

Measuring Upward Mobility*

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ABSTRACT. We conceptualize and measure upward mobility over income or wealth. At the core of our exercise is the *Growth Progressivity Axiom*: transfers of instantaneous growth rates from relatively rich to poor individuals increases upward mobility. This axiom, along with mild auxiliary restrictions, identifies an “upward mobility kernel” with a single free parameter, in which mobility is linear in individual growth rates, with geometrically declining weights on baseline incomes. We extend this kernel to trajectories over intervals. The analysis delivers an upward mobility index that does not rely on panel data. That significantly expands our analytical scope to data-poor settings.

1. Introduction

Social mobility refers to the ease of transition between socioeconomic categories. To the extent that those categories (e.g. income or wealth) are vertically ranked, mobility — or at least *upward* mobility — is linked to *directed* movement, either upward or downward. In this sense, upward mobility is closely linked to economic growth. Yet the two terms are not synonymous: upward mobility is higher when the relatively worse-off enjoy greater upward movement than their better-off counterparts. In this latter sense, mobility is connected to economic *equality*, without necessarily being identical to it.

These observations connecting mobility, growth and equality lead to a view of upward mobility as *pro poor growth*, a concept that aggregates growth across individuals, weighted by their baseline economic characteristics. (See Section 2 for relevant literature.) It explicitly sets aside the “pure movement” component of mobility, which allows mobility to increase with sheer volatility across categories.

We follow this line of reasoning to propose a new measure of upward mobility. We axiomatize this measure from first principles. A central axiom that runs through the exercise is *Growth Progressivity*, which states that a transfer of growth rates from richer

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to poorer individuals increases upward mobility. This axiom is the dynamic analogue of the Transfers Principle for inequality measurement, which states that an income transfer from richer to poorer individuals decreases inequality; see, e.g. Fields and Fei (1978).

Growth progressivity raises the thorny question of whether and for how long we want to continue to reward the higher growth of someone who was originally poorer, but is no longer so by virtue of that growth. With this issue in mind, we begin by defining an upward mobility *kernel* — an index that captures a snapshot of mobility at an *instant* in time. For that instant, the growth progressivity axiom is unambiguous, as no crossing of distinct incomes can occur no matter what their instantaneous growth rates are.

The kernel has as its domain vectors of observations, each consisting of a baseline individual endowment (we call it *income*) and an instantaneous growth rate of that endowment. Under mild auxiliary restrictions, we exploit properties of multiaffine functions to show that Growth Progressivity implies the *linearity* of the upward mobility kernel in individual growth rates, with weights declining in baseline incomes (Theorem 1). This central connection between Growth Progressivity and linearity in growth rates is both of intrinsic interest and crucial to empirical implementation, as we shall see.

Additional restrictions then pin down the specific weighting function that appears in Theorem 1. One of these axioms is Growth Alignment, which states that instantaneous upward mobility increases when all individuals experience higher growth. Together, these axioms identify a one-parameter family of upward mobility kernels. Each is a weighted sum over individual growth rates, with weights that geometrically decline in income and are indexed by a single parameter to mark the speed of that decline (Theorem 2). Section 3.3 discusses a corresponding one-parameter family of *relative* upward mobility kernels, which nets out overall economic growth from the absolute kernels.

The upward mobility kernel is our starting point. Our goal is to use it to define a measure of upward mobility over domains typically available to the researcher, populated by income *trajectories* across intervals of time. It is entirely possible that two such trajectories could cross, so we cannot apply Growth Progressivity based on overall growth rates over the interval. Instead, we approach this extension by imposing two substantive conditions.

First, we ask that upward mobility for a collection of income trajectories be fully pinned down by the collection of all upward mobility kernels (at every instant of time) precipitated by those trajectories. We call this property *reducibility*. Our second condition asks that our measure of upward mobility defined on income trajectories should be *additive* over time; specifically, that upward mobility over an interval of time should be the average of the upward mobilities over any two sub-intervals composing that interval.

Theorem 3 combines reducibility, additivity and our earlier axioms to generate the main measure of upward mobility that we take to the data. This convenient discrete-time formula also has a welfarist interpretation as the annualized growth of Atkinson's "equivalent income," or growth in the monetized value of the Atkinson welfare function.

A central outcome of our exercise is a measure that can be constructed *without the need for panel data*. This conclusion questions the need for estimating transition probabilities, which is typically accomplished with much difficulty, as the data is often proprietary; see, for instance, Chetty et al. (2014b) and Acciari et al. (2022). Such exercises are near-impossible to conduct (with currently available data) in the majority of countries. Sections 4 and 5 develop panel independence in detail, and we ask for the reader's patience in postponing a final judgment until the material in these sections is absorbed.

Section 6 discusses several aspects of our measure, including its connections to growth and inequality, its distinct separation from "exchange mobility" (the component of mobility that tracks pure movement), as also how other measures fare under our axioms. We mention Section 6.7 in particular, which discusses an extension of our measure to accommodate the presence of social groupings in the population.

Section 7 conducts three preliminary empirical exercises. First, we compare our measure to the panel-dependent index used by Chetty et al. (2017). They estimate the fraction of children who earn more than their parents (in US birth cohorts 1940–1984) and document a secular decline in this fraction. Our panel-independent index tracks theirs very closely; see Figure 5.² This empirical concordance is not of intrinsic importance for our argument, which we firmly make on conceptual grounds. (For instance, the Chetty et al. 2017 measure fails Growth Progressivity.) But we hope it will encourage the more empirically-minded researcher who might not find conceptual arguments alone to be entirely convincing.

Second, we use *repeated cross-sectional data* from the World Inequality Database (2021) to study the evolution of 10-year mobility in Brazil, India and France since 1980. The exercise reveals new trends for these countries that may be of independent interest. Indeed, while we do not conduct such a study here, our panel-independent measure significantly expands the measurement of upward mobility to a large set of countries.

Finally, Section 7.3 uses our measure of mobility to revisit the so-called "Great Gatsby Curve," a positive cross-sectional relationship between income inequality and inter-generational income persistence identified by Corak (2013), and made famous by Alan Krueger in a 2012 speech at the White House. To match the intergenerational exercise, we

²This exercise is in the spirit of Berman (2022), who also attempts to circumvent the panel structure, though from an empirical perspective. See Section 7 for more discussion.

use the 30-year version of our upward mobility measure. Indeed, that measure supports the Corak-Krueger hypothesis for the sample of countries in the original study. But the resemblance ends there. Very different patterns appear among a larger set of countries.

These empirical exercises are not meant to be definitive. Rather, they serve as empirical proof of concept. They demonstrate the applicability of our measure in broad settings and hopefully pave the way for future research on income mobility, especially in countries for which the scarcity of panel surveys has hindered such research.

2. Related Literature

Different approaches have been taken to the measurement of mobility reflecting the variety of opinions on just what the term means (Fields 2010).

A large literature studies “exchange mobility,” with no distinction drawn between gains and losses. Many exchange-mobility measures are based solely on transition probabilities for population distributions across categories; see Prais (1955), Atkinson (1981), Bartholomew (1982), Conlisk (1974), Dardanoni (1993), Hart (1976), or Shorrocks (1978). In Shorrocks (1978), for instance, any increase in an off-diagonal entry of the transition matrix for incomes increases mobility. Other examples include the measures of King (1983) and Chakravarty (1984) that track rerankings in the distribution (McClendon 1977 and Markandya 1982), the income elasticity of progeny income with respect to parental income (Solon 1999, Jäntti and Jenkins 2015), and rank-rank correlations across generations (Dahl and DeLeire 2008, Chetty, Hendren, Kline and Saez 2014a).

This view of mobility can be summarized as the converse of immobility; mobility *is* movement. Even in several papers that value absolute gains and not just ranks, e.g., Fields and Ok (1996), Mitra and Ok (1998) and Cowell and Flachaire (2018), individual losses and gains both contribute positively to mobility. In this sense, these are also measures of “movement” rather than “upward mobility.” See Section 6.9 for more discussion.

“Mobility as movement” divests itself of the ethical connotations that should undergird the concept (Fields and Ok 1999a; Jäntti and Jenkins 2015). We all agree that gains are better than losses. When empirical studies emphasize a specific part of the transition matrix, say, transfers from bottom to upper ranks, they implicitly provide this welcome sense of direction (see, e.g., Bhattacharya and Mazumder 2011, Chetty, Hendren, Kline and Saez 2014a and Berman 2022). However, by looking at ranks, *overall* growth is typically normalized away, so the absolute aspect of overall income growth is removed.

A related approach aims to measure mobility as a cross-person equalizer of income or income compared to initial distribution (see Chakravarty, Dutta and Weymark 1985, Maasoumi and Zandvakili 1986, or Fields 2010). In these measures too, overall growth is typically normalized away. However, as discussed in Section 6.4, we will connect closely in spirit with these authors, in that our measure of mobility can be interpreted as the change in a social welfare function.

Two well-known measures of mobility are both directional and absolute. Fields and Ok (1999b) propose “directional mobility measures” that sum individual growth rates. More recently, the absolute mobility measure in Chetty, Grusky, Hell, Hendren, Manduca and Narang (2017) records the fraction of children who earn more than their parents. We discuss both measures in more detail in Section 6.8. For now, we note that both measures throw away information about who gains and who loses. In contrast, as already discussed, the relative mobility literature is sensitive to such matters. Genicot & Ray & Concha-Arriagada (2022) review the literature on mobility, with an emphasis on directional measures and on empirical evidence from developing countries.

Our derived measure connects us to a literature on *pro-poor growth*. Chenery, Jolly, Ahluwalia, Bell and Duloy (1974) introduced an index of economic performance as a weighted sum of group growth rates. This led to a literature on pro-poor growth using a variety of weights decreasing with income (see Dardanoni 1993, Essama-Nssah 2005, and Ravallion and Chen 2003) and proposing graphical tools such as the growth incidence curve (Grimm 2007, Bourguignon 2011, Ravallion and Chen 2003, Dhongde and Silber 2016, Palmisano and de Gaer 2016, Creedy and Gemmell 2018, Palmisano 2018). These authors observe that the difference between “anonymous” and “non-anonymous” growth incidence curves corresponds to pure exchange mobility (see Berman and Bourguignon 2022). They also connect pro-poor growth to the literature on convergence (O’Neill and Kerm 2008, Wodon and Yitzhaki 2005, Bourguignon 2011 and Dhongde and Silber 2016). Using non-anonymous growth incidence curves, indices of directional mobility have been proposed that place more weight on the growth rates of lower-ranked individuals (Jenkins and Van Kerm 2016, Palmisano and de Gaer 2016 and Berman 2022).

Our analysis builds on these insights. As noted in the Introduction, we impose Growth Progressivity as an axiom, and show that this restriction (along with some other mild conditions) precipitates a measure that is linear in growth rates, with weights that decline geometrically in income. Our exercise therefore establishes the foundational principles of upward mobility measures. But more than that, it leads to a discrete-time measure which applies to non-panel data. As a by-product, our axioms accomplish the task of

filtering out the exchange mobility component from our measure of *upward* mobility. The development of these ideas are the central themes of the paper.

Note that the same measures are used to measure both intergenerational and intragenerational mobility. As emphasized by Jäntti and Jenkins (2015), similar conceptual issues arise when studying either type of mobility. However, it is worth noting that possible changes in population size have been seldom discussed, though they could make a difference. Section 6.6 demonstrates that our measure can accommodate potential alterations in population size, and thus can be utilized to analyze both inter- and intra-generational mobility.

3. Prelude: Instantaneous Upward Mobility

Our aim is to develop a measure of *upward mobility* that is:

1. *Directional*: it rewards growth, and punishes decay; and
2. *Progressive*: it rewards “growth transfers” from higher to lower incomes.

Of course, rewarding growth to lower incomes raises the specter of income crossings as a result of differential growth. To avoid this issue, we begin with the concept of an “instantaneous” upward mobility kernel — an index that captures a snapshot of mobility at a moment in time. No crossing of distinct incomes can occur in that instant, so that the idea of growth progressivity is unambiguous. Section 4 will then use this kernel to provide a foundation for measuring upward mobility over income *trajectories*.

We refer to our central variable as “income”. What we have in mind is some measure of permanent income, a variable which serves as a reasonable state variable for well-being at any moment in time. We acknowledge that such a statistic is often not observed; a problem common to other measurement exercises pertaining to inequality, poverty, GDP and so on. See Section 5 for more discussion. For the formal analysis, however, all that matters is that we have some variable for which continuous changes can be envisaged.

When thinking of income trajectories, it will be useful to think of a single unit (individual, household) moving over time. But it will soon become clear that our framework applies not just to unchanging units but also to intergenerational units across dynasties. For more on these matters and more generally on evolving populations, see Section 6.6.

3.1. Axiomatic Development. Each person i is linked to a pair $z_i = (y_i, g_i)$, where $y_i > 0$ is baseline income and $g_i = \dot{y}_i(t)/y_i(t)$ is the instantaneous growth rate of that income. Denote by $\mathbf{z} = \{z_i\}$ the population collection of incomes and growth rates, including

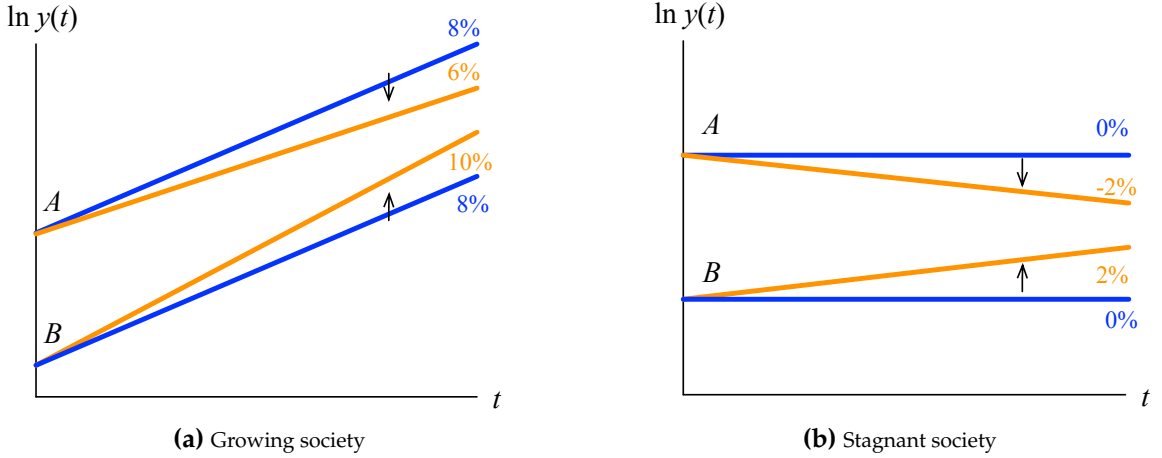


Figure 1. Illustration of the Growth Progressivity Axiom. If growth *rates* are transferred from relatively rich to relatively poor without income crossings, upward mobility rises.

repetitions. An *upward mobility kernel* is a continuous function $M(\mathbf{z})$, defined over all finite populations, anonymous to permutations of indices within $\mathbf{z}$. We place two mild background restrictions on M , the first being little more than a normalization.

1. **Zero Growth Anchoring.** If under both $\mathbf{z}$ and $\mathbf{z}'$, *every* individual has a zero growth rate, then $M(\mathbf{z}) = M(\mathbf{z}')$. Normalize this common value to zero.

The second restriction connects the measure across varying populations, and also gives it cardinal meaning. For any pair $\mathbf{z}$ and $\mathbf{z}'$, let $\mathbf{z} \oplus \mathbf{z}'$ denote the merged situation which contains the *union* of all income-growth pairs in $\mathbf{z}$ and $\mathbf{z}'$. For instance, $\mathbf{z} \oplus \mathbf{z}$ means that $\mathbf{z}$ has been duplicated. We place a restriction on “locally merged” situations that have identical sets of incomes and growth rates except for just one individual k :

2. **Local Merge.** Suppose $\mathbf{z}$, $\mathbf{z}'$ and $\mathbf{z}''$ are identical except for one index k , with $g'_k = g_k - \epsilon$ and $g''_k = g_k + \epsilon$ for some $\epsilon > 0$. Then $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$ if $\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$.

This axiom demands that if average upward mobility is altered by moving one person’s growth rate up while her clone’s growth rate is moved down, then mobility is also altered when both persons coexist and experience these same changes simultaneously. It injects cardinal meaning into the mobility measure, in that it involves an average across two situations, but overall it is a mild restriction. What comes next is of more substantive significance. We wish to formalize the idea that across two societies, the one in which the poor grow relatively faster (and the rich slower) is more upwardly mobile.

For instance, Figure 1(a) considers a two-person society with baseline incomes (5000, 10000) and corresponding growth rates (8%, 8%). We would like to say that it is more upwardly

mobile than another society with the same baseline incomes, but growing at (6%, 10%). Or if, as in Figure 1(b), the growth rates are (2%, -2%) then it is more upwardly mobile than a stagnant society, with the same starting incomes, growing at (0%, 0%). More generally:

3. Growth Progressivity. For any $\mathbf{z}$, i and j with $y_i < y_j$, and for $\epsilon > 0$, form $\mathbf{z}'$ by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then instantaneous upward mobility goes up: $M(\mathbf{z}') > M(\mathbf{z})$.

Growth Progressivity “rewards” a transfer of *growth points* from rich to poor. Note that because growth rates are instantaneous, there is no immediate “income crossing” when baseline incomes are distinct. We will address income crossings in Section 4.

Despite its resemblance to the well-known Transfers Principle for inequality comparisons (Fields and Fei 1978), Growth Progressivity is distinct in that it involves transfers of growth *rates*, not incomes. It implies that “upward mobility” is willing to accept a sacrifice of aggregate growth in some circumstances. This is a central feature. Mobility is not devoid of social meaning. Progress among the relatively poor should be favored, where the inherently dynamic nature of “progress” means that it is measured by growth rates. We emphasize that upward mobility is not to be regarded as synonymous with overall welfare, which would include many other considerations, such as per-capita income, inequality, or poverty. Any aggregate growth sacrifice will need to enter that final assessment.

That said, the reader interested in weakening Growth Progressivity is directed to Section 6.2. While the same methods will apply (and our formal proofs are written to make that clear), a different range of measures is thereby characterized.

