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Discrete optimization

A computationally efficient approach to optimizing offers in centrally committed electricity markets

Yuzhou Jiang [a], Ramteen Sioshansi [a,b,c,d,e,*]

[a] Department of Integrated Systems Engineering, The Ohio State University, 1971 Neil Avenue, Columbus, OH 43210-1271, United States of America
[b] Department of Engineering and Public Policy, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213-3815, United States of America
[c] Carnegie Mellon Electricity Industry Center, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213-3815, United States of America
[d] Department of Electrical and Computer Engineering, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213-3815, United States of America
[e] Wilton E. Scott Institute for Energy Innovation, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213-3815, United States of America

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ABSTRACT

We study the incentive properties of the two primary approaches to incorporating unit-commitment decisions in wholesale electricity markets. One approach is centralized unit commitment, wherein generating firms provide complex multi-part offers that specify their non-convex fixed and variable operating costs. The market operator uses these offers to co-optimize unit-commitment and economic-dispatch decisions. The second approach is self-commitment, whereby firms determine unit-commitment decisions for their generating units individually and submit simple offers for the provision of energy. Operators of self-committed markets determine generator dispatch based on the merit order of the simple offers.

Comparing the incentive properties of the two market designs is challenging because the offer-optimization problem for a firm that participates in a centrally committed market is a bi-level model with binary variables in the lower-level problem. To address this challenge, we develop a computationally efficient approach to solve such a problem and illustrate the method with examples. We use the examples to compare the incentive properties of the two market designs. Our examples show that the profit of the profit-maximizing firm does not differ significantly between the two market designs but that system costs can be higher under a self-committed design. These cost differences are because the complex offers and discriminatory payment schemes that are used under centrally committed designs can mitigate incentives for the profit-maximizing firm to exercise market power.

1. Introduction

Designing wholesale electricity markets raises the question of how unit-commitment decisions are co-ordinated amongst generators. Existing markets use two primary approaches. Some markets, especially those in United States of America, employ a centrally committed design. With such a design, unit-commitment decisions are made by a market operator (MO), which receives complex offers from firms that specify the non-convex costs and constraints of their generating units. Muckstadt and Koenig (1977) give a formative application of Lagrangian relaxation (LR) to solve the model that underlies a centrally committed market. Baldick (1995) gives a generalized formulation of the model, which accounts for a broad array of constraints, and refines the LR algorithm. Hobbs et al. (2001) survey such models and algorithmic developments. Scarf (1990, 1994) notes a pricing challenge with centrally committed designs—uniform linear pricing can be economically confiscatory, due to the non-convex costs and constraints that govern generator operations. O'Neill et al. (2005), Sioshansi (2014) discuss make-whole payments as a common mechanism to address this challenge. Make-whole payments are discriminatory transfers from the MO to each generator to ensure that the latter earns non-negative profit (on the basis of the offers that the generator submits to the market). In most cases, the costs of make-whole payments are socialized to customers.

Another common design, especially in Europe and Australia, are self-committed wholesale electricity markets. Generating firms in such markets determine individually the commitments of their units and

* Corresponding author at: Department of Integrated Systems Engineering, The Ohio State University, 1971 Neil Avenue, Columbus, OH 43210-1271, United States of America.
E-mail addresses: jiang.762@buckeyemail.osu.edu (Y. Jiang), rsioshan@andrew.cmu.edu (R. Sioshansi).

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submit simple offers that specify prices at which the units provide energy. Operators of self-committed markets determine the dispatch of generating units based on the merit order of the simple offers. Because self-committed markets rely upon simple offers being submitted into the market, the onus is upon generating firms to ensure that the offers yield revenues that recover their non-convex costs. Imran and Kockar (2014) compare the designs of European and North American electricity markets, including with respect to the treatment of unit-commitment decisions.

There are works that examine these two market designs. Johnson et al. (1997), Sioshansi et al. (2008a) show that using LR to solve the market model that underlies a centrally committed design can impact the profits of individual generating units. Profit differences arise because solutions that yield similar total cost can give different prices and unit-commitment and dispatch decisions. Sioshansi and Tignor (2012) demonstrate that these profit impacts under a centrally committed design can be most pronounced for generators with greater operational flexibility. Elmaghraby and Oren (1999) consider the treatment of intertemporal constraints and non-convex costs under a self-committed design. They propose a horizontal market-clearing procedure to address these aspects of the market-clearing problem. Sioshansi et al. (2008b) show that even with truthful cost revelation, a self-committed design yields productive-efficiency losses. Ahlqvist et al. (2022) compare the two designs, including their relative merits *vis-à-vis* supply and demand flexibility, resource remuneration, and market power and efficiency.

These works that examine the two market designs assume truthful revelation by the participating generating firms. We know of two works that compare the market designs while considering strategic behavior with asymmetric information between generating firms and the MO. Sioshansi and Nicholson (2011) characterize and compare Nash equilibria under the two market designs assuming two symmetric firms that compete during a single operating period. Duggan, Jr. and Sioshansi (2019) extend the work of Sioshansi and Nicholson (2011) by relaxing the duopoly assumption, assuming a symmetric oligopoly with an arbitrary number of firms.

This paper relaxes the assumptions of these previous works and explores strategic behavior by a single firm that participates in a market with one of the two designs, *i.e.*, the offers of the other firms are fixed. Specifically, we relax the symmetric-firm assumption by assuming that generating firms have the same capacities but can have different production costs. We consider also multiple as opposed to a single operating period. Multiple operating periods complicate the profit-maximizing behavior of the strategic firm under a centrally committed design in two ways. First, it is typical for the MO to use the same so-called long-lived offers to clear the market during multiple operating periods (*e.g.*, 24 hour-long periods is common of many day-ahead wholesale electricity markets). Second, the determination of any make-whole payment that a generator receives depends upon revenue and cost across multiple operating periods. Our modeling approach allows the strategic firm to account for these complexities.

We cast our problem as a bi-level optimization model, which has the strategic firm's profit maximization as the upper-level and the MO's market clearing as the lower-level problem. There is an extensive literature that applies bi-level optimization to model strategic behavior in markets, especially when the lower-level problem is convex. Fampa et al. (2008) propose a penalty-based heuristic and a mixed-integer-linear program (MILP), which uses binary expansion, to solve a bi-level problem with economic dispatch in the lower level. Gabriel and Leuthold (2010) consider a quadratic lower-level model and employ a linearization technique, which is based on a set of discrete generation levels, to obtain an MILP formulation for their problem. Hobbs et al. (2000) model profit-maximizing firms competing in a transmission-constrained market and solve the model using a penalty-based interior-point method. Barroso et al. (2006) discretize the strategy sets of market participants and compute Nash equilibria of the resultant game. Bakirtzis et al. (2007) assume that generating firms use stepped offers to participate in an energy market. Ruiz and Conejo (2009) model the lower level of a bi-level model as a multi-period optimal-power-flow problem that considers uncertainty in consumers' bids and rival generators' offers. Kozanidis et al. (2013) incorporate unit-commitment decisions into the lower-level market clearing of a bi-level model. They use parametric integer optimization to develop a problem-specific algorithm that exploits the relationship between the offers that are submitted by the strategic firm and total system cost.

Our paper has two primary contributions to this existing literature. First, we provide an efficient algorithm to compute optimal offers by a strategic market participant under a centrally committed design. This is our primary technical contribution, which addresses the challenge of having binary variables in the lower level of a bi-level model. Second, we use numerical examples to compare centrally and self-committed designs for wholesale electricity markets. In particular, we examine the extent to which a strategic generating firm can manipulate its offers under the two market designs to impact prices, profits, settlement costs, and other market-performance metrics.

The remainder of the paper is organized as follows. Section 2 presents the formulations of bi-level models for a profit-maximizing generator under the two market designs and assumptions that are common to the market designs. Section 3 applies standard approaches to convert the bi-level model for the self-committed market into an equivalent single-level optimization problem. Section 4 examines the properties of a centrally committed market and develops an algorithm to solve the bi-level model that corresponds to this case. The appendix provides a proof of a proposition that underlies this analysis. Section 5 extends the algorithm that is developed in Section 4 to the case of a centrally committed design with make-whole payments. Section 6 presents numerical examples that we use to compare the two market designs. Section 7 concludes.

2. Model assumptions and formulations

This section provides assumptions and formulations of bi-level optimization models for a profit-maximizing firm that participates in centrally and self-committed markets.

2.1. Model notation

Under both designs, we assume that the market consists of a set, $\mathcal{G}$, of generating firms and let $j \in \mathcal{G}$ be a generic generating-firm index. We let $i \in \mathcal{G}$ denote the index of the strategic generator, the offers of which are optimized in our models. For notational ease, we assume that each generating firm owns a single generator. As is common, the MO's market model is assumed to use hourly operating periods. We let $\mathcal{T}$ and $t \in \mathcal{T}$ denote the set of hours in the MO's model horizon and the time index, respectively.

We introduce the following assumption regarding the treatment of transmission constraints in the MO's model.

Assumption 1. The market model includes no binding transmission constraints and treats all generators and load as being at a single transmission-network node.

Assumption 1 is needed for model tractability. Including transmission constraints would hamper significantly our analysis of a centrally committed market design. In addition, Yao et al. (2004) note that transmission constraints can complicate the derivation of profit-maximizing strategies by strategic generating firms. In some cases it can be optimal for a firm to follow a strategy that congests or decongests a transmission line infinitesimally. Such strategies can be difficult to capture using the bi-level modeling approach that we employ. An implication of Assumption 1 is that our work does not capture added firm-behavior complexities that are associated with the use of locational marginal pricing in some wholesale electricity markets.

Next, we define the following parameters, which are common to both market designs.

$\bar{b}^v$	maximum energy offer (\$/MWh)
c_j^f	fixed operating cost of firm j (\$/hour)
c_j^v	variable operating cost of firm j (\$/MWh)
D_t	hour-t demand (MW)
K	generator production capacity (MW)

Generators are assumed to have the same capacities but different costs. Generation costs are non-convex, because $\forall j \in \mathcal{G}$, firm j incurs a fixed cost of c_j^f if it is committed during a given hour. Otherwise, if it is shutdown during that hour, it incurs no cost during that hour but its production during that hour must be zero.

2.2. Model of self-committed design

To formulate firm i's profit-maximization under a self-committed design, first we define b_j^v as firm j's energy offer (\$/MWh). For all $j \in \mathcal{G}, t \in \mathcal{T}$ we let $u_{j,t}$ denote firm j's hour-t unit-commitment status. $u_{j,t}$ is equal to 1 is firm j is online during hour t and is equal to zero otherwise. For all $j \in \mathcal{G}$, b_j^v and $u_{j,t}$ are treated as fixed parameters in the MO's lower-level problem. b_i^v and $u_{i,t}$, $\forall t \in \mathcal{T}$ are variables in firm i's upper-level profit-maximization problem, whereas $\forall j \in \mathcal{G}, j \neq i, t \in \mathcal{T}$, b_j^v and $u_{j,t}$ are treated as parameters by firm i. We define also $x_{j,t}$ as firm j's hour-t power output (MW). For all $j \in \mathcal{G}, t \in \mathcal{T}$, $x_{j,t}$ is a variable that is determined in the MO's lower-level model, which is formulated as:

$$\min \sum_{j \in \mathcal{G}, t \in \mathcal{T}} b_j^v x_{j,t} \tag{1}$$

$$\text{s.t.} \sum_{j \in \mathcal{G}} x_{j,t} = D_t; \forall t \in \mathcal{T} \qquad (\omega_t) \tag{2}$$

$$0 \le x_{j,t} \le K u_{j,t}; \forall j \in \mathcal{G}, t \in \mathcal{T} \qquad (\alpha_{j,t}^-, \alpha_{j,t}^+); \tag{3}$$

where the decision variables are $x_{j,t}$, $\forall j \in \mathcal{G}, t \in \mathcal{T}$ and the Lagrange-multiplier set that is associated with each constraint set is in parentheses to its right. Objective function (1) minimizes the cost of operating the system, based on the supply offers that are submitted by the generating firms (*i.e.*, it may be that $b_j^v \neq c_j^v$ for some $j \in \mathcal{G}$). Constraint set (2) ensures hourly load balance and (3) enforces production limits. If $u_{j,t} = 0$ for some $j \in \mathcal{J}, t \in \mathcal{T}$, then $x_{j,t}$ must equal zero as well.

Firm i's bi-level profit-maximization problem is:

$$\max \sum_{t \in \mathcal{T}} \left[(\omega_t - c_i^v) x_{i,t} - c_i^f u_{i,t} \right] \tag{4}$$

$$\text{s.t.} 0 \le b_i^v \le \bar{b}^v \tag{5}$$

$$u_{i,t} \in \{0,1\}; \forall t \in \mathcal{T} \tag{6}$$

$$(1)\text{–}(3); \tag{7}$$

where the decision variables are $u_{i,t}$, $\forall t \in \mathcal{T}$, b_i^v, and all of the variables of (1)–(3). Objective function (4) maximizes firm i's profit. We use the standard convention that $\forall t \in \mathcal{T}$, the Lagrange multiplier, ω_t, that is associated with hour-t load-balance requirement (2) sets the hour-t energy price. Constraint (5) imposes standard restrictions on firm i's energy offer. Constraint set (6) requires that firm i's unit-commitment decisions be binary. Because we are modeling a self-committed design, firm i makes its own unit-commitment decisions as opposed to those decisions being made by the MO. Constraint set (7) embeds the MO's market-clearing model within firm i's profit maximization. This is necessary, because the MO's model determines the values of $x_{i,t}$ and ω_t, $\forall t \in \mathcal{T}$.

2.3. Model of centrally committed design

To formulate a model for a centrally committed design, we retain the same notation as is used for the self-committed market. In addition, $\forall j \in \mathcal{G}$, we define b_j^f as firm j's fixed-cost offer (\$/hour), which is treated as a parameter in the MO's model, and $\bar{b}^f$ as the maximum fixed-cost offer (\$/hour), which is a parameter. Under a centrally committed design, the MO's model is:

$$\min \sum_{j \in \mathcal{G}, t \in \mathcal{T}} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) \tag{8}$$

$$\text{s.t.} \sum_{j \in \mathcal{G}} x_{j,t} = D_t; \forall t \in \mathcal{T} \tag{9}$$

$$0 \le x_{j,t} \le K u_{j,t}; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{10}$$

$$u_{j,t} \in \{0,1\}; \forall j \in \mathcal{G}, t \in \mathcal{T}; \tag{11}$$

where the decision variables are $x_{j,t}$ and $u_{j,t}$, $\forall j \in \mathcal{G}, t \in \mathcal{T}$. Objective function (8) minimizes the cost of operating the system, which includes non-convex fixed-cost offers. Although they impose an ancillary cost upon customers, make-whole payments are not included normally in the MO's objective function. Make-whole payments are excluded from (8) because including them would cause the MO to clear the market as a monopsonist, which is economically inefficient. Constraint set (9) ensures hourly load balance, (10) enforces production limits, and (11) imposes integrality on unit-commitment decisions.

Firm i's profit-maximization problem is:

$$\max \sum_{t \in \mathcal{T}} \left[(\eta_t - c_i^v) x_{i,t} - c_i^f u_{i,t} \right] \tag{12}$$

$$\text{s.t.} 0 \le b_i^v \le \bar{b}^v \tag{13}$$

$$0 \le b_i^f \le \bar{b}^f \tag{14}$$

$$(8)\text{–}(11); \tag{15}$$

where the decision variables are b_i^v, b_i^f, and all of the variables of (8)–(11). Objective function (12) maximizes firm i's profit. For all $t \in \mathcal{T}$, we let η_t denote the hour-t energy price. Problem (8)–(11) does not have well defined dual variables or Lagrange multipliers, because of the integrality restrictions. We impose a standard assumption in Section 4 on how to set marginal prices, which behave similarly to ω_t, $\forall t \in \mathcal{T}$ under a self-committed design. Constraints (13) and (14) impose standard limits on firm i's offer and (15) embeds the MO's model as the lower-level problem.

2.4. Additional assumptions

We conclude this section by introducing the following two additional assumptions that underlie our analysis.

Assumption 2. Under a self-committed market design firm i knows the values of b_j^v and $u_{j,t}$, $\forall j \in \mathcal{G}, j \neq i, t \in \mathcal{T}$, with certainty. Under a centrally committed market design firm i knows the values of b_j^v and b_j^f, $\forall j \in \mathcal{G}, j \neq i$, with certainty.

Assumption 2 is needed to make analysis of a centrally committed design tractable. Assumption 2 can be viewed as related to the definition of a Nash equilibrium. Nash, Jr. (1950) defines a Nash equilibrium as assuming that each player predicts perfectly the strategies of its rivals and selecting a strategy from which it has no profitable unilateral deviation. So long as firm i's rival select b_j^v and $u_{j,t}$, $\forall j \in \mathcal{G}, j \neq i, t \in \mathcal{T}$ under a self-committed design to maximize their individual profits, a solution to (4)–(7) satisfies this definition. The same can be said of a solution to (12)–(15), so long as b_j^v and b_j^f, $\forall j \in \mathcal{G}, j \neq i$ maximize the individual profits of firm i's rivals.

