Std. dev.	0.005	0.006	0.001	0.003	0.001	0.004	0.001
			Optimized CP (*N*2)				
Mean	−21.4	−8.4	5.9	−39.2	−23.8	15.4	–
Std. dev.	0.004	0.006	0.001	0.003	0.002	0.005	–

time cost is reduced by 30.7%, the schedule delay cost is increased by 76.5%, and the social welfare per capita is improved by 6.0 (DKK). As expected, when congestion is more severe, the improvement in terms of social welfare by imposing the optimized TCS is higher.

6.4. Comparison with time of day pricing

Under the time of day pricing, travellers' behaviour is simulated based on the same travel behaviour model described in Section 3.2. The time of day pricing also uses a distance-based tariff, which is paid

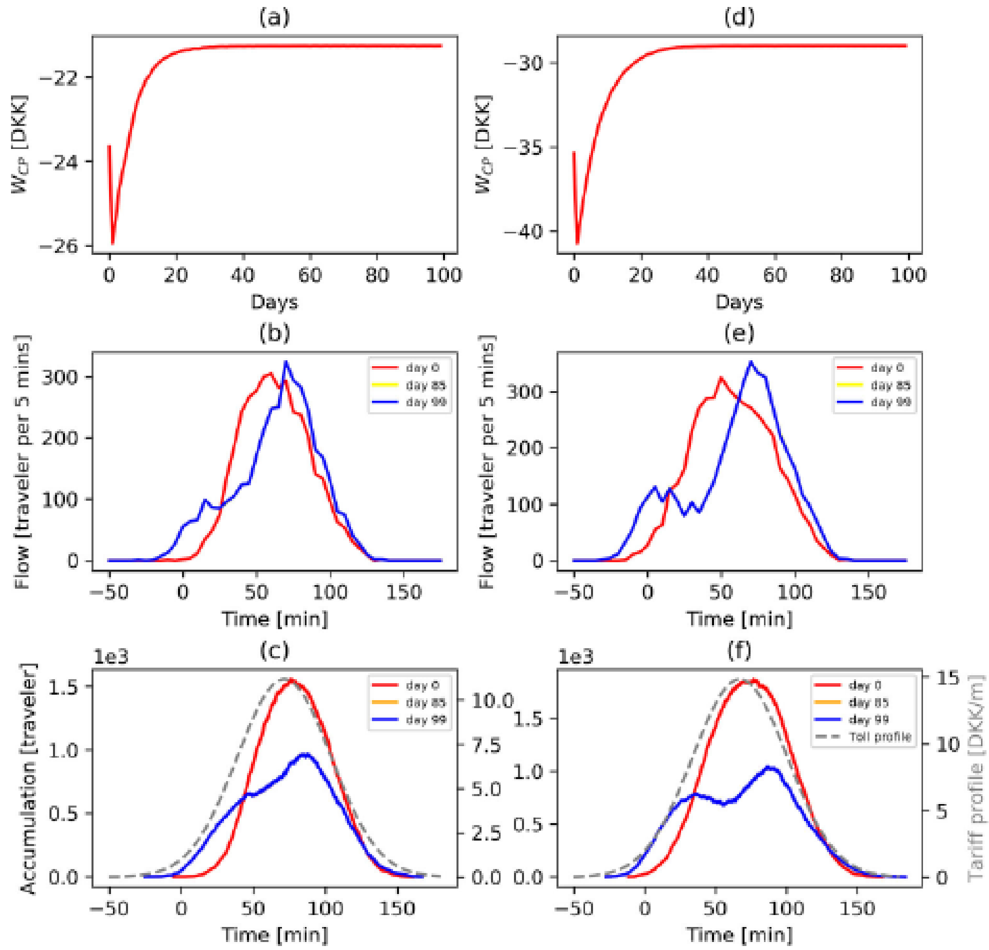

Figure 5. The evolution process in the moderate (left) and high (right) congestion scenarios with optimized tariff profiles.

at the beginning of the trip. The only difference is that the tariff is set in DKK instead of credits. Thus, the experienced (or estimated) travel cost $c^{CP}_{i,d}(t)$ for traveller i on day d departing at time t can be written as $c^{CP}_{i,d}(t) = -\theta_i \cdot T_{i,d}(t) - \delta_i \cdot SDE_i \cdot (T_i^* - t - T_{i,d}(t)) - (1-\delta_i) \cdot SDL_i \cdot (t + T_{i,d}(t) - T_i^*)$, and $C^{CP}_{i,d}(t) = \tilde{c}^{CP}_{i,d}(t) - g^{CP}(t) \cdot L_i \cdot w$, where $g^{CP}(t)$ is the tariff in dollars at time t.