Theorem 1. *Zero Growth Anchoring, Growth Progressivity, and Local Merge hold if and only if*

$$M(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) g_i \quad (1)$$

for some continuous collection $\{\phi_i\}$, with $\phi_i(\mathbf{y}) = \phi_j(i\mathbf{y}_j)$ whenever i and j are permuted in $\mathbf{y}$ to get $i\mathbf{y}_j$, and $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$ when $y_i < y_j$.

While Theorem 1 is proved in the Appendix, we illustrate here the power of the Growth Progressivity axiom. It is responsible for precipitating the additivity and linearity of our measure in individual growth rates.³ Without linearity, the measure must exhibit different local sensitivities to the growth rate g_i for some i . We are then able to exploit this variation in local sensitivities to construct a society in which a transfer of growth rates from one “near-clone” of i to another violates the Growth Progressivity Axiom. But these

³The other axioms also play their part. Axiom 1 removes any intercept term that depends on incomes. Axiom 2 takes us across populations of varying size. Without it, Growth Progressivity would still limit the curvature of the measure, but allow for some nonlinearity depending on cross-individual income gaps.

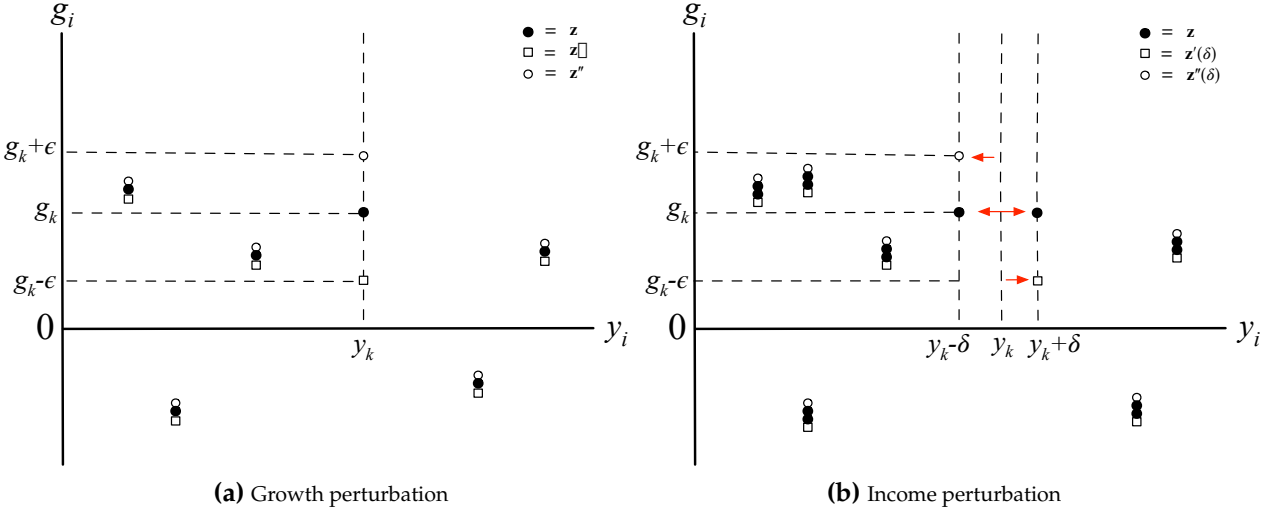


Figure 2. Illustration of proof that M is multi-affine. Part (a) shows two situations (filled and hollow dots) which are identical except for the growth rates at y_k , which are different. Part (b) perturbs the incomes in both situation by generating incomes $y_k - \delta$ and $y_k + \delta$ instead of y_k . The perturbation δ is to be thought of as small.

observations are both imprecise and incomplete. The main argument runs in two central steps. The first of these is the assertion that $M(\mathbf{y}, \mathbf{g})$ must be *multi-affine* in $\mathbf{g}$; i.e., for every k , mobility as a function of g_k conditional on $\mathbf{y}$ and $\mathbf{g}_{-k}$, denoted by $m(g_k)$, is affine in g_k :

$$m(g_k) = Ag_k + B$$

for constants A and B that could depend on $(\mathbf{y}, \mathbf{g}_{-k})$. Because m is continuous, it is enough to show that for every $\epsilon > 0$,

$$m(g_k) = \frac{1}{2} [m(g_k - \epsilon) + m(g_k + \epsilon)]. \quad (2)$$

Suppose the claim is false, so that (2) fails for some g_k and $\epsilon > 0$. Let $\mathbf{g}$ be a vector of growth rates with growth g_k for individual k and $\mathbf{g}_{-k}$ for the others, and let $\mathbf{z} = (\mathbf{y}, \mathbf{g})$. The filled dots in Part (a) of Figure 2 depicts this situation. It also shows two other situations, $\mathbf{z}'$ (represented by the squares) and $\mathbf{z}''$ (represented by the hollow dots). The proximity of dots and squares away from y_k is meant to imply that these three situations are identical in incomes and growths, except at y_k , where $\mathbf{z}'$ exhibits a lower growth rate than $\mathbf{z}$ (by ϵ) and $\mathbf{z}''$ a higher growth rate (also by ϵ). Because (2) fails, we have

$$m(g_k + \epsilon) - m(g_k) \neq m(g_k) - m(g_k - \epsilon),$$

but using the definition of m , this means $M(\mathbf{z}'') - M(\mathbf{z}) \neq M(\mathbf{z}) - M(\mathbf{z}')$, or:

$$M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z}) \quad (3)$$

by the Local Merge Axiom. Suppose that “<” holds in (3); the opposite “>” has a parallel argument. Part (b) of Figure 2 perturbs $\mathbf{z}$, $\mathbf{z}'$ and $\mathbf{z}''$ to separate y_k into $y_k - \delta$ and $y_k + \delta$, as shown, with the perturbed $\mathbf{z}'(\delta)$ having $y_k + \delta$ and the perturbed $\mathbf{z}''(\delta)$ having $y_k - \delta$. Part (b) also perturbs $\mathbf{z}$ in two ways: $\mathbf{z}^-(\delta)$ replaces y_k by $y_k - \delta$ while $\mathbf{z}^+(\delta)$ replaces y_k by $y_k + \delta$. Let $\mathbf{z}(\delta) \equiv \mathbf{z}^+(\delta) \oplus \mathbf{z}^-(\delta)$. Using the continuity of M and “<” in (3) and the fact that $\mathbf{z}'(\delta) \oplus \mathbf{z}''(\delta) \rightarrow \mathbf{z}' \oplus \mathbf{z}''$ and $\mathbf{z}(\delta) \rightarrow \mathbf{z} \oplus \mathbf{z}$, we must conclude that for $\delta > 0$ and small,

$$M(\mathbf{z}'(\delta) \oplus \mathbf{z}''(\delta)) < M(\mathbf{z}^+(\delta) \oplus \mathbf{z}^-(\delta)).$$

But this contradicts Growth Progressivity, for $\mathbf{z}'(\delta) \oplus \mathbf{z}''(\delta)$ can be achieved from $\mathbf{z}^+(\delta) \oplus \mathbf{z}^-(\delta)$ by transferring a growth rate of $\epsilon > 0$ from $y_k + \delta$ to $y_k - \delta$.

A parallel argument applies when “>” holds in (3), by perturbing $\mathbf{z}''(\delta)$ to the higher income $y_k + \delta$ and $\mathbf{z}'(\delta)$ to the lower income $y_k - \delta$.

So M is multiaffine, and therefore it is expressible as follows: for every $\mathbf{y} \gg 0$, there is a collection $\phi_S(\mathbf{y})$ for every nonempty subset S of $\{1, \dots, n\}$, such that

$$M(\mathbf{z}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right], \quad (4)$$

where n is the population, the sum ranges over all nonempty⁴ index subsets S of $\{1, \dots, n\}$, and the $\phi_S(\mathbf{y})$ are income-vector-dependent coefficients (see, e.g., Gallier 1999, Chapter 4.5). We argue that all nontrivial product terms *must have zero coefficients*. Otherwise, for some $\mathbf{g}$, it is possible to transfer growth rates from relatively poor to relatively rich while increasing M , so violating the Growth Progressivity Axiom (see Appendix). The only terms that can have zero coefficients are the linear terms in (4). That smaller index terms among $\{\phi_i(\mathbf{y})\}$ have larger values than the larger-index terms is also an immediate consequence of Growth Progressivity. That outlines our proof of Theorem 1.

3.2. A One-Parameter Family for Instantaneous Upward Mobility. Expression (1) is a key implication of the Growth Progressivity Axiom, and illustrates the power of that Axiom. To highlight this, we have (so far) placed no restrictions on how the upward mobility kernel changes with growth, or on baseline weights apart from the property derived in Theorem 1. We now impose further axioms that bring the weights $\phi_i(\mathbf{y})$ into sharper focus.

4. Income Neutrality. Given $\mathbf{z} = (\mathbf{y}, \mathbf{g})$, form $\mathbf{z}' = (\lambda \mathbf{y}, \mathbf{g})$ by scaling all baseline incomes by the same positive constant λ . Then $M(\mathbf{z}) = M(\mathbf{z}')$.

⁴The empty product can be excluded by the Zero Growth Anchoring Axiom.

5. **Growth Alignment.** For any $\mathbf{y}$, if $\mathbf{g} > \mathbf{g}'$, then $M(\mathbf{y}, \mathbf{g}) > M(\mathbf{y}, \mathbf{g}')$. And if $\mathbf{g} = (g, g, \dots, g)$, then for every $\mathbf{y}$ and $\mathbf{y}'$, $M(\mathbf{y}, \mathbf{g}) = M(\mathbf{y}', \mathbf{g})$.

6. **Binary Growth Tradeoffs.** For any ij , any (y_i, y_j) , and any two growth pairs (g_i, g_j) and (g'_i, g'_j) , the comparison of $\mathbf{z} = ((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$ and $\mathbf{z}' = ((y_i, g'_i), (y_j, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$ is insensitive to the value of $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$.

Axiom 4 asserts that only relative baseline incomes matter. Axiom 5 (partially) aligns upward mobility with growth: if *every* income grows faster, then mobility is deemed to be higher. This steers us towards absolute upward mobility; see Section 3.3 for a parallel development of relative upward mobility.

Axiom 6 declares that any tradeoffs across a pair of growth rates depends only on the characteristics of just *that* pair. This is in the spirit of “independence of irrelevant alternatives.” There are well-known misgivings about that axiom (see the critical assessment in Pearce 2021). One qualification concerns the choice of domain for this Axiom. Under absolute upward mobility, individual growth rates matter *per se*, so that the domain of Axiom 6 is reasonable. But if the context is one of relative upward mobility, then the Axiom more properly applies to the *excess* (positive or negative) of individual growth against overall growth. There is more discussion of this point in Section 3.3.

Theorem 2. *Axioms 1-6 hold if and only if*

$$M_\alpha(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}}, \text{ for some } \alpha > 0. \quad (5)$$

whenever the population size $n \geq 3$.

The family of instantaneous kernels characterized here will form the nucleus of our main analysis, to be developed in Section 4. It will be useful to see the marginal roles played by the new axioms in precipitating it. When $n \geq 3$, Binary Growth Tradeoffs along with anonymity allow us to write the ϕ -functions from Theorem 1 as

$$\phi_i(\mathbf{y}) = \psi(y_i)h(\mathbf{y})$$

for functions ψ and h . By Growth Alignment, both functions are strictly positive-valued, and we can normalize $M(\mathbf{z}) = g$ when all growth rates equal g . Using (1), $\sum_i \psi(y_i)h(\mathbf{y}) = 1$. Substituting this information in (1),

$$M(\mathbf{z}) = \frac{\sum_i \psi(y_i)g_i}{\sum_i \psi(y_i)}.$$

The Appendix shows that Income Neutrality must then imply $\psi(y) = y^{-\alpha}$, where $\alpha > 0$ by Growth Progressivity. That establishes (5), and Theorem 2.

3.3. The Relative Mobility Kernel. The derived upward mobility kernel has both absolute and relative features, the former embodied in Growth Alignment and the latter in Growth Progressivity. We might want to “net out” aggregate growth and view what remains as a relative measure of mobility. We call this the *relative upward mobility kernel*:

$$R_\alpha(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}} - g = \frac{\sum_{i=1}^n y_i^{-\alpha} e_i}{\sum_{i=1}^n y_i^{-\alpha}}, \quad (6)$$

where g is the overall rate of growth and $e_i \equiv g_i - g$ is the individual *excess* growth rate. Noting that $g = (\sum_j y_j g_j) / (\sum_j y_j)$, this can be rewritten as

$$R_\alpha(\mathbf{z}) = \sum_{i=1}^n \phi_i^*(\mathbf{y}) g_i, \text{ where } \phi_i^*(\mathbf{y}) = \frac{y_i^{-\alpha}}{\sum_{j=1}^n y_j^{-\alpha}} - \frac{y_i}{\sum_{j=1}^n y_j}. \quad (7)$$

Because $\phi_i^*(\mathbf{y}) > \phi_j^*(\mathbf{y})$ whenever $y_i < y_j$, the relative upward mobility kernel satisfies Growth Progressivity (it satisfies Local Merge and Zero Growth Anchoring as well). It is therefore one of the measures accommodated by Theorem 1. But growth is not the variable of central interest in the relative upward mobility kernel. Rather, it is *excess growth* over and above the aggregate growth rate. From that perspective, we must change our domain to pairs $(\mathbf{y}, \mathbf{e})$, where $\mathbf{y}$ is a vector of baseline incomes, and $\mathbf{e}$ is the vector of excess growth rates $e_i = g_i - g$. Income Neutrality is maintained, while Growth Alignment must be removed. Axiom 6, on Binary Growth Tradeoffs, changes its meaning: the independence condition on binary tradeoffs across g_i and g_j is not the same as the independence condition on binary tradeoffs across e_i and e_j . Finally, Axiom 1 is extended to

1'. Zero Excess Growth Anchoring. If in two situations $\mathbf{z}$ and $\mathbf{z}'$, *every* individual has the same growth rate, then $M(\mathbf{z}) = M(\mathbf{z}')$; normalize this common value to zero.

These reconfigured axioms fully characterize the relative upward mobility kernel in (6). We omit the proof, which follows that of Theorem 2.

4. Upward Mobility Over Time Intervals

Growth data are typically generated *across* intervals of time, and not *at* instants of time. The latter fully allow Growth Progressivity to be applied. After all, the ranking of two distinct incomes cannot be reversed in an instant, no matter how disparate the growth rates. But over intervals, income crossings can and do occur. Individual 1 might initially be poorer than 2, and then richer. Growth progressivity cannot, therefore, be unambiguously applied “in favor of” individual 1: we do not want to ascribe a higher weight to the income growth of individual 1 over the entire interval, just because she was initially poorer.

To address this issue, suppose (just temporarily) that we could fully observe all income trajectories over some given time interval. We will view upward mobility as a functional defined on such trajectories. To place structure on this mapping, we first break up the trajectories into smaller sub-trajectories — in the limit, into instants of time. Overall upward mobility will be derived from all the instantaneous kernels thus generated.

Specifically, for every starting date s and ending date t , denote a trajectory by $\mathbf{y}[s, t] = \{y_i[s, t]\}$, where i indexes individuals. Upward mobility is then a mapping $\mathbf{y}[s, t] \mapsto \mu(\mathbf{y}[s, t])$. We presume translation invariance in calendar time as a self-evident restriction: for all pairs of trajectories $(\mathbf{y}[s, t], \tilde{\mathbf{y}}[0, t - s])$ with $\tilde{\mathbf{y}}(\tau) = \mathbf{y}(s + \tau)$ for $\tau \in [0, t - s]$,

$$\mu(\mathbf{y}[s, t]) = \mu(\tilde{\mathbf{y}}[0, t - s]). \quad (8)$$

Assume that income paths are strictly positive and continuously differentiable everywhere. Then instantaneous growth rates are well-defined and continuous everywhere (but see Section 6.5). We now impose two conditions on the upward mobility measure.

4.1. Reducibility. Return to our individuals 1 and 2, with incomes $\{y_1(0), y_2(0)\}$ at date 0 and $\{y_1(T), y_2(T)\}$ at date T . Recall that 1 is initially poorer — $y_1(0) < y_2(0)$ — but that eventually $y_1(T) > y_2(T)$. See Figure 3a. Growth Progressivity cannot be indiscriminately applied over the entire interval. The higher growth experienced by individual 1 should not necessarily be viewed as conferring greater upward mobility just because 1 was poorer than 2 to start with. After all, 1 is richer than 2 in the second phase of the process.

As mentioned earlier, a natural resolution is to examine the trajectories in “pieces”. If 1 is initially poorer, let Growth Progressivity act in favor of 1, but ask that Growth Progressivity act in favor of 2 once a crossing occurs. (Such switches could occur on multiple occasions, as in Figure 3b.) That leads to the notion of *reducibility*: regard our measure as being fully determined by the upward mobility kernels at every instant of time during the interval. More formally, reducibility asks that μ be expressible as

$$\mu(\mathbf{y}[s, t]) = \Psi(\{M(\mathbf{z}(\tau))\}_s^t) \quad (9)$$

for some pointwise, nondecreasing “aggregator” Ψ , where M is our kernel, and $\{\mathbf{z}(\tau)\}$ the collection of income and growth rate pairs induced by the right-hand derivatives of $\mathbf{y}[s, t]$. We normalize Ψ by asking that if instantaneous upward mobility is constant at any $m \in \mathbb{R}$ over the entire time interval, then upward mobility over that interval also equals m .