Assumption 2 makes (4)–(7) and (12)–(15) static in nature, due to the assumed sequence of events. Specifically, under a self-committed design, firm i's rivals are assumed to choose b_j^v and $u_{j,t}$, $\forall j \in \mathcal{G}, j \neq i, t \in \mathcal{T}$, which is followed by firm i's profit-maximizing choice of b_i^v and $u_{i,t}$, $\forall t \in \mathcal{T}$ and then the MO determining prices and production levels. A centrally committed design has a similar sequence of events. Firm i's rivals choose b_j^v and b_j^f, $\forall j \in \mathcal{G}, j \neq i$, which is followed by firm i's profit-maximizing choice of b_i^v and b_i^f, and the MO determining prices, commitments, and production levels.

Assumption 3. Under a self- and centrally committed market design, respectively, if (1)–(3) or (8)–(11) have multiple optimal solutions, one that is preferable to firm i is chosen.

Assumption 3 states that if there are multiple optimal solutions, the MO chooses a market-clearing solution that maximizes firm i's profit. This assumption is key to the standard approach that we employ to analyze firm i's profit-maximization under a self-committed design (*cf.* Section 3 for details). As such, it is natural to adopt this assumption for analysis of a centrally committed design.

3. Equivalent single-level formulation of (4) –(7)

The standard approach to make (4)–(7) computationally tractable is to recast it as an equivalent single-level model, which can be solved using off-the-shelf optimization software. Problem (1)–(3) is a convex linear optimization. Thus, Sioshansi and Conejo (2017) note that an optimal solution to (1)–(3) can be characterized by its necessary and sufficient Karush–Kuhn–Tucker (KKT) conditions, which are:

$$b_j^v - \omega_t - \alpha_{j,t}^- + \alpha_{j,t}^+ = 0; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{16}$$

(2) (17)

$$0 \le x_{j,t} \perp \alpha_{j,t}^- \ge 0; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{18}$$

$$x_{j,t} \le K u_{j,t} \perp \alpha_{j,t}^+ \ge 0; \forall j \in \mathcal{G}, t \in \mathcal{T}. \tag{19}$$

Conditions (18)–(19) include complementary-slackness requirements that are non-convex. Hereafter, we assume that (18)–(19) are convexified using the approach that Fortuny-Amat and McCarl (1981) propose, which entails the use of auxiliary specially-ordered-set variables.

Thus, we can convert (4)–(7) into an equivalent single-level problem which consists of (4)–(6) and (16)–(19). The replacement of (7) with (16)–(19) relies, implicitly, upon Assumption 3. This reliance stems from the property that if (7) has multiple optimal solutions, (4)–(6) and (16)–(19) necessarily selects one that maximizes firm i's profit. Constraints (5)–(6) and (16)–(19) yield a convex feasible region, but (4) is non-convex because it contains bi-linear terms in which ω_t and $x_{i,t}$ are multiplied. We obtain an exactly equivalent convexification of (4) by noting that by (16):

$$\sum_{t\in\mathcal{T}} \left[(\omega_t - c_i^v)x_{i,t} - c_i^f u_{i,t}\right] = \sum_{t\in\mathcal{T}} \left[\left(b_i^v - \alpha_{i,t}^- + \alpha_{i,t}^+ - c_i^v\right) x_{i,t} - c_i^f u_{i,t}\right];$$

and that by (18) and (19):

$$\sum_{t\in\mathcal{T}} \left[\left(b_i^v - \alpha_{i,t}^- + \alpha_{i,t}^+ - c_i^v\right) x_{i,t} - c_i^f u_{i,t}\right] = \sum_{t\in\mathcal{T}} \left(b_i^v x_{i,t} + K u_{i,t}\alpha_{i,t}^+ - c_i^v x_{i,t} - c_i^f u_{i,t}\right). \tag{20}$$

The strong-duality equality for (1)–(3) is:

$$\sum_{j\in\mathcal{G},t\in\mathcal{T}} b_j^v x_{j,t} = \sum_{t\in\mathcal{T}} \left(D_t\omega_t - \sum_{j\in\mathcal{G}} K u_{j,t}\alpha_{j,t}^+\right); \tag{21}$$

Substituting (21) into (20) gives:

$$\sum_{t\in\mathcal{T}} \left[(\omega_t - c_i^v)x_{i,t} - c_i^f u_{i,t}\right] = \sum_{t\in\mathcal{T}} \left[D_t\omega_t - \sum_{j\in\mathcal{G},j\ne i} \left(b_j^v x_{j,t} + K u_{j,t}\alpha_{j,t}^+\right) - c_i^v x_{i,t} - c_i^f u_{i,t}\right]; \tag{22}$$

which is convex and linear in the decision variables of (4)–(7). Thus, a computationally efficient approach to solving (4)–(7) is to solve the single-level mixed-integer linear optimization problem:

$$\max \sum_{t\in\mathcal{T}} \left[D_t\omega_t - \sum_{j\in\mathcal{G},j\ne i} \left(b_j^v x_{j,t} + K u_{j,t}\alpha_{j,t}^+\right) - c_i^v x_{i,t} - c_i^f u_{i,t}\right] \tag{23}$$

s.t. (5)–(6), (16)–(19). (24)

Because (23) is equal exactly to (4), solving (23)–(24) gives an exact solution to (4)–(7).

4. Properties of and solution algorithm for centrally committed design

The analysis of (12)–(15) is more complicated than what is presented in Section 3. The complexity arises because (8)–(11) has binary variables, meaning that there are no computationally tractable optimality conditions with which to replace (15). Thus, our approach to analyzing (12)–(15) is to prove characteristics of an optimal solution to (8)–(11), which are used to develop a solution algorithm for (12)–(15).

We begin with the following assumption, which eliminates cases wherein demand is an exact integer multiple of the firms' capacities, which would raise technical challenges without added insights.

Assumption 4. For all $t \in \mathcal{T}$, D_t is not an integer multiple of K.

For all $t \in \mathcal{T}$, we define $r_t = D_t - K \cdot \lfloor D_t/K \rfloor$ as the hour-t residual demand. By Assumption 4, $r_t \in (0, K)$, $\forall t \in \mathcal{T}$.

We let (x^*, u^*) denote an optimal solution to (8)–(11) and $\forall t \in \mathcal{T}$ we partition the generating firms into the sets, $\mathcal{G}_t^I$, $\mathcal{G}_t^M$, and $\mathcal{G}_t^V$. $\mathcal{G}_t^I$ is the set of hour-t-inframarginal generators and $x_{j,t}^* = K$, $\forall j \in \mathcal{G}_t^I$. $\mathcal{G}_t^M$ is the set of hour-t-marginal generators and $x_{j,t}^* \in (0, K)$, $\forall j \in \mathcal{G}_t^M$. By Assumption 4, $\mathcal{G}_t^M$ is non-empty $\forall t \in \mathcal{T}$. Finally, $\mathcal{G}_t^V$ is the set of hour-t-inactive generators and $x_{j,t}^* = 0$, $\forall j \in \mathcal{G}_t^V$.

Next, we prove the following lemma, which shows that all hour-t marginal generators have the same energy offer.

Lemma 1. *For all $t \in \mathcal{T}$ and $j, h \in \mathcal{G}_t^M$, we have that $b_j^v = b_h^v$.*

Proof. Suppose for contradiction that $\exists t \in \mathcal{T}$ and $j, h \in \mathcal{G}_t^M$ with $j \ne h$ and that, without loss of generality, j and h are labeled such that $b_j^v < b_h^v$. The value of (8) is reduced if $x_{h,t}^*$ is decreased by ϵ and $x_{j,t}^*$ is increased by ϵ where ϵ is sufficiently small that $x_{h,t}^* - \epsilon \ge 0$ and $x_{j,t}^* + \epsilon \le K$. This gives the desired contradiction. □

We add the following assumption that hourly energy prices are equal to the energy offer of the marginal unit(s).

Assumption 5. For all $t \in \mathcal{T}$, η_t is equal to b_j^v for some $j \in \mathcal{G}_t^M$.

The following lemma shows the relationship between the energy offers of inframarginal and marginal generators.

Lemma 2. *For all $t \in \mathcal{T}$, $l \in \mathcal{G}_t^I$, and $h \in \mathcal{G}_t^M$ we must have $b_l^v \le b_h^v$.*

Proof. Following from Lemma 1 we have that $\forall t \in \mathcal{T}$, the optimized value of:

$$\sum_{j\in\mathcal{G}} \left(b_j^v x_{j,t} + b_j^f u_{j,t}\right);$$

equals:

$$\sum_{j\in\mathcal{G}_t^I} \left(b_j^v K + b_j^f\right) + b_h^v \cdot \left(D_t - K \cdot \left|\mathcal{G}_t^I\right|\right) + \sum_{j\in\mathcal{G}_t^M} b_j^f;$$

where $h \in \mathcal{G}_t^M$. Suppose for contradiction that $\exists l \in \mathcal{G}_t^I$ such that $b_l^v > b_h^v$. This contradicts the optimality of (x^*, u^*), because the value of (8) is reduced by making generator l marginal and generator h inframarginal. □

Now we show the following key property of an optimal solution to firm i's bi-level profit-maximization problem.

Proposition 1. *Suppose that* (b_i^{f*}, b_i^{v*}) *is an optimum of* (12)–(15). *Then* $\exists(x^*, u^*)$ *that is an optimum of* (8)–(11), *satisfies* Assumption 3, *and has* $x_{i,t}^* \in \{0, r_t, K\}$, $\forall t \in \mathcal{T}$.

Proof. Suppose for contradiction that $\exists \tau \in \mathcal{T}$ such that $x_{i,\tau}^* \notin \{0, r_\tau, K\}$. We can consider the following two cases.

First, suppose that $x_{i,\tau}^* \in (0, r_\tau)$. In such a case, we must have $|\mathcal{G}_\tau^M| \geq 2$ to satisfy (9). We define $\epsilon = r_\tau - x_{i,\tau}^*$ and by (9) we must have:

$$\sum_{j \in \mathcal{G}_\tau^M, j \neq i} x_{j,\tau}^* = K \cdot \left(\left\lfloor \frac{D_\tau}{K} \right\rfloor - \left|\mathcal{G}_\tau^I\right| \right) + \epsilon.$$

Thus, we must have that:

$$\left|\mathcal{G}_\tau^M \setminus \{i\}\right| \geq \left\lfloor \frac{D_\tau}{K} \right\rfloor - \left|\mathcal{G}_\tau^I\right| + 1. \tag{25}$$

We let $\tilde{\mathcal{G}}_\tau \subset \mathcal{G}_\tau^M$ be a (possibly empty) proper subset of $\mathcal{G}_\tau^M$ such that $i \notin \tilde{\mathcal{G}}_\tau$ and:

$$|\tilde{\mathcal{G}}_\tau| = \left\lfloor \frac{D_\tau}{K} \right\rfloor - \left|\mathcal{G}_\tau^I\right|.$$

Such a subset is guaranteed to exist by (25). Consider an alternative solution, $(\tilde{x}, \tilde{u})$, to (8)–(11) where $\tilde{x}_{j,t} = x_{j,t}^*$ and $\tilde{u}_{j,t} = u_{j,t}^*$, $\forall t \neq \tau$, $\tilde{x}_{j,\tau} = K$ and $\tilde{u}_{j,\tau} = 1$, $\forall j \in \mathcal{G}_\tau^I \cup \tilde{\mathcal{G}}_\tau$, $\tilde{x}_{i,\tau} = r_\tau$ and $\tilde{u}_{i,\tau} = 1$, and $\tilde{x}_{j,\tau} = 0$ and $\tilde{u}_{j,\tau} = 0$, $\forall j \notin \mathcal{G}_\tau^I \cup \tilde{\mathcal{G}}_\tau \cup i$. Such a solution improves the value of (8) by:

$$\sum_{j \in \mathcal{G}_\tau^M \setminus \tilde{\mathcal{G}}_\tau, j \neq i} b_j^f;$$

which is non-negative.

Next, suppose that $x_{i,\tau}^* \in (r_\tau, K)$, in which case we must have $|\mathcal{G}_\tau^M| \geq 2$ to satisfy (9). Firm i's hour-τ profit is:

$$(\omega_\tau - c_i^v) x_{i,\tau}^* - c_i^f.$$

We can consider two cases, which depend upon the sign of $\omega_\tau - c_i^v$. If $\omega_\tau - c_i^v \geq 0$, consider an alternative solution to (8)–(11) in which $x_{i,\tau}^*$ is increased by ϵ and:

$$\sum_{j \in \mathcal{G}_\tau^M, j \neq i} x_{j,\tau}^*;$$

is decreased by ϵ, where ϵ is sufficiently small so as not to violate (10). Such an alternative solution does not change the value of (8), because by Lemma 1 we have that $b_i^{v*} = b_j^v$, $\forall j \in \mathcal{G}_\tau^M$. However, this alternative solution increases firm i's profit. This contradicts (x^*, u^*) satisfying Assumption 3. If $\omega_\tau - c_i^v < 0$, consider an alternative solution to (8)–(11) in which $x_{i,\tau}^*$ is decreased by ϵ and:

$$\sum_{j \in \mathcal{G}_\tau^M, j \neq i} x_{j,\tau}^*;$$

is increased by ϵ, where ϵ is sufficiently small so as not to violate (10). Such an alternative solution does not change the value of (8). However, this alternative solution increases firm i's profit, which contradicts (x^*, u^*) satisfying Assumption 3. □

Proposition 1, which relies upon the assumption that all generators have the same K-MW capacity, reduces the complexity of (12)–(15) considerably. Problem (12)–(15) is simplified, because by Proposition 1, (12)–(15) has at most $3^{|\mathcal{T}|}$ candidate optimal solutions. Next, we introduce the following lexicographic assumption on an optimal solution, (x^*, u^*), to (12)–(15), which simplifies the subsequent analysis without loss of generality.

Assumption 6. The firms are ordered lexicographically by fixed-cost offer and then by index so that an optimal solution, (x^*, u^*), to (12)–(15) has $|\mathcal{G}_t^M| = 1$, $\forall t \in \mathcal{T}$.

Assumption 6 and its lexicographic ordering can be satisfied without loss of generality. To see this, suppose that a solution, $(\tilde{x}, \tilde{u})$, with $|\mathcal{G}_t^M| \geq 2$ for some $t \in \mathcal{T}$, is optimal to (12)–(15). By Lemmata 1 and 2 we must have $b_j^v = b_h^v$, $\forall j, h \in \mathcal{G}_t^M$ and $b_j^v \leq b_h^v$, $\forall j \in \mathcal{G}_t^I$ and $h \in \mathcal{G}_t^M$. Place the elements of $\mathcal{G}_t^M$ into a lexicographic order by the values of b_j^f, $\forall j \in \mathcal{G}_t^M$ and break ties by indices. Consider an alternative solution in which the first $|\mathcal{G}_t^M| - 1$ members of $\mathcal{G}_t^M$ are assigned to $\mathcal{G}_t^I$ and the final member remains in $\mathcal{G}_t^M$ but produces r_t MW during hour t. Such a solution can be no more costly than $(\tilde{x}, \tilde{u})$ is.

Next, we define the following metric with which to compare the offers of two firms and state the subsequent lemma, which we do not prove, because it follows trivially from the definition and the principle of optimality.

Definition 1. We say that firm j is weakly less expensive than firm h when producing m MW, which we denote as $j \leq_m h$, if $b_j^v m + b_j^f \leq b_h^v m + b_h^f$. We define firm j being cost-equivalent to and weakly more expensive than firm h, which are denoted as $j =_m h$ and $j \geq_m h$, respectively, analogously.

Lemma 3. *Suppose that* (x^*, u^*) *is optimal to* (12)–(15) *and gives the partitions,* $(\mathcal{G}_t^I, \mathcal{G}_t^M, \mathcal{G}_t^V)$, $\forall t \in \mathcal{T}$. *Then* $\forall t \in \mathcal{T}$ *we have* $j \leq_K h$, $\forall j \in \mathcal{G}_t^I$ *and* $h \in \mathcal{G}_t^V$ *and* $j \leq_{r_t} h$, $\forall j \in \mathcal{G}_t^M$ *and* $h \in \mathcal{G}_t^V$.

Next, we consider optimal solutions to two variants of (8)–(11). The first removes firm i from the set of candidate generators and $\forall t \in \mathcal{T}$, we let $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$ denote the resultant hour-t partition of the firms. The second has firm i with arbitrarily small offers among the set of candidate generators (*i.e.*, firm i is inframarginal $\forall t \in \mathcal{T}$ such that $D_t > K$ and is marginal during other hours) and $\forall t \in \mathcal{T}$, we let $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$ denote the resultant hour-t partition of the firms. For notational ease, we give the following definition of the indices of the unique marginal generators during each hour under these two sets of partitions.

Definition 2. For all $t \in \mathcal{T}$ and the resultant partitions, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$ and $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$, we define $j_t^{M,-}$ and $j_t^{M,+}$, respectively, as the index of the unique firms that are members of $\mathcal{G}_t^{M,-}$ and $\mathcal{G}_t^{M,+}$.

We prove now the following two lemmata, which show important relationships between the two sets of partitions.