Similar to Section 5, we define the social welfare W_{CP} of congestion pricing, which consists of consumer surplus and regulator revenue, as $W_{CP} = \frac{1}{N}[CS + RR] = \frac{1}{N}\sum_{i=1}^{N}[\tilde{c}_{i,d}(t^{dep}_{i,d}) + \epsilon_i(t^{dep}_{i,d})]$.

The domains of the tariff profile function parameter A are the same as before. The BO is used to optimize the tariff profile, utilizing the LHS sampling method to generate initial points. Figure 5(a–c) show the evolution process in the moderate congestion scenario with optimized tariff profile, the parameters of which are $A = 11.2$, $\xi = 71.5$ and $\sigma = 32.6$. It can be seen from the results presented in Figure 5 and Table 3, the time of day pricing reaches an equilibrium of social welfare and flow pattern close to that of TCS case, with a higher tariff rate. Figure 5(d–f) present the evolution process of high congestion scenario, with tariff profile parameters $A = 14.8$, $\xi = 67.3$ and $\sigma = 33.5$. Combined with Figure 4 and Table 3, we conclude that by optimizing the tariff profiles, TCS and CP have the same performance in terms of social welfare and flow pattern at equilibrium.

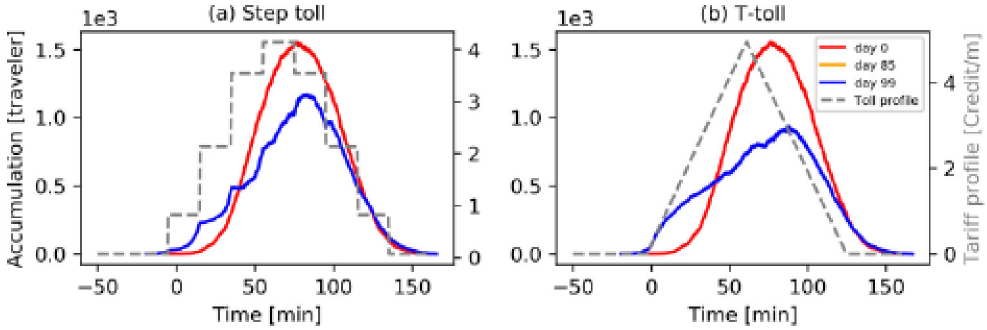

Figure 6. Accumulation for the optimized step tariff and T-toll profiles.

Table 4. Comparisons among the optimized Gaussian tariff, T-toll and step tariff TCS.

Unit: (DKK/cap)	Travel time cost	Schedule delay	Random utility	Consumer surplus	Social welfare	Tariff payment	Credit price (DKK)
			No tariff case ($N1$)				
Mean	−26.7	−3.3	5.5	−24.5	−24.5	–	–
Std. dev.	0.020	0.004	0.004	0.013	0.013	–	–
			Optimized Gaussian tariff ($N1$)				
Mean	−20.9	−6.2	5.8	−34.0	−21.3	12.8	2.6
Std. dev.	0.005	0.005	0.001	0.002	0.001	0.003	0.001
			Optimized T-toll ($N1$)				
Mean	−20.6	−6.6	5.8	−33.5	−21.4	12.1	2.4
Std. dev.	0.001	0.001	0.000	0.001	0.001	0.001	0.000
			Optimized step tariff ($N1$)				
Mean	−22.1	−5.2	5.4	−31.8	−21.8	10.0	2.0
Std. dev.	0.017	0.006	0.002	0.015	0.010	0.006	0.001

6.5. Comparison of alternative credit tariff mechanisms

6.5.1. Comparison between different credit tariff profiles

Since the performance of the TCS ultimately depends on the choice of the tariff profile functional form, we compare the performance of the above Gaussian profile with two alternatives: a step tariff and a triangular tariff (T-toll) profile. Both are inspired by Zheng, Rérat, and Geroliminis (2016) and Daganzo and Lehe (2015), respectively, with the caveat that the comparison is limited to the scenario of $N = 3700$ and the symmetric profile assumption is made as before. For the step tariff, there are six parameters including tariff charges of five steps and a position parameter indicating the center of the symmetric tariff profile. For the T-toll and similarly to the Gaussian case, there are three parameters including the height, length of the base and, again, a position parameter.