4.2. Additivity. Our second condition concerns the time separability of our upward mobility measure. Divide a time interval into two subintervals of equal lengths, and suppose that measured mobility is μ_1 and μ_2 in each interval. We would then like to say

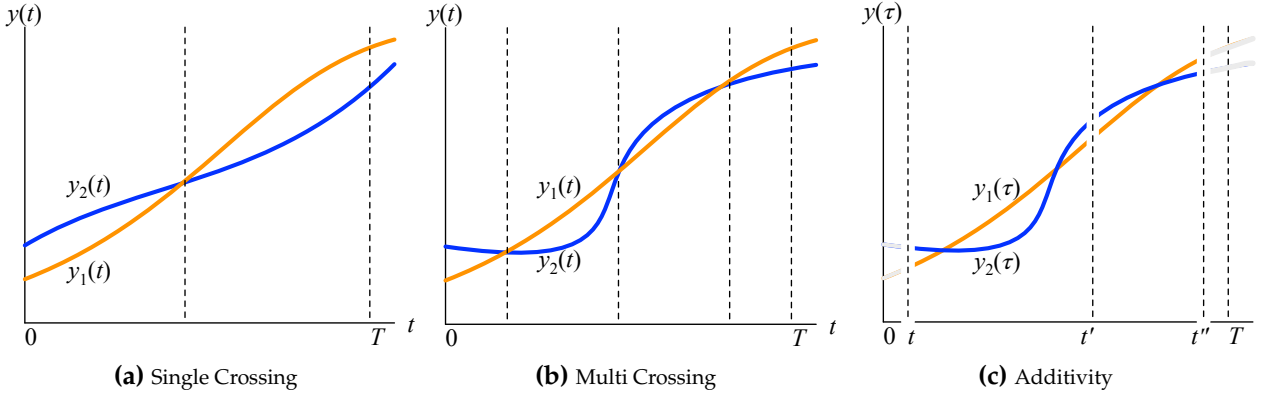


Figure 3. Possible Income Trajectories.

that overall mobility across the entire interval is just $(\mu_1 + \mu_2)/2$, just as we would average logarithmic growth. Adjusting in the obvious way for intervals of unequal size⁵ — as in Figure 3(c), for instance — say that an upward mobility measure is *additive* if for every collection of income trajectories of the form $\mathbf{y}[s, t]$ and every intermediate date $u \in (s, t)$,

$$(u - s)\mu(\mathbf{y}[s, u]) + (t - u)\mu(\mathbf{y}[u, t]) = (t - s)\mu(\mathbf{y}[s, t]). \quad (10)$$

Additivity is transparent when the trajectories are differentiable, so that mobility kernels are defined at every moment in time. When they are not, any discontinuous jumps will have to be fully accounted for. In Section 6.5 we show how to accommodate those discontinuities.

It should be noted that additivity, or reducibility, or indeed the joint imposition of the two, are compatible with a variety of mobility measures, including those that simply describe “pure movement.” But in conjunction with our earlier axioms for instantaneous upward mobility, the allowable class of measures has a particularly narrow form.

4.3. Upward Mobility on Trajectories.

Theorem 3. *Axioms 1–6, reducibility (9), and additivity (10) hold if and only if over any collection $\mathbf{y}[s, t]$ of continuous and right-differentiable income trajectories on $[s, t]$, $\mu(\mathbf{y}[s, t])$ has a representation of the form:*

$$\mu_\alpha(\mathbf{y}[s, t]) = \frac{1}{t - s} \ln \left[\frac{\sum_{i=1}^n y_i(t)^{-\alpha}}{\sum_{i=1}^n y_i(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0. \quad (11)$$

⁵We do not even need to make this adjustment. Under continuity, unweighted additivity over two sub-intervals of equal size would actually *imply* the weighted definition in (10), but we assume the weighted version so as not to add technical arguments.

We mention the main lines of the argument here. It is not hard to see that (11) satisfies all the conditions of the Theorem, so we focus on “only if.” By Theorem 2 — and therefore Axioms 1–6 — the upward mobility kernel at any date is $M_\alpha(\mathbf{z}) = \sum_{i=1}^n \left[y_i^{-\alpha} / \sum_{j=1}^n y_j^{-\alpha} \right] g_i$ for some $\alpha > 0$, where α is independent of calendar time by the translation invariance condition (8). Therefore, by reducibility,

$$\mu_\alpha(\mathbf{y}[s, t]) = \Psi \left(\left\{ \sum_{i=1}^n \frac{y_i(\tau)^{-\alpha} g_i(\tau)}{\sum_{i=1}^n y_i(\tau)^{-\alpha}} \right\}_s^t \right), \quad (12)$$

and by the additivity of μ , (12) further simplifies to

$$\mu_\alpha(\mathbf{y}[s, t]) = \int_s^t h \left(\sum_{i=1}^n \frac{y_i(\tau)^{-\alpha} g_i(\tau)}{\sum_{i=1}^n y_i(\tau)^{-\alpha}} \right) d\tau \quad (13)$$

for some function h . (See Steps 1 and 2 of the formal proof.) The normalization on the aggregator Ψ tells us that $\mu(\mathbf{y}[s, t]) = m$ if the upward mobility kernel is constant at m over $[s, t]$. Applying this restriction to (13), we must conclude that for every such $m \in R$,

$$(t - s)h(m) = m, \quad (14)$$

and substituting (14) into (13), and integrating (see Step 3 of the formal proof for details), we obtain (11), which is the form in which we wish to take our mobility measure to the data. Notice that (11) divides by the normalization term $t - s$, and so picks out “average mobility” over the period, expressible as, say, an annual percentage rate.

Theorem 3 also applies to the relative mobility measure in Section 3.3. Impose reducibility just as we did for absolute mobility, but using relative mobility kernels. Then Theorem 3 asks us to integrate the kernel described in Section 3.3 over a income trajectory to generate a corresponding relative mobility measure; call it ρ . That measure is independent of the particular trajectory, for the same reason that the upward mobility measure and the overall growth rate both are. Letting $\bar{y}$ denote per-capita income, we have:

$$\rho_\alpha(\mathbf{y}[s, t]) = \mu_\alpha(\mathbf{y}[s, t]) - \frac{1}{t - s} [\ln(\bar{y}(t)) - \ln(\bar{y}(s))] = \frac{1}{t - s} \left\{ \ln \left[\sum_i \left(\frac{y_i(t)}{\bar{y}(t)} \right)^{-\alpha} \right]^{-\frac{1}{\alpha}} - \ln \left[\sum_i \left(\frac{y_i(s)}{\bar{y}(s)} \right)^{-\alpha} \right]^{-\frac{1}{\alpha}} \right\}. \quad (15)$$

5. Panel Independence

Theorem 3 has the implication that our upward mobility measure does not need panel data for its implementation. Even though both reducibility and additivity rely conceptually on the full observability of income trajectories, that observability is discarded in the sequel. Equation (11) makes it clear that only information on starting and terminal incomes is needed for our upward mobility computation. That is, while i is an index that sums

weighted incomes in both the numerator and denominator of the expression in (11), there is no presumption that i stands for the same person at the beginning and the end of the interval. Because the identity connection between starting and terminal incomes is thereby broken, we see that reducibility already begins to de-emphasize the need for panel data. Additivity and our derived linearity of the upward mobility kernels do the rest.⁶

Panel independence significantly expands the scope of a mobility measure (see Section 7 for some applications). High-quality panel income data at the individual or household level is a scarce commodity in most countries. Even in the United States or Europe, the construction of a satisfactory intergenerational panel has necessitated access to proprietary data (Chetty et al. 2017 and Acciari et al. 2022),⁷ so as to estimate a transition probabilities over incomes. For the vast majority of developing countries, it is unclear whether even that proprietary access is feasible, and there is certainly no data in the public domain. The panel-independence property questions the very need for that data.

Ease of use apart, there are conceptual aspects of panel-independence that we now discuss.

First, panel-independence may appear counterintuitive. After all, the researcher may feel that mobility is fundamentally a dynamic construct for dynasties or lineages.

It is certainly true that to assess the fortunes of a family over time, *that* family must be tracked. (For instance, from the perspective of the “mobility of an individual dynasty,” it does indeed matter whether the trajectories in question are given by Figure 3a or by Figure 3b.) But to assess upward mobility overall, it is not an individual family that the researcher is after, but the contributions of all families to upward mobility at every point of time. A family located at two different points in the cross-sectional distribution will receive different weights under the mobility measure. By reducibility, the impact on overall mobility must feed through the impact on mobility kernels.

Such nimble weight switches are central to our argument. The application of Growth Progressivity over an entire interval of time would directly invoke the upward mobility measure in equation (5) of Theorem 2, replacing instantaneous growth by the discrete growth rate of income over that interval. The resulting measure would involve a high weight on an agent *even after* that agent has become richer. In contrast, the instantaneous approach that undergirds our measure assigns greater social weight *at any instant* to a poorer individual, but re-adjusts those weights continuously — certainly once rankings are reversed, but even when income differentials narrow or widen.

⁶So Growth Progressivity along with the other conditions is also instrumental in precipitating panel-independence, because it implies the linearity of instantaneous upward mobility in growth rates.

⁷Chetty et al. (2017) use their transition matrix to estimate the fraction of children who fare better than their parents. This leads to a measure that we will discuss in Sections 6.8 and 7.

Second, upward mobility filters out the component of mobility that is pure movement. Such movement or “exchange mobility” may well be important in other contexts, such as geographical relocation or seasonal migration. But the researcher interested in upward mobility must trade upward against downward movement. Under pure movement, both contribute to mobility. As far as upward mobility is concerned, it is the “net” directional movement that matters. In short, upward mobility is *not all* one might mean by mobility. And it is upward mobility — not exchange mobility — that admits panel-independent measurement. This conceptual separation of pure movement and directional movement is important. For more discussion, see Section 6.9.

Third, there may be a reluctance to give up the panel, because it is felt that “income” is not a sufficient statistic for lifetime welfare. We suggest that the researcher should choose the variable they deem most appropriate *before* conducting the measurement, such as current consumption, current income, permanent income, or wealth. In a world of imperfect data, consumption is usually considered the preferred indicator to measure living standards (see Deaton and Zaidi 2002). Or short panels could greatly help in income-averaging. Indeed, a similar recommendation also applies to the measurement of poverty or inequality. It all has to do with what we are measuring the mobility (inequality, poverty) *of*, and not with the measure *per se*.

Fourth, and even if we accept the third point, an individual’s socioeconomic position might also be driven by stigma or status for some identifiable social group to which that individual belongs, such as religion, caste or ethnicity.

This is an important concern. We would need to approach social variables from first principles, incorporate such variables into the analysis, and see what that implies for measurement. The point is important enough that we have devoted a separate section to it; see Section 6.7. We show there that whereas *group* variables may need to be tracked over time, the panel-independence of *individual* trajectories continues to apply.

One might still object to panel-independence on the grounds of some visceral intuition that it is a tall order for mobility measurement. This is where the axiomatic approach might help. It permits us to examine each condition separately for its palatability. First, from the axioms that characterize the mobility kernel (principally, Growth Progressivity), we extract the logical implication that instantaneous upward mobility is linear in individual growth rates. Second, we posit *reducibility*: an assumption that asserts that all the information for overall upward mobility is coded inside the grand collection of instantaneous kernels.

This condition is already discussed in Section 4.1, but one difficulty with reducibility is that instantaneous kernels may not be well-defined at every moment of time (recall that

we presumed that trajectories are continuously differentiable). Discrete, lumpy events may well occur: an inheritance, a job loss or gain, a promotion, and so on. Section 6.5 shows us how to extend the analysis to this case, with no change in the results.

Finally, *additivity* states that upward mobility across two contiguous time intervals add up to overall upward mobility on the union of those intervals. We find it difficult to argue against this assumption. (It would hold for exchange mobility as well, or for overall growth rates.⁸) It is really how these axioms interact that is at the heart of the exercise. All three elements — linearity in growth rates, reducibility and additivity — jointly conspire to precipitate panel-independence, but it goes without saying that such interaction is a matter of logical necessity and does not constitute an additional assumption.

6. Aspects of Upward Mobility

6.1. Upward Mobility, and Growth. Our measure connects upward mobility to *pro-poor growth* (Chenery, Jolly, Ahluwalia, Bell and Duloy 1974, Dardanoni 1993, Ravallion and Chen 2003, Essama-Nssah 2005, Jenkins and Van Kerm 2006, 2011, Palmisano and de Gaer 2016 and Berman 2022). Theorem 2 declares that the weights on different incomes must be powers of the inverses of those incomes. For instance, $\alpha = 0.5$ doubles the weight on a \$40,000 baseline relative to \$160,000. As $\alpha \simeq 0$, our measure converges to the unweighted average of growth rates of individual income,⁹ and as $\alpha \rightarrow \infty$, it becomes Rawlsian.

The lower boundary of our class of measures as $\alpha \downarrow 0$ is of special interest. To our understanding, it first appears in Fields and Ok (1999b) as a directional measure of mobility. Saez and Zucman (2020) refer to it as “the people’s growth rate,” noting that each person’s growth rate is given equal weight. In itself, it fails growth progressivity (see Section 6.8 for more) though, of course, all positive values of α do satisfy that axiom. But it is an important boundary. Below it, individual growth rates garner larger weight at *higher* baseline incomes, a feature that might be viewed as uncomfortably regressive. Indeed, the familiar measure of aggregate growth over the period $[s, t]$ is given by

$$\text{Average Growth} = \mu_{-1}(\mathbf{y}[s, t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^m y_j(t)}{\sum_{j=1}^m y_j(s)} \right] = \frac{1}{t-s} \int_s^t g(\tau) d\tau, \quad (16)$$

which is well into regressive territory, given that the α -value corresponding to it is -1 ! No wonder that in stark contrast to the people’s growth rate, the aggregate growth rate has been referred to as the *plutocratic* growth rate (Milanovic 2005). Nevertheless, our

⁸The growth *rate* connects to log incomes and therefore generates additivity; see, for instance, equation (25).

⁹This can be seen by applying L’Hôpital’s Rule to $-\ln \left(\frac{\sum_i y_i(t)^{-\alpha}}{\sum_i y_i(s)^{-\alpha}} \right) / \alpha(t-s)$, as $\alpha \rightarrow 0$.

measure can be viewed as a “growth rate equivalent”: when all growth rates are the same, our measure *is* that growth rate. Otherwise it corrects for the progressivity of growth.

6.2. Weakening Growth Progressivity. The foregoing discussion suggests a weakening of the Growth Progressivity axiom that allows for a wider class of ethical stances and — by implication — a correspondingly broader collection of mobility measures:

3'. Income Progressivity. For any $\mathbf{z}$, i and j with $y_i < y_j$, and for $\epsilon > 0$, form $\mathbf{z}'$ by altering $\dot{y}_i$ to $\dot{y}_i + \epsilon$ and $\dot{y}_j$ to $\dot{y}_j - \epsilon$. Then instantaneous upward mobility goes up.

Given Axiom 5 (Growth Alignment), Income Progressivity is a substantial weakening of Growth Progressivity.¹⁰ When the latter axiom holds, the former perforce applies. But the former axiom holds in situations where the latter does not. Indeed — and this connects to the discussion in Section 6.1 — Income Progressivity additionally holds for all negative α -weights between -1 and 0 , or between the plutocratic growth rate and the people's growth rate, while Growth Progressivity does not. (For $\alpha > 0$, both axioms are satisfied.)

Interestingly, if we replace Axiom 3 by 3', the above measures with $\alpha \in (-1, 0]$ are the *only* additional measures thereby obtained. In other words, the analysis so far, *including the linearity of the upward mobility kernel in growth rates*, fully extends, but with a weaker constraint on α — that it should exceed -1 instead of 0 . So technically, this change neither strengthens or weakens the results. We retain a strong preference for Growth Progressivity rather than Income Progressivity on intrinsic grounds, but nothing in our methods hinders an analysis of the latter axiom. The formal proofs in the Appendix are written to explicitly accommodate both the results for Growth Progressivity and Income Progressivity.

6.3. Upward Mobility and Inequality. Upward mobility rewards equalization, but only the equalization implicit in the *change* of incomes, as a consequence of rewarding differential growth for the relatively poor. Therefore upward mobility in itself is not a measure of equality. For instance, if all incomes grow at the same rate, our measure returns the same answer irrespective of the initial distribution. Additionally, the measure values growth all around relative to zero-growth situations, even if that growth is disequalizing.

6.4. Upward Mobility as Change in Welfare. The Atkinson welfare function is given by

$$a_\alpha(\mathbf{y}) = \left(\frac{1}{n} \sum_i y_i^{-\alpha} \right)^{-\frac{1}{\alpha}}, \text{ where } \alpha > 0. \quad (17)$$

¹⁰Axiom 3' looks very close to the Transfers Principle for inequality measurement, but it is placed on an entirely distinct domain, applying as it does to changes in incomes and not in their levels. The resulting measure continues to be silent on baseline inequality, whereas the Transfers Principle would not be

We can think of $a_\alpha(\mathbf{y})$ as the *Atkinson equivalent income* of an income vector $\mathbf{y}$, when the welfare (or curvature) parameter is α . It is then trivial to see that our instantaneous mobility kernel is precisely the instantaneous rate of growth of Atkinson equivalent income; that is,

$$M_\alpha(\mathbf{z}) = \sum_{i=1}^n \frac{\partial a_\alpha(\mathbf{y})}{\partial y_i} \frac{1}{y_i(t)} \frac{dy_i(t)}{dt} \quad (18)$$

along any differentiable trajectory of incomes. And if we then turn to the discrete measure, we have the expected implication that for any $s < t$ and differentiable trajectory $\mathbf{y}[s, t]$,

$$\ln a_\alpha(\mathbf{y}(t)) - \ln a_\alpha(\mathbf{y}(s)) = (t - s)\mu(\mathbf{y}[s, t]). \quad (19)$$

Equation (19) implies that upward mobility can be viewed as the “average percentage change” in Atkinson welfare (or Atkinson equivalent income).

A similar interpretation applies to relative mobility. The measure in (15) can be written as

$$\rho_\alpha(\mathbf{y}(s, t)) = \frac{1}{t - s} \left[\ln(W_\alpha(\mathbf{y}(t))) - \ln(W_\alpha(\mathbf{y}(s)G)) \right],$$

where $G = \bar{y}(t)/\bar{y}(s)$ is the overall growth factor. That is, ρ_α can be seen as a net adjustment in welfare experienced in from going from initial to final distribution, relative to moving to a hypothetical distribution $\mathbf{y}(s)G$, where everyone experiences the average growth rate of the economy. This view of relative mobility as a net change in social welfare over and above balanced growth underlies the ethical measures of mobility of Chakravarty et al. (1985), though in our case it has emerged endogenously from more primitive axioms.

It is worth noting, however, that our interpretation of mobility as the change in Atkinson equivalent income is *restricted* to coefficients of inequality aversion that exceed one, or to welfare curvatures exceeding that of the logarithmic function. This is an implication of growth progressivity, which imposes a strong preference for equalization. So only a subclass of the Atkinson family can be invoked for our interpretation. Under Income Progressivity (see Section 6.2), we would indeed span the entire Atkinson class.

