COMPARING TEACHER GOALS FOR STUDENT FOCUSING AND NOTICING WITH STUDENT OUTCOMES FOR FOCUSING AND NOTICING

Charles Hohensee, Sara Gartland, Yue Ma, Srujana Acharya

University of Delaware

The topic of study in this report is student focusing and noticing. Specifically, we examined a teacher's goals for student focusing and noticing and the student outcomes for focusing and noticing. The mathematics context for this research was quadratic functions and covariational reasoning. Two whole-class discussion episodes were analyzed. Results showed ways that the teacher's goals and student outcomes were aligned and three ways that they were misaligned. These results could inform how quadratic functions are taught and how teachers can improve the alignment between their goals for student focusing and noticing and student outcomes for focusing and noticing.

INTRODUCTION

Focusing and noticing are important aspects of learning ("noticing, or perceiving, provides the rich backdrop of experience on which learning depends;" Mason, 2002, p. 33). However, as teachers know, "any human adult who interacts with another has opportunities to notice that in many situations the other perceives and attends to things that are different from those one attends to oneself" (von Glasersfeld, 1995, p. 179). Thus, what mathematics students focus on and notice may not align with what mathematics teachers want them to focus on and notice. In this report, we examine how a teacher's goals for student focusing and noticing compare to student outcomes for focusing and noticing. We situated our study in the context of quadratic functions (QF).

Student Focusing and Student Noticing

According to our conceptualization, student focusing and noticing are two related processes. Student focusing itself has two parts. The first part of student focusing is when students direct one or more senses toward one or more features of a perceptual or conceptual field ("focused attention picks a chunk of experience, isolates it from what came before and from what follows;" von Glasersfeld, 1995; p. 91). The second part of focusing is when students make a mental record of the feature or features their senses are directed toward ("For the mind, then, 'to posit it as object against itself', is to re-present it," von Glasersfeld, p. 91). In other words, student focusing goes beyond simply directing senses toward a feature of a perceptual or conceptual field.

Student noticing follows and builds on student focusing. Specifically, noticing is when students identify properties, regularities and/or irregularities, or concepts about the features of a perceptual or conceptual field that are being focused on ("establish[ing]

2023. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.). *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 75-82). PME 46.

regularities in the flow of experience," von Glasersfeld, p. 144). In other words, student noticing goes beyond students simply focusing on something.

Teacher Goals for Student Focusing and Noticing Versus Student Outcomes for Focusing and Noticing

We define *teacher goals for student focusing and noticing* as those features of a perceptual or conceptual field that the teacher wants students to focus on and the properties, regularities/irregularities, or concepts the teacher wants students to notice. We define *student outcomes for focusing and noticing* as those features of a perceptual or conceptual field that students actually focus on and the properties, regularities/irregularities, or concepts students actually notice. Prior research on student noticing has looked at what students focus on and how the classroom interactions bring about what gets focused on and noticed (Lobato et al., 2013).

Importance of Student Focusing and Noticing for Reasoning About Mathematics

We conceptualize student focusing and noticing as of key importance for reasoning about mathematics. Previously we stated that noticing is dependent on focusing (i.e., only what students focus on can they noticing something about). In a similar way, reasoning is dependent on focusing and noticing, because only what students focus on and notice can they reason about ("what we fail to notice is unlikely to have much influence upon our [mental] actions;" Mason, 2002, p. 29, parenthetical added). In other words, student focusing and noticing "provide the perceptual and/or conceptual material on which learning processes operate" (Hohensee, 2016, p. 71). Thus, student focusing and noticing are at the leading edge of cognition.

Student Focusing and Noticing About Quadratic Functions That Supports Covariational Reasoning

Student focusing and noticing are important for reasoning about QFs (Lobato et al., 2012). An important feature of QFs that students could focus on is the quantities involved in QFs. For example, students could direct their eyes toward and make a mental record of the rows of a distance-time (DT) QF table. Important regularities of QFs that students could notice are the regularity that the changes in the dependent variable are changing by constant amounts when the changes in the independent variable are constant (Lobato et al., 2012).

Covariational reasoning is defined as reasoning about "the way the dependent and independent variables [of a function] change together" (Ayalon et al., 2016, p. 381). Student focusing and noticing which, as argued above, could support mathematical reasoning in general, could also support covariational reasoning in particular. For example, students could use the noticed regularity described above to reason covariationally about generating additional values of a DTQF table.