Lemma 4. *For all* $t \in \mathcal{T}$ *such that* $\mathcal{G}_t^{I,-} \neq \emptyset$, $\exists j \in \mathcal{G}_t^{I,-}$ *such that:*

$$\mathcal{G}_t^{I,+} = \left(\mathcal{G}_t^{I,-} \setminus \{j\}\right) \cup \{i\}.$$

Proof. If for any given $t \in \mathcal{T}$ we have $\mathcal{G}_t^{I,-} \neq \emptyset$ then we must have $D_t > K$, which means that $\mathcal{G}_t^{I,+} \neq \emptyset$. By its definition, i is an element of $\mathcal{G}_t^{I,+}$. Thus, all we must show is that $\forall j \in \mathcal{G}_t^{I,+}$ with $j \neq i$, we must have that $j \in \mathcal{G}_t^{I,-}$.

Suppose, for contradiction, that $\exists j \in \mathcal{G}_t^{I,+}$ with $j \neq i$ and $j \notin \mathcal{G}_t^{I,-}$. By Assumption 6 we must have:

$$\left|\mathcal{G}_t^{I,-}\right| = \left|\mathcal{G}_t^{I,+}\right|.$$

Thus, there must be $h, k \in \mathcal{G}_t^{I,-}$ with $k \neq h$ and $h, k \notin \mathcal{G}_t^{I,+}$. At least one of h or k must be a member of $\mathcal{G}_t^{V,+}$, because by Assumption 6, $\mathcal{G}_t^{M,+}$ is a singleton. Without loss of generality, we assume that $h \in \mathcal{G}_t^{V,+}$, meaning that either $k \in \mathcal{G}_t^{V,+}$ or $k = j_t^{M,+}$. Let us assume that $j = j_t^{M,-}$. If $k = j_t^{M,+}$, the lexicographic ordering of Assumption 6 is violated. Conversely, if $k \in \mathcal{G}_t^{V,+}$, we know that $j_t^{M,+} \neq j_t^{M,-}$, because of our assumption that $j = j_t^{M,-}$. Thus, we have either that $j_t^{M,+} \in \mathcal{G}_t^{I,-}$ or $j_t^{M,+} \in \mathcal{G}_t^{V,-}$. However, we cannot have $j_t^{M,+} \in \mathcal{G}_t^{I,-}$, as this would violate the lexicographic ordering of Assumption 6. On the other hand, if $j_t^{M,+} \in \mathcal{G}_t^{V,-}$ the principle of optimality requires:

$$b_j^v K + b_j^f + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f < b_h^v K + b_h^f + b_j^v r_t + b_j^f; \tag{26}$$

where the strict inequality is required by imposing the lexicographic-ordering requirement of Assumption 6 on the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$. However, (26) violates the optimality of the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$. Thus, we can have neither $j_t^{M,+} \in \mathcal{G}_t^{I,-}$ nor $j_t^{M,+} \in \mathcal{G}_t^{V,-}$, which means that $k \notin \mathcal{G}_t^{V,+}$ and that we must have $j \in \mathcal{G}_t^{V,-}$. However, because $h \in \mathcal{G}_t^{I,-}$ and $h \in \mathcal{G}_t^{V,+}$, we must have $j =_K h$, which violates the lexicographic ordering of Assumption 6. □

Lemma 5. *For all $t \in \mathcal{T}$ exactly one of following holds: either $j_t^{M,+} \in \mathcal{G}_t^{I,-}$ or $\mathcal{G}_t^{M,-} = \mathcal{G}_t^{M,+}$.*

Proof. For any $t \in \mathcal{T}$ both statements cannot hold simultaneously, because by Assumption 6 both $\mathcal{G}_t^{M,-}$ and $\mathcal{G}_t^{M,+}$ are singletons. Thus, we must show that it is impossible for neither statement to hold.

Suppose for contradiction that $\exists t \in \mathcal{T}$ for which neither statement holds. Then, $j_t^{M,+} \in \mathcal{G}_t^{V,-}$. By Lemma 4 this requires that $j_t^{M,-} \notin \mathcal{G}_t^{I,+}$, which implies that $j_t^{M,-} \in \mathcal{G}_t^{V,+}$. However, having $j_t^{M,-} \in \mathcal{G}_t^{V,+}$ contradicts the lexicographic ordering of Assumption 6. □

Lemma 4 shows that if $\mathcal{G}_t^{I,-}$ is non-empty for a given $t \in \mathcal{T}$, then $\mathcal{G}_t^{I,-}$ and $\mathcal{G}_t^{I,+}$ differ exactly by one firm (*i.e.*, firm i becomes a member of $\mathcal{G}_t^{I,+}$ and one member of $\mathcal{G}_t^{I,-}$ becomes a member of $\mathcal{G}_t^{M,+}$ or $\mathcal{G}_t^{V,+}$). Lemma 5 states that $\forall t \in \mathcal{T}$, a firm that is inactive under the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, cannot become marginal under the partition, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$.

Proposition 1 provides a theoretical foundation with which to develop an algorithm to solve (12)–(15). Specifically, one can compute firm i's profit under each of the $3^{|\mathcal{T}|}$ candidate solutions that Proposition 1 characterizes and select the optimal one. Enumerating completely all $3^{|\mathcal{T}|}$ solutions is computationally costly. Moreover, this technique does not connect firm i's offers to the resultant production allocation. Thus, we derive necessary and sufficient conditions that provide this linkage.

To do so, we begin with the following definition.

Definition 3. For all $t \in \mathcal{T}$ and the resultant partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, such that $\mathcal{G}_t^{I,-} \neq \emptyset$, we define:

$$j_t^{I,-} = \arg\max_{j \in \mathcal{G}_t^{I,-}} b_j^v K + b_j^f.$$

Next, we let:

$$\hat{x}_i = (\hat{x}_{i,1}, \ldots, \hat{x}_{i,|\mathcal{T}|});$$

denote a candidate set of production levels for firm i, which is of the form that is given by Proposition 1. We define the following constraint set, $\mathcal{B}_{\hat{x}_i}$, that relates firm i's offer to $\hat{x}_i$. The subsequent proposition shows that $\mathcal{B}_{\hat{x}_i}$ is necessary and sufficient for the offer, (b_i^f, b_i^v), to yield $\hat{x}_i$ as firm i's production levels that are given by (8)–(11).

For each $t \in \mathcal{T}$, there are three possible hour-t production levels. We add different constraints to $\mathcal{B}_{\hat{x}_i}$ depending upon the desired value of $\hat{x}_{i,t}$. The totality of these constraints $\forall t \in \mathcal{T}$ gives the set, $\mathcal{B}_{\hat{x}_i}$. We begin with the case of $\hat{x}_{i,t} = 0$. In such a case, we add the inequalities:

$$b_i^v r_t + b_i^f \geq \max\left\{ \left(b^v_{j_t^{I,-}} - b^v_{j_t^{M,-}} \right) K + b^f_{j_t^{I,-}} - b^f_{j_t^{M,-}}, 0 \right\} + b^v_{j_t^{M,-}} r_t + b^f_{j_t^{M,-}} \quad (27)$$

$$b_i^v K + b_i^f \geq \max\left\{ \left(b^v_{j_t^{M,-}} - b_j^v \right) r_t + b^f_{j_t^{M,-}} - b_j^f, 0 \right\} + b_j^v K + b_j^f; \forall j \in \mathcal{G}_t^{I,-}; \quad (28)$$

to $\mathcal{B}_{\hat{x}_i}$.

If $\hat{x}_{i,t} = r_t$, there are two possible cases. First, if $j_t^{M,-} \geq_K j_t^{I,-}$, we add the inequalities:

$$b_i^v r_t + b_i^f \leq b^v_{j_t^{M,-}} r_t + b^f_{j_t^{M,-}} \quad (29)$$

$$b_i^v \geq b_j^v; \forall j \in \mathcal{G}_t^{I,-} \quad (30)$$

$$b_i^v K + b_i^f + b^v_{j_t^{M,-}} r_t + b^f_{j_t^{M,-}} \geq b^v_{j_t^{I,-}} K + b^f_{j_t^{I,-}} + b_i^v r_t + b_i^f; \quad (31)$$

to $\mathcal{B}_{\hat{x}_i}$. Otherwise, if $j_t^{M,-} <_K j_t^{I,-}$, we add the inequalities:

$$b_i^v r_t + b_i^f \leq b^v_{j_t^{M,-}} r_t + b^f_{j_t^{M,-}} + \left(b^v_{j_t^{I,-}} - b^v_{j_t^{M,-}} \right) K + b^f_{j_t^{I,-}} - b^f_{j_t^{M,-}} \quad (32)$$

$$b_i^v \geq b^v_{j_t^{M,-}} \quad (33)$$

$$b_i^v K + b_i^f + N \geq M + b_i^v r_t + b_i^f; \quad (34)$$

to $\mathcal{B}_{\hat{x}_i}$, where:

$$M = \max_{j \in \left(\mathcal{G}_t^{I,-} \cup \mathcal{G}_t^{M,-}\right) \setminus \left\{ j_t^{I,-} \right\}} \left(b_j^v K + b_j^f \right);$$

and:

$$N = \min_{j \in \left\{ j_t^{I,-} \right\} \cup \mathcal{G}_t^{V,-}} \left(b_j^v r_t + b_j^f \right).$$

Finally, if $\hat{x}_{i,t} = K$, there are two possible cases. First, if $j_t^{M,-} \neq j_t^{M,+}$, we add the inequalities:

$$b_i^v K + b_i^f + b^v_{j_t^{M,+}} r_t + b^f_{j_t^{M,+}} \leq b^v_{j_t^{M,+}} K + b^f_{j_t^{M,+}} + b^v_{j_t^{M,-}} r_t + b^f_{j_t^{M,-}} \quad (35)$$

$$b_i^v \leq b^v_{j_t^{M,+}}; \quad (36)$$

to $\mathcal{B}_{\hat{x}_i}$. Otherwise, if $j_t^{M,-} = j_t^{M,+}$, we add the inequalities:

$$b_i^v K + b_i^f \leq b_p^v K + b_p^f \quad (37)$$

$$b_i^v \leq b^v_{j_t^{M,+}} \quad (38)$$

$$b_p^v K + b_p^f + b_i^v r_t + b_i^f \geq b_i^v K + b_i^f + b^v_{j_t^{M,+}} r_t + b^f_{j_t^{M,+}}; \quad (39)$$

to $\mathcal{B}_{\hat{x}_i}$, where $p \in \mathcal{G}_t^{I,-}$ and $p \notin \mathcal{G}_t^{I,+}$. Such a firm, p, is guaranteed to exist by Lemma 4.

Proposition 2. *For any set of firm-i production levels, $\hat{x}_i$, that is of the form that is characterized by Proposition 1, the constraint set, $\mathcal{B}_{\hat{x}_i}$, which is defined by (27)–(39), is necessary and sufficient to have $\hat{x}_i$ be optimal in (8)–(11).*

The proof of Proposition 2 is provided in the appendix in three parts, which correspond to showing the necessity and sufficiency of $\mathcal{B}_{\hat{x}_i}$ for hours, $t \in \mathcal{T}$, such that $\hat{x}_{i,t} = 0$, $\hat{x}_{i,t} = r_t$, and $\hat{x}_{i,t} = K$.

Proposition 2 connects firm i's upper-level variables, which constitutes its offer, to a commitment and dispatch schedule that it wants to receive from the lower-level MO problem. However, Proposition 2 provides constraints that the offers must satisfy, not an optimal set of offers. Hence, we introduce the following auxiliary problem:

$$\max_{b_i^f, b_i^v} b_i^v$$
$$\text{s.t.} \left(b_i^f, b_i^v \right) \in \mathcal{B}_{\hat{x}_i}$$
$$(13)\text{–}(14);$$

which we denote as $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$. We prove in the following lemma that for a given $\hat{x}_i$, $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ generates firm-i offers that result in those production levels from the MO's lower-level problem and maximize firm i's profit.

Lemma 6. *For any $\hat{x}_i$ that is of the form that is characterized by Proposition 1, an optimal solution to $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ generates firm-i offers that result in $\hat{x}_i$ being optimal in (8)–(11) and (12) being maximized.*

Proof. Objective function (12) is additively separable in t and we can consider separately hours such that $\hat{x}_{i,t} = 0$, $\hat{x}_{i,t} = K$, and $\hat{x}_{i,t} = r_t$. By definition, firm i's profit is zero during all hours $t \in \mathcal{T}$ such that $\hat{x}_{i,t} = 0$.

For hours $t \in \mathcal{T}$, such that $\hat{x}_{i,t} = K$, we have from Assumption 5 that η_t depends upon neither b_i^v nor b_i^f. Thus, for all $t \in \mathcal{T}$ such that $\hat{x}_{i,t} = 0$ or $\hat{x}_{i,t} = K$, firm i's hour-t profit does not depend upon firm i's offer. For the final case of all $t \in \mathcal{T}$ such that $\hat{x}_{i,t} = r_t$, Assumptions 5 and 6 imply that firm i's hour-t profit is:

$$(\eta_t - c_i^v)x_{i,t} - c_i^f u_{i,t} = (b_i^v - c_i^v)x_{i,t} - c_i^f u_{i,t};$$

which is strictly increasing in b_i^v. Thus, the objective function of $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ maximizes the terms in firm i's profit function that are dependent upon firm i's offers. □

The final step before developing our solution algorithm for (12)–(15) is to introduce the following variant of the MO's lower-level problem. This variant takes as an input a partition set, $(\mathcal{G}_t^I, \mathcal{G}_t^M, \mathcal{G}_t^V)$, $\forall t \in \mathcal{T}$ and is formulated as:

$$\min \sum_{j \in \mathcal{G}, t \in \mathcal{T}} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) \tag{40}$$

$$\text{s.t.} \sum_{j \in \mathcal{G}} x_{j,t} = D_t; \forall t \in \mathcal{T} \qquad (\lambda_t) \tag{41}$$

$$0 \le x_{j,t} \le K u_{j,t}; \forall j \in \mathcal{G}, t \in \mathcal{T} \qquad (\rho_{j,t}^-, \rho_{j,t}^+) \tag{42}$$

$$0 \le u_{j,t} \le 1; \forall j \in \mathcal{G}, t \in \mathcal{T} \qquad (\beta_{j,t}^-, \beta_{j,t}^+) \tag{43}$$

$$u_{j,t} \ge 1; \forall t \in \mathcal{T}, j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M \qquad (\zeta_{j,t}) \tag{44}$$

$$u_{j,t} \le 0; \forall t \in \mathcal{T}, j \in \mathcal{G}_t^V \qquad (\zeta_{j,t}). \tag{45}$$

Problem (40)–(45) is the same as (8)–(11) except that integrality restriction (11) is relaxed and replaced with restrictions that fix the values of the commitment variables based on the given partition set. Problem (40)–(45) is a convex linear optimization, meaning that it has well defined Lagrange multipliers, which are given in parentheses to the right of each constraint set. Following Assumption 5, $\forall t \in \mathcal{T}$, λ_t can be used as the hour-t energy price.

We conclude our theoretical analysis of a centrally committed design by presenting pseudocode in Algorithm 1 for our technique to solve (12)–(15). Lines 1 and 2 begin by determining the two sets of partitions, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$ and $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$, $\forall t \in \mathcal{T}$, respectively. Line 3 initializes the main iterative loop, which is in Lines 4–14. z^* stores the incumbent best value of firm i's objective function and b_i^{f*} and b_i^{v*} represent the offer that achieves z^*.

Algorithm 1 Solution Technique for (12)–(15)

1: Solve (8)–(11) with i removed from $\mathcal{G}$ to obtain the partitions, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, $\forall t \in \mathcal{T}$
2: Solve (8)–(11) with $b_i^f \leftarrow 0$ and $b_i^v \leftarrow 0$ to obtain the partitions, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$, $\forall t \in \mathcal{T}$
3: $z^* \leftarrow -\infty$, $b_i^{f*} \leftarrow \bar{b}^f$, $b_i^{v*} \leftarrow \bar{b}^v$
4: **for** $\hat{x}_i \in \Xi$ **do**
5: $\quad(\hat{b}_i^v, \hat{b}_i^f) \leftarrow \operatorname{argmax} \mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$
6: $\quad$**if** $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ is feasible and bounded **then**
7: $\quad\quad$Solve (8)–(11) with $b_i^f \leftarrow \hat{b}_i^f$ and $b_i^v \leftarrow \hat{b}_i^v$ to obtain the partitions, $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$, $\forall t \in \mathcal{T}$
8: $\quad\quad$Solve (40)–(45) with partitions, $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$, $\forall t \in \mathcal{T}$, to obtain energy prices, $\hat{\lambda}_t$, $\forall t \in \mathcal{T}$
9: $\quad\quad\hat{z} \leftarrow \sum_{t \in \mathcal{T}} [(\hat{\lambda}_t - c_i^v)\hat{x}_{i,t} - c_i^f \hat{u}_{i,t}]$
10: $\quad\quad$**if** $\hat{z} > z^*$ **then**
11: $\quad\quad\quad z^* \leftarrow \hat{z}$, $b_i^{f*} \leftarrow \hat{b}_i^f$, $b_i^{v*} \leftarrow \hat{b}_i^v$
12: $\quad\quad$**end if**
13: $\quad$**end if**
14: **end for**

Lines 4–14 loop through the elements of, Ξ, which is the set of possible values of $\hat{x}_i$ in accordance with Proposition 1, *i.e.*, $\forall \hat{x}_i \in \Xi$ we have $\hat{x}_{i,t} \in \{0, r_t, k\}$, $\forall t \in \mathcal{T}$. For each $\hat{x}_i \in \Xi$, Line 5 solves $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ to determine optimal offers that attain the dispatch level $\hat{x}_i$. $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ may be infeasible for some $\hat{x}_i$, *e.g.*, the inequalities that define $\mathcal{B}_{\hat{x}_i}$ may be inconsistent. Such $\hat{x}_i$ are excluded from further consideration. If $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ is feasible and bounded (*cf.* Line 6), the optimized value of $(\hat{b}_i^f, \hat{b}_i^v)$ that is found in Line 5 is used to find the partitions, $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$, $\forall t \in \mathcal{T}$, in Line 7. Line 8 solves (40)–(45) to determine the resultant energy prices, which are used in Line 9 to compute firm i's profit, where we assume that $\hat{u}_{i,t} = 0$, $\forall t \in \mathcal{T}$ such that $\hat{x}_{i,t} = 0$ and $\hat{u}_{i,t} = 1$ for all other t. If the profit that firm i earns from $\hat{x}_i$ is greater than z^* (*cf.* Line 10) then z^* and (b_i^{f*}, b_i^{v*}) are updated in Line 11.