6.5. Trajectories With Jumps. Theorem 3 assumes that income trajectories are continuously differentiable in time, so that growth rates are everywhere well-defined (and continuous) and we can apply our instantaneous measure. But discontinuous events might well occur, such as an inheritance, a sudden loss of job, or a promotion. If incomes are stationary except at these crucial events, instantaneous upward mobility would be zero at every date except at the isolated jumps. But of course, overall upward mobility is not zero.

It is, however, not difficult to modify the analysis so that they apply to paths with simple jump discontinuities at finitely many dates. Every such continuous trajectory,

differentiable or not, can be approximated by continuously differentiable trajectories, and they all generate the same answer as in (11). The reason is that (11) is independent of the exact path of intermediate trajectories as long as they are continuously differentiable.

6.6. Population Shares. Income-growth observations could be repeated, so there is no need for population weighting in (11). But this presumes that the population is constant. In practice, data on income-specific growth rates are provided by m quantiles, and are available as $\{y_i(\tau), n_i(\tau)\}_{i=1}^m$, where $n_i(\tau)$ is the population share in quantile i at date τ . To incorporate the data in this form, with varying populations, we need to take a stand on what happens when a new individual is added to or removed from the set of observations.

To do this, we need an obvious Population Neutrality principle, analogous to Income Neutrality (Axiom 4): when populations at any instant are replicated by some positive integer, keeping their distributions over (y, g) unchanged, the mobility kernel is unchanged.

Now proceed as follows. With finite populations, demographic changes will occur at discrete instants in time. Suppose that between s and t there are J consecutive time intervals $I_1, \dots, I_J$ such that in interval j , the population is constant at n_j . Define n^* to be the lowest common multiple of $(n_1, \dots, n_J)$, and scale population in each interval I_j from n_j to n^* . Mobility kernels are unchanged at every instant that they are defined, by population neutrality. Moreover, population will now remain stationary at n^* over the entire interval.

All that remains is to connect the trajectories so that each of the n^* individuals have incomes that are fully defined on $[s, t]$. Our earlier discussion implies that it won't matter how we match the different trajectories. Of course, any connection rule will generally entail jump discontinuities in the trajectories, but these can be taken care of exactly as we did in Section 6.5. With this procedure in hand, and explicitly keeping track of identical observations at each income our upward mobility measure becomes:

$$\mu_\alpha(y[s, t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^J n_j(t) y_j^{-\alpha}(t)}{\sum_{j=1}^J n_j(s) y_j^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}}, \quad (20)$$

where n_j is the share of the population earning y_j . This is the form in which we take our measures to the data in Section 7. Observe that once in this format, we need pay no special attention to *inter-* versus *intra-*generational mobility. To mimic intergenerational mobility, we could simply look at mobility over large periods of time, such as 30-year intervals — indeed, this is precisely what we do in Section 7.1.

6.7. Upward Mobility and Social Groupings. We return to a discussion initiated in Section 5. Suppose that we do have a good individual- or household-level measure of permanent

income. Even then, that does not take care of other social variables that might confer status or stigma. The fact that person B is currently richer than person A might not detract from the reality that B belongs to a low-income social group, perhaps demarcated by ethnicity, race, gender or religion. If such groupings are salient, our measures of upward mobility in (11) and (15) may need to incorporate this fact. We indicate one such approach here.

Suppose that there are K social groups that partition society, with generic element k . Each person i belongs to one such group $k(i)$.¹¹ Returning temporarily to the case of instantaneous upward mobility, the data now form a collection $(\mathbf{z}, \mathbf{w})$, with $z_i = (y_i, g_i)$ just as before for every i and where $w_{k(i)}$ is the mean income of the group $k(i)$ to which i belongs. So each individual i is labeled by her baseline income y_i , but also by her group income $w_{k(i)}$. We extend Growth Progressivity to incorporate this:

Social Growth Progressivity. For any $\mathbf{z}$, i and j with $y_i \leq y_j$ and $w_{k(i)} \leq w_{k(j)}$ with at least one strict inequality, and for $\epsilon > 0$, form $\mathbf{z}'$ by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then instantaneous upward mobility goes up: $M(\mathbf{z}') > M(\mathbf{z})$.

We additionally extend Income Neutrality to

Social Income Neutrality. For any $(\mathbf{z}, \mathbf{w})$ and any $\lambda > 0$, $M(\lambda \mathbf{y}, \mathbf{g}, \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$, and $M(\mathbf{y}, \mathbf{g}, \lambda \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$.

Observe that we require separate scale neutrality relative to each set of incomes (individual and social). Of course, in the sequel, group income will be connected to the individual incomes of group members in the obvious way; that is,

$$w_k(t) = \frac{1}{n_k} \sum_{j \in k} y_j(t) \quad (21)$$

where n_k is the number of members in group k . But conceptually, our measure is defined on *all* pairs $(\mathbf{z}, \mathbf{w})$. Note also that the growth rates of group incomes are irrelevant for the instantaneous kernel. But once we consider upward mobility over intervals, we must account for the growth of group incomes as they evolve according to (21).

Finally, we extend Binary Growth Tradeoffs to account for the presence of social groups:

Social Binary Growth Tradeoffs. For any i, j , any $(y_i, y_j, w_{k(i)}, w_{k(j)})$, and any two growth pairs (g_i, g_j) and (g'_i, g'_j) , the comparison of $((y_i, w_{k(i)}, g_i), (y_j, w_{k(j)}, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ and $((y_i, w_{k(i)}, g'_i), (y_j, w_{k(j)}, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ is insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)})$.

The following proposition is established using a straightforward extension of the argument for Theorem 2, and so we omit the proof.

¹¹There may be multiple group identities. We take it that one such partition is salient for the analyst.

Proposition (Extension of Theorem 2). *Social Growth Progressivity, Social Income Neutrality, Social Binary Growth Tradeoffs and Growth Alignment hold if and only if for every population of size $n \geq 3$ and groupings K ,*

$$\mu_{\alpha,\beta}(\mathbf{z}, \mathbf{w}) = \frac{\sum_{i=1}^n y_i^{-\alpha} w_{k(i)}^{-\beta} g_i}{\sum_{i=1}^n y_i^{-\alpha} w_{k(i)}^{-\beta}}, \text{ for some } (\alpha, \beta) \gg 0. \quad (22)$$

As before, we can use this instantaneous kernel and apply it to income trajectories over time intervals. For any interval $[s, t]$, we are given a collection of individual trajectories $\mathbf{y}[s, t]$. In addition, there is a partition of the population into sets $k \in K$, where each k contains individuals. An upward mobility measure is then a mapping

$$(\mathbf{y}[s, t], K) \mapsto \mu(\mathbf{y}[s, t], K).$$

Recalling (21), we note that $(\mathbf{y}[s, t], K)$ generates accompanying group trajectories $\mathbf{w}[s, t]$. It also generates higher moments of those trajectories. It will turn out that we will need panel data on these summary statistics at the group level, but — in parallel with Theorem 3 — *not* panel data at the individual level. The theorem that follows illustrates this idea:

Theorem 4. *Social Growth Progressivity, Social Income Neutrality, Social Binary Growth Tradeoffs, Growth Alignment, Reducibility and Additivity hold if and only if for every population of size $n \geq 3$ and groupings K ,*

$$\mu_{\alpha,\beta}(\mathbf{y}[s, t], K) = \frac{1}{t-s} \left\{ \ln \left[\frac{\sum_{i=1}^n y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^n y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}} \right]^{-1/\alpha} - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau \right\}, \quad (23)$$

for some $(\alpha, \beta) \gg 0$, where for each k and τ , $a_k(\tau)$ is Atkinson group income as defined in (17).

This extended measure is a conceptual extension of our baseline measure, but it is also a consistent mathematical extension: convergence occurs to the baseline measure as $\beta \rightarrow 0$. Specifically, the first term on the right hand side of (23) is the analogue of our baseline measure. It is entirely panel-independent. The second term is the “correction” created in the presence of group incomes. This correction varies over time as group incomes change, and does depend on the knowledge of trajectories, *but only at the group level*. Specifically, we would need information on the mean income trajectory and Atkinson inequality at the group level, but none of this is individual panel information.

It could be argued that the empirical researcher would not be lucky enough to possess Atkinson group inequality measures for precisely the value of α for which she chose to measure mobility. But it is possible to approximate the Atkinson measure by standard measures of inequality, such as the coefficient of variation, provided we are willing to make suitable assumptions about the higher-order moments of group distribution. As this is

not entirely germane to the flow of the current discussion, we relegate this observation to the Appendix, following the proof of Theorem 4.

We can go further without specific information regarding group dispersion if we are willing to make some structural assumption about those dispersions *across* groups. For instance, if relative inequality or the *ratio* of Atkinson incomes to average group incomes is the same across groups (though that common ratio may be time-varying), then we can eliminate a_k from (23) to obtain

$$\mu_{\alpha,\beta}(\mathbf{y}[s, t], K) = \frac{1}{t-s} \left\{ \ln \left[\frac{\sum_{i=1}^n y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^n y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}} \right]^{-1/\alpha} - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-(\alpha+\beta)} g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-(\alpha+\beta)} d\tau} \right\},$$

whereupon even the second term can be shown to be independent of group trajectories. A similar observation holds if group inequalities differ but are all constant over time.

6.8. Some Comparisons with Alternative Measures of Mobility. Before turning to empirical applications, we briefly discuss how our measure conceptually relates to some other mobility measures. Given our focus, we limit ourselves to measures that are directional and/or relative. First consider the measure

$$\mu^C(\mathbf{y}[0, 1]) = \sum_{i=1}^n I(y_i(0), y_i(1)). \quad (24)$$

where $I(y_i(0), y_i(1))$ is an indicator for $y_i(0) < y_i(1)$. Of course, (24) can be written as:

$$\mu^C(\mathbf{y}[0, 1]) = \text{Population share under } \mathbf{z} \text{ for whom the future improves on the present.}$$

This measure is used in Chetty et al. (2017) and Berman (2022), and we've already encountered it in the context of our discussion on panels. By writing it in the slightly convoluted form (24), we uncover two essential contrasts. First, the growth experiences of the poor are treated on par with those of the rich: μ^C counts only the unweighted share of those families whose absolute fortunes improved. Second, the indicator I is a step function, and cannot be expressed as a linear combination of growth rates. Both these differences can be traced back to a failure of μ^C to satisfy Growth Progressivity.¹²

As an example, suppose that there are two income or lifetime income groups of equal size, at levels \$19,000 and \$20,000. Suppose that the children of each group earn less than their parents, say a continuous time equivalent of $g = -0.04$ less cumulated over one period (these amounts would be approximately \$18,255 and \$19,216 respectively). Call this situation 1; then $\mu_1^C = 0$; no one earns more than their parents. Now suppose that

¹²Growth Alignment is also not satisfied, as the measure only seeks to know if future prospects improved or deteriorated without asking by how much, but this is a minor issue which can easily be rectified.

we alter the situation so that the children of the first group decay by a still higher rate $g' = -0.10$, and “transfer” this loss of 0.06 over to the children of the richer group, so that they now earn approximately \$21,237. (The poorer children now earn approximately \$17,192.) Call this situation 2. One would be hard-pressed to argue that upward mobility is higher under 2 compared to 1, but all the same, $\mu_2^C = 0.5 > \mu_1^C$.

Or consider a growing society with two equally-sized groups at incomes \$10,000 and \$20,000, and with $g = 0.01$ for each group. Then, of course, $\mu_1^C = 1$. If we transfer 0.02 growth points from the rich group to the poor, the poor now catch up with the rich (without overtaking them). Growth Progressivity states that upward mobility must go up. But $\mu_1^C = 1 > \mu_2^C = 0.5$. This sort of example can be constructed for *any* nonlinear h function. Indeed, that is why the linearity of mobility in instantaneous individual growth rates is implied by Theorem 1.

One might respond that the fault lies in our axioms and not the measure μ^C . We disagree. As just noted, the new situation in the second example has poorer families actually *catching up* with their richer counterparts. Upward mobility rewards — and in our opinion *should* reward — this narrowing of inequalities.¹³ It is the fact that μ^C actually falls instead that is problematic. In this case it comes from a psychological anchor built into the zero-improvement threshold. Cross that threshold, and policymakers are presumably delighted. Fail to cross it, and they are not. This knife-edge preoccupation with the zero threshold is not warranted, especially in a world where granular data is increasingly available, and indeed, already available to some of the authors who have used μ^C .¹⁴

As already noted in Section 6.1, Fields and Ok (1999b) provide an axiomatic derivation for a mobility measure that (a) rewards growth and (b) is sensitive to inequality. Without going into detail about the setting or the axioms, we simply record the measure that they obtain:¹⁵

$$\mu^{FO}(\mathbf{y}[0, 1]) = \frac{1}{n} \sum_{i=1}^n [\ln(y_i(0)) - \ln(y_i(1))] = \frac{1}{n} \sum_{i=1}^n \left[\int_0^1 g_i(\tau) d\tau \right], \quad (25)$$

where $y_i(0)$ and $y_i(1)$ are initial and final incomes at dates 0 and 1. The kernel corresponding to this measure has $\alpha = 0$. This measure — dubbed the people’s growth rate by Saez and Zucman (2020) — sits on the “left edge” of our axiomatized family, though that edge is not included in our class. Growth Alignment, Income Neutrality and Binary Growth

¹³This parallel between pro-poor growth and convergence has been emphasized by O’Neill and Kerm (2008), Wodon and Yitzhaki (2005), Bourguignon (2011) and Dhongde and Silber (2016), among others.

¹⁴Our points are echoed in a different context in the critique of the head count measure of poverty, in which a disequalizing transfer from the poor to the less poor could result in a fall in the head count, which is an unsatisfactory property. See Sen (1976) and Foster, Greer and Thorbecke (1984).

¹⁵They call this a “directional measure” to emphasize that “upward” changes in income are preferred to downward changes; as already discussed, their other measures do not have this property.

Tradeoffs are easily seen to be satisfied. But μ^{FO} fails Growth Progressivity in the sense of being neutral to growth rate transfers between rich and poor. That said, it is still more progressive than the “plutocratic” notion of aggregate growth, and it satisfies the weaker axiom of Income Progressivity (see the discussion in Sections 6.1 and 6.2).

We now turn to rank-based measures. Recall that under our measure, the weights are a function of baseline incomes. In contrast, measures of pro-poor growth (see, e.g., Jenkins and Van Kerm 2016) possess weights that depend on the relative ranking of agents. These measures fail our axioms in a seemingly technical way: they are not continuous in the data. But this technicality is important both conceptually and practically: tiny changes in initial incomes can cause rank reversals and thereby cause discrete jumps in upward mobility. Requiring upward mobility to be continuous in income implies that the weights $\phi_i(\mathbf{y})$ in (1) must depend on cardinal income values and not on ordinal ranks.

A subclass of ranked-weighted mobility measures abandons cardinal changes altogether. Such measures *only* register a change in the event of a change in relative positions: individuals must switch ranks for any notice to be taken of them. This extreme fixation with ranking certainly suffers from the lack of continuity noted earlier. Rank mobility can be generated by tiny changes in income. Worse still, there could be large changes in *relative* income that go entirely unnoticed. As an obvious example, suppose that there are two individuals with incomes \$10,000 and \$20,000. The poorer individual grows at 50% over some period, the richer at 10%. Pure rank-based mobility would be zero.

In contrast, consider our relative mobility measure; see Section 3.3 and equation (15). Tiny changes in incomes do not significantly affect that measure, whether or not ranks are switched. At the same time, and because our measure is sensitive to growth experiences *in a cardinal way*, relative mobility could rise or fall even if there is no switch in ranks, and would certainly be sensitive to the changes described in the previous example.

Our measure rewards higher growth to the relatively poor as long as they are poor, but “loses interest” in those individuals as soon as they have attained higher ranks, focusing instead on individuals that now occupy the lower echelons of the distribution. That would generate a positive correlation with rank-based measures: after all, to generate a change in ranks, it must be that the relatively poor grew faster than the relatively rich to begin with. But relative mobility will pick up that process at every step of the way, while remaining unaffected by switches of rank. A rank-based measure, in contrast, would *only* be affected by switches of rank, and not by the growth processes that led up to those switches.

6.9. Upward Mobility and Exchange Mobility. Our measure of upward mobility (both in its absolute and relative incarnations) excludes exchange mobility. This is not to negate

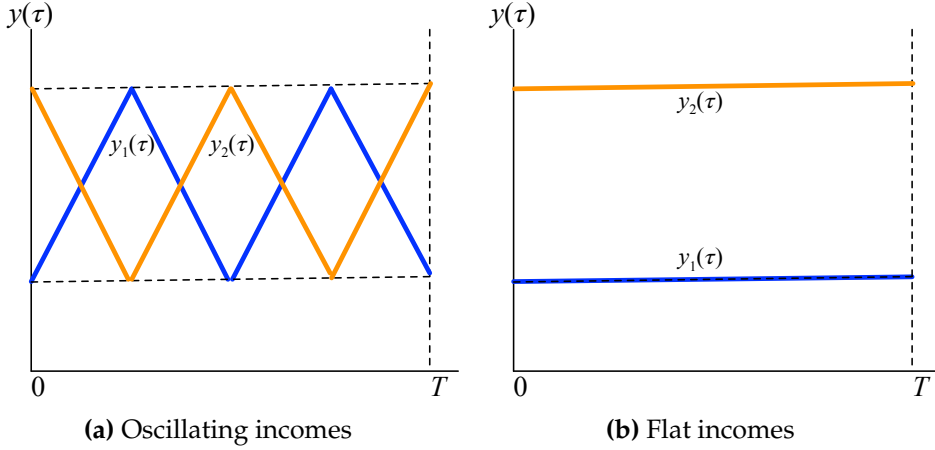


Figure 4. EXCHANGE MOBILITY. Panel (a) shows two dynasties oscillating between two income levels. Panel (b) has the same dynasties stationary at each income level. Exchange mobility is positive in panel (a) but zero in (b). Upward mobility is zero in both panels.

exchange mobility as an object of attention — after all, one might well be interested in movement across horizontal categories, such as location. But our contention is that when the categories in question are *ranked*, a separation of the two mobility notions is warranted.