Purpose and Research Question

Our study is the first, to our knowledge, that examines together the teacher goals for student focusing and noticing and the student outcomes for focusing and noticing. If, as explained earlier, focusing and noticing are necessary for reasoning about mathematics, then the lack of research on this topic means our study is poised to make a significant contribution to the field. Our ongoing work for this study is guided by the following question: *In the context of quadratic functions instruction, how do the teacher goals for student focusing and noticing that supports covariational reasoning during instruction compare to student outcomes for focusing and noticing?*

METHODS

Context

This study took place during a summer mathematics program for secondary students in the Mid-Atlantic region of the United States. The program focused on DTQFs and the DTQF instruction provided during the program intentionally promoted students' covariational reasoning. During the program, the students, the teacher, and a research team met for two 1-hour instructional sessions per day. Sessions were held every weekday for two weeks. Each 1-hour instructional session typically focused on a single instructional activity. The instructional activities involved the use of, or referenced, at least one DTQF animation created in SimCalc. Students often explored the DTQF animations on laptop computers through small-group activities and then participated in whole-group discussions about their mathematical reasoning on the activities.

Participants

The participants were students recruited from a youth organization (N = 18) that supports students from underrepresented populations academically. A research team of five conducted this study. The team was comprised of the summer program teacher, who was also the first author on this report, two mathematics education graduate students, and two high school mathematics teachers who taught in public secondary schools in the United States and had often taught QFs. The research team met at the end of each day to debrief the day's lessons and to plan for the next day.

Data Collection and Analysis

Data collection also occurred during the 2-week summer program. Specifically, all instructional sessions were video- and audio-recorded. Recordings were made of whole-group discussions and small-group activities. Artifacts (e.g., student responses on instructional activity worksheets) were also collected. However, only recordings of whole-group discussions were analysed for this report.

Transcripts of audio-recordings were analysed using qualitative coding methods. Analysis focused on the whole-group discussions, which we called *episodes*. To code the episodes, we first transcribed the recordings of the whole-group discussions. Then, the first, third, and fourth authors cooperatively developed inductive codes (Strauss & Corbin, 1985) to capture the teacher's goals for student focusing and noticing and student outcomes for focusing and noticing. Finally, the first and second author used the coded transcripts to answer the research question.

FINDINGS

Initial analysis has revealed that for the QF instruction in our study, the teacher goals for student focusing and noticing and student outcomes for focusing and noticing had some alignment and some misalignment. By *alignment*, we mean the teacher's goals for student focusing and noticing were consistent with student outcomes for focusing and noticing. By *misalignment*, we mean the teacher's goals were not consistent with the student outcomes. Our overarching finding is that ongoing cycles of alignment and misalignment happen as the teacher and students co-construct understandings of QFs that support covariational reasoning. Moreover, we found three kinds of misalignment. To illustrate these findings, we present two back-to-back episodes, Episode 2B and 3A, that show alignment and misalignment.

Episode 2B: Alignment and Misalignment when Discussing Two DTQF Animations

Episode 2B occurred during the second instructional session, on the second day of the summer program. This session involved an activity in which students compared two characters, a clown and a frog, represented in two separate DTQF animations. In this activity, students worked in small groups to record what they noticed about each individual animation and to reason about which character was faster relative to the other animation. The animations could not be played simultaneously. However, each animation included a numbered horizontal axis from which distance measurements could be determined and a clock from which time measurements could be determined.

Teacher goals for student focusing and noticing. During this episode, the feature of the DTQF the teacher primarily tried to direct students to focus on was particular pairs of accumulated quantities of distance and time, one DT pair from each animation. Moreover, the specific DT pairs the teacher wanted students to focus on were those that could be used to determine which animation was faster. Focusing on these features could support covariational reasoning because reasoning covariationally about a DTQF requires reasoning with distances *and* times, not just distances or just times.

The property of those DT pairs that the teacher wanted students to notice was the property (or properties) that indicated which animation was going faster relative to the other animation. The following question from the teacher illustrates their attempts to get students to notice properties of the DT pairs that would indicate which animation was faster:

> If Clown and Frog are going the same time, like you had 4 seconds, and you said Clown was faster. What does that mean in terms of the distance? . . . If you find that the distance is the same, what does the time tell you about the two characters?