Under both the step tariff and T-toll, we observe a similar convergence pattern. Therefore, only the accumulations are shown in Figure 6. Table 4 summarizes the detailed information of all the performance measures considered. We relied on 200 Bayesian Optimization iterations for the step-tariff case since there are six parameters to optimize while the smaller number of parameters for the two other cases relied on 40 iterations only. It was found that Gaussian tariff and T-toll have a similar performance in terms of welfare. Interestingly, the *CS* and time-related performance measures show differences between the Gaussian and the T-toll, with a lower travel time, higher schedule delay and lower credit value flow in the latter. Such credit market differences may justify a careful look into efficient market design under more realistic market-related behaviours and equity aspects in TCS-related policy decisions.

Nevertheless, both Gaussian and T-Toll outperformed the step tariff. This gap could naturally be reduced with a higher number of steps in the step tariff functional form.

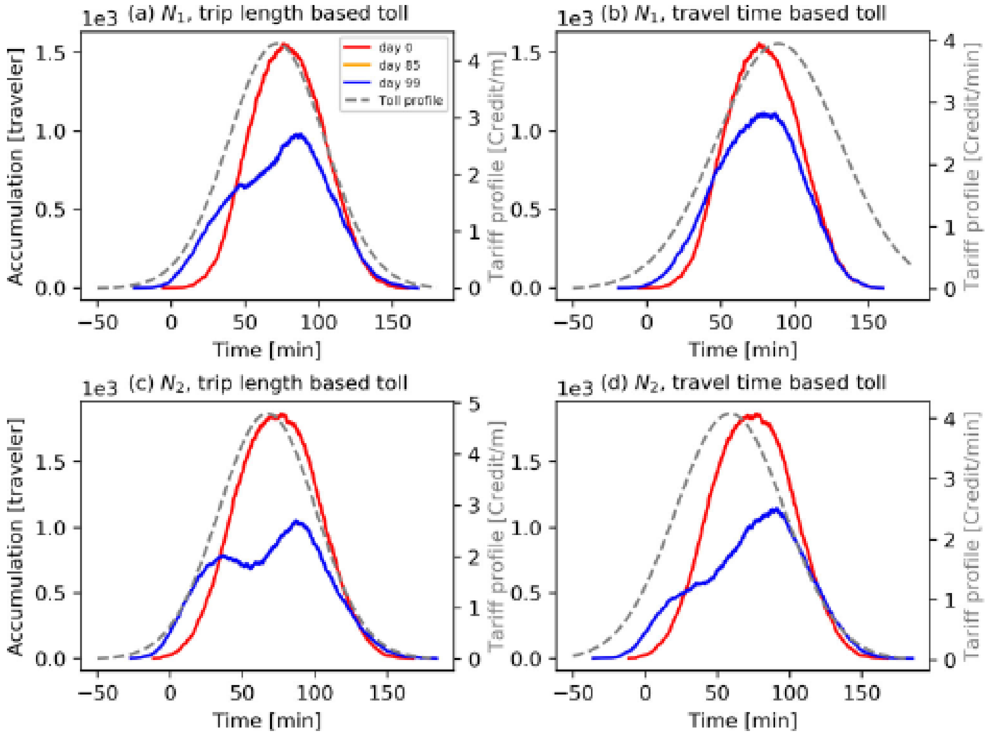

Figure 7. The accumulations under different optimized tariff profiles.

6.5.2. Comparison among trip-length, travel-time based and standard area-based credit tariff mechanisms

One could formulate a credit tariff as a function of travel time instead of trip length, allowing a direct accounting of the contribution to congestion. Here, we consider a travel time-based tariff and compare it with the already presented trip length-based tariff. We simply change the term $p_d \cdot g(t) \cdot L_i \cdot w$ in Equation (4) to $p_d \cdot g(t) \cdot T_{i,d}(t) \cdot w'$, where $w' = 0.08$ and the other parameters are kept the same. Note that for the trip length-based tariff, the unit of $g(t)$ was (Credit/meter) while for the new travel time-based tariff, the unit of $g(t)$ is (Credit/minute). In addition, we also tested the (trip agnostic) standard area-based (zonal) mechanism for both scenarios, wherein the unit of $g(t)$ is (Credit]. Figure 7 presents the accumulations for the different demand scenarios considered ($N_1 = 3700$ and $N_2 = 4500$) for the optimized two types credit tariff mechanisms. The detailed performance measures are summarized in Table 5.