To make the point as sharply as possible, Figure 4a displays two dynasties that oscillate back and forth across two levels of income. Figure 4b shows them both stationary at each of their incomes. There is substantial exchange mobility under the former situation; none under the latter. Because income categories are ranked, we could also agree that — at least ex post — Figure 4a shows substantially less inequality across the two dynasties, though ex ante, as the diagram unfolds in time, this is less clear if the measurement of permanent income is accurate (there would be inequality in permanent incomes throughout, looking “forward”). Be that as it may, *upward mobility* is zero in both panels. The upward movement exhibited by one dynasty is exactly nullified by the downward movement in the other.

Put another way, upward mobility subtracts downward movements from upward movement, and returns a net value. In contrast, exchange mobility *adds* those movements. The following discussion, while not derived from a rigorous axiomatic foundation, might help to make the distinction. Rewrite the mobility kernel from Theorem 1 as

$$M_\alpha(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) g_i = M_\alpha^+(\mathbf{z}) - M_\alpha^-(\mathbf{z}), \quad (26)$$

where

$$M_\alpha^+(\mathbf{z}) = \sum_{i=1}^n \phi_i^+(\mathbf{y}) \max\{g_i, 0\} \text{ and } M_\alpha^-(\mathbf{z}) = \sum_{i=1}^n \phi_i^-(\mathbf{y}) \max\{-g_i, 0\}$$

record “movement up” and “movement down” respectively, with weighting functions $\{\phi_i^+\}$ and $\{\phi_i^-\}$ that might conceivably depend on the sign of growth. Equation (26) captures the idea that our upward mobility measure subtracts movement-down from movement-up, interested as it is in *net* upward movement. If instead we were to add these two objects, we would obtain a measure of *overall* movement in the society, whether up or down:

$$E_\alpha(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y})|g_i| = M_\alpha^+(\mathbf{z}) + M_\alpha^-(\mathbf{z}). \quad (27)$$

While not the subject of our paper, this would be our preferred approach to exchange mobility, and not a measure based on rank switches. (Any combination of M_α and E_α might also be entertained.) However, it is not to be noted that even under additivity and reducibility, the extension of E to income trajectories *must* retain information about the full shape of those trajectories. So that extension would not be panel-independent.

7. Upward Mobility in the Data

A central feature of our measure, discussed in Section 5 and elsewhere, is that it does not rely on panel data for its implementation. In this section, we apply our measure of upward mobility to the United States, Brazil, India and France using repeated cross-sectional data from the World Inequality Lab (World Inequality Database 2021, see Appendix 2.1 for more on the data). The exercise that follows demonstrates the applicability of our measure, and also contributes to a growing literature comparing upward mobility across regions (among others Ayala and Sastre 2002, Fields and Ok 1999a, Jenkins and Van Kerm 2011, Chetty et al. 2014a) with the added advantage that we are using a measure of mobility with explicit conceptual foundations.

7.1. An Initial Comparison with Existing Empirical Studies. Perhaps the most popular measure of directional mobility (deployed empirically) is the share of families whose absolute fortune has improved across generations.¹⁶ In Section 6.8, we discussed how our measure of upward mobility differs from this “absolute mobility” measure. In this section, we are interested in comparing how these measures behave empirically. As pointed out by Deutscher and Mazumder (2020), in practice the trends exhibited by various mobility measures do tend to be similar, differing mainly depending on whether they are directional or not. This is quite apart from the conceptual considerations highlighted in this paper.

¹⁶This measure is more often used in the context of parents and children to measure inter-generational mobility but a similar measure can also be used to measure absolute intra-generational mobility over time.

In well-known work, Chetty et al. (2017) estimated this absolute mobility measure — the fraction of children who earn more than their parents — for US birth cohorts from 1940 to 1984 and documented its decline. They estimate the transition matrix of the parent-child income distribution from a unique panel of tax records for more recent cohorts (Chetty et al. 2014a), and combine this with estimates of the marginal income distributions by generation using the CPS and decennial Census data. As noted by Chetty et al. (2014b) and Berman and Bourguignon (2022), income rank correlations in the US have remained fairly stable over time. Chetty et al. (2017)’s absolute mobility estimates are plotted in both panels of Figure 5.

This exercise relies on panel data, of course, albeit estimated off a more recent copula. Indeed, in practice, such estimates of absolute mobility depend largely on the *marginal* income distributions, and relatively little on the exact numbers in the estimated transition matrix, as long as certain ordinal correlations hold steady. Berman (2022) approximates Chetty et al. (2017)’s measure of absolute mobility using available transition matrices estimated for other countries or periods. The World Inequality Database 2021 (WID) provides yearly percentile distributions of income for the adult US population:

$$\mathbf{y}^c(\tau) \equiv \{y_1^c(\tau), y_2^c(\tau), \dots, y_{100}^c(\tau)\} \text{ for } c = \text{US and year } \tau \in [1940, 1984].$$

Using these, Berman (2022) estimates the mean and variance of each marginal distribution which, under a log-normality restriction, suffices to characterize the entire marginals at 30-year intervals with starting year ranging from 1945 to 1985. Applying his empirical approximation, he then obtains estimates of absolute income mobility. Figure 5(a) plots our replication of these estimates using Berman’s approach.¹⁷

Figure 5(a) also displays annualized growth rates, one series from the dataset in Chetty et al. (2017) and the other from the WID. Finally, the Figure displays upward mobility $\mu_\alpha[\mathbf{y}(t, t + 30)]$ in annualized percentage form, setting $\alpha = 0.5$ and running over the same 30-year intervals.¹⁸ All estimates of growth and mobility are tagged by their *starting* year.

We see that despite the difference in data and approaches, all three sets of measures capture the overall large decline in mobility in the generations that followed World War II. Clearly, the Chetty *et al.* estimates, the Berman approximation, and our panel-free measure move closely with one another. Some differences do arise, but they appear to stem largely from differences in the growth patterns recorded by the two datasets. Figure 5(b) compares our measure and that of Chetty et al. (2017), using their dataset rather than the WID. To do so,

¹⁷We thank Yonathan Berman for sharing his code with us. Our estimates and Berman (2022) differ slightly due to updates to the WID database.

¹⁸Recall that our upward mobility measure is the continuous growth rate of Atkinson equivalent income, so that $e^\mu - 1$ corresponds to the annualized growth rate. Typically, μ is small and so $\mu \approx e^\mu - 1$.

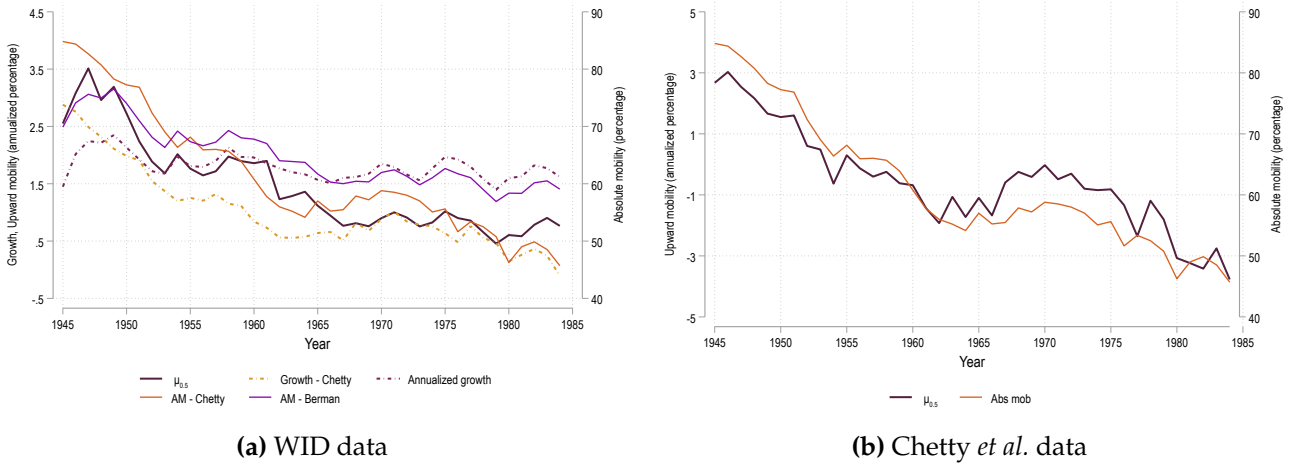


Figure 5. MOBILITY TRENDS IN THE UNITED STATES. This figure displays trends in mobility over thirty-year intervals for the United States, indexed by starting years. Panel (a) presents two estimates of annualized per-capita growth (from Chetty et al. 2017 and from the World Inequality Database 2021), the Chetty et al. (2017) absolute mobility measure, a replication of Berman (2022) using the WID data, and finally upward mobility $\mu_{0.5}$ using percentile data from WID and expressed in annualized percentage form. (Upward mobility and growth rates are measured on the left vertical axes, while the Chetty et al and Berman measures are on the right vertical axis.) Panel (b) displays $\mu_{0.5}$ (annualized percentage) using decile distribution from Chetty et al. (2017), as well as the Chetty et al. (2017) measure. Sources: Berman (2022), Chetty et al. (2017) and World Inequality Database (2021).

we aggregate the income data from Chetty et al. (2017) into deciles,¹⁹ and then measure upward mobility using formula (20) for a 30-year interval. Figure 5 then plots this measure of upward mobility using the data from Chetty et al. (2017), and also reproduces Chetty et al. (2017)'s absolute mobility as shown in Panel (a). This exercise confirms even more strongly that the two measures are very closely aligned in their ordinal movements.

That said, the similarity between upward mobility and absolute mobility is not guaranteed in theory. If the US experienced large changes in exchange mobility, the measures would display less harmony over the period. In that counterfactual case, the empirical researcher would need to be more thoroughly convinced by the conceptual arguments made here.

We used $\alpha = 0.5$ as a benchmark for this exercise,²⁰ but the reader may be interested in the robustness of our findings to different values of the pro-poor factor. Figure 10 in Appendix 2.2 shows that very similar patterns of decline in upward mobility are observed

¹⁹The sample in Chetty et al. (2017) has negative and zero income entries among the poorest percentiles. Our measure is sensitive to imputation assumptions for these low values, especially for higher values of α . We resolve the issue by measuring upward mobility on decile data. See the Appendix for more discussion.

²⁰Choosing a pro-poor factor is a question of judgment. A pro-poor factor of $\alpha = 0.5$ doubles the weight in the instantaneous mobility measure on someone earning \$40,000 relative to someone earning \$160,000, while a pro-poor factor of $\alpha = 1$ doubles the weight on someone earning \$40,000 relative to someone earning \$80,000.

for various values of the pro-pooriness factor α ranging from 0.1 to 5, as well as $\alpha \simeq 0$, which corresponds to Fields and Ok (1999b). At the same time, increasing α predictably puts more weight on growth at the lowest quantiles. See Appendix 2.2 for more discussion.

Finally, notice that these measures do not merely track overall growth — something that will become even more apparent in the next section. There is a good reason for this. Figure 9 in Appendix 2.2 shows how starting in the early 1950s, the upper income quintile has experienced higher than average 30-year growth while the bottom two quintiles have seen their real growth almost vanish. These trends are naturally reflected in upward mobility.

7.2. Upward Mobility in Brazil, India and France. Encouraged by the comparison above and by our earlier axiomatic development, we now study upward mobility in settings where panel data are not available. This brings developing countries into focus.

Specifically, we apply our measures to study 10-year upward mobility in Brazil, India and France using decile data from the WID.²¹ Our benchmark measure $\mu_{0.5}[\mathbf{y}(t, t + 10)]$ sets $\alpha = 0.5$, and is computed for all t ranging from 1980 to 2010.²² (Figure 12b in Appendix 2.3 shows robustness to different values of α .) For this exercise, we also bring on board our measure of relative upward mobility, which nets out growth. Recall that these measures are akin to some equivalent growth rate. They can take positive or negative values and can be expressed as annual percentages, just as growth rates are.

Figure 6 plots these measures, with — it is fair to say — striking effect.

After the debt crisis of 1980, Brazil entered a long decade of stagnation. Figure 6(a) shows that 10-year upward mobility fluctuated between -1% and 1% over this decade, co-moving closely with overall growth. Relative mobility is therefore negligible over the period. Figure 12a in the Appendix confirms that all quintiles experienced similar growth throughout the 1980s. By the mid-1990s, however, Brazil had been transformed by trade liberalization, a series of privatizations and several pro-business policies. Growth reappeared between 1997 and 2007, but the quintiles diverged significantly in their growth experiences. The second to the fourth quintile did sustain positive growth, but incomes of the lowest quintile essentially decayed for most of these years except for the mid 90s. Finally, income growth in the top quintile significantly surpassed those in the other quintiles between 1999 and 2003. This is mirrored in a dramatic drop-off in upward mobility even as growth rose, with an even more severe plunge in relative mobility. Indeed, upward mobility is negative between 1993 and 2003. The implementations of

²¹Panel data do exist for France (European Community Household Panel ECHP) and India (India Human Development Survey IHDS), though not for all years.

²²The first year for which the data are available for all three countries is 1980.

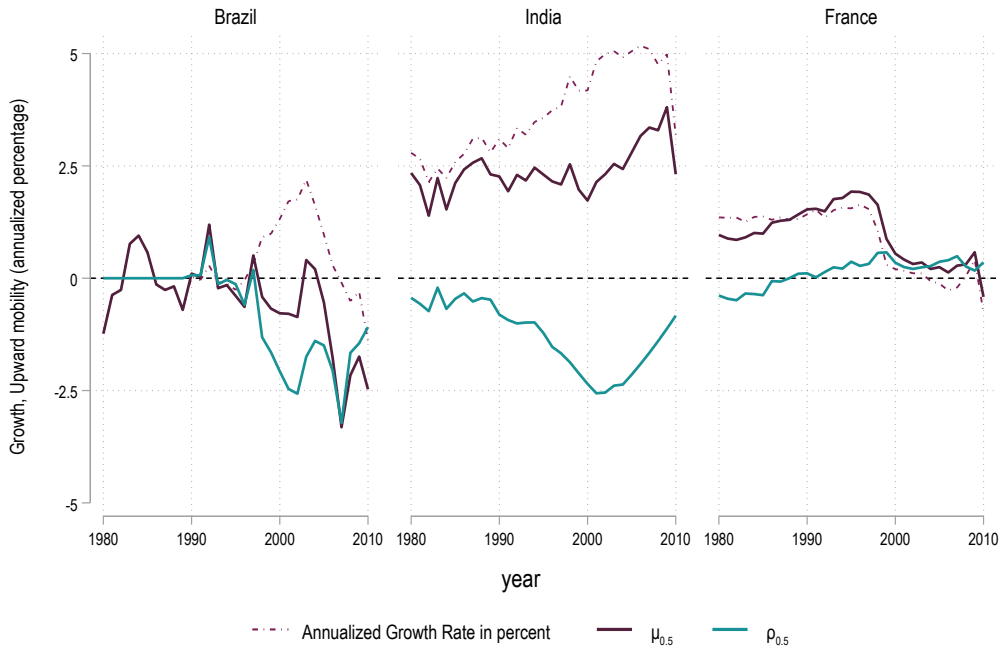


Figure 6. UPWARD MOBILITY IN BRAZIL, INDIA AND FRANCE. These diagrams show growth, upward mobility and relative mobility for $\alpha = 0.5$, all expressed as annualized percentages over ten-year intervals, and indexed by starting years. Data source: World Inequality Database (2021).

strong social programs in 2003 may have helped to partially reverse the trend then. In 2007, Brazil's growth was negative and upward mobility was at its lowest at -3.35%.

The Indian story is equally dramatic, albeit on a different growth scale. Unlike Brazil, the overall period is one of steady growth. Following deregulation in the early 1990s, India's per-capita growth rate experienced a steady acceleration, from 2.75% to an impressive 5% in 2005. But upward mobility, already short of growth after 1980, increasingly departs from it after 1990. So relative mobility trends sharply downward into the 2000s (recall: our mobility estimates are indexed by *starting* years), though a later recovery is visible. The overall picture is consistent with a post-1990s reform regime that is unambiguously pro-business. Separately, Figure 12a in Appendix 2.3 makes it abundantly clear that this acceleration of growth is purely concentrated in the top quintile. Our findings are in line with the inequality estimates of Chancel and Piketty (2019), who showed that the share of income of the top 1% rose from 6% in 1980 to over 22% in 2005. In the late 2000s, India suffered from the severe contraction in global trade when the financial meltdown morphed into a worldwide economic downturn. This shock particularly affected the top quintile (Figure 12a) which explains the upward trend in relative upward mobility.

Finally, France paints a very different picture. Despite growth stagnating at about 1.5 % until 1997, upward mobility has risen to about 2% in 1995. Figure 12a in Appendix 2.3

reveals how the growth has been systematically higher among the lowest quintiles. As a result, upward mobility *exceeded* income growth from 1988 onwards, so relative mobility is positive. In fact, even though the great recession made resulted in negative growth rates, we see that upward mobility remained positive at around 0.5% between 2000 and 2009, which is in striking contrast to the experiences of India and Brazil.

These vignettes are not a substitute for a detailed study of mobility trends, but rather serve as proof of concept for our measure of upward mobility.²³

7.3. The Great Gatsby Curve. Coined in a speech by Alan Krueger in 2012,²⁴ the “Great Gatsby curve” plots the relationship between income inequality and intergenerational income mobility. Krueger used graphs akin to those in Figure 7 to show how the US is both especially unequal *and* exhibits low mobility. That observation comes hand in hand with an irresistible prediction, which didn’t escape Krueger: that because we are more unequal now than we were a generation ago, we should expect even less social mobility going forward. Janet Yellen, then Chair of the Federal Reserve, reiterated this concern two years later during the Conference on Economic Opportunity and Inequality.²⁵

Theory suggests that this relationship could go either way (see Durlauf et al. 2021 for a review), which makes the empirical question all the more interesting. Arguments that emphasize convergence, or any form of regression to the mean, would indicate that a condition of high inequality could be followed by higher mobility, as the relatively poor grow faster than the relatively rich. On the other hand, arguments based on non-convexities or poverty traps suggest that high inequality could depress subsequent mobility, as the relatively poor are held back by low rates of returns or frustrated aspirations (Genicot and Ray 2017, 2020). Moreover, there is the distinction to be drawn between absolute and relative mobility. The above arguments apply to the latter, while arguments regarding the former would additionally need to factor in the implications for aggregate economic growth.