We conclude this section with the following lemma, which states that Algorithm 1 is guaranteed to find an optimal solution to (12)–(15).

Lemma 7. *Algorithm 1 is guaranteed to produce an optimal solution to (12)–(15).*

Proof. By Proposition 1, an optimal solution to (12)–(15) occurs at an element of Ξ. By Lemma 6, for each $\hat{x}_i \in \Xi$, $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ finds firm-i offers that maximize firm-i profit. □

5. Centrally committed market with make-whole payments

Make-whole payments are a common approach to address potential economic confiscation under centrally committed designs. Extending (12)–(15) to include make-whole payments is straightforward, because such payments change only firm i's objective function. With make-whole payments, (12)–(15) changes to:

$$\max \sum_{t \in \mathcal{T}} \left[(\eta_t - c_i^v)x_{i,t} - c_i^f u_{i,t} \right] + \max \left\{ 0, \sum_{t \in \mathcal{T}} \left[(b_i^v - \eta_t)x_{i,t} + b_i^f u_{i,t} \right] \right\} \tag{46}$$

$$\text{s.t.} (13)\text{–}(15); \tag{47}$$

and retains the same decision variables.

Importantly, the MO's lower-level problem does not change with make-whole payments, meaning that most of the analysis that we present in Section 4 applies to (46)–(47). Extending our analysis to include make-whole payments requires three steps. First, we prove a result that is analogous to Proposition 1. Next, we linearize (46). Third, we develop an algorithm that is akin to Algorithm 1.

Proposition 3. *Suppose that (b_i^{f*}, b_i^{v*}) is an optimum of (46)–(47). Then $\exists (x^*, u^*)$ that is an optimum of (8)–(11), satisfies Assumption 3, and has $x_{i,t}^* \in \{0, r_t, K\}$, $\forall t \in \mathcal{T}$.*

Proof. The sole difference between (46)–(12) and (47)–(15) is the $\max\{\cdot\}$ operator that is in (46). If the $\max\{\cdot\}$ operator equals zero, then Proposition 3 follows immediately from Proposition 1. Thus, we consider the other case, wherein the $\max\{\cdot\}$ operator is strictly positive. In such a case, firm i's profit is:

$$\sum_{t \in \mathcal{T}} \left[(b_i^{v*} - c_i^v)x_{i,t} + \left(b_i^{f*} - c_i^f \right) u_{i,t} \right].$$

Suppose for contradiction that $\exists \tau \in \mathcal{T}$ such that $x_{i,\tau}^* \notin \{0, r_\tau, K\}$. We consider two cases.

First, consider a case wherein $x_{i,\tau}^* \in (0, r_\tau)$. We can use the exact same argument as in the proof of Proposition 1 to show that there is an alternative feasible solution that is less costly in (8)–(11).

Next, consider a case wherein $x_{i,\tau}^* \in (r_\tau, K)$, in which case we must have $|\mathcal{G}_\tau^M| \ge 2$ to satisfy (9). By Lemma 1 we have that $b_i^{v*} = b_j^v$, $\forall j \in \mathcal{G}_\tau^M$ and by Assumption 5 we have that $\eta_\tau = b_i^{v*}$. We can consider two cases, which differ by the sign of $b_i^{v*} - c_i^v$. If $b_i^{v*} - c_i^v \ge 0$, consider an alternative solution in which $x_{i,\tau}^*$ is increased by ϵ and:

$$\sum_{j \in \mathcal{G}_\tau^M, j \ne i} x_{j,\tau}^*;$$

is decreased by ϵ, where ϵ is sufficiently small so as not to violate (10). Such an alternative solution does not change the value of (8) but weakly increases the value of (46), which contradicts (x^*, u^*) satisfying Assumption 3. For the other case, wherein $b_i^{v*} - c_i^v < 0$, consider an alternative solution in which $x_{i,\tau}^*$ is decreased by ϵ and:

$$\sum_{j \in \mathcal{G}_\tau^M, j \neq i} x_{j,\tau}^*;$$

is increased by ϵ, where ϵ is sufficiently small so as not to violate (10). This alternative solution does not change the value of (8) but increases strictly the value of (46), which contradicts the optimality of (b_i^{f*}, b_i^{v*}) in (46)–(47). □

As is the case without make-whole payments, Proposition 3 allows us to limit our attention to a finite set of production levels for firm i. Next, to linearize (46), we begin with the KKT conditions for (40)–(45), which are:

$$b_j^v - \lambda_t - \rho_{j,t}^- + \rho_{j,t}^+ = 0; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{48}$$

$$b_j^f - K\rho_{j,t}^+ - \beta_{j,t}^- + \beta_{j,t}^+ - \zeta_{j,t} = 0; \forall t \in \mathcal{T}, j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M \tag{49}$$

$$b_j^f - K\rho_{j,t}^+ - \beta_{j,t}^- + \beta_{j,t}^+ + \zeta_{j,t} = 0; \forall t \in \mathcal{T}, j \in \mathcal{G}_t^V \tag{50}$$

(41) (51)

$$0 \le x_{j,t} \perp \rho_{j,t}^- \ge 0; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{52}$$

$$x_{j,t} \le K u_{j,t} \perp \rho_{j,t}^+ \ge 0; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{53}$$

$$0 \le u_{j,t} \perp \beta_{j,t}^- \ge 0; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{54}$$

$$u_{j,t} \le 1 \perp \beta_{j,t}^+ \ge 0; \forall j \in \mathcal{G}, t \in \mathcal{T} \tag{55}$$

$$u_{j,t} \ge 1 \perp \zeta_{j,t} \ge 0; \forall t \in \mathcal{T}, j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M \tag{56}$$

$$u_{j,t} \le 0 \perp \zeta_{j,t} \ge 0; \forall t \in \mathcal{T}, j \in \mathcal{G}_t^V; \tag{57}$$

and its strong-duality equality, which is:

$$\sum_{j \in \mathcal{G}, t \in \mathcal{T}} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) = \sum_{t \in \mathcal{T}} \left[D_t \lambda_t - \sum_{j \in \mathcal{G}} \beta_{j,t}^+ + \sum_{j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{j,t} \right]. \tag{58}$$

We break (46) into two parts to linearize it. First, we have from (48) and Assumption 5 that:

$$\sum_{t \in \mathcal{T}} \left[(\eta_t - c_i^v) x_{i,t} - c_i^f u_{i,t} \right] = \sum_{t \in \mathcal{T}} \left[(b_i^v - \rho_{i,t}^- + \rho_{i,t}^+ - c_i^v) x_{i,t} - c_i^f u_{i,t} \right]. \tag{59}$$

From (52) and (53), the right-hand side of (59) simplifies to:

$$\sum_{t \in \mathcal{T}} \left[b_i^v x_{i,t} + K u_{i,t} \rho_{i,t}^+ - c_i^v x_{i,t} - c_i^f u_{i,t} \right];$$

which becomes:

$$\sum_{t \in \mathcal{T}} \left[D_t \lambda_t - \sum_{j \in \mathcal{G}} \beta_{j,t}^+ + \sum_{j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{j,t} - \left(b_i^f - K\rho_{i,t}^+ \right) u_{i,t} - \sum_{j \in \mathcal{G}, j \neq i} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) - c_i^v x_{i,t} - c_i^f u_{i,t} \right]; \tag{60}$$

by (58). Conditions (49) and (50) imply that (60) becomes:

$$\sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \left[D_t \lambda_t - \sum_{j \in \mathcal{G}} \beta_{j,t}^+ + \sum_{j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{j,t} - \left(\beta_{i,t}^- - \beta_{i,t}^+ + \zeta_{i,t} \right) u_{i,t} - \sum_{j \in \mathcal{G}, j \neq i} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) - c_i^v x_{i,t} - c_i^f u_{i,t} \right] + \sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^V} \left[D_t \lambda_t - \sum_{j \in \mathcal{G}} \beta_{j,t}^+ + \sum_{j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{j,t} - \left(\beta_{i,t}^- - \beta_{i,t}^+ - \zeta_{i,t} \right) u_{i,t} - \sum_{j \in \mathcal{G}, j \neq i} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) - c_i^v x_{i,t} - c_i^f u_{i,t} \right];$$

which by (54)–(57) simplifies further to:

$$\sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \left[D_t \lambda_t - \sum_{j \in \mathcal{G}} \beta_{j,t}^+ + \sum_{j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{j,t} + \beta_{i,t}^+ - \zeta_{i,t} - \sum_{j \in \mathcal{G}, j \neq i} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) - c_i^v x_{i,t} - c_i^f u_{i,t} \right] + \sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^V} \left[D_t \lambda_t - \sum_{j \in \mathcal{G}} \beta_{j,t}^+ + \sum_{j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{j,t} + \beta_{i,t}^+ - \sum_{j \in \mathcal{G}, j \neq i} \left(b_j^v x_{j,t} + b_j^f u_{j,t} \right) - c_i^v x_{i,t} - c_i^f u_{i,t} \right];$$

and can be rewritten as:

$$\sum_{t \in \mathcal{T}} \left[D_t \lambda_t - \sum_{j \in \mathcal{G}, j \neq i} \left(\beta_{j,t}^+ + b_j^v x_{j,t} + b_j^f u_{j,t} \right) + \sum_{j \in \mathcal{G}_t^I \cup \mathcal{G}_t^M, j \neq i} \zeta_{j,t} - c_i^v x_{i,t} - c_i^f u_{i,t} \right];$$

which is linear in the decision variables of (46)–(47).

The second part of (46) has the $\max\{\cdot\}$ operator. Using (48) and Assumption 5, the term in the $\max\{\cdot\}$ operator in (46) can be written as:

$$\sum_{t \in \mathcal{T}} \left[(b_i^v - \eta_t) x_{i,t} + b_i^f u_{i,t} \right] = \sum_{t \in \mathcal{T}} \left[\left(\rho_{i,t}^- - \rho_{i,t}^+ \right) x_{i,t} + b_i^f u_{i,t} \right];$$

which simplifies further to:

$$\sum_{t \in \mathcal{T}} \left(-K\rho_{i,t}^+ + b_i^f \right) u_{i,t};$$

by (52) and (53). This expression becomes:

$$\sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \left(\beta_{i,t}^- - \beta_{i,t}^+ + \zeta_{i,t} \right) u_{i,t} + \sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^V} \left(\beta_{i,t}^- - \beta_{i,t}^+ - \zeta_{i,t} \right) u_{i,t};$$

by (49) and (50), which simplifies further to:

$$\sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{i,t} - \sum_{t \in \mathcal{T}} \beta_{i,t}^+; \tag{61}$$

by (54)–(57). We introduce an auxiliary binary variable, ϕ, which equals 1 if the term in the $\max\{\cdot\}$ operator in (46) is positive and equals 0 otherwise. With this definition, we can write the $\max\{\cdot\}$ operator in (46) as:

$$\max \left\{ 0, \sum_{t \in \mathcal{T}} \left[(b_i^v - \eta_t) x_{i,t} + b_i^f u_{i,t} \right] \right\} = \sum_{t \in \mathcal{T}} \left[(b_i^v - \eta_t) x_{i,t} + b_i^f u_{i,t} \right] \phi;$$

the right-hand side of which becomes the linear expression:

$$\left(\sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \zeta_{i,t} - \sum_{t \in \mathcal{T}} \beta_{i,t}^+ \right) \phi = \sum_{t \in \mathcal{T}: i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M} \hat{\zeta}_{i,t} - \sum_{t \in \mathcal{T}} \hat{\beta}_{i,t}^+;$$

by (61), where we define $\hat{\zeta}_{i,t} = \zeta_{i,t}\phi$ and $\hat{\beta}_{i,t}^+ = \beta_{i,t}^+\phi$, $\forall t \in \mathcal{T}$. For all $t \in \mathcal{T}$, $\hat{\zeta}_{i,t}$ and $\hat{\beta}_{i,t}^+$ are bilinear, however because these are the products of continuous and binary variables, they can be linearized by adding the constraints:

$$\phi \in \{0, 1\} \tag{62}$$

$$0 \le \hat{\zeta}_{i,t} \le M\phi; \forall t \in \mathcal{T} : i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M \tag{63}$$

$$\zeta_{i,t} - (1 - \phi)M \le \hat{\zeta}_{i,t} \le \zeta_{i,t}; \forall t \in \mathcal{T} : i \in \mathcal{G}_t^I \cup \mathcal{G}_t^M \tag{64}$$

$$0 \le \hat{\beta}_{i,t}^+ \le M\phi; \forall t \in \mathcal{T} \tag{65}$$

$$\beta_{i,t}^+ - (1 - \phi)M \le \hat{\beta}_{i,t}^+ \le \beta_{i,t}^+; \forall t \in \mathcal{T}; \tag{66}$$

where M is a sufficiently large constant (*cf.* the work of Sioshansi and Conejo (2017) for further details).

We define now an auxiliary problem, which plays a similar role in our solution methodology to that of $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ in the case without make-whole payments. The auxiliary problem takes as an input a set of firm-i production levels, $\hat{x}_i$, and an associated partition set, $(\mathcal{G}_t^I, \mathcal{G}_t^M, \mathcal{G}_t^V)$, $\forall t \in \mathcal{T}$, and is formulated as:

$$\max \sum_{t\in\mathcal{T}} \left[D_t\lambda_t - \sum_{j\in\mathcal{G}, j\neq i} \left(\beta_{j,t}^+ + b_j^v x_{j,t} + b_j^f u_{j,t}\right) + \sum_{j\in\mathcal{G}_t^I\cup\mathcal{G}_t^M, j\neq i} \zeta_{j,t} - c_i^v x_{i,t} - c_i^f u_{i,t} - \hat{\beta}_{i,t}^+ \right] + \sum_{t\in\mathcal{T}: i\in\mathcal{G}_t^I\cup\mathcal{G}_t^M} \hat{\zeta}_{i,t} \tag{67}$$

$$\text{s.t.} \left(b_i^f, b_i^v\right) \in \mathcal{B}_{\hat{x}_i} \tag{68}$$

$$(48)\text{–}(57),\ (62)\text{–}(66); \tag{69}$$

where the decision variables include all of the variables of (46)–(47), the Lagrange multipliers of (48)–(57), and the auxiliary variables, $\hat{\zeta}_{i,t}$ and $\hat{\beta}_{i,t}^+$, $t \in \mathcal{T}$ and ϕ. We show now the following two lemmata. The first shows that there is a one-to-one correspondence between $\hat{x}_i$ and a partition set, $(\mathcal{G}_t^I, \mathcal{G}_t^M, \mathcal{G}_t^V)$, $\forall t \in \mathcal{T}$. The second is akin to Lemma 6, and shows that for a given $\hat{x}_i$, (67)–(69) yields offers that solve (46)–(47).

Lemma 8. *For any $(b_i^f, b_i^v) \in \mathcal{B}_{\hat{x}_i}$ there is a unique resultant partition set that satisfies the lexicographic ordering that underlies Assumption 6.*

Proof. Suppose for contradiction that for a given $(b_i^f, b_i^v) \in \mathcal{B}_{\hat{x}_i}$, $\exists \tau \in \mathcal{T}$ such that there are two partitions, which we denote as $(\mathcal{G}_\tau^I, \mathcal{G}_\tau^M, \mathcal{G}_\tau^V)$ and $(\hat{\mathcal{G}}_\tau^I, \hat{\mathcal{G}}_\tau^M, \hat{\mathcal{G}}_\tau^V)$. Consider, first, the case wherein $\hat{x}_{i,\tau} = K$. In such a case, we have that:

$$\sum_{j\in\mathcal{G}_\tau^I\setminus\{i\}} \left(b_j^v K + b_j^f\right) + b_{j_\tau^M}^v r_\tau + b_{j_\tau^M}^f = \sum_{j\in\hat{\mathcal{G}}_\tau^I\setminus\{i\}} \left(b_j^v K + b_j^f\right) + b_{\hat{j}_\tau^M}^v r_\tau + b_{\hat{j}_\tau^M}^f;$$

where j_τ^M and $\hat{j}_\tau^M$ denote the unique elements of $\mathcal{G}_\tau^M$ and $\hat{\mathcal{G}}_\tau^M$, respectively. This equality, which holds because if it does not one of the partitions is not optimal in (8)–(11), implies that the two partitions are identical, otherwise the lexicographic ordering of Assumption 6 is violated.