It can be seen that the three types of tariff converge to an acceptable degree. In both demand scenarios, the peak accumulation and departure rate with the travel time-based tariff are close to the ones with trip length-based tariff. There is a slightly better welfare performance for the travel time-based tariff only under the high congestion scenario, thanks to small benefits in schedule delay. Indeed, the travel time-based tariff may reflect the contribution to the congestion more directly. Note that, even with a fixed trip length, when a traveller considers changing departure time, the associated credit tariff payment will also change. Here, the direct contribution to congestion is taken care of by the optimized fixed tariff rate and the credit market. Yet, from the traveller's perspective, when a traveller's evaluates departure times, the trip length-based tariff allows for a clear information on credit payment while the travel time needed for the user's travel time-based tariff estimation is uncertain in practice. This falls under information provision and perception modelling research which, while related, is outside of the scope of this manuscript. In addition, both trip-based TCS show the superiority compared to standard (trip agnostic) area-based TCS.

Table 5. Comparisons among the optimized trip length-based TCS, optimized travel time-based TCS and optimized standard area-based TCS.

Unit: (DKK/cap)	Travel time cost	Schedule delay	Random utility	Consumer surplus	Social welfare	Tariff payment	Credit price (DKK)
			No tariff case (*N*1)				
Mean	−26.7	−3.3	5.5	−24.5	−24.5	–	–
Std. dev.	0.020	0.004	0.004	0.013	0.013	–	–
			Optimized standard area-based TCS (*N*1)				
Mean	−21.2	−5.8	6.0	−46.6	−22.0	24.5	4.9
Std. dev.	0.010	0.007	0.001	0.039	0.004	0.040	0.010
			Optimized trip length-based TCS (*N*1)				
Mean	−20.9	−6.2	5.8	−34.0	−21.3	12.8	2.6
Std. dev.	0.005	0.005	0.001	0.002	0.001	0.003	0.001
			Optimized travel time-based TCS (*N*1)				
Mean	−22.5	−5.0	5.9	−32.1	−21.6	10.4	2.1
Std. dev.	0.010	0.009	0.001	0.001	0.005	0.006	0.001
			No tariff case (*N*2)				
Mean	−31.0	−4.7	5.9	−29.8	−29.8	–	–
Std. dev.	0.080	0.050	0.010	0.120	0.120	–	–
			Optimized standard area-based TCS (*N*2)				
Mean	−24.1	−7.2	6.3	−38.8	−25.0	13.8	2.8
Std. dev.	0.005	0.002	0.001	0.005	0.003	0.002	0.000
			Optimized trip length-based TCS (*N*2)				
Mean	−21.5	−8.3	6.0	−39.1	−23.8	15.3	3.1
Std. dev.	0.005	0.006	0.001	0.003	0.001	0.004	0.001
			Optimized travel time-based TCS (*N*2)				
Mean	−21.9	−7.7	5.9	−38.0	−23.7	14.3	2.9
Std. dev.	0.003	0.001	0.001	0.005	0.003	0.002	0.001

7. Conclusions

This paper proposes a tradable credit scheme (TCS) to manage urban transport network congestion considering the day-to-day evolution of traffic flow. The properties of the TCS were examined via both analytical and simulation approaches. The properties were analysed in the light of recent generic TCS formulations, namely Bao, Verhoef, and Koster (2019), Brands et al. (2020), applied to the case of area-based road traffic control, and extended for heterogeneous trip lengths, i.e. a distance-based tariff instead of access-based tariff. The TCS here at stake relies on a daily fixed credit price, a time-of-day varying tariff charging (or credit tariff), a morning commute control policy and heterogeneous decision makers (in terms of choice preferences, trip length, and preferred arrival times). Meanwhile, a network simulation model is developed to capture the day-to-day evolution of traffic flow. The model is built upon the *trip-based MFD* (Arnott 2013; Daganzo and Lehe 2015) and its efficient implementation proposed in Lamotte and Geroliminis (2016) which allowed us to study fundamental properties of the TCS. Finally we integrate this overall simulation model that combines the TCS and network simulation model with a Bayesian optimization framework for determining the optimal credit tariff charging scheme that maximizes the total social welfare.