The left panel of Figure 7 reproduces the Great Gatsby curve from Corak (2013), Figure 1, with mobility measured by the negative of the intergenerational elasticity of income (IGE).²⁶ Countries with a Gini coefficient higher by ten percentage points have on average a value of $1 - \text{IGE}$ that is lower by 0.2. Now, a lower “lack of persistence” — as measured

²³Indeed, it is easy to compute 10-year upward mobility over 1990–2018, both absolute and relative, for all countries available in the WID. See Genicot et al. (2022).

²⁴Krueger, Alan, “The Rise and Consequences of Inequality in the United States”, 12 January 2012.

²⁵<https://www.federalreserve.gov/newsevents/speech/yellen20141017a.htm> Krueger’s graph came from work by Corak and is published in Corak (2013).

²⁶We thank Miles Corak for sharing his data with us. The original figure plotted IGE, a measure of immobility, on the vertical axis. The left panel of Figure 7 displays $1 - \text{IGE}$ instead to make it a measure of mobility.

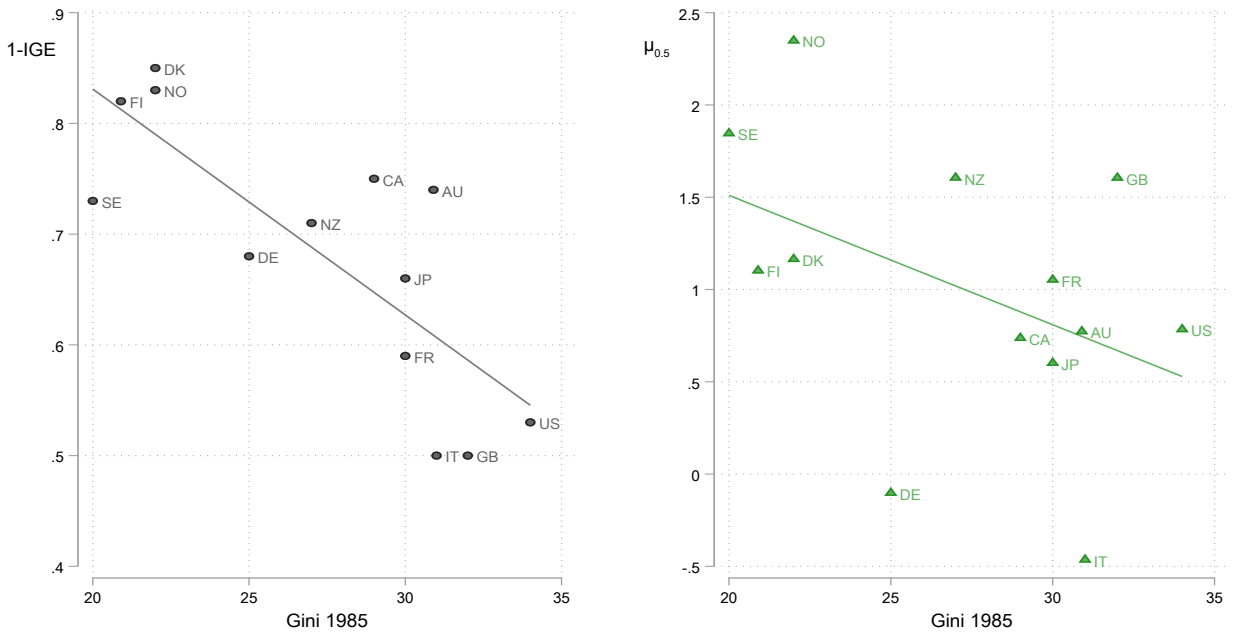


Figure 7. GREAT GATSBY CURVE. These figures plot mobility on the vertical axis and the Gini coefficients of inequality around the baseline year for mobility on the horizontal axis for OECD countries (the Ginis are taken from Corak 2013). The left panel reproduces the Great Gatsby curve from Krueger (2012)/Corak (2013), with “lack of intergenerational persistence,” or $1 - \text{IGE}$ where IGE stands for the intergenerational elasticity of income, on the vertical axis. The right panel displays upward mobility $\mu_{0.5}$ for the same countries, expressed in annual percentage terms, over the 1985–2015 interval constructed using income deciles from the World Inequality Database (2021).

by $1 - \text{IGE}$ — does not necessarily mean less upward mobility in some conceptually unambiguous sense. But we’ve already seen some empirical concordance across measures and the current instance is no exception. In the right panel of Figure 7, we apply our measure $\mu_{0.5}$ to show that 30-year upward mobility and inequality continue to be inversely correlated for the countries considered by Krueger/Corak (2013).²⁷ Gini coefficients ten percentage points higher tend to be 0.7 percentage points less upwardly mobile. Figure 13 in Appendix shows that a negative albeit insignificant correlation between inequality and *relative* upward mobility is also observed in the same sample.

It is almost a truism that high income countries are outliers in multiple dimensions (Henrich 2021), so it is hard to stop there, especially with a measure that allows us to move to a far broader set of countries. Figure 8 extends the analysis to all the 71 countries²⁸ for which the WID data allows us to calculate upward mobility over the 1985–2015 interval and for which we have the Gini coefficients. Different symbols represent the different

²⁷The Gini coefficients from Corak (2013) use disposable household income around 1985 as provided by the OECD. The downward relationship between the 30-year upward mobility and inequality holds albeit in a less pronounced way, if we use the World Bank Gini coefficients instead.

²⁸Country distributions are: 23 in Africa, 8 in Latin America, 17 in Asia and 23 in the rest of the world.

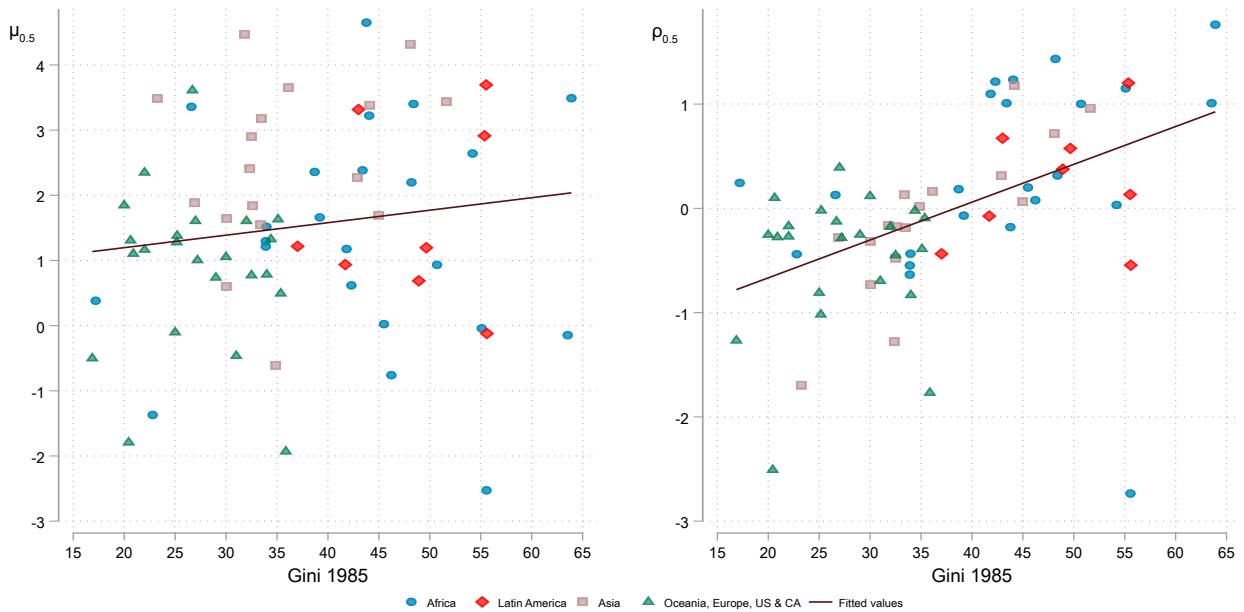


Figure 8. GREAT GATSBY CURVE. Following Krueger (2012), these panels plot mobility on the vertical axis and the Gini coefficient of inequality in the base year on the horizontal axis. The left panel displays upward mobility $\mu_{0.5}$ and the right panel relative mobility $\rho_{0.5}$ over the 1985–2015 intervals expressed as annualized percentages, along with linear fits for all the countries. Sources: World Inequality Database (2021) and the World Income Inequality Database.

regions of the world. The left panel of Figure 8 shows that the relationship between upward mobility and inequality is noisy, and if anything it is positive. The right panel of Figure 8 shows a tighter and positive correlation between relative mobility and inequality. The positive correlation appears to be driven by African and Asian countries where more convergence in income seem to have occurred in relatively more unequal countries.

Two remarks by way of qualification end this section. First, the original Gatsby curve relates the intergenerational elasticity of income to inequality. It is entirely possible — the consensus between the two panels of Figure 8 notwithstanding — that the Gatsby curve *as measured by IGE* will look different in the expanded set of countries. (There are no data to examine this possibility.) Second, what we described so far are raw correlations across the cross-section of countries, and does not necessarily reflect changes within countries. This is indeed feasible to examine with existing data. The existence (or not) of the Gatsby curve requires more careful investigation, and it is the subject of our ongoing research. But the cross-country correlations upon which Krueger proposed his hypothesis do not stand up to scrutiny over an expanded set of countries.

8. Conclusion

We have proposed an axiomatic approach to measuring upward mobility. The approach rests fundamentally on the notion of *Growth Progressivity*: that a transfer of growth rates from relatively rich to relatively poor increases upward mobility. This seemingly innocuous assumption must be applied with care, so as to avoid crossings of income that might result from differential growth. So our argument proceeds by first axiomatizing a *kernel*, one that records instantaneous upward mobility at any point of time, based on a vector of incomes and instantaneous growth rates. In this setting, Growth Progressivity and a few mild auxiliary restrictions force that kernel to be linear in individual growth rates, with geometrically declining weights on baseline incomes.

The linearity is crucial when we pass to the main setting of interest, which is upward mobility over time intervals. Two substantive but intuitive restrictions guide the measure here. The first is *reducibility*, which asks that overall upward mobility over an interval of time be pinned down by the collection of all instantaneous mobilities throughout that interval. The second is *additivity*, which requires upward mobility along an interval to be the sum of upward mobilities on any split of that interval into two sub-intervals. The linearity of the mobility kernel in growth works with these two restrictions in a powerful way, precipitating a measure of upward mobility that is free of the need for panel data.

We discuss this panel independence in detail, and relate it to the distinction we have drawn throughout between upward mobility and exchange mobility.

Reliable panel data on income trajectories are available for very few countries. Almost all of them are high-income economies. Because our measure is panel-free, a significant widening of scope is thereby achieved. Our index faithfully tracks existing panel-based indices that document the significant drop in upward mobility in the United States after World War II. It also corroborates the existence of a negative correlation between inequality and upward mobility in the small cross-section of 13 developed countries, on which Alan Krueger based his well-known hypothesis of the Great Gatsby curve. But more than that, it permits an immediate extension of these investigations to a far larger set of countries.

In this paper, we touch on these themes in a very preliminary way. We show upward mobility trends for Brazil, India and France between 1980–2010, uncovering patterns that are of interest and surely merit further detailed investigation. And we extend the Great Gatsby cross-section to over 70 countries, and argue for the *absence* of a negative correlation between inequality and upward mobility on this larger set. In ongoing research, we are particularly interested in taking this second line of analysis much further, by exploiting

the panel structure at the country level and by incorporating the essential linearities that must underly any theory that delivers the Great Gatsby hypothesis.

In summary, upward mobility incorporates certain intuitive notions about mobility *as social progress*, and not mobility as mere movement across categories. In this paper, these notions have led us to a measure of upward mobility that is minimally reliant on panel data. Our arguments are developed in detail, and hopefully in some satisfactory way from first principles, as embodied in the axioms that we use. If convincing, our measure expands the measurement and analysis of upward mobility to a far larger set of societies.

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Appendix

A. Proofs

We will prove all our results for both Axioms 1-6, and for the case in which Growth Progressivity (Axiom 3) is replaced by Income Progressivity (Axiom 3'). In what follows, we note that the analogous characterization of Theorem 1 for the latter set of axioms is identical to that of Theorem 1, except that

$$\phi_i(\mathbf{y}) > \phi_j(\mathbf{y}) \text{ when } y_i < y_j \text{ is replaced by } \phi_i(\mathbf{y})/y_i > \phi_j(\mathbf{y})/y_j \text{ when } y_i < y_j. \quad (28)$$

Proof of Theorem 1. Certainly, (1), along with the stated restrictions on $\{\phi_i\}$, satisfies Axioms 1–3. It also satisfies Axioms 1, 2 and the weaker Axiom 3' when the replacement (28) is made. We now establish the converse.

STEP 1. Assume Axioms 1, 2 and 3'. For some $\mathbf{z}^*$ suppose $y_i = y_j$ for some i and j . For $\epsilon > 0$, define $\mathbf{z}^{**}$ identical to $\mathbf{z}^*$ except that $g'_i = g_i - \epsilon$ and $g'_j = g_j + \epsilon$. Then $M(\mathbf{z}^*) = M(\mathbf{z}^{**})$.

Proof. Define $\eta \equiv \epsilon y_i = \epsilon y_j > 0$. For $\delta > 0$ but small, define $\mathbf{z}^*(\delta)$ and $\mathbf{z}^{**}(\delta)$ as follows: each has the same set of incomes and the same growth rates for every individual as $\mathbf{z}^*$ and $\mathbf{z}^{**}$ respectively, except that y_i and y_j are replaced by $y_i - \delta$ and $y_j + \delta$. Note that $\mathbf{z}^*(\delta)$ is converted into $\mathbf{z}^{**}(\delta)$ by transferring η of income from i (poorer) to j (richer). By Axiom 3', $M(\mathbf{z}^*(\delta)) > M(\mathbf{z}^{**}(\delta))$ for every $\delta > 0$. Passing to the limit as $\delta \rightarrow 0$ and using the continuity of M , we have $\lim_{\delta \rightarrow 0} M(\mathbf{z}^*(\delta)) = M(\mathbf{z}^*)$ and $\lim_{\delta \rightarrow 0} M(\mathbf{z}^{**}(\delta)) = M(\mathbf{z}^{**})$, so that

$$M(\mathbf{z}^*) \geq M(\mathbf{z}^{**}). \quad (29)$$

Next, define a new situation $\mathbf{z}'^{**}(\delta)$ which is exactly like $\mathbf{z}^{**}(\delta)$ except that the growth rates are flipped: income $y_i - \delta$ now has the growth rate $g_i + \epsilon$, while income $y_j + \delta$ has the growth rate $g_j - \epsilon$. Applying Axiom 3' again, we now have $M(\mathbf{z}^*(\delta)) < M(\mathbf{z}'^{**}(\delta))$ for every $\delta > 0$. Passing to the limit as $\delta \rightarrow 0$ just as we did before, we must now conclude that

$$M(\mathbf{z}^*) \leq M(\mathbf{z}^{**}). \quad (30)$$

Combining (29) and (30), we obtain Step 1.

STEP 2. $M(\mathbf{y}, \mathbf{g})$ is multiaffine in $\mathbf{g}$; i.e., for every k , $M(\mathbf{y}, \mathbf{g})$ is affine in g_k , or

$$M(\mathbf{y}, \mathbf{g}) = A(\mathbf{y}, \mathbf{g}_{-k})g_k + B(\mathbf{y}, \mathbf{g}_{-k}) \quad (31)$$

for two functions A and B .

Proof. Because M is continuous, it is enough to show that for every $(\mathbf{y}, \mathbf{g})$ and $\epsilon > 0$,

$$M(\mathbf{y}, \mathbf{g}) = \frac{1}{2}[M(\mathbf{y}, \mathbf{g}_{-k}, g_k - \epsilon) + M(\mathbf{y}, \mathbf{g}_{-k}, g_k + \epsilon)]. \quad (32)$$

To this end, fix any $\mathbf{z} = (\mathbf{y}, \mathbf{g})$ and $\epsilon > 0$. Define $\mathbf{z}'$ and $\mathbf{z}''$, both identical to $\mathbf{z}$ for all income-growth pairs other than at y_k : under $\mathbf{z}'$, $g'_k = g_k - \epsilon$, and under $\mathbf{z}''$, $g''_k = g_k + \epsilon$. Now suppose that (32) fails; then

$$\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$$

so that by Local Merge (Axiom 2),

$$M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z}) \quad (33)$$

But that contradicts Step 1 once we set $\mathbf{z}^* = \mathbf{z} \oplus \mathbf{z}$ and $\mathbf{z}^{**} = \mathbf{z}' \oplus \mathbf{z}''$.

A well-known consequence of multiaffine real-valued functions (see, e.g., Gallier 1999, Chapter 4.5) is that M has the following representation: for every $\mathbf{y} \gg 0$, there is $\phi_S(\mathbf{y})$ for every nonempty $S \subseteq \{1, \dots, n\}$, such that

$$M(\mathbf{z}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right].$$

noting that the empty product can be excluded by the Zero Growth Anchoring Axiom. Remembering that $g_j = \dot{y}_j/y_j$ for all j , define $\Psi_S(\mathbf{y}) \equiv \phi_S(\mathbf{y})/(\prod_{j \in S} y_j)$ for all S to rewrite

$$M(\mathbf{z}) = \sum_S \Psi_S(\mathbf{y}) \left[\prod_{j \in S} \dot{y}_j \right]. \quad (34)$$

STEP 3. $\Psi_S(\mathbf{y}) = 0$ for any $\mathbf{y}$ and any S with $|S| \geq 2$.