The other cases wherein $\hat{x}_{i,\tau} = r_t$ and $\hat{x}_{i,\tau} = 0$ yield the equalities:

$$\sum_{j\in\mathcal{G}_\tau^I} \left(b_j^v K + b_j^f\right) = \sum_{j\in\hat{\mathcal{G}}_\tau^I} \left(b_j^v K + b_j^f\right);$$

and:

$$\sum_{j\in\mathcal{G}_\tau^I} \left(b_j^v K + b_j^f\right) + b_{j_\tau^M}^v r_\tau + b_{j_\tau^M}^f = \sum_{j\in\hat{\mathcal{G}}_\tau^I} \left(b_j^v K + b_j^f\right) + b_{\hat{j}_\tau^M}^v r_\tau + b_{\hat{j}_\tau^M}^f;$$

respectively, which yield the same conclusions that the two partitions must be equivalent. □

Lemma 9. *For any $\hat{x}_i$ that is of the form that is characterized by Proposition 3, an optimal solution to (67)–(69) generates firm-i offers that result in $\hat{x}_i$ being optimal in (8)–(11) and maximizing (46).*

Proof. By Proposition 2, the constraint set, $\mathcal{B}_{\hat{x}_i}$, is necessary and sufficient for (b_i^f, b_i^v) to yield $\hat{x}_i$ as an optimum of (8)–(11). By Lemma 8, for a given offer, $(b_i^f, b_i^v) \in \mathcal{B}_{\hat{x}_i}$, there is a unique partition that is associated with $\hat{x}_i$. Thus, fixing this partition, (8)–(11) can be replaced with (40)–(45), which, in turn, can be replaced with necessary and sufficient KKT conditions (48)–(57). Finally, (62)–(66) are needed to linearize the $\max\{\cdot\}$ in (46), which yields the equivalent objective function that is given by (67). □

To conclude our analysis of a centrally committed design with make-whole payments, we provide pseudocode in Algorithm 2 for our technique to solve (46)–(47). Algorithms 1 and 2 are similar, with the key difference in Line 8 of the former. For each given firm-i production schedule, Algorithm 1 determines firm-i profit in Line 9 by determining first prices in Line 8. Line 8 of Algorithm 2 does this in one step by solving (67)–(69), which incorporates make-whole payments.

Algorithm 2 Solution Technique for (46)–(47)

1: Solve (8)–(11) with i removed from $\mathcal{G}$ to obtain the partitions, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, $\forall t \in \mathcal{T}$
2: Solve (8)–(11) with $b_i^f \leftarrow 0$ and $b_i^v \leftarrow 0$ to obtain the partitions, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$, $\forall t \in \mathcal{T}$
3: $z^* \leftarrow -\infty$, $b_i^{f*} \leftarrow \bar{b}^f$, $b_i^{v*} \leftarrow \bar{b}^v$
4: **for** $\hat{x}_i \in \Xi$ **do**
5: $\quad (\hat{b}_i^v, \hat{b}_i^f) \leftarrow \operatorname{argmax} \mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$
6: $\quad$ **if** $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ is feasible and bounded **then**
7: $\quad\quad$ Solve (8)–(11) with $b_i^f \leftarrow \hat{b}_i^f$ and $b_i^v \leftarrow \hat{b}_i^v$ to obtain the partitions, $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$, $\forall t \in \mathcal{T}$
8: $\quad\quad$ $\hat{z} \leftarrow \max$ (67) s.t. (68)–(69)
9: $\quad\quad$ **if** $\hat{z} > z^*$ **then**
10: $\quad\quad\quad$ $z^* \leftarrow \hat{z}$, $b_i^{f*} \leftarrow \hat{b}_i^f$, $b_i^{v*} \leftarrow \hat{b}_i^v$
11: $\quad\quad$ **end if**
12: $\quad$ **end if**
13: **end for**

The following lemma shows that Algorithm 2 is guaranteed to find an optimal solution to (46)–(47).

Lemma 10. *Algorithm 2 is guaranteed to produce an optimal solution to (46)–(47).*

Proof. By Proposition 3, an optimal solution to (46)–(47) occurs at an element of Ξ. By Lemma 9, for each $\hat{x}_i \in \Xi$, solving (67)–(69) yields a set of firm-i offers that maximizes firm i's profit. □

6. Numerical example

This section presents two numerical examples in which firm i has two rivals, which are denoted as firms 1 and 2, and there are $|\mathcal{T}| = 3$ operating hours with demands, $D_1 = 25$ MW, $D_2 = 34$ MW, and $D_3 = 38$ MW. Firms i, 1, and 2 have $K = 20$ MW capacities in both examples. The fixed costs of firms 1 and 2, c_1^f and c_2^f, are non-zero in the first example and are zero in the second example.

We consider these two examples because having non-zero c_1^f or c_2^f complicates comparing the two market designs. Under a centrally committed design, firms can signal and recover their fixed costs through their multi-part offers and make-whole payments. Under a self-committed design, firms must account for their fixed costs in their energy offers. As such, we use the first example to examine how firm i structures its fixed- and variable-cost offers to compete against its two rivals under a centrally committed design. The second example is used to contrast firm i's offering behavior between centrally and self-committed designs.

Another issue in comparing the two market designs is that Assumption 6 guarantees that a single marginal generator is operated during each operating period. Our approach to solving firm i's profit-maximization problem under a self-committed design has no such guarantee. In comparing the two market designs with the second example, we assume that the generating firms and MO behave in a manner to yield a similar result to Assumption 6.

6.1. Non-zero c_1^f and c_2^f

6.1.1. Data

Columns two and three of Table 1 provide the assumed cost parameters of firms 1 and 2. Our base case assumes that firm i has a fixed cost of $c_i^f = 10$ and considers cases wherein c_i^v varies from 0.50 to 5.00 at increments of 0.25. Section 6.1.3 presents a parametric analysis, in which we examine the sensitivity of firm i's profit-maximizing behavior and resultant market operations with different values of c_i^f.

Table 1
Cost data for Firms 1 and 2 with Non-Zero c_1^f and c_2^f.

j	c_j^v	c_j^f
1	4	10
2	5	10

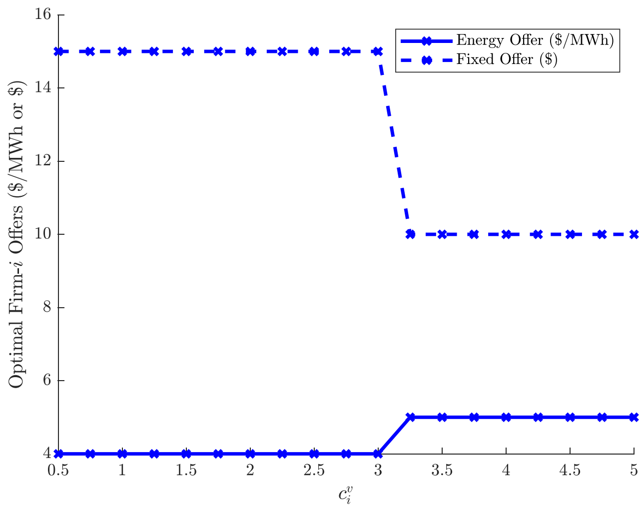

Fig. 1. Optimized firm-i offers under centrally committed design with non-zero c_1^f and c_2^f in the base case.

6.1.2. Base-case results

Fig. 1 shows optimized firm-i offers in the base case under a centrally committed design with make-whole payments for different values of c_i^v. Firm i's strategy differs for high and low values of c_i^v. If $c_i^v \leq 3.00$, firm i submits an offer with $b_i^v = 4.00$, so it matches firm 1's energy cost, and $b_i^f = 15$, which is the highest value that allows firm i to be an inframarginal generator during all three hours. In following such a strategy, the energy price is set by firm 1, which is the marginal generator, to $4.00/MWh during each hour and firm i's make-whole payments are maximized. Conversely, if $c_i^v \geq 3.25$, firm i submits an offer with $b_i^v = 5.00$ and $b_i^f = 10$, which allows firm i to match firm 2 on cost and to become the marginal generator during all three hours, yielding $5.00/MWh energy prices during each hour.

Fig. 2 summarizes the optimized profit, which is broken into two components, that firm i earns under the centrally committed design with different values of c_i^v. The first profit component is operating profit from energy sales, *i.e.*, total revenue from selling energy less the sum of variable and fixed operating costs. This profit component is negative for $c_i^v \geq 4.25$. The second profit component is the make-whole payment. Fig. 2 shows that total profit is decreasing in c_i^v. Moreover, for sufficiently high values of c_i^v, firm i's profit-maximizing strategy is to submit an offer that yields an *actual* profit loss, which is recovered through make-whole payments (firm i's optimal profit is exactly zero for the boundary case of $c_i^v = 5.00$). Firm i receives make-whole payments for all values of c_i^v that we examine, despite earning positive rents from energy payments for $c_i^v \leq 4.00$.

Fig. 1 shows that $c_i^v = 3.25$ is the threshold beyond which it is profitable under a centrally committed design for firm i to submit offers that result in its being the marginal generator. For instance, if $c_i^v = 4.00$ an optimal offer that results in firm i being inframarginal yields a profit loss of $30 from energy sales, which is supplemented by a $45 make-whole payment, for a total profit of $15. Conversely, the optimized offers that are shown in Fig. 1 that result in firm i being marginal during each hour yields $7 of profit from energy sales, which is supplemented by a $30 make-whole payment. This result demonstrates the trade-off between selling more energy at a lower price as an inframarginal

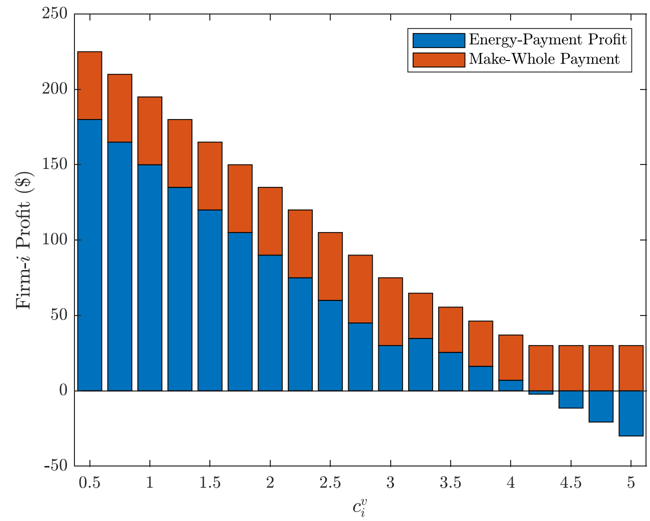

Fig. 2. Optimized firm-i profit under centrally committed design with non-zero c_1^f and c_2^f in the base case.

generator versus selling less energy at a higher price as a marginal supplier. Make-whole payments affect this trade-off.

Fig. 3 summarizes three different cost metrics from the optimized firm-i offers under a centrally committed design. The first is actual cost, which is defined as:

$$\sum_{j\in\mathcal{G},t\in\mathcal{T}} \left(c_j^v x_{j,t}^* + c_j^f u_{j,t}^*\right);$$

where $\forall j \in \mathcal{G}, t \in \mathcal{T}$ we let $x_{j,t}^*$ and $u_{j,t}^*$ denote the values of $x_{j,t}$ and $u_{j,t}$ that are obtained from solving the bi-level problem for the centrally committed design. The second is as-offered cost, which is defined as the value of (8). The distinction between actual and as-offered costs are that the former are actual costs that are incurred by the generating firms, whereas the latter are what the MO believes their costs to be, based upon offers that it receives. The final cost metric is settlement cost, which is defined as the sum of energy and make-whole payments to the generating firms. Finally, Fig. 3 shows the true cost minimum. The distinction between actual and true-minimal cost is that the former is based on commitment and dispatch decisions that are made using offers, which may not reflect actual cost and may be suboptimal with true cost information. Thus, the latter may be viewed as the cost of a perfectly competitive benchmark.

Fig. 3 shows that if $2.50 \leq c_i^v \leq 3.00$, the centrally committed design yields the true cost minimum. If $c_i^v < 4.00$ and $4.00 < c_i^v < 5.00$, firm i is strictly less costly than firm 1 and 2, respectively. However, firm i's optimal offering strategies (*cf.* Fig. 1) result in firm i appearing to be as costly as its more-expensive rival. Under a centrally committed design, $3.25 \leq c_i^v$ is the threshold, beyond which firm i increases its offer to match firm 2's cost. As such, a centrally committed design is not cost optimal if $3.25 \leq c_i^v \leq 3.75$. Productive-efficiency losses arise for these values of c_v^i because it is profit-maximizing for firm i to submit offers that result in its being operated as a marginal generator, whereas it is cost-minimal for it operate as an inframarginal unit. A centrally committed design yields cost-minimal solutions if $4.00 \leq c_i^v$. This result stems from firm i's behavior during hour 1. Specifically, the make-whole payment provides a financial incentive for firm i to operate as the marginal generator during all three hours.

System operations appear considerably more costly, on the basis of the offers that are submitted, than actual costs. This is because firm i's optimal offers result in its appearing to be as costly as its more-expensive rival. Specifically, if $c_i^v \leq 3.25$, firm i submits offers that makes it appear as costly as firm 1. Above this threshold, firm i submits

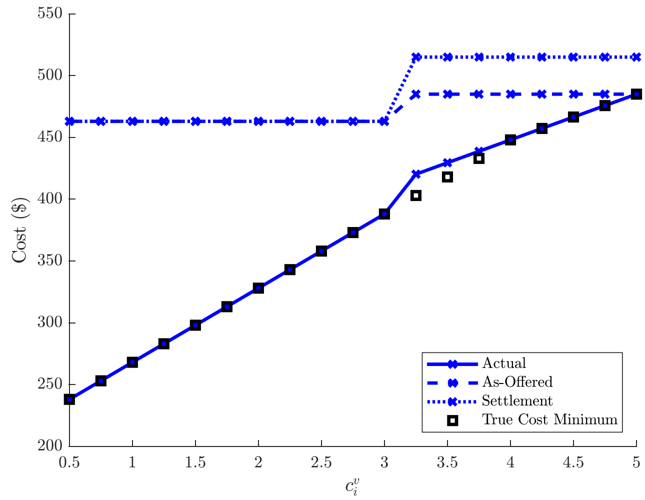

Fig. 3. Resultant actual, as-offered, and settlement costs from optimized firm-i offers under centrally committed design and true cost minimum with non-zero c_1^f and c_2^f in the base case.

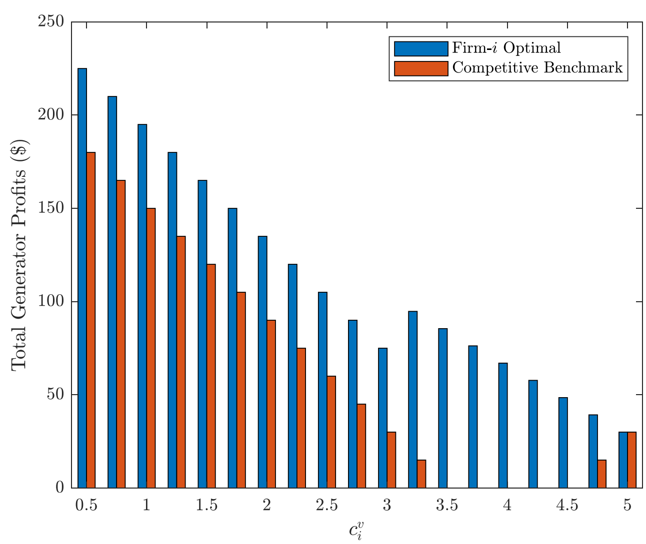

Fig. 4. Total generator profits resulting from optimized firm-i offers under centrally committed design with non-zero c_1^f and c_2^f in the base case assuming firm-i optimal and competitive offers.

offers to appear as costly as firm 2. Settlement cost is higher, also, than actual and as-offered costs.

We conclude our analysis of the base case with non-zero c_1^f and c_2^f with two profit comparisons. First, Fig. 4 summarizes total generator profits under a centrally committed design in two cases. The first, which is labeled 'Firm-i Optimal', uses firm i's optimal offers. The second, which is labeled 'Competitive Benchmark' assumes that firm i submits offers that are equal to its actual costs. As expected, generator profits are higher under if firm i optimizes its offer as compared to a competitive benchmark.