Analytically, this paper presents conditions for existence of the market and network equilibrium, and the uniqueness of the credit market price. In contrast, establishing the uniqueness of departure flows and convergence of the day-to-day model is challenging due to the absence of a closed-form expression for travel times from the trip-based MFD model. Consequently, one has to resort to numerical experiments under a wide range of demand and supply inputs to examine equilibrium and convergence properties (e.g. Arnott 2013; Lamotte and Geroliminis 2015; Mariotte, Leclercq, and Laval 2017), or apply more tractable congestion models for which uniqueness can be established (Yang and Wang 2011; Ye and Yang 2013). Numerically, the experiments demonstrate convergence of the day-to-day model, and examine network performance and welfare for three comparative polices: a no-control case, time-of-day pricing and the proposed TCS.

Note that the MFD model has its limitations. Specifically, for a single reservoir network with endogenous traffic, if the demand (or departure rate) is excessively large, the system will decay towards a gridlock (Daganzo 2007). This issue has been discussed in the literature. For example, Mahmassani, Saberi, and Zockaie (2013) showed that when a sufficient proportion of drivers are adaptive and the drivers do not switch routes when time improvement is below a predetermined threshold, the gridlock eventually recovers. Nevertheless, this problem requires more investigation and we defer it to future research. Therefore, only two demand scenarios, a moderate congestion and a high congestion scenarios, are examined in this study. The numerical results showed convergence of the credit price and demonstrated stable network patterns, verifying the analytical properties of price uniqueness and its inverse proportionality with the endowment. Notably, the proposed TCS improves the social welfare compared to the no-control case and demonstrates the expected theoretical equivalence with time-of-day pricing. The framework proposed also allowed for a comparison of different credit charging mechanisms. While testing different functional forms for the credit tariff profile, the optimized Gaussian-shaped tariff had a similar performance (in terms of welfare) to the triangular tariff; both outperformed a simple step tariff. Moreover, the results suggest that a travel time-based credit tariff mechanism outperforms a trip length-based tariff mechanism in terms of reducing the schedule delay and enhancing the social welfare in congested scenarios. Yet, while both mechanisms would have to rely on advanced technology for possible implementation, the trip length-based tariff scheme may have advantages in terms of behavioural uncertainty during the traveller decision making process. The results also show the superiority of a trip-based TCS compared to traditional (trip agnostic) area-based (zonal) TCS.

The above developments and findings contribute to the increasing body of knowledge on mobility-related TCS, both in terms of insights into the properties of area-based TCS as well as key modelling and implementation frameworks for the design of future TCS.

Avenues for extension of the proposed framework include the consideration of day-to-day variability in demand and supply, and the design of individual- and group- specific credit allocation schemes that can guarantee Pareto improvement (Seilabi et al. 2020). In the path for increased knowledge on the feasibility of TCS, the design of TCS markets that accommodate detailed and individual market interactions along with different buying and selling strategies should also be analyzed. The buying and selling behaviours are not considered in this study, while they are required for investigating potential market operation models for practical implementation of the TCS, both from a theoretical (Dogterom, Ettema, and Dijst 2017; Chen et al. 2020) and empirical viewpoint (Brands et al. 2020). In this study we also assumed that the credit endowment and credit price are constant within a day. Nevertheless, it is acknowledged that adaptive credit charging, sporadic endowment and quantity control interventions by the regulator and real-time / within-day credit price adjustment may bring the TCS closer to efficient operations, especially under the non-recurrent conditions of a real transportation system. Yet, detailed simulation and behavioural experiments approaches may again be required to overcome the common simplifying assumptions employed for analytical tractability. Nonetheless, the aforementioned findings of this paper bring insights into possible modelling techniques to include in the design and real-time operations of practice ready area-wide TCS. Finally, the consideration of additional and combined choice dimensions in future TCS efficiency analysis (such as mode, route, departure time and trip cancellation) is currently lacking in the current literature (Akamatsu and Wada 2017), yet it is in much need for bringing TCS closer to practice.