Proof. Suppose the assertion is false. Then there are $\mathbf{y}$, indices i and j , and $S \subseteq \{1, \dots, n\}$ such that $\{ij\} \subset S$ and $\Psi_S(\mathbf{y}) \neq 0$. Fix any numbers $\{\bar{y}_k\}$, for $k \neq i, j$, such that

$$\zeta \equiv \sum_{T: i, j \in T} \Psi_T(\mathbf{y}) \left[\prod_{k \neq i, j} \bar{y}_k \right] \neq 0. \quad (35)$$

Also define

$$\beta \equiv \sum_{T: i \in T, j \notin T} \Psi_T(\mathbf{y}) \left[\prod_{k \in T-i} \bar{y}_k \right], \quad \gamma \equiv \sum_{T: i \notin T, j \in T} \Psi_T(\mathbf{y}) \left[\prod_{k \in T-j} \bar{y}_k \right], \text{ and } \delta \equiv \sum_{T: i \notin T, j \notin T} \Psi_T(\mathbf{y}) \left[\prod_{k \in T} \bar{y}_k \right], \quad (36)$$

where the numbers are to be interpreted as zero in case any of the above products are empty. For our given $\mathbf{y}$ and any $D > 0$, consider any growth vector $\mathbf{g}$ such that the

corresponding absolute income-change vector $\dot{\mathbf{y}}$ satisfies $\dot{y}_k = \bar{y}_k$ for all $k \neq i, j$, and such that $\dot{y}_i + \dot{y}_j = D$. (We will be placing more restrictions on $\dot{y}_i$, $\dot{y}_j$ and D below.) Then, combining (35) and (36), it is easy to see that

$$M(\mathbf{y}, \mathbf{g}) = \zeta \dot{y}_i \dot{y}_j + \beta \dot{y}_i + \gamma \dot{y}_j + \delta.$$

Differentiating with respect to $\dot{y}_i$ and $\dot{y}_j$ and using $\dot{y}_j = D - \dot{y}_i$, we see that

$$\frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial \dot{y}_i} - \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial \dot{y}_j} = \zeta D - 2\zeta \dot{y}_i + \beta - \gamma.$$

In what follows, recall from (35) that $\zeta \neq 0$. Now we consider the following cases. First, if $y_i = y_j$, we know from Step 1 that the above derivative should be zero, but that clearly cannot hold for arbitrary values of D and $\dot{y}_i$, both of which we are absolutely free to choose. Second, if $y_i < y_j$, Income Progressivity implies that the above derivative is positive. Again, we are free to choose D and $\dot{y}_i$. If $\zeta > 0$, choose $D > 0$ and large and $\dot{y}_i$ smaller than D but close to it; then the derivative is negative a contradiction. Finally, if $\zeta < 0$, choose $D > 0$ and large, but choose $\dot{y}_i$ to be small; then the derivative is again negative, a contradiction. (The case $y_i > y_j$ is similarly dealt with.)

It follows that $\zeta = 0$, which establishes Step 3. Therefore (34) reduces to

$$M(\mathbf{z}) = \sum_i \Psi_i(\mathbf{y}) \dot{y}_i = \sum_i \phi_i(\mathbf{y}) g_i, \quad (37)$$

where the subscript “ $\{i\}$ ” has been changed to “ i ” in a slight abuse of notation.

The continuity of each ϕ_i follows from that of M . By anonymity, $M(\mathbf{z})$ is unchanged when the data for i and j are exchanged, so (37) implies that $\phi_i(\mathbf{y}) = \phi_j({}^i\mathbf{y}_j)$. By Income Progressivity, $\Psi_i(\mathbf{y}) > \Psi_j(\mathbf{y})$ when $y_i < y_j$, which means that $\phi_i(\mathbf{y})/y_i > \phi_j(\mathbf{y})/y_j$, as in (28).

Finally, if Growth Progressivity holds, then (37) implies that $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$ whenever $y_i < y_j$. It is only at this stage that we invoke the stronger Axiom 3 instead of 3'. $\square$

Proof of Theorem 2. (5) satisfies Axioms 1–6 when $\alpha > 0$, with Axiom 3 replaced by 3' when $\alpha > -1$. We prove the converse in several steps. Just as in the proof of Theorem 1, Growth Progressivity will only be invoked at the very end to impose the tighter restriction on α .

STEP 1. Under $n \geq 3$, Growth Alignment, Binary Growth Tradeoffs, and the anonymity and continuity of M , Theorem 2 in Chatterjee $\textcircled{R}$ Ray $\textcircled{R}$ Sen (2021) implies that M can be written as

$$M(\mathbf{z}) = f\left(\sum_{i=1}^n h(y_i, g_i), \mathbf{y}\right), \quad (38)$$

where h is strictly increasing in g_i for each y_i , and $x \mapsto f(x, \mathbf{y})$ is strictly increasing over x in the range of $\sum_i h(y_i, g_i)$ as we range over all $(\mathbf{y}, \mathbf{g})$.²⁹

STEP 2. We claim that $h(y, g)$ is affine in g , i.e., there exist $\phi(y) > 0$ and $v(y)$ such that

$$h(y, g) = \phi(y)g + v(y). \quad (39)$$

If (39) is true, then it must be that $\phi(y) > 0$ because h is increasing in g . Now, if (39) is false, then there is some $y > 0$, $g \in \mathbb{R}$ and $\epsilon > 0$ such that

$$h(y, g + \epsilon) - h(y, g) \neq h(y, g) - h(y, g - \epsilon). \quad (40)$$

Consider any pair $\mathbf{z}^*$ and $\mathbf{z}^{**}$ such that $\mathbf{y} = \mathbf{y}'$, and for two indices i and j , $y_i = y'_i = y_j = y'_j = y$, while $g_i = g_j = g$, $g'_i = g_i - \epsilon$, $g'_j = g_j + \epsilon$, and $\mathbf{g}_{-ij} = \mathbf{g}'_{-ij}$. Observe that

$$\sum_{i=1}^n h(y_i, g_i) - \sum_{i=1}^n h(y'_i, g'_i) = [h(y, g + \epsilon) - h(y, g)] - [h(y, g) - h(y, g - \epsilon)] \neq 0,$$

and so, because f is strictly increasing in its first argument, it must be that

$$M(\mathbf{z}^*) - M(\mathbf{z}^{**}) = f\left(\sum_{i=1}^n h(y_i, g_i), \mathbf{y}\right) - f\left(\sum_{i=1}^n h(y'_i, g'_i), \mathbf{y}\right) \neq 0. \quad (41)$$

But this contradicts Step 1 in the proof of Theorem 1.

STEP 3. By Growth Alignment, whenever $g_1 = \dots = g_n = g$, then $f(\sum_{i=1}^n h(y_i, g), \mathbf{y}) = \lambda(g)$ for every income vector $\mathbf{y}$, for some strictly increasing function λ . By Theorem 1, $\lambda(g) = ag$, for some $a > 0$, so that

$$f\left(\sum_{i=1}^n h(y_i, g), \mathbf{y}\right) = ag \text{ for every income vector } \mathbf{y}, \text{ whenever } g_1 = \dots = g_n = g. \quad (42)$$

STEP 4. With (39) of Step 2 in mind, define for any $\mathbf{z} = (\mathbf{y}, \mathbf{g})$,

$$g(\mathbf{z}) \equiv \frac{\sum_{i=1}^n \psi(y_i)g_i}{\sum_{i=1}^n \psi(y_i)}. \quad (43)$$

This is always well-defined because $\psi(y) > 0$ for every y . Using (39) of Step 2, (42) of Step 3 and (43), we see that

$$\begin{aligned} M(\mathbf{z}) &= f\left(\sum_{i=1}^n h(y_i, g), \mathbf{y}\right) = f\left(\sum_{i=1}^n [\psi(y_i)g_i + v(y_i)], \mathbf{y}\right) = f\left(\sum_{i=1}^n [\psi(y_i)g(\mathbf{z}) + v(y_i)], \mathbf{y}\right) \\ &= ag(\mathbf{z}) = \frac{\sum_{i=1}^n \psi(y_i)g_i}{\sum_{i=1}^n \psi(y_i)}, \end{aligned} \quad (44)$$

²⁹Chatterjee © Ray © Sen (2021) build on Debreu (1960), Gorman (1968) and Wakker (1988).

where by Growth Alignment and the continuity of M , ψ is a positive-valued, continuous function.

STEP 5. We prove that $\psi(y)$ is proportional to $y^{-\alpha}$ for some $\alpha \in \mathbb{R}$. To this end, we first show that for every strictly positive (y_1, y_2, λ) ,

$$\frac{\psi(y_1)}{\psi(y_2)} = \frac{\psi(\lambda y_1)}{\psi(\lambda y_2)}. \quad (45)$$

Suppose that this is false for some (y_1, y_2, λ) . Without loss, suppose that $y_1 < y_2$ and that “>” holds in (45).³⁰ Pick values g_1, g'_1, g_2, g'_2 such that $g_1 > g'_1$ and $g'_2 > g_2$, and such that

$$\frac{\psi(y_1)}{\psi(y_2)} > \frac{g'_2 - g_2}{g_1 - g'_1} > \frac{\psi(\lambda y_1)}{\psi(\lambda y_2)}. \quad (46)$$

Now consider two situations $\mathbf{z}$ and $\mathbf{z}'$. Under $\mathbf{z}$, the values for persons 1 and 2 are (y_1, g_1) and (y_2, g_2) , while under $\mathbf{z}'$, the corresponding values are (y_1, g'_1) and (y_2, g'_2) . Otherwise, the two situations are identical. Manipulating the left inequality in (46), we must conclude that

$$\psi(y_1)g_1 + \psi(y_2)g_2 > \psi(y_1)g'_1 + \psi(y_2)g'_2,$$

and consequently, that $M(\mathbf{z}) > M(\mathbf{z}')$. Now scale every income in $\mathbf{y}$ and $\mathbf{y}'$ by the common factor λ in (45) and call the new situations $\mathbf{z}_\lambda = (\mathbf{y}_\lambda, \mathbf{g}_\lambda)$ and $\mathbf{z}'_\lambda = (\mathbf{y}'_\lambda, \mathbf{g}'_\lambda)$. Manipulating the right inequality in (46), we must conclude that

$$\psi(\lambda y_1)g_1 + \psi(\lambda y_2)g_2 < \psi(\lambda y_1)g'_1 + \psi(\lambda y_2)g'_2,$$

so that now we have $M(\mathbf{z}'_\lambda) > M(\mathbf{z}_\lambda)$. But this reversal contradicts Income Neutrality. Therefore (45) must be true.

By defining $w = y_1$, $w' = y_2/y_1$, and $\lambda = 1/y_1$, we see from (45) that ψ satisfies the fundamental Cauchy equation

$$\psi(w)\psi(w') = \psi(w w')\psi(1) \quad (47)$$

for every $(w, w') \gg 0$. The class of solutions to (47) (that also satisfy continuity and $\psi(w) > 0$ for $w > 0$) must be proportional to $\psi(p) = p^{-\alpha}$ for some constant α (see Aczél 1966, p.41, Theorem 3).

To complete the proof, we note that under Growth Progressivity, α must be positive. And under Income Progressivity, $\alpha > -1$. □

³⁰Asking for $y_1 < y_2$ is without loss. Suppose, however, that “<” holds in (45). Then simply rename λy_1 to w'_1 , λy_2 to w'_2 and set $\lambda' = 1/\lambda$. Then the assertion in the main text holds: $w'_1 < y'_2$ and “>” holds in (45).

Proof of Theorem 3. In this proof, we will slightly abuse notation by using $M_\alpha(\tau)$ to denote $M_\alpha(\mathbf{z}(\tau))$. By Theorem 2 and reducibility, we know that (12) holds; that is:

$$\mu_\alpha(\mathbf{y}[s, t]) = \Psi\left(\{M_\alpha(\tau)\}_s^t\right), \quad (48)$$

for every s, t with $s < t$, where we recall that the kernel M_α is given by

$$M_\alpha(\tau) = \sum_{i=1}^n \frac{y_i(\tau)^{-\alpha} g_i(\tau)}{\sum_{i=1}^n y_i(\tau)^{-\alpha}},$$

and is continuous in τ , by our assumptions on $\mathbf{y}[s, t]$. We now proceed in steps.

STEP 1. For any $u \geq 0$, the limit expression

$$\lim_{v \rightarrow u} \Psi(\{M_\alpha(\tau)\}_u^v), \text{ is well-defined and equals } M_\alpha(u). \quad (49)$$

Proof. Fixing u and picking any $v > u$ (the case $v < u$ is symmetric), define $\bar{m}(v)$ and $\underline{m}(v)$ to be, respectively, the supremum and infimum of $M_\alpha(\tau)$ for $\tau \in [u, v]$. Let C^v be the function on $[u, v]$ that takes constant value $\bar{m}(v)$, and c^v the function on $[u, v]$ that takes constant value $\underline{m}(v)$. Then, because Ψ is nondecreasing and normalized,

$$\underline{m}(v) = \Psi(\{c^v\}_u^v) \leq \Psi(\{M_\alpha(\tau)\}_u^v) \leq \Psi(\{C^v\}_u^v) = \bar{m}(v). \quad (50)$$

Because $M_\alpha(\tau)$ is continuous, both $\underline{m}(v)$ and $\bar{m}(v)$ converge to $M_\alpha(u)$ as $v \downarrow u$. Using this information in (50), we must conclude that (49) holds.

STEP 2. for any s and t with $s < t$, and every strictly positive and continuously differentiable trajectory $\mathbf{y}[s, t]$,

$$\mu(\mathbf{y}[s, t]) = \frac{1}{t-s} \int_s^t M_\alpha(u) du = \frac{1}{t-s} \int_s^t \sum_{i=1}^n \frac{y_i(u)^{-\alpha} g_i(u)}{\sum_{i=1}^n y_i(u)^{-\alpha}} du. \quad (51)$$

To prove (51), fix $0 \leq s < t$ and some strictly positive and continuously differentiable trajectory $\mathbf{y}[s, t]$. For any $s \leq u < t$, define

$$L(u) \equiv (t-u)\mu_\alpha(\mathbf{y}(u, t)). \quad (52)$$

By additivity, we know that for any $0 \leq u < v < t$, $(v-u)\mu_\alpha(\mathbf{y}(u, v)) + (t-v)\mu_\alpha(\mathbf{y}(v, t)) = (t-u)\mu_\alpha(\mathbf{y}(u, t))$. Equivalently, using (48) and (52),

$$(v-u)\Psi(\{M_\alpha(\tau)\}_u^v) + L(v) = L(u),$$

so that

$$\frac{L(v) - L(u)}{v - u} = -\Psi(\{M_\alpha(\tau)\}_u^v). \quad (53)$$

Using Step 1, we must conclude that L is differentiable at u . Using (49) in (53), we have

$$L'(u) = -M_\alpha(u) \text{ for all } s \leq u < t. \quad (54)$$

Integrating the formula in (54) over all u between s and t , we must conclude that

$$(t-s)\Psi\left(\{M_\alpha(\tau)\}_s^t\right) = L(s) = L(t) - \int_s^t L'(u)du = L(t) + \int_s^t M_\alpha(u)du = \int_s^t M_\alpha(u)du,$$

where the very last equality uses the fact that $L(t) = 0$ (use Step 1 and the definition of $L(u)$). Therefore

$$\mu_\alpha(\mathbf{y}[s, t]) = \frac{1}{t-s} \int_s^t M_\alpha(u)du = \frac{1}{t-s} \int_s^t \sum_{i=1}^n \frac{y_i(u)^{-\alpha} g_i(u)}{\sum_{i=1}^n y_i(u)^{-\alpha}} du,$$

which establishes (51).

STEP 3. To finally establish (11), consider any collection of positive-valued, continuously differentiable income trajectories $\mathbf{y}[s, t]$ that connect $\mathbf{y}(s)$ and $\mathbf{y}(t)$.

$$\begin{aligned} \mu_\alpha(\mathbf{y}[s, t]) &= \frac{1}{t-s} \int_s^t \frac{\sum_i y_i(\tau)^{-\alpha} g_i(\tau)}{\sum_i y_i(\tau)^{-\alpha}} d\tau \\ &= \frac{1}{t-s} \int_s^t \frac{\sum_i y_i(\tau)^{-\alpha-1}}{\sum_i y_i(\tau)^{-\alpha}} \dot{y}_i(\tau) d\tau \\ &= -\frac{1}{\alpha(t-s)} \left[\ln \left(\sum_i y_i(\tau)^{-\alpha} \right) \right]_s^t \\ &= \frac{1}{t-s} \ln \left[\frac{\sum_i y_i(t)^{-\alpha}}{\sum_i y_i(s)^{-\alpha}} \right]^{\frac{1}{\alpha}}, \end{aligned}$$

which yields (11) as desired. $\square$

Proof of Theorem 4. Reducibility and additivity imply, just as in the arguments leading up to Theorem 3, that our upward mobility measure μ must be given by

$$\mu_{\alpha,\beta}(\mathbf{y}[s, t], K) = \frac{1}{t-s} \int_s^t M_{\alpha,\beta}(\mathbf{z}(\tau), \mathbf{w}(\tau)) d\tau, \quad (55)$$

where $\mathbf{z}(\tau)$ is induced from $\mathbf{y}[s, t]$ as before, and $\mathbf{w}(\tau)$ is induced from $(\mathbf{y}[s, t], K)$ by (21).

For any $(\alpha, \beta) \in \mathbb{R}^2$, define

$$\sigma_{\alpha,\beta}(\mathbf{y}[s, t], K) = -\ln \left(\frac{\sum_{i=1}^n y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^n y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}} \right) = -\left[\ln \left(\sum_{i=1}^n y_i(\tau)^{-\alpha} w_{k(i)}(\tau)^{-\beta} \right) \right]_s^t.$$

By differentiating the last expression with respect to τ , it is easy to see that

$$\begin{aligned}
\sigma_{\alpha,\beta}(\mathbf{y}[s, t], K) &= \int_s^t \frac{\alpha \left[\sum_{i=1}^n y_i(\tau)^{-\alpha-1} w_{k(i)}(\tau)^{-\beta} \dot{y}_i(\tau) \right] + \beta \left[\sum_{i=1}^n y_i(\tau)^{-\alpha} w_{k(i)}(\tau)^{-\beta-1} \dot{w}_{k(i)}(\tau) \right]}{\sum_{i=1}^n y_i(\tau)^{-\alpha} w_{k(i)}(\tau)^{-\beta}} d\tau \\
&= \int_s^t \frac{\alpha \left[\sum_{i=1}^n y_i(\tau)^{-\alpha} w_{k(i)}(\tau)^{-\beta} g_i(\tau) \right] + \beta \left[\sum_{i=1}^n y_i(\tau)^{-\alpha} w_{k(i)}(\tau)^{-\beta} g_{k(i)}(\tau) \right]}{\sum_{i=1}^n y_i(\tau)^{-\alpha} w_{k(i)}(\tau)^{-\beta}} d\tau \\
&= \alpha \int_s^t M_{\alpha,\beta}(\mathbf{z}(\tau), \mathbf{w}(\tau)) d\tau + \beta \int_s^t \frac{\sum_{k \in K} w_k(\tau)^{-\beta} [\sum_{i \in k} y_i(\tau)^{-\alpha}] g_k(\tau)}{\sum_{k \in K} w_k(\tau)^{-\beta} [\sum_{i \in k} y_i(\tau)^{-\alpha}]} d\tau \\
&= \alpha \int_s^t M_{\alpha,\beta}(\mathbf{z}(\tau), \mathbf{w}(\tau)) d\tau + \beta \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau, \tag{56}
\end{aligned}$$

where we recall that $a_k(\tau)$ is Atkinson equivalent group income; see (17).