Our second profit comparison relaxes the requirement that firm i submit the same set of long-lived offers for each of the $|\mathcal{T}| = 3$ operating hours and instead allows for so-called short-lived offers, which can be different for each hour. Fig. 5 summarizes optimized firm-i profit under a centrally committed design with long- and short-lived offers. The figure shows that allowing short-lived offers yields profit increases, especially if c_i^v is relatively low. As an example, consider the case with $c_i^v = 0.50$. With long-live offers, it is profit-maximizing for firm i to submit $b_i^v = 4.00$, which is the highest offer that allows firm i to be

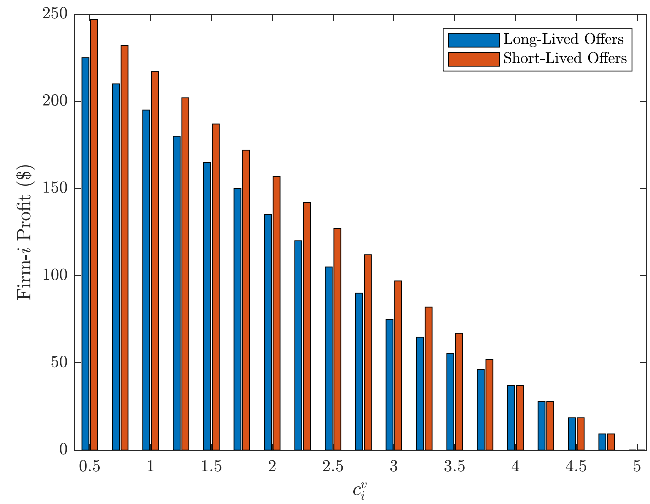

Fig. 5. Optimized Firm-i profit under centrally committed design with non-zero c_1^f and c_2^f in the base case with long- and short-lived offers.

inframarginal, and $b_i^f = 15$, which is the highest offer that prevents firm 2 being committed and dispatched in firm i's place. With short-lived offers, it is profit-maximizing for firm i to submit the same offers during hour 1 but to offer instead $b_i^f = 24$ and $b_i^f = 28$ during hours 2 and 3, respectively. Although the energy price remains \$4.00/MWh during all three hours with the short-lived offers, firm i receives greater make-whole payments, due to the higher values of b_i^f.

6.1.3. Sensitivity of results to fixed cost

We examine cases with $c_i^f = 5$ and $c_i^f = 15$ to determine how firm i's fixed cost impacts its offers and the resultant impact on dispatch, profit, and costs under a centrally committed design. Both cost levels yield the same type of threshold offering behavior. If $c_i^f = 5$, firm i submits offers under a centrally committed design that match firm 1 so long as $c_i^v \le 3.00$. Otherwise, it submits offers to match firm 2. With $c_i^f = 15$, firm i matches firm 1 up to the same threshold value of $c_i^v \le 3.00$. Thereafter, if $3.25 \le c_i^v \le 4.50$, firm i submits offers to match firm 2. If $c_i^v \ge 4.75$, firm i submits an offer at the price ceiling, thereby withdrawing its capacity from the market completely. This capacity withholding stems from firm i being unable to compete profitably with firm 2 if firm-i costs are too high.

With $c_i^f = 5$, firm i is an inframarginal generator when its offer matches firm 1 and is the marginal generator when its offer matches firm 2. With $c_i^f = 15$, firm i is an inframarginal generator when its offer matches firm 1 and its dispatch is mixed when its offer matches firm 2. Specifically, firm i is the marginal generator if $3.25 \le c_i^v \le 3.75$. If $4.00 \le c_i^v \le 4.50$, firm i is not dispatched during hour 1 and firm 2 is the marginal generator during hour 1 instead.

Figs. 6 and 7 summarize for the two cases with $c_i^f = 5$ and $c_i^f = 15$, respectively, the cost metrics that Fig. 3 provides for the base case. The results are qualitatively similar in these sensitivity cases to the base case. There are productive-efficiency losses in cases wherein firm i should be an inframarginal generator under the cost minimum but firm i is the marginal generator instead due to its offering strategy. Settlement costs are higher than actual costs.

Figs. 8 and 9 summarize optimized firm-i profit with $c_i^f = 5$ and $c_i^f = 15$, respectively. As under the base case, firm i is able to extract a make-whole payment in all cases in which it is dispatched, regardless of whether it earns a strictly positive rent from energy payments only.

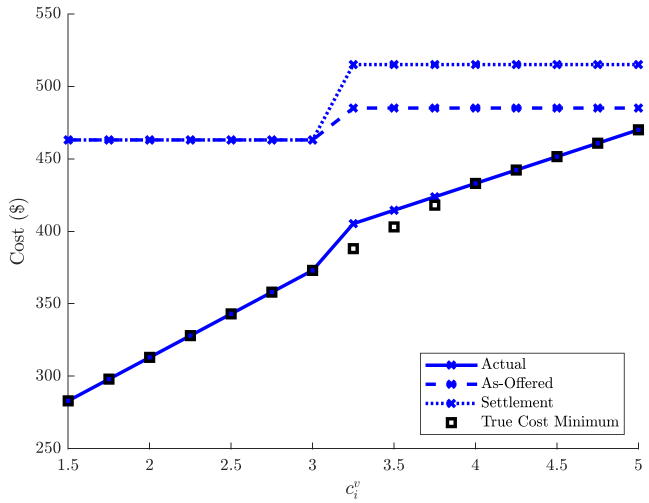

Fig. 6. Resultant actual, as-offered, and settlement costs from optimized firm-i offers under centrally committed design and true cost minimum with non-zero c_1^f and c_2^f and $c_i^f = 5$.

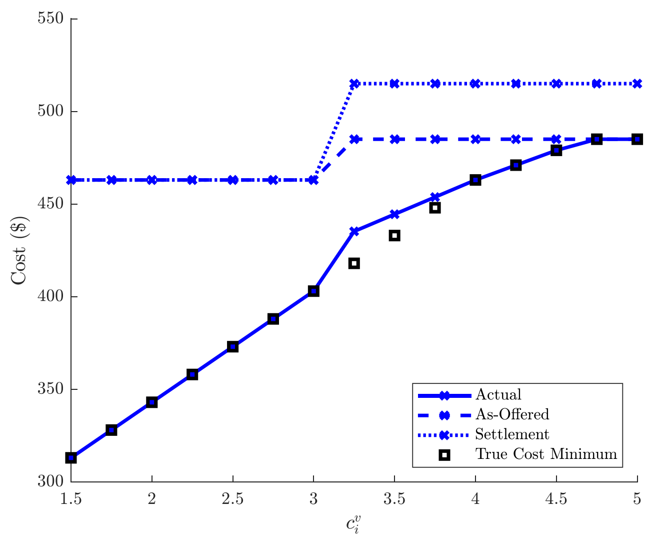

Fig. 7. Resultant actual, as-offered, and settlement costs from optimized firm-i offers under centrally committed design and true cost minimum with non-zero c_1^f and c_2^f and $c_i^f = 15$.

6.2. $c_1^f = c_2^f = 0$

Having $c_1^f = c_2^f = 0$ simplifies the comparison of the two market designs, because it obviates the need for making a behavioral assumption regarding how firms 1 and 2 incorporate their fixed costs into their energy offers under a self-committed design. Instead, with $c_1^f = c_2^f = 0$, we assume simply that firms 1 and 2 submit their per-unit energy-generation costs under the two market designs. This example assumes that $c_1^v = 5$ and $c_2^v = 6$.

Figs. 10–12 summarize the same information for the case with $c_1^f = c_2^f = 0$ that Figs. 1–3 do for the base case with non-zero c_1^f and c_2^f. Overall, the results for the centrally committed design are qualitatively similar between cases with zero and non-zero c_1^f and c_2^f. Moreover, firm i's optimal offering strategies under centrally and self-committed designs exhibit the same threshold-type behavior. Thus, market outcomes under the two designs have qualitative similarities.

Specifically, with $c_1^f = c_2^f = 0$, if $c_i^v \leq 3.25$ firm i matches firm 1's offer and is the inframarginal generator under a self-committed design.

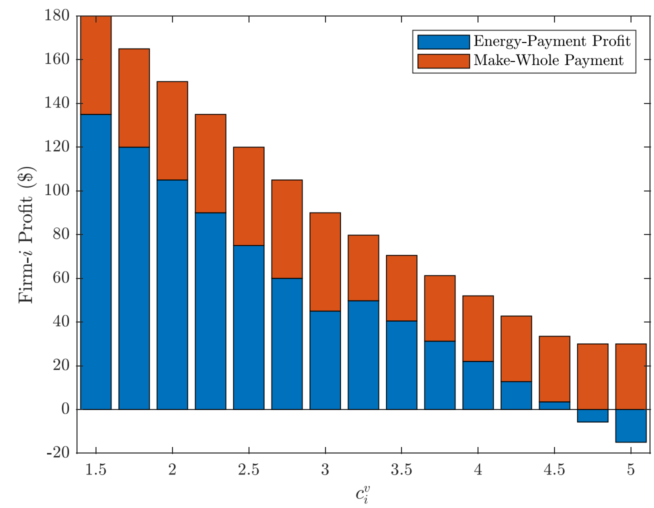

Fig. 8. Optimized firm-i profit under centrally committed design with non-zero c_1^f and c_2^f and $c_i^f = 5$.

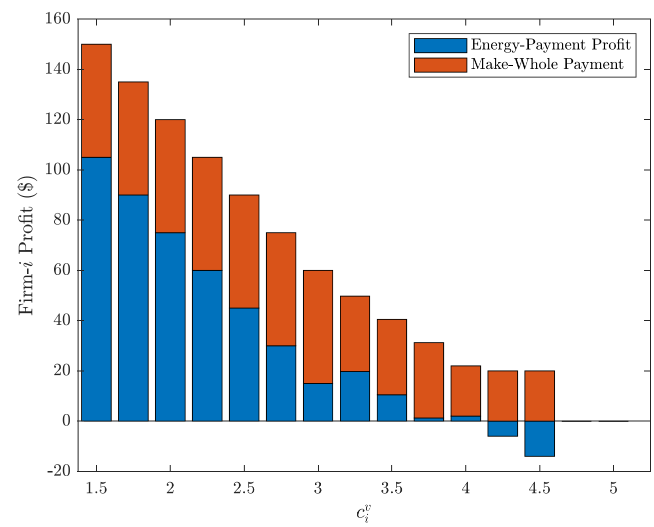

Fig. 9. Optimized firm-i profit under centrally committed design with non-zero c_1^f and c_2^f and $c_i^f = 15$.

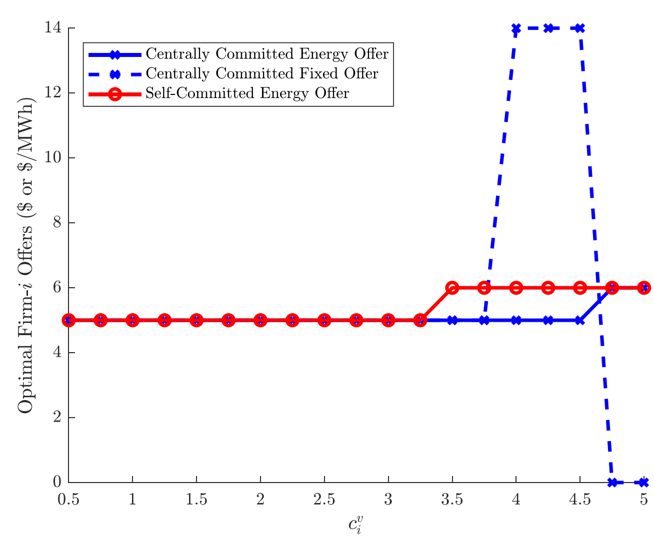

Fig. 10. Optimized firm-i offers under centrally and self-committed designs with $c_1^f = c_2^f = 0$.

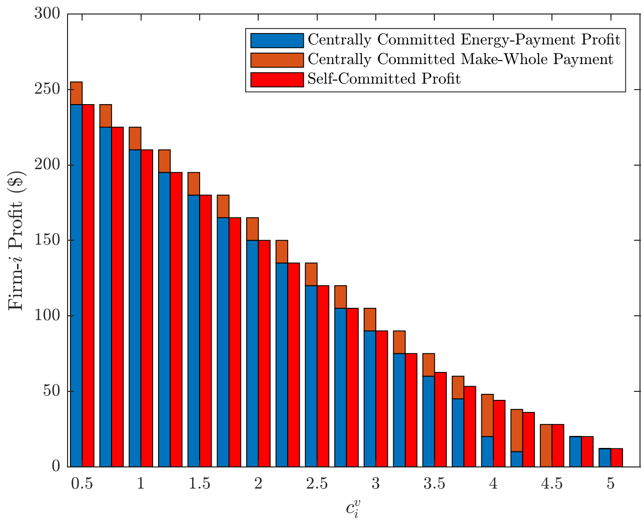

Fig. 11. Optimized firm-i profit under centrally and self-committed designs with $c_1^f = c_2^f = 0$.

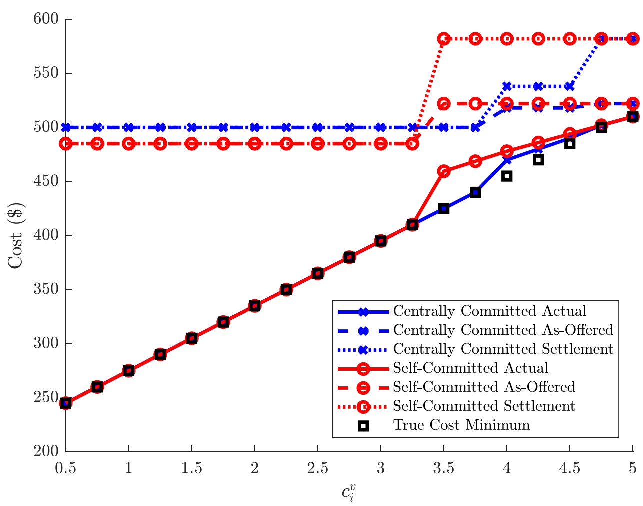

Fig. 12. Resultant actual, as-offered, and settlement costs from optimized firm-i offers under centrally and self-committed designs and true cost minimum with $c_1^f = c_2^f = 0$.

If $c_i^v \geq 3.50$, firm i raises its offer under a self-committed design to match firm 2's cost and is the marginal generator. Firm i's behavior under a centrally committed differs slightly with zero compared to non-zero c_1^f and c_2^f if c_i^v is relatively high. With $c_1^f = c_2^f = 0$ and $c_i^v \leq 3.75$, firm i submits offers under a centrally committed design to be the inframarginal generator during all three hours. If $c_1^f = c_2^f = 0$ and $4.00 \leq c_i^v \leq 4.50$, firm i submits offers that make it inactive during hour 1 and inframarginal during hours 2 and 3. If $c_1^f = c_2^f = 0$ and $4.75 \leq c_i^v$, firm i submits the same offers under a centrally and self-committed design and is the marginal generator during hours 2 and 3 with centralized commitment.

Fig. 11 shows that firm i's profit under a centrally committed design is slightly superior to that under self commitment for all values of c_i^v that we consider. This result arises from the principle of optimality. Because $c_1^f = c_2^f = 0$, we assume that firms 1 and 2 submit the same offers under the two market designs. As such, the additional degrees of freedom that are provided by multi-part offers and a make-whole payment under centralized commitment imply that firm-i profit can be no less under such a design.

Fig. 12 shows that centralized commitment yields lower productive-efficiency losses compared to self commitment. This result stems from $c_i^v = 4.50$ and $c_i^v = 3.50$ being the thresholds under a centrally and self-committed design, respectively, beyond which its optimal behavior results in firm i matching the offer of its more expensive rival (firm 2). By matching firm 2's offer, firm i switches from being an inframarginal to the marginal generator. This switch yields productive-efficiency losses, because the MO makes inefficient operational decisions on the basis of the incorrect cost information that firm i submits. The availability of the make-whole payment under a centrally committed design makes it preferable for firm i to remain as an inframarginal as unit for a larger range of values of c_i^v.

Fig. 11 shows firm i's profit is lower under a self-committed design as compared to centralized commitment. However, for relatively high values of c_i^v, settlement cost is much higher under self commitment (*cf.* Fig. 12). This higher settlement cost implies that total generator profits are higher under self commitment for high values of c_i^v. The lack of a make-whole payment under such a design means that firm i's sole mechanism to increase profit is through the uniform energy price, which is paid also to the inframarginal firm 1. Under a centrally committed design, firm i has an additional degree of freedom in collecting a make-whole payment, which is discriminatory and does not affect settlements that are paid to firm 1. This finding suggests that the discriminatory nature of the make-whole payment may mitigate the cost of the exercise of market power.

Finally, we examine the impact of relaxing the long-lived-offer requirement. Fig. 10 shows that if $c_i^v = 4.75$, firm i's profit-maximizing long-lived offers under centralized commitment are $b_i^{f*} = 0$ and $b_i^{v*} = 6.00$. These offers result in firm i being inactive during hour 1 and marginal during hours 2 and 3 and earning hourly energy profits of \$0.00, \$7.50, and \$12.50, respectively. Conversely, if firm i is able to submit short-lived offers, it submits arbitrarily high offers during hour 1 (thereby remaining inactive). For the other two hours it submits $b_i^{v*} = 5.00$ (thereby being dispatched as the inframarginal generator) and $b_i^{f*} = 14$ and $b_i^{f*} = 18$ during hours 2 and 3, respectively, which maximize make-whole payments. These offers yield losses of \$5.00 during each of hours 2 and 3 from energy payments, which are supplemented by make-whole payments of \$14.00 and \$18.00, respectively, and give a total net profit of \$22.00.