Notes

1. The open-source code for the simulation is available at https://github.com/RM-Liu/MFD_TCS
2. The sharp change from day 0 to day 1 is due to setting the initial perceived travel time on day 0 as free flow travel time, which differs greatly from the realized one. For the remaining scenarios, the equilibrium travel-times of this base case are used in the initial perceived conditions.
3. For this experiment, we keep the time window TW_i fixed across days to enable the computation of $I_{\min}$.

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Appendices

Appendix 1. Proof of Theorem 1

This proof begins with

Lemma A.1: *c_i and p solve*

$$Q(p, Z) = 0 \tag{A1}$$

$$\textit{iff } p = [p + \rho \cdot Z]_+ \tag{A2}$$

where ρ is a constant larger than 0.

Sufficiency: from (9), Equation (A1) holds iff

$$p \cdot Z = 0, \quad p \geq 0, \quad Z \leq 0 \tag{A3}$$

If c_i and p satisfy (A3), then either

$$p = 0, \quad Z \leq 0 \quad \text{or} \quad p \geq 0, \quad Z = 0 \tag{A4}$$

holds. Obviously, (A4) satisfies (A2).

Necessity: if c_i and p satisfy (A2), then either

$$\begin{cases} p = 0, \\ \rho \cdot Z \leq 0 \end{cases} \quad \text{or} \quad \begin{cases} p > 0, \\ Z = 0 \end{cases} \tag{A5}$$

holds. And as (A5) satisfies (A3), then (A1) holds. Therefore, Lemma A.1 holds.

By Lemma A.1, the equilibrium condition (12) is equivalent to

$$\begin{cases} \bar{c}_i(t) = c_i(t), \quad \forall t \in TW_i \\ p = [p + \rho \cdot Z]_+ \end{cases} \tag{A6}$$

Then, we introduce the fixed point theorem from Khamsi and Kirk (2011):

Theorem A.2: *Let Ω be a bounded closed convex subset of $\mathbb{R}^m$ and let $\boldsymbol{g}: \Omega \to \Omega$ be continuous. Then $\boldsymbol{g}$ has a fixed point.*

According to Lamotte and Geroliminis (2018), let $f(t) = \int_0^t V(n(s))\mathrm{d}s$, which associates to a time t the distance travelled by a traveller from the origin of time to t, let $\chi = f(t)$ denote a particular distance travelled and let $\mathcal{V}$ be the speed function in the χ space: $\mathcal{V}(\chi) = V(n(f^{-1}(\chi)))$. Then the travel time $T_i(t, L_i)$ that associates to an departure time t and a trip length L_i has an explicit expression:

$$T_i(t, L_i) = \int_{f(t)}^{f(t)+L_i} \frac{1}{\mathcal{V}(s)}\,\mathrm{d}s, \quad \forall t \in TW_i$$

Since there is no gridlock, we have the speed $\mathcal{V}$ is always higher than a minimal speed strictly positive, and f is continuous, $T_i(t, L_i)$ is continuous. Then, assuming a non-positive and continuous travel cost $\boldsymbol{c}_i(\cdot)$ and $\bar{\boldsymbol{c}}_i(\cdot)$ also for the flow domain, $\boldsymbol{c}_i(\cdot)$ is represented in the domain of the latter for our fixed point solution.

Let $\boldsymbol{t} = [t_1, \ldots, t_m]^{\mathrm{T}}$, where $t_i \in [t_{i,0}^{\mathrm{dep}} - \tau \cdot \Delta t, t_{i,0}^{\mathrm{dep}} + \tau \cdot \Delta t]$, $\bar{\boldsymbol{c}}(\boldsymbol{t}) = [\bar{c}_1(t_1), \ldots, \bar{c}_m(t_m)]^{\mathrm{T}}$. Then there exists $\boldsymbol{y} = (y_1, \ldots, y_m)^T \leq \boldsymbol{0}$, such that $\boldsymbol{y} \leq \bar{\boldsymbol{c}}(\boldsymbol{t}) \leq \boldsymbol{0}$. Therefore, a compact and convex set can be defined as $\Omega_{\bar{\boldsymbol{c}}} = [y_1, 0] \times \cdots \times [y_m, 0]$, then for all $\bar{\boldsymbol{c}} \in \Omega_{\boldsymbol{C}}$. Denote $Z_d = \sum_i \sum_{t \in TW_i} \Pr_i(C_i(t|p)) \cdot g(t) \cdot L_i \cdot w - I \cdot N$, then

$$\begin{aligned} \lim_{p \to \infty} Z_d(\bar{\boldsymbol{c}}, p) &= \lim_{p \to \infty} \sum_i \sum_{t \in TW_i} \Pr_i(C_i(t|p)) \cdot g(t) \cdot L_i \cdot w \\ &\quad - I \cdot N \\ &= (I_{\min} - I)N < 0, \quad \forall \bar{\boldsymbol{c}} \in \Omega_{\bar{\boldsymbol{c}}} \end{aligned} \tag{A7}$$