Noticing properties of DT pairs that indicate which animation was faster relative to the other animation could supported covariational reasoning because noticing that kind of property (e.g., noticing that the distances are the same but the times are different) could become the perceptual/conceptual material with which students could reason covariationally to decide which animation was going faster (e.g., reasoning covariationally that if both animations ran for 4 second, the animation with the greater distance is faster).

Student focusing and noticing outcomes. During this episode, some features students focused on and properties they noticed were aligned with the teacher's goals for student focusing and noticing. Students did focus on DT pairs that could be used to determine which animation would win a head-to-head race. For example, Bob said:

> To get to 70 meters, [Frog] took 4.5, 4 point 50 seconds . . . Now with the [Clown], we tried stopping it around 70 meters and, we got pretty close. And 70 meters, [Clown] took about 4.2, 4 20 seconds . . . It tells us that the Clown from the jump to 70 meters is faster.

In this example, Bob illustrated a focus on DT pairs that had the same distance (i.e., 70 meters) and noticed that one DT pair had a lower time (i.e., Clown with a time of 4.2 s), which aligned with the teacher's goals for student focusing and noticing.

Other features students focused on and noticed were misaligned with what the teacher wanted students to focus on and notice. For example, sometimes students focused on a different quantity, *changes in distance*. Halima talked about this quantity in the following quote:

> Another thing that I noticed with the Clown is, each time it moves, it travels way greater than it did last frame. So I think that's a very important, because if you looked at the last jump, from like 35 to 65 [meters], and then from 65 to 100 . . . It travels greater distance each frame the Clown . . . compared to the Frog.

In this example, Halima focused on the changes in distance from 35 to 65 meters and from 65 to 100 meters, which was misaligned with the teacher's goals for student focusing for the activity. In this case, the misalignment was that the students were focusing on and noticing a quantity the teacher did not have as a goal for students to notice. Therefore, something this episode shows is that one kind of misalignment between the teacher's goals for student focusing and noticing for DTQFs and the student outcomes for focusing and noticing for DTQFs is when students focus on and/or notice something about a different quantity than the quantity or quantities the teacher wants them to focus on and notice. This is relevant for the teaching of functions because functions involve numerous different quantities to notice (e.g., distance, time, change in distance, change in time, etc.).

An additional observation we made was that students who focused on changes in distance did not appear to also focus on changes in time. Thus, this focus did not yet support covariational reasoning, which requires focusing on two quantities.

Episode 3A: Alignment and Misalignment when Discussing One DT Quantity

Episode 3A occurred during the first instructional session on the third day of the summer program. This instructional session was about an activity in which students viewed a single DTQF animation of a dog entering a forest, turning around, and leaving the forest. In this activity, the distance measurements were hidden from view, but the clock displaying the time remained visible. Additionally, the time took on negative, zero, and positive values.

Teacher goals for student focusing and noticing. In this episode, the feature of the DTQF the teacher primarily tried to direct students to focus on was just the independent variable, namely the time quantity. For QF data, the independent variable values are often presented in a constant pattern, which then means that the corresponding values of the dependent variable will be presented as a growing and/or shrinking pattern. For this reason, the distances in a DTQF may draw more of students' attention than the times. To support a more balanced focus on distance *and* time in later lessons so that students could engage in covariational reasoning, the teacher intended with this lesson to first establish student focusing on and noticing of time in a DTQF context.

Properties of time in a DTQF (i.e., the independent variable of the quadratic function) that the teacher wanted students to focus on and notice were how time was being measured, how time was changing, the sign of the time was (i.e., positive or negative), and the changing sign of the time. The teacher made the following comments during this episode that reveal this goal for student noticing:

> He enters the forest when it hits zero, so right here, right at this point here, it's zero? Okay, time is zero, seconds . . . Mmm, so that's good, so it starts at -4.5, this is the start, and then the end, time, is, what is it? 8.70 seconds . . . Ooh, oh, that's interesting. Total time was 13.2 . . . so in this activity we focused on the time, we're trying to attend specifically to the time.

Student Focusing and Noticing Outcomes. In Episode 3A, students did focus on features of time and did notice properties of time in the DTQF, which aligned with the teacher's goals for student focusing and noticing. For example, Natasha commented, "We noticed that the time continues as he turns around to go away from the forest," and Demarcus noticed "So, ah we noticed that time starts at a negative number."