Fig. 13 summarizes the same cost information for the case of long-lived offers that Fig. 12 does for the case of short-lived offers. Fig. 13 shows that centralized commitment continues to outperform a self-committed design with short-lived offers. Indeed, there are no productive-efficiency losses with centralized commitment and short-lived offers, because actual cost equals the true cost minimum for all values of c_i^v that we consider. Productive-efficiency losses under centralized commitment that are shown in Fig. 12 for $4.00 \leq c_i^v \leq 4.50$ arise because firm i's profit-maximizing long-lived offers result in its being inactive during hour 1. Such offers maximize firm-i profit, despite firm i being lower-cost to operate than firm 2. As such, firm 2 is dispatched in firm i's place, which yields the cost increases. Fig. 13 shows that short-lived offers can alleviate these productive-efficiency losses, because firm i can submit offers during hour 1 that result in its being dispatched without unduly impacting its earnings during the remaining hours.

6.3. Computational performance

Algorithms 1 and 2 examine a set of $3^{|\mathcal{T}|}$ candidate optimal firm-i production levels and solve subproblems for each. As such, it may scale poorly. We benchmark the computational performance of Algorithm 2 to the method that is proposed by Huppmann and Siddiqui (2018), which solves (12)–(15) by solving an auxiliary mixed-integer non-linear optimization problem. Although this method requires solving only one problem, the problem includes auxiliary binary variables and constraints, the numbers of which grow exponentially in $|\mathcal{G}|$ and $|\mathcal{T}|$.

To conduct such a comparison, we use sets of randomly generated instances of (12)–(15). Each set has different-sized sets, $\mathcal{G}$ and $\mathcal{T}$.

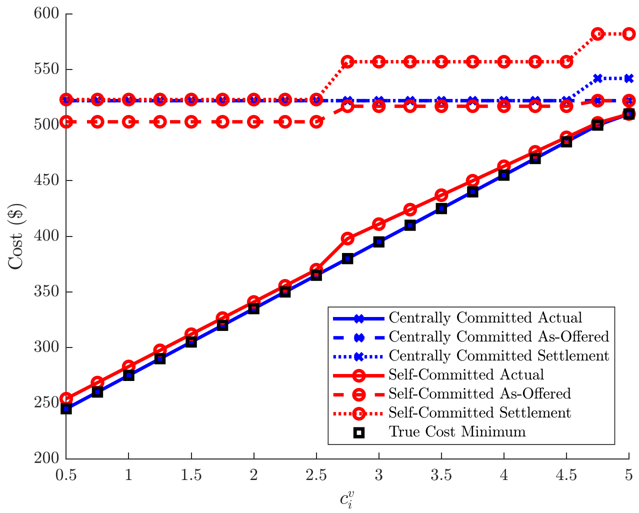

Fig. 13. Resultant actual, as-offered, and settlement costs from optimized firm-i offers under centrally and self-committed designs and true cost minimum with $c_1^f = c_2^f = 0$ if firm i submits short-lived offers.

Table 2
Average computational performance of Algorithm 2 and method of Huppmann and Siddiqui (2018) (Denoted as HS).

$\lvert\mathcal{G}\rvert$	$\lvert\mathcal{T}\rvert$	HS	Algorithm 2	
		Time (s)	Time (s)	Infeasible $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$
2	2	0.176	0.066	0.06
2	3	2.572	0.098	0.21
2	4	70.073	0.212	0.66
2	5	5827.641	0.319	1.63
3	2	5.167	0.086	0.83
3	3	1159.978	0.153	4.61
3	4	∞	0.296	17.99
3	5	∞	0.660	59.89
4	2	∞	0.061	1.50
4	3	∞	0.135	8.93
4	4	∞	0.275	33.00
4	5	∞	0.531	104.15
5	2	∞	0.068	1.97
5	3	∞	0.146	10.51
5	4	∞	0.302	37.73
5	5	∞	0.651	119.49

The sets of instances are solved using the two methods, which are programmed using Python 3.7. The optimization problems are solved using Gurobi 9.1.1. All of the computations are conducted on a system with a processor with two 2.90-GHz cores and 16.0 GB of memory. We impose a 43200-s (12-hour) time limit on the computations. Table 2 reports the average time to solve (12)–(15) across each set of random instances using the two methods. Computation times of ∞ indicate that the instances are not solvable within the 12-hour time limit.

Overall, Algorithm 2 scales better than the method of Huppmann and Siddiqui (2018). Algorithm 2 has two properties that provide better scaling performance. First, the number of optimization problems that Algorithm 2 requires solving grows only with the size of $\mathcal{T}$. Conversely, the method of Huppmann and Siddiqui (2018) entails solving an optimization problem that grows with the size of both $\mathcal{G}$ and $\mathcal{T}$. As such, none of the instances with $\lvert\mathcal{G}\rvert \geq 4$ and only some instances with $\lvert\mathcal{G}\rvert = 3$ can be solved by that method within 12 hours. If $\mathcal{G}$ is held fixed, computation times for Algorithm 2 roughly double each time an additional hour is added to the model horizon. On the other hand, if $\mathcal{T}$ is held fixed, computation times for Algorithm 2 see relatively small changes as $\mathcal{G}$ grows.

Another property of Algorithm 2 that provides for favorable scaling is that many of the subproblems are infeasible. Any firm-i allocation, $\hat{x}_i$, that yields an infeasible $\mathcal{P}_C^{\mathcal{A}}(\hat{x}_i)$ can be eliminated from consideration. The final column of Table 2 shows that as $\mathcal{G}$ and $\mathcal{T}$ increase, an increasing number of these subproblems are infeasible. For instance, with $\mathcal{G} = 4$ and $\mathcal{T} = 5$, there are $3^5 = 243$ candidate firm-i allocations to examine. However, nearly half of these yield infeasible subproblems and do not require further consideration.

The methodology of Huppmann and Siddiqui (2018) is more general and can solve any bi-level optimization problem with binary variables in the lower level. Conversely, Algorithm 2 is tailored for our specific problem and assumptions. Thus, the work of Huppmann and Siddiqui (2018) has an advantage relative to our work, in its more broad applicability.

7. Concluding remarks

This paper explores profit-maximizing strategic behavior in wholesale electricity markets, considering two common archetypal market designs—self- and centrally committed. We analyze both designs using a bi-level model, whereby the strategic firm determines profit-maximizing offers in the upper level and the MO clears the market in the lower level. We incorporate make-whole payments, which are a commonly used remuneration scheme under centrally committed designs, into the bi-level model of the centrally committed market.

The bi-level model of the self-committed design is computationally tractable using standard techniques, because its lower-level problem is a linear optimization. Thus, an optimum of the MO's problem can be characterized using optimality conditions, allowing the bi-level problem to be converted to a single-level problem. This approach cannot be taken with the model of the centrally committed design, because its lower-level problem includes binary variables. Huppmann and Siddiqui (2018) propose a general methodology, which can be applied to any bi-level model with binary variables in its lower level. However, their approach does not exploit model structure, which can make their method computationally expensive. A major contribution of our work is developing an efficient solution algorithm, which exploits model structure, for the model of the centrally committed design. Importantly, our solution algorithm is exact, insomuch as it introduces no approximations. We prove that our methodology is guaranteed to find an optimal solution and demonstrate the model and its computational efficiency using numerical examples.

Our examples demonstrate trade-offs between the two market designs. A self-committed design is relatively simple, with firms internalizing their non-convex costs into energy offers. This simplicity and the lack of a discriminatory make-whole payment through which firms can recover their non-convex costs yield two undesirable properties. For one, the strategic firm is incentivized to submit higher offers under a self-committed design. This property is reflected in the threshold value of c_i^v beyond which firm i matches firm 2's offer being lower under a self-committed design. This property gives rise to greater productive efficiency losses under a self-committed design. The second property is that under a self-committed design, firms must rely upon increasing the uniform energy price to increase their profits. The discriminatory nature of make-whole payments under a centrally committed design reduces firm i's incentives to increase the energy price. This property is reflected by the higher settlement costs under a self-committed design with higher values of c_i^v. A centrally committed design allowing firm i to increase its profit with a reduced cost impact appears to be consistent with the theory of two-part tariffs. Tirole (1988) shows that a monopolist can use a two-part tariff to extract surplus without reducing social welfare. On the other hand, firm i is able to manipulate its offers under a centrally committed design to receive make-whole payments, even when it earns a positive rent from energy payments. We study the effect on market-participant behavior of long- versus short-lived offers. Strategic-firm profit increases with the latter, but productive-efficiency losses under a centrally committed design are eliminated. It is unclear whether this is a general result or specific to our example and further study is needed before drawing broad market-design conclusions.

We compare the computational performance of our solution method to that of Huppmann and Siddiqui (2018). Both methods grow exponentially in the problem size, but our method scales better in our computational test due to its having two inherent advantages. First, our method grows only with the number of time periods. The method of Huppmann and Siddiqui (2018) grows with the number of time periods and firms. Moreover, as the problem size increases, more of the subproblems that must be solved under our methodology are infeasible and can be excluded from consideration.

Our model is a stylized simplification of actual wholesale electricity markets. Relaxing our simplifying assumptions provides many avenues for further research. We assume that firms have equal capacities and we neglect intertemporal constraints and costs, *e.g.*, ramping constraints or a fixed cost to keep a generator online between one hour and the next. These assumptions facilitate the development of Algorithms 1 and 2, because they limit firm i to three possible production levels during each hour. A second simplification is that we compute a static partial equilibrium, whereby a single firm optimizes its offer, taking the offers and behavior of its rivals as fixed. As such, it is challenging to compare the two market designs if firm i's rivals have non-zero fixed costs, because we must make a behavioral assumption about how those costs are incorporated into energy offers under a self-committed design. Relaxing these assumptions, for instance by finding a Nash equilibrium with multiple strategic firms, may reveal additional insights into the relative merits of the two market designs. An alternative avenue for further work is to relax Assumption 2 and optimize firm i's offers under uncertainty regarding its rivals' behavior. Another simplifying assumption is that the profit-maximizing firm has only price as a strategic variable. Allowing the profit-maximizing firm to determine the quantity that it offers could provide an additional degree of freedom to exercise market power. Tirole (1988) discusses such a finding in comparing the stylized Bertrand and Nash–Cournot models of competition.

Another limitation of our work is that it compares the two market designs solely from the perspective of short-run system and market operations and on the basis of cost and operational efficiency alone. There are other considerations, which are beyond the scope of our work, but which may be valuable areas of future study and comparison of the two market designs. One consideration is the allocation of the cost of make-whole (or any other discriminatory) payments that are necessitated by non-convexities in unit-commitment decisions. Most centrally committed markets socialize these costs to customers in a *pro rata* or similar simple fashion, which may be neither efficient, individually rational, nor incentive compatible. O'Neill et al. (2017) propose a pricing scheme, which ensures non-negative economic gains to each agent that clears the market. The use of such a pricing scheme likely would change profit-maximizing behavior by the strategic generator (and our findings). Moreover, the complexity of such a pricing scheme may make the simplicity of the self-committed design desirable. As another example of a consideration that is beyond the scope of our work, Mays et al. (2021) examine the impacts of discriminatory payments in centrally committed designs upon generator-entry and -exit decisions. They demonstrate that such payments can distort these decisions and the resultant capacity mix, whereas pricing schemes that increase uniform prices to reduce discriminatory payments may support higher capacity levels. Including these considerations in a comparison between self- and centrally committed designs would be an important extension of our work. While our model assumes fixed capacity levels, it may be possible to extend it by including capacity decisions into the upper-level problems of the two market designs.

Acknowledgments

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Appendix A. Proof of Proposition 2

Proof $\forall t \in \mathcal{T}$ **such that** $\hat{x}_{i,t} = 0$**.** We let $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$ denote a partition that is optimal in (8)–(11) and let $\hat{j}_t^M$ denote the unique member of $\hat{\mathcal{G}}_t^M$. We show that we must have:

$$\sum_{j \in \hat{\mathcal{G}}_t^I} \left(b_j^v K + b_j^f \right) + b_{\hat{j}_t^M}^v r_t + b_{\hat{j}_t^M}^f = \sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f \right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f ;$$

otherwise at least one of the partitions, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$ or $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$, violates its definition. The lexicographic ordering of Assumption 6 requires that $\mathcal{G}_t^{I,-} = \hat{\mathcal{G}}_t^I$ and $\mathcal{G}_t^{M,-} = \hat{\mathcal{G}}_t^M$. Thus, by the principle optimality, minimizing (8) requires that:

$$\sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f \right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f \leq \sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f \right) + b_i^v r_t + b_i^f ;$$

which implies (27) if $j_t^{I,-} \leq_K j_t^{M,-}$. Otherwise, if $j_t^{I,-} >_K j_t^{M,-}$, we have that:

$$\sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f \right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f \leq \sum_{j \in \left(\mathcal{G}_t^{I,-} \cup \left\{ j_t^{M,-} \right\} \right) \setminus \left\{ j_t^{I,-} \right\}} \left(b_j^v K + b_j^f \right) + b_i^v r_t + b_i^f ;$$

which implies (27) as well. In addition, $\forall g \in \mathcal{G}_t^{I,-}$ such that $j_t^{M,-} \leq_{r_t} g$ we have that:

$$\sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f \right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f \leq \sum_{j \in \left(\mathcal{G}_t^{I,-} \cup \{i\} \right) \setminus \{g\}} \left(b_j^v K + b_j^f \right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f ;$$

which implies (28)$\forall g \in \mathcal{G}_t^{I,-}$ such that $j_t^{M,-} \leq_{r_t} g$. For the other case of all $g \in \mathcal{G}_t^{I,-}$ such that $j_t^{M,-} >_{r_t} g$ we have that:

$$\sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f \right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f \leq \sum_{j \in \left(\mathcal{G}_t^{I,-} \cup \{i\} \right) \setminus \{g\}} \left(b_j^v K + b_j^f \right) + b_g^v r_t + b_g^f ;$$

which implies (28) $\forall g \in \mathcal{G}_t^{I,-}$ such that $j_t^{M,-} >_{r_t} g$.

To prove the sufficiency of (27)–(28) we show that if the inequalities hold, $\hat{x}_{i,t} = 0$ is weakly preferable to $\hat{x}_{i,t} = r_t$ and $\hat{x}_{i,t} = K$ in terms of minimizing (8). To show this, consider the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, which by definition excludes firm i. Suppose, for contradiction, that when firm i is included it is optimal for i to replace $j_t^{M,-}$ as the marginal generator. For this to be true, $\exists j \in \mathcal{G}_t^{I,-}$ such that $j >_K j_t^{M,-}$, because by Lemma 3 we know that $g \leq_K h, \forall g \in \mathcal{G}_t^{I,-}$ and $h \in \mathcal{G}_t^{V,-}$. Thus, by the definition of $j_t^{I,-}$, we have that $j_t^{I,-} \geq_K j_t^{M,-}$. Therefore, (27) implies that replacing $j_t^{I,-}$ (or any member of $\mathcal{G}_t^{I,-}$) with $j_t^{M,-}$ and making $j_t^{M,-}$ an element of $\mathcal{G}_t^{I,-}$ increases the value of (8) weakly, which contradicts the optimality of replacing $j_t^{M,-}$ with i as the marginal generator.

Finally, suppose for contradiction that when firm i is added to the candidate set of generators that it is optimal for i to replace a member of $\mathcal{G}_t^{I,-}$ as an inframarginal generator. For this to be optimal, $\exists j \in \mathcal{G}_t^{I,-}$ such that $j <_{r_t} j_t^{M,-}$, because by Lemma 3 we have that $j_t^{M,-} \leq_{r_t} h$, $\forall h \in \mathcal{G}_t^{V,-}$ and $j \leq_K h$, $\forall j \in \mathcal{G}_t^{I,-}$ and $h \in \mathcal{G}_t^{V,-}$. However, (28) implies that replacing $j \in \mathcal{G}_t^{I,-}$ such that $j <_{r_t} j_t^{M,-}$ with $j_t^{M,-}$ increases the value of (8) weakly, which contradicts the optimality of having i replace a member of $\mathcal{G}_t^{I,-}$. Thus, (27)–(28) are sufficient to ensure that $\hat{x}_{i,t} = 0$ is optimal in (8)–(11). □

Proof $\forall t \in \mathcal{T}$ such that $\hat{x}_{i,t} = r_t$. We let $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$ denote a partition that is optimal in (8)–(11) and assume that i is the unique member of $\hat{\mathcal{G}}_t^M$. We know that:

$$\left|\hat{\mathcal{G}}_t^I\right| = \left|\mathcal{G}_t^{I,-}\right|;$$

otherwise (9) is violated by at least one of $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$ or $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$. We consider now the two cases that depend upon the relationship between $j_t^{M,-}$ and $j_t^{I,-}$.