For some $\hat{\boldsymbol{c}} \in \Omega_{\bar{\boldsymbol{c}}}$, if

$$Z(\hat{\boldsymbol{c}}, p) \leq 0, \quad \forall p \geq 0 \tag{A8}$$

then $[p + \rho Z(\hat{\boldsymbol{c}}, p)]_+ \leq p, \forall p \geq 0$. Let us define $\Omega_{\bar{\boldsymbol{c}}}^-$ as the set of $\hat{\boldsymbol{c}} \in \Omega_{\bar{\boldsymbol{c}}}$ satisfying condition (A8) and define $\Omega_{\bar{\boldsymbol{c}}}^+ = \Omega_{\bar{\boldsymbol{c}}} \backslash \Omega_{\bar{\boldsymbol{c}}}^-$. Then there exists $\hat{\boldsymbol{c}} \in \Omega_{\bar{\boldsymbol{c}}}^+$, such that $Z(\hat{\boldsymbol{c}}, p) > 0$ for some $p \geq 0$. According to (A7) and (A8), there exists $\bar{p} \geq 0$, such that $Z(\hat{\boldsymbol{c}}, p) \leq 0$, $\forall p \geq \bar{p} \Rightarrow [p + \rho Z(\hat{\boldsymbol{c}}, p)]_+ \leq p, \forall p \geq \bar{p}$. Let $p^+ = \max_{p \leq \bar{p}}[p + \rho Z(\hat{\boldsymbol{c}}, p)]_+$, then $\forall p \in [0, p^+]$, $[p + \rho Z(\hat{\boldsymbol{c}}, p)]_+ \leq p^+$. Therefore $\forall p \in \Omega_p \triangleq [0, \max_{\hat{\boldsymbol{c}} \in \Omega_{\bar{\boldsymbol{c}}}^+} p^+]$ and $\hat{\boldsymbol{c}} \in \Omega_{\bar{\boldsymbol{c}}}$, $[p + \rho Z(\hat{\boldsymbol{c}}, p)]_+ \in \Omega_p$.

Based on the analysis above, $\Omega_{\bar{\boldsymbol{c}}} \times \Omega_p$ is compact and convex. Since travel cost $\boldsymbol{c}(\cdot)$ and $[\cdot]$ are continuous, (A6) has at least one fixed point by Theorem A.2, implying there exists an equilibrium solution of the proposed dynamic system.

Appendix 2. Proof of Theorem 2

Assume there are two equilibrium credit prices p_1 and p_2, then by (7) and (9), for $k = 1, 2$, we have

$$\begin{aligned} & p_k \left[\sum_i \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_k)) \cdot g(t) \cdot L_i \cdot w - l \cdot N \right] = 0, \quad p_k \geq 0, \\ & \sum_i \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_k)) \cdot g(t) \cdot L_i \cdot w - l \cdot N \leq 0 \end{aligned} \tag{A9}$$

Thus,

$$\begin{aligned} & (p_1 - p_2) \left(\sum_i \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_1)) \cdot g(t) \cdot L_i \cdot w \right. \\ & \quad \left. - \sum_i \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_2)) \cdot g(t) \cdot L_i \cdot w \right) \\ & = p_1 \left[l \cdot N - \sum_i \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_2)) \cdot g(t) \cdot L_i \cdot w \right] \\ & \quad + p_2 \left[l \cdot N - \sum_i \sum_{t \in TW_i} \Pr_i(C_i(t \mid p_1)) \cdot g(t) \cdot L_i \cdot w \right] \\ & \geq 0 \end{aligned} \tag{A10}$$

By (13), the equality in (A10) holds if and only if $p_1 = p_2$. Hence, the equilibrium credit price is unique.