Combining (55) and (56), we must conclude that

$$\begin{aligned}
(t-s)\mu_{\alpha,\beta}(\mathbf{y}[s, t], K) &= \int_s^t M_{\alpha,\beta}(\mathbf{z}(\tau), \mathbf{w}(\tau)) d\tau \\
&= \frac{1}{\alpha} \sigma_{\alpha,\beta}(\mathbf{y}[s, t], K) - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau, \tag{57}
\end{aligned}$$

and the proof is complete. $\square$

Modification of (23) when Atkinson group inequality data are unavailable. For any $\lambda > 0$, a second-order Taylor series expansion of $\lambda^{-\alpha}$ around 1 yields

$$\lambda^{-\alpha} = 1 - \alpha(\lambda - 1) + \frac{\alpha(1 + \alpha)}{2}(\lambda - 1)^2 + o(\lambda - 1)^2. \tag{58}$$

where o has the usual meaning. For every τ and i , define $\lambda_i(\tau) = y_i(\tau)/w_k(i)(\tau)$. Then, using (58), we see that for any group k ,

$$\begin{aligned}
\frac{1}{n_k} \sum_{i \in k} \lambda_i(\tau)^{-\alpha} &= 1 - \frac{\alpha}{n_k} \sum_{i \in k} [\lambda_i(\tau) - 1] + \frac{\alpha(1 + \alpha)}{2n_k} \sum_{i \in k} [\lambda_i(\tau) - 1]^2 + o\left(\frac{1}{n_k} \sum_{i \in k} [\lambda_i(\tau) - 1]^2\right) \\
&= 1 + \frac{\alpha(1 + \alpha)}{2} \text{NV}_k(\tau) + o(\text{NV}_k(\tau)), \tag{59}
\end{aligned}$$

where we use the identity $\sum_{i \in k} [\lambda_i(\tau) - 1] = 0$, and define

$$\text{NV}_k(\tau) = \frac{1}{n_k w_k(\tau)^2} \left(\sum_{i \in k} [y_i(\tau) - w_k(\tau)]^2 \right)$$

to be the *normalized variance* of group k incomes at time τ , or equivalently the square of its coefficient of variation. Opening $a_k(\tau)$ back up in (57) and using (59),

$$\begin{aligned} \mu_{\alpha,\beta}(\mathbf{y}[s,t], K) &= \frac{1}{t-s} \left\{ \frac{1}{\alpha} \sigma_{\alpha,\beta}(\mathbf{y}[s,t], K) - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} w_k(\tau)^{-(\alpha+\beta)} [\sum_{i \in k} \lambda_i(\tau)^{-\alpha}] g_k(\tau)}{\sum_{k \in K} w_k(\tau)^{-(\alpha+\beta)} [\sum_{i \in k} \lambda_i(\tau)^{-\alpha}]} d\tau \right\} \\ &\simeq \frac{1}{t-s} \left\{ \ln \left[\frac{\sum_{i=1}^n y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^n y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}} \right]^{-1/\alpha} - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-(\alpha+\beta)} \left[1 + \frac{\alpha(1+\alpha)}{2} \text{NV}_k(\tau) \right] g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-(\alpha+\beta)} \left[1 + \frac{\alpha(1+\alpha)}{2} \text{NV}_k(\tau) \right]} d\tau \right\}, \quad (60) \end{aligned}$$

where “ $\simeq$ ” is an approximation that is sensible if skewness and other higher moments are small relative to the normalized variance of each group. Certainly, the approximation in (58) can be taken further to accommodate skewness, kurtosis, and so on, if these are held to be significant (and if the relevant data are at hand). Equation (60) would then be adjusted in the appropriate way.

2. Upward Mobility using the WID

2.1. Data. We use the World Inequality Database 2021. It combines fiscal, survey and national accounts data. In countries with small informal sectors and high-quality tax microdata, that tax data is the main source. Surveys and imputation methods are used to make minor adjustments in order to account for non-filers and certain tax-exempt incomes. In contrast, income surveys are the main sources for most emerging economies, and tax datasets are only used to correct the top of the income distribution. Income surveys come mainly from the World Bank (via [PovcalNet](#)). The income data are pre-tax total incomes, computed using the equal-split assumption (that is, if the tax unit has more than one income-contributing individual contributing, the assumption is that everyone contributes in equal part to the total income of that tax unit). All incomes are expressed in PPP and in real terms, with a base year of 2021. A detailed description of the methodology is available on the [WID website](#).

In terms of inequality, we use the Gini coefficients from the [World Bank \(WB\)](#) supplemented with data from the [World Income Inequality Database \(WIID\)](#) when missing. For each country-year observation missing in the World Bank database, we select from the WIID the Gini values that 1. have been estimated either by the World Bank or alternatively the OECD; 2. are of average or high quality; and 3. are similar to our primary source.³¹ Finally, for countries for which the Gini in 1985 was missing but surrounding years were available, we averaged the Ginis from the nearest years available in a window around 1975-1995 (this procedure applied to 28 out of the 71 countries).

³¹We use Ginis in gross/net income and that cover urban or all areas in the computation.

To replicate the analysis presented in this paper, you can access the necessary data and code at Genicot and Ray (2023).

2.2. Upward Mobility in the United States: 30 Year Intervals.

Growth Incidence Curves. The upward mobility measures do not merely track overall growth, and there is a good reason for this. Figure 9 shows how starting in the early 1950s, the upper income quintile has experienced higher than average 30-year growth while the bottom two quintiles of the distribution have seen their real growth almost vanish.

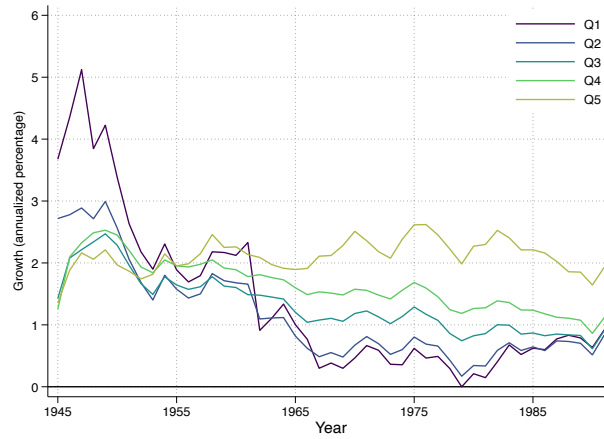


Figure 9. GROWTH INCIDENCE CURVES FOR THE UNITED STATES. This diagram shows the annualized growth rate of each income quintile for 30-year intervals, indexed by starting years.

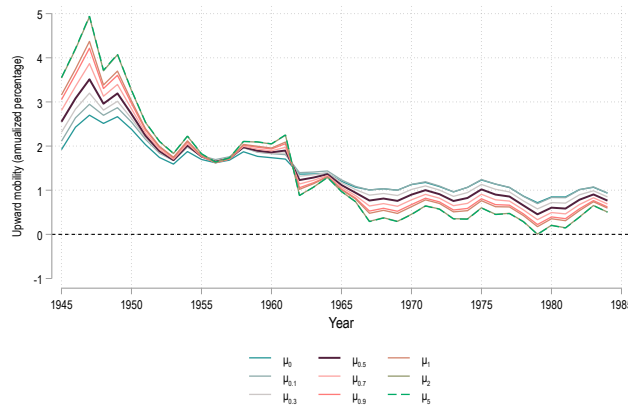


Figure 10. UPWARD MOBILITY IN THE US OVER 30-YEAR INTERVALS. This figure displays upward mobility for different values of α , indexed by starting years.

Sensitivity to α . Chetty et al. (2017)'s sample has negative and zero income entries among the poorest percentiles. As observed in Section 6.1, our measure converges to the growth

rate of the lowest percentile as $\alpha \rightarrow \infty$. Negative or zero values are therefore problematic, especially for large α , and our measure could be sensitive to imputation assumptions. We therefore measure upward mobility on the Chetty et al. (2017) data aggregated into deciles. Figure 10 plots 30-year upward mobility (expressed in annualized percentage terms) in the US for a range of values of α ranging from 0 (Fields and Ok 1999b) to 5. We see that the exact value of α does not affect the pattern to any large degree.

Censoring Low Incomes in Chetty et al. (2017)'s Sample. Aggregating the data into deciles is our preferred approach to deal with low income values. Alternatively, we could “censor” Chetty et al. (2017)'s sample and set all income values to some minimum; e.g., \$100. However, the level of upward mobility can be sensitive to these imputations, and we avoid them.

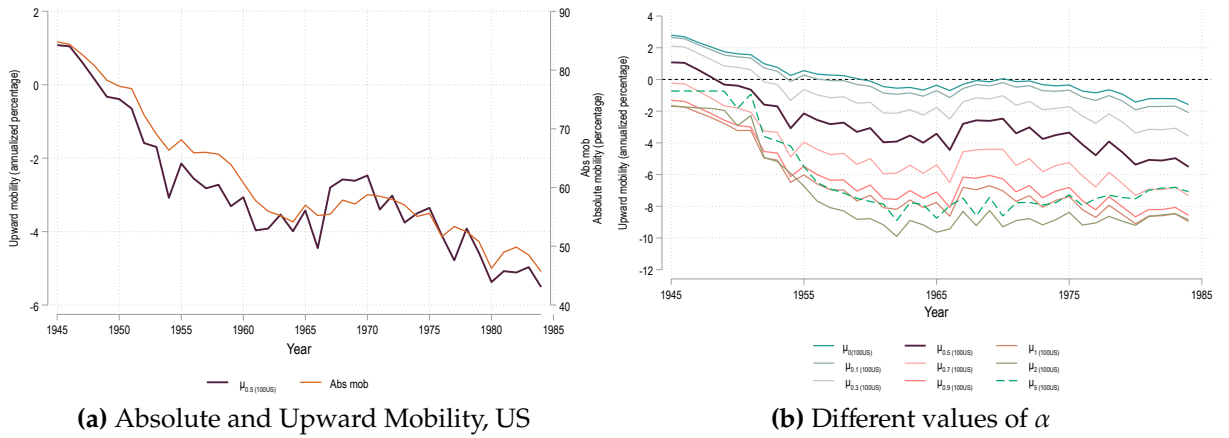


Figure 11. CHETTY *et al* DATA CENSORED AT \$100. This figure displays trends in mobility over 30-year intervals for the United States, indexed by starting years. Panel (a) builds on Chetty et al. (2017), censored at \$100 to remove negative and zero incomes, and displays $\mu_{0.5}$ along with the Chetty et al. (2017) measure. Panel (b) displays upward mobility for a range of values of α .

2.3. Brazil, India and France.

Growth Incidence Curves. Panel (a) of Figure 12 shows the ten-year growth rates by quintile for each country.

Sensitivity to α As we did for the US data, we explore the robustness to α . Panel (b) of Figure 12 plots 10-year upward mobility in Brazil, India and France for values of α ranging from 0 to 5. We see that the measure is not very sensitive to the exact value of α for India where similar growth was experienced by the bottom four quintiles (see Panel (a)). In Brazil and France, where the growth patterns among the lower quintiles are more

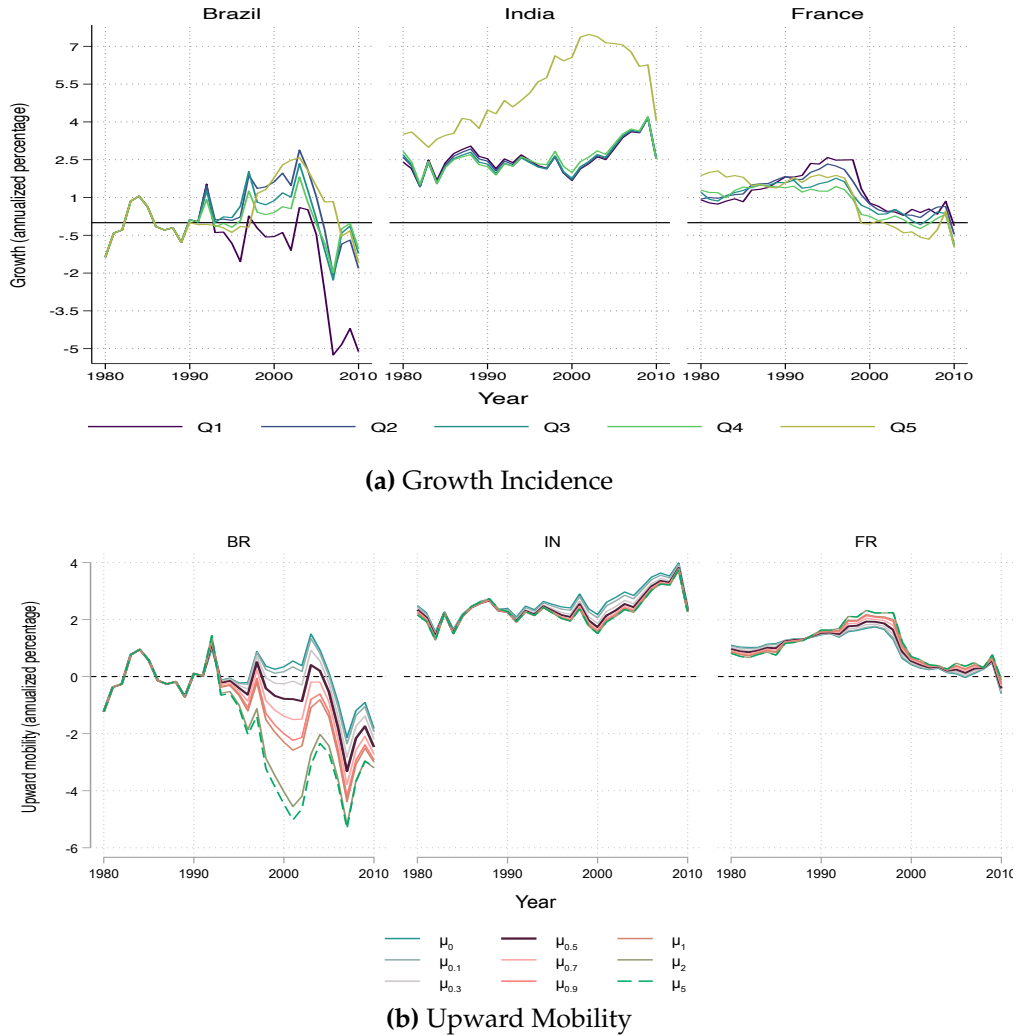


Figure 12. GROWTH INCIDENCE AND UPWARD MOBILITY OVER 10-YEAR INTERVALS, VARIOUS α . Panel (a) shows annualized growth rates for each quintile over ten-year intervals, indexed by starting years. Panel (b) displays trends in upward mobility (in annualized percentages) for different values of α , indexed by starting years.

differentiated, increasing α affects upward mobility in a predictable manner: it lowers it in Brazil where the bottom quintile fared relatively poorly and increases it in France when the bottom quintiles outperform the others.

2.4. The Great Gatsby Curve .

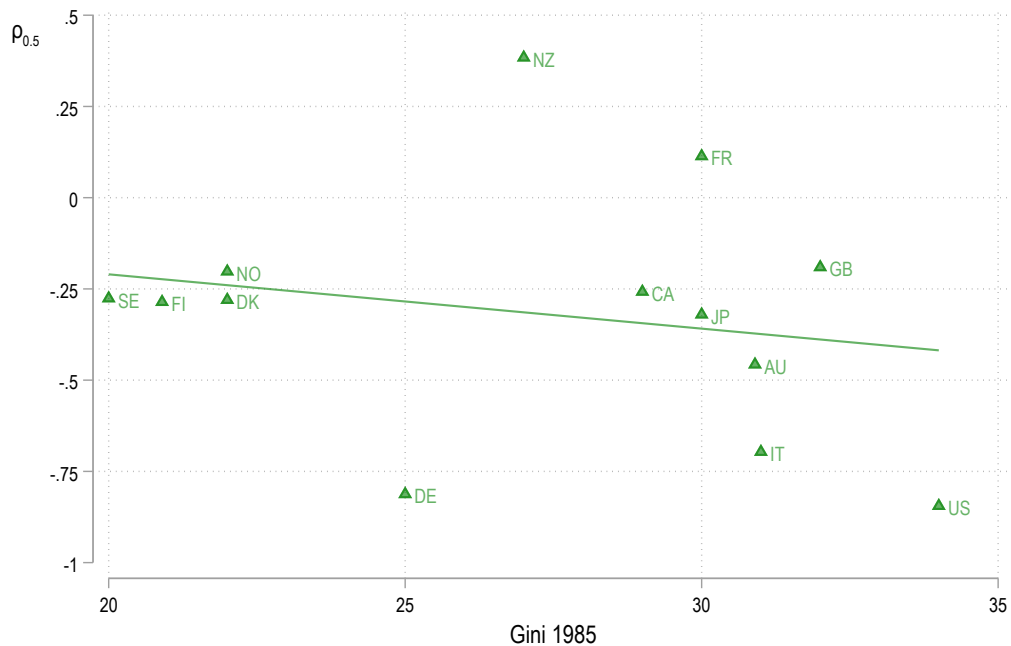


Figure 13. GREAT GATSBY CURVE. This figure plots, for the same countries as Corak (2013), relative upward mobility $\rho_{0.5}$ in annual percentage terms over 1985-2015 on the vertical axis, and the Gini coefficient of inequality around the mobility baseline year on the horizontal axis, for OECD countries. Mobility is measured using income deciles from the World Inequality Database (2021) and the Ginis are taken from Corak 2013.