First, consider the case wherein $j_t^{M,-} \geq_K j_t^{I,-}$, which implies that $j_t^{M,-} \geq_K j$, $\forall j \in \mathcal{G}_t^{I,-}$. In this case we have that $j_t^{M,-} \notin \hat{\mathcal{G}}_t^I$. Otherwise, if $j_t^{M,-} \in \hat{\mathcal{G}}_t^I$, $\exists j \in \mathcal{G}_t^{I,-}$ with $j \notin \hat{\mathcal{G}}_t^I$ and $j_t^{M,-} \geq_K j$, which violates the lexicographic requirement of Assumption 6 if the inequality holds as an equality and violates the optimality of $\hat{\mathcal{G}}_t^I$ if the inequality is strict. To prove that (29)–(31) is necessary, we begin by showing by contradiction that we must have:

$$\sum_{h \in \hat{\mathcal{G}}_t^I} \left(b_h^v K + b_h^f\right) = \sum_{h \in \mathcal{G}_t^{I,-}} \left(b_h^v K + b_h^f\right). \tag{A.1}$$

Suppose to the contrary that the left-hand side of (A.1) is smaller. In such a case, given that $j_t^{M,-} \notin \hat{\mathcal{G}}_t^I$, the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, violates its definition. Conversely, suppose that the left-hand side of (A.1) is larger. This case yields a contradiction as well, because the partition, $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$, violates its definition. Thus, by (A.1) we have that $\hat{\mathcal{G}}_t^I = \mathcal{G}_t^{I,-}$. Applying the principle of optimality to (8)–(11) gives:

$$\sum_{h \in \mathcal{G}_t^{I,-}} \left(b_h^v K + b_h^f\right) + b_i^v r_t + b_i^f \leq \sum_{h \in \mathcal{G}_t^{I,-}} \left(b_h^v K + b_h^f\right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f;$$

which yields (29). We have also that:

$$\sum_{h \in \mathcal{G}_t^{I,-}} \left(b_h^v K + b_h^f\right) + b_i^v r_t + b_i^f \leq \sum_{h \in \left(\mathcal{G}_t^{I,-} \cup \{i\}\right) \setminus \{g\}} \left(b_h^v K + b_h^f\right) + b_g^v r_t + b_g^f;$$

for all $g \in \mathcal{G}_t^{I,-}$, which gives (30). Finally, we have:

$$\sum_{h \in \mathcal{G}_t^{I,-}} \left(b_h^v K + b_h^f\right) + b_i^v r_t + b_i^f \leq \sum_{h \in \left(\mathcal{G}_t^{I,-} \cup \{i\}\right) \setminus \left\{j_t^{I,-}\right\}} \left(b_h^v K + b_h^f\right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f;$$

which gives (31).

To prove sufficiency of (29)–(31) for the case wherein $j_t^{M,-} \geq_K j_t^{I,-}$, consider a partition in which $\mathcal{G}_t^{I,-}$ are the inframarginal units, i is marginal, and $\mathcal{G}_t^{V,-} \cup \mathcal{G}_t^{M,-}$ are the inactive units. By (29) we know that this partition gives a weakly smaller value of (8) than $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$ does. This means that having $\hat{x}_{i,t} = r_t$ is better than having $\hat{x}_{i,t} = 0$. If the MO has $\hat{x}_{i,t} = K$, then $\exists j \in \mathcal{G}_t^{M,-} \cup \mathcal{G}_t^{V,-}$ that is selected to be the marginal unit. Such a j is selected to be the marginal unit because by (30) switching i and h, $\forall h \in \mathcal{G}_t^{I,-}$ weakly increases the value of (8). Similarly, $\forall j \in \mathcal{G}_t^{M,-} \cup \mathcal{G}_t^{V,-}$ and $g \in \mathcal{G}_t^{I,-}$, switching j and g weakly increases the value of (8) by Lemma 3 and due to the assumption that $j_t^{M,-} \geq_K j_t^{I,-}$. Finally, we know from Lemma 3 that $j_t^{M,-} \leq_{r_t} g$, $\forall g \in \mathcal{G}_t^{V,-}$. Inequality (31) implies that having i as an inframarginal unit and $j_t^{M,-}$ as a marginal unit gives a weakly higher value of (8) compared to having $j_t^{I,-}$ as an inframarginal unit and i as a marginal unit. Thus, by appealing to the definition of $j_t^{I,-}$, we can argue that having $\hat{x}_{i,t} = K$ is suboptimal in (8)–(11).

We consider now the other case wherein $j_t^{M,-} <_K j_t^{I,-}$. We show the necessity of (32)–(34) by arguing that we must have:

$$\hat{\mathcal{G}}_t^I = \left(\mathcal{G}_t^{I,-} \cup \mathcal{G}_t^{M,-}\right) \setminus \left\{j_t^{I,-}\right\}. \tag{A.2}$$

To show this equivalence, note that by Lemma 3 we have that $j \leq_K h$, $\forall j \in \mathcal{G}_t^{I,-}$ and $h \in \mathcal{G}_t^{V,-}$, by assumption we have that $j_t^{M,-} <_K j_t^{I,-}$, and by definition we have that $j_t^{I,-} \geq_K j$, $\forall j \in \mathcal{G}_t^{I,-}$. As such, we conclude that having $\hat{\mathcal{G}}_t^I$ as is given by (A.2) is an inframarginal-generator set that minimizes (8). Therefore, applying the principle of optimality to (8)–(11) gives:

$$\sum_{j \in \hat{\mathcal{G}}_t^I} \left(b_j^v K + b_j^f\right) + b_i^v r_t + b_i^f \leq \sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f;$$

which yields (32) after rearranging terms. Furthermore, we have that:

$$\sum_{j \in \hat{\mathcal{G}}_t^I} \left(b_j^v K + b_j^f\right) + b_i^v r_t + b_i^f \leq \sum_{j \in \left(\hat{\mathcal{G}}_t^I \cup \{i\}\right) \setminus \mathcal{G}_t^{M,-}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f;$$

which implies (33). Finally, using the definitions of M and N that are given following the definition of (34), we have:

$$\sum_{j \in \hat{\mathcal{G}}_t^I} \left(b_j^v K + b_j^f\right) + b_i^v r_t + b_i^f \leq \sum_{j \in \left(\hat{\mathcal{G}}_t^I \cup \{i\}\right) \setminus \{M^*\}} \left(b_j^v K + b_j^f\right) + N;$$

where:

$$M^* = \operatorname*{arg\,max}_{j \in \left(\mathcal{G}_t^{I,-} \cup \mathcal{G}_t^{M,-}\right) \setminus \left\{j_t^{I,-}\right\}} \left(b_j^v K + b_j^f\right);$$

from which (34) follows.

To show the sufficiency of (32)–(34) if $j_t^{M,-} <_K j_t^{I,-}$, we assume that (32)–(34) and consider a partition wherein:

$$\left(\mathcal{G}_t^{I,-} \cup \mathcal{G}_t^{M,-}\right) \setminus \left\{j_t^{I,-}\right\};$$

is the inframarginal-generator set, i is marginal, and:

$$\mathcal{G}_t^{V,-} \cup \left\{j_t^{I,-}\right\};$$

is the inactive-generator set. If (32) holds, then this partition gives a weakly lower value of (8) than the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, does. By definition, the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, corresponds to a case wherein $\hat{x}_{i,t} = 0$. To show that $\hat{x}_{i,t} = r_t$ yields a weakly lower value of (8) compared to $\hat{x}_{i,t} = K$, we note that by Lemma 2 we have that $b_{j_t^{M,-}}^v \geq b_j^v$, $\forall j \in \mathcal{G}_t^{I,-}$. Thus, (33) implies that $b_i^v \geq b_j^v$, $\forall j \in \mathcal{G}_t^{I,-}$. As such, for any:

$$j \in \left(\mathcal{G}_t^{I,-} \cup \mathcal{G}_t^{M,-}\right) \setminus \left\{j_t^{I,-}\right\};$$

it cannot improve the value of (8) to switch i and j by having i as an inframarginal generator and j as a marginal generator. Thus, for $\hat{x}_{i,t} = K$ to be optimal in (8)–(11), there must exist:

$$h \in \mathcal{G}_t^{V,-} \cup \left\{j_t^{I,-}\right\}; \tag{A.3}$$

and:

$$j \in \left(\mathcal{G}_t^{I,-} \cup \mathcal{G}_t^{M,-}\right) \setminus \left\{j_t^{I,-}\right\}; \tag{A.4}$$

such that it is optimal in (8)–(11) for h to become marginal and j to become inactive, because by Lemma 3, the definition of $j_t^{I,-}$, and the assumption that $j_t^{M,-} <_K j_t^{I,-}$ we have that $j \leq_K h$ for all:

$$j \in \left(\mathcal{G}_t^{I,-} \cup \mathcal{G}_t^{M,-}\right) \setminus \left\{j_t^{I,-}\right\};$$

and for all:

$$h \in \mathcal{G}_t^{V,-} \cup \left\{j_t^{I,-}\right\}.$$

However, by (34), there is no pair, j and h, that satisfies:

$$b_i^v K + b_i^f + b_h^v r_t + b_h^f < b_j^v K + b_j^f + b_i^v r_t + b_i^f;$$

meaning that there is no pair, j and h, that satisfies (A.3) and (A.4) and which does not increase the value of (8) if h becomes marginal and j becomes inactive. Thus, having $\hat{x}_{i,t} = r_t$ weakly reduces the value of (8) relative to having $\hat{x}_{i,t} = K$. □

Proof $\forall t \in \mathcal{T}$ such that $\hat{x}_{i,t} = K$. We let $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$ denote a partition that is optimal in (8)–(11) and let $\hat{j}_t^M$ denote the unique member of $\hat{\mathcal{G}}_t^M$. By assumption we have that $i \in \hat{\mathcal{G}}_t^I$. We know that:

$$\left|\hat{\mathcal{G}}_t^I\right| = \left|\mathcal{G}_t^{I,+}\right|;$$

otherwise (9) is violated by at least one of $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$ or $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$. Indeed, we can argue that the partitions, $(\hat{\mathcal{G}}_t^I, \hat{\mathcal{G}}_t^M, \hat{\mathcal{G}}_t^V)$ and $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$, are equal exactly to one another. To do so, we begin by noting that by the definitions of the two partitions, we have:

$$\sum_{j \in \hat{\mathcal{G}}_t^I \setminus \{i\}} \left(b_j^v K + b_j^f\right) + b_{\hat{j}_t^M}^v r_t + b_{\hat{j}_t^M}^f = \sum_{j \in \mathcal{G}_t^{I,+} \setminus \{i\}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f.$$

As such, from the lexicographic requirement of Assumption 6 we have that $\hat{\mathcal{G}}_t^I \setminus \{i\} = \mathcal{G}_t^{I,+} \setminus \{i\}$ and $\hat{j}_t^M = j_t^{M,+}$. Thus, we must have that $\hat{\mathcal{G}}_t^I = \mathcal{G}_t^{I,+}$ and $\hat{\mathcal{G}}_t^V = \mathcal{G}_t^{V,+}$. We consider now the two possible cases that are implied by Lemma 5, and which differ based on whether $j_t^{M,-}$ and $j_t^{M,+}$ are equal or not.

We consider first the case wherein $j_t^{M,-} \neq j_t^{M,+}$. By Lemmata 4 and 5 we must have that $j_t^{M,+} \in \mathcal{G}_t^{I,-}$ and $\mathcal{G}_t^{V,+} = \mathcal{G}_t^{M,-} \cup \mathcal{G}_t^{V,-}$. To show the necessity of (35)–(36), we note from applying the principle of optimality to (8)–(11) that:

$$\sum_{j \in \mathcal{G}_t^{I,+}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f \le \sum_{j \in \mathcal{G}_t^{I,-}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,-}}^v r_t + b_{j_t^{M,-}}^f.$$

Combining this inequality with the fact that $j_t^{M,+} \in \mathcal{G}_t^{I,-}$ gives (35). Moreover, we have that:

$$\sum_{j \in \mathcal{G}_t^{I,+}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f \le \sum_{j \in \left(\mathcal{G}_t^{I,+} \cup \left\{j_t^{M,+}\right\}\right) \setminus \{i\}} \left(b_j^v K + b_j^f\right) + b_i^v r_t + b_i^f;$$

which implies (36).

To show the sufficiency of (35)–(36), suppose that they hold and consider the partition, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$. If (35) holds then the value of (8) is weakly lower under this partition as opposed to under the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, which is equivalent to having $\hat{x}_{i,t} = 0$. Thus, having $\hat{x}_{i,t} = K$ is a weak improvement over having $\hat{x}_{i,t} = 0$. On the other hand, for the MO to select $\hat{x}_{i,t} = r_t$ then $\exists j \in \mathcal{G}_t^{M,+} \cup \mathcal{G}_t^{V,+}$ that is selected to be an inframarginal generator. If $j = j_t^{M,+}$ is selected to be inframarginal, (36) implies that the value of (8) increases weakly compared to having i as an inframarginal generator. We can show also that selecting a firm, $j \in \mathcal{G}_t^{V,+}$, to be inframarginal weakly increases the value of (8) by arguing that $\forall j \in \mathcal{G}_t^{M,-} \cup \mathcal{G}_t^{V,-}$ we have that $j_t^{M,+} \le_K j$. To show this, recall that by Lemmata 4 and 5, $\mathcal{G}_t^{V,+} = \mathcal{G}_t^{M,-} \cup \mathcal{G}_t^{V,-}$. Based on the definition of the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, and because we know that $j_t^{M,+} \in \mathcal{G}_t^{I,-}$, Lemma 3 implies that $j_t^{M,+} \le_K j, \forall j \in \mathcal{G}_t^{V,-}$. Furthermore, Lemma 2 implies that $b_{j_t^{M,+}}^v \le b_{j_t^{M,-}}^v$. Thus, from the definition of the partition, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$, and Lemma 3 we have that $j_t^{M,+} \le_{r_t} j_t^{M,-}$, because $j_t^{M,-} \in \mathcal{G}_t^{V,+}$. Thus, we have that $j_t^{M,+} \le_K j_t^{M,-}$, which completes the argument that $j_t^{M,+} \le_K j, \forall j \in \mathcal{G}_t^{M,-} \cup \mathcal{G}_t^{V,-}$.

We consider now the other case wherein $j_t^{M,-} = j_t^{M,+}$. In this case, by Lemma 4 $\exists p \in \mathcal{G}_t^{I,-}$ such that:

$$\mathcal{G}_t^{I,+} = \left(\mathcal{G}_t^{I,-} \cup \{i\}\right) \setminus \{p\}.$$

Thus, $\mathcal{G}_t^{V,+} = \mathcal{G}_t^{V,-} \cup \{p\}$. To show the necessity of (37)–(39), we note that by applying the principle of optimality to (8)–(11) we have:

$$\sum_{j \in \mathcal{G}_t^{I,+}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f \le \sum_{j \in \left(\mathcal{G}_t^{I,+} \cup \{p\}\right) \setminus \{i\}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f;$$

which gives (37). We have also that:

$$\sum_{j \in \mathcal{G}_t^{I,+}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f \le \sum_{j \in \left(\mathcal{G}_t^{I,+} \cup \left\{j_t^{M,+}\right\}\right) \setminus \{i\}} \left(b_j^v K + b_j^f\right) + b_i^v r_t + b_i^f;$$

which yields (38). Finally, we have:

$$\sum_{j \in \mathcal{G}_t^{I,+}} \left(b_j^v K + b_j^f\right) + b_{j_t^{M,+}}^v r_t + b_{j_t^{M,+}}^f \le \sum_{j \in \left(\mathcal{G}_t^{I,+} \cup \{p\}\right) \setminus \{i\}} \left(b_j^v K + b_j^f\right) + b_i^v r_t + b_i^f;$$

which implies (39).

To show the sufficiency of (37)–(39), we assume that they hold and consider the partition, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$. Due to (37) and the assumption for this case that $j_t^{M,-} = j_t^{M,+}$, we know that the value of (8) is weakly lower under the partition, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$, as opposed to the partition, $(\mathcal{G}_t^{I,-}, \mathcal{G}_t^{M,-}, \mathcal{G}_t^{V,-})$, which corresponds to having $\hat{x}_{i,t} = 0$. Thus, having $\hat{x}_{i,t} = K$ is a weak improvement from the MO's perspective over having $\hat{x}_{i,t} = 0$. For it to be optimal for the MO to select $\hat{x}_{i,t} = r_t$ then $\exists j \in \mathcal{G}_t^{M,+} \cup \mathcal{G}_t^{V,+}$ that is selected to be an inframarginal generator. If (38) holds, then selecting $j = j_t^{M,+}$ to become an inframarginal generator increases weakly the value of (8) compared to having $\hat{x}_{i,t} = K$. Consider the other case in which $j \in \mathcal{G}_t^{V,+} = \mathcal{G}_t^{V,-} \cup \{p\}$ is selected to become an inframarginal generator. We can show making such a j an inframarginal generator increases the value of (8) weakly by showing that selecting $j = p$ to become inframarginal increases the value of (8) weakly. Considering the case of $j = p$ is sufficient, because $p \in \mathcal{G}_t^{I,-}$ and by Lemma 3 we have that $p \le_K g, \forall g \in \mathcal{G}_t^{V,-}$. Inequality (39) implies that having $\hat{x}_{i,t} = r_t$ and $j = p$ as an inframarginal generator increases the value of (8) weakly compared to the partition, $(\mathcal{G}_t^{I,+}, \mathcal{G}_t^{M,+}, \mathcal{G}_t^{V,+})$. □

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