However, in this episode, like Episode 2B, some of the features students focused on and properties they noticed were misaligned with the teacher's goals for student focusing and noting. Specifically, some of the features focused on and properties noticed were not about time. Two types of misalignment emerged.

The first type of misalignment that emerged was that the students' focusing on and noticing of time sometimes co-occurred with a focus on and noticing of the direction of the dog's travel in the animation. For example, Natasha notice "like when he turns around, he pauses, but the time still continues." Similarly, Halima said "But it's only the time before because once the dog goes back out [of the forest], it's still positive. So, we just think it's the time before the dog enters the forest first." Therefore, something new this episode shows is that another kind of misalignment between the teacher's goals for student focusing and noticing of a DTQF and the student outcomes for focusing and noticing of a DTQF is that students' focus and noticing of a DTQF that is aligned with the teacher's goals for of a DTQF (e.g., time) might co-occur with some extra focus and noticing that is not aligned with the teacher's goals (e.g., direction of travel).

The second type of misalignment that emerged was when students sometimes focused on and noticed properties of speed during Episode 3A. This is an example of students being distracted from focusing on and noticing of a more basic features of the perceptual or conceptual field of a DTQF (i.e., time) that the teacher wanted students to focus on and notice properties about in favour of focusing instead on and noticing of a more complex feature of the perceptual or conceptual field of a DTQF (i.e., speed). For example, in the following exchange the teacher asked about time and the student responded with what they noticed about speed:

Teacher: We are talking about time. Okay give us another observation.

Natasha: It seemed like his speed was consistent.

Teacher: What did you mean by that?

Natasha: Like, looking at it, we think he's like moving at a consistent pace

Althought the speed in the DTQF animation was not actually constant, the transcript shows that Natasha was focused on speed and noticed a property of speed that appeared to distract them from focusing on and noticing properties of the time. Therefore, something new this episode shows is that another kind of misalignment between the teacher's goals for student focusing and noticing of a DTQF and the student outcomes for focusing and noticing of a DTQF is when students are distracted by a more complex feature of the perceptual or conceptual field of DTQFs (e.g., speed) from focusing on and noticing a more basic feature of the perceptual/conceptual field of DTQFs that the teacher wants students to focus on (e.g., time).

Both types of misalignment could distract students from focusing on and noticing what the teacher wants them to focus on and notice. Moreover, both types of misalignment in the context of DTQFs may not support students with covariational reasoning. In the former case of misalignment, a focus on the direction of travel of the animation may simply distract students from fully focusing on time, which is one of the quantities needed for covariational reasoning in DTQF contexts. In the latter case of misalignment, the focus on speed bypassed a focus on time, which is one of the quantities needed for covariational reasoning in at DTQF context.

DISCUSSION

This study examined the teacher's goals and the student outcomes for focusing and noticing in the context of DTQFs. The motivation for this research was that (a) focusing and noticing are important for mathematical learning, and (b) what teachers want students to focus on and notice may not always in fact be what students focus on and notice. Thus, it seemed important to us to better understand the relationship between what mathematics teachers want students to focus on and notice and what students actually focus on and notice. Also, because we are interested in researching ways to promote covariational reasoning in quadratic functions contexts, it seemed important for that research to better understand the focusing and noticing that does and does not support covariational reasoning.

Our results showed alignment and misalignment between what the teacher wanted students to focus on and notice and what students actually focused on and noticed. A contribution our research makes is uncovering different ways teacher goals and student outcomes for focusing and noticing might be misaligned. Our study found three types of misalignments, (a) when students focus on and notice different quantities than intended, (b) when student focusing and noticing of the intended features co-occurs with focusing on and noticing unintended features, and (c) when focusing and noticing bypasses more basic intended features in favour of more complex features. Better understanding misalignments could inform the design of instructional activities for teaching QFs and may have relevance for teaching other mathematics concepts. Our ongoing research on other episodes in the current data set, and on other data sets will further explore this issues.

CONCLUSION

There is a need in the field of mathematics education for more research on student focusing and noticing. With a coordinated analysis of teacher goals and student outcomes, we have uncovered ways teacher's goals and student outcomes are aligned and misaligned. This is a line of research that offers promise of new insights for teaching quadratic functions and also for teaching other mathematics concepts.

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