Accurate Representation of Signal Power Spectral Density in the Optical Network Emulation (ONE) Engine

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ABSTRACT

The Optical Network Emulation (ONE) engine is a software tool that offers students the opportunity to learn how to control and operate open optical (wavelength division multiplexing) transport networks, such as those based on the Open ROADM MSA standards. This paper describes multiple modelling techniques that are implemented in the ONE engine to represent the signal power spectral density at any link/fiber section of the emulated transport network. These techniques make use of polynomial fitting and deconvolution computation methods.

Keywords: Super Gaussian Modelling, Polynomial Fitting, Deconvolution, Fourier Transform, WSS

1. INTRODUCTION

The Optical Network Emulation (ONE) engine is designed to digitally recreate both data and control planes of optical transport networks. Among other advantages ONE enables (i) students to operate an emulated optical transport network without requiring access to expensive equipment and (ii) engineers to test the correct functionality of software defined networking (SDN) controllers and orchestrators [1]. In the ONE engine, the power spectral density (PSD) of a signal (e.g., generated by a transceiver) is modelled using different degrees of polynomial fitting. The advantages offered by using polynomial fitting to represent the signal PSD include the availability of a library of functions in the ONE engine that can handle most PSD shapes (e.g., Super Gaussian and erf) at any sampling resolution. By varying the polynomial degree one can also choose the desired level of accuracy while at the same time contain the dataset size (i.e., the number of coefficients) that is required to model the signal PSD. With an accurate representation of the signal PSD, it is possible to closely estimate the effects caused by specific devices that the signal goes through, e.g., wavelength selective switch (WSS).

The contribution of this paper is twofold. We first assess the achievable accuracy when using polynomial fitting to represent a signal PSD that is modelled using Super Gaussian (SG) functions of various orders. We then present a procedure (named Noise-Tolerant Deconvolution or NTD for short) which aims to compute the polynomial fitting of a signal PSD (generated by a commercial-grade transceiver) from the experimental data collected through a low-resolution optical spectrum analyzer (OSA). The NTD procedure applies a conventional deconvolution combined with data noise reduction to compute the polynomial fitting of the signal spectrum with virtually infinite frequency resolution. Combined, the NTD procedure and polynomial fitting are shown to yield numerical estimates of the signal PSD at the output of a WSS device that closely match experimental data.

2. TECHNICAL CONTRIBUTIONS

Researchers make often use of Super Gaussian (SG) functions to model various optical signal PSDs, as they produce spectral shapes that match experimental data well [2]. The SG model depends on parameters relating to the signal PSD and can be express as

$$S_{sg}(f) = \frac{1}{\sigma_{sg}\sqrt{2\pi}} \exp\left[-\left(f^2/2\sigma_{sg}^2\right)^n\right], \tag{1}$$

where n is the order of the SG function, f the frequency, and σ_{sg} the signal bandwidth given by,

$$\sigma_{sg} = \frac{BW_{mdB}}{2\left[2\left(ln\sqrt{10^{m/10}}\right)^{1/n}\right]^{1/2}}, \tag{2}$$

where BW_{mdB} represents the $m\ dB$ bandwidth of the signal spectrum. The SG functions are used as benchmark to assess the accuracy of polynomial fitting (Section 2.1) and for computing the deconvolved spectrum (Section 2.2).

2.1 Polynomial Fitting

Polynomial fitting of a function is the process of representing the function using polynomial coefficients. There are well established implemented procedures to compute the coefficients of polynomial fitting for either an analytical expression or a set of data points [3]. The remaining key questions are the definition of the objective function to be optimized and the trade-off that is achievable by varying the degree of the polynomial term. Let the *cut-off power* level define the domain over which the polynomial fitting is expected to operate. For a signal PSD the cut-off power can be conveniently defined by the $m\ dB$ bandwidth of the signal. For example, a -50 dB cut-off power represents the range of frequencies over which the signal PSD is at least -50 dB with respect to the peak power. Let the objective function be either the Least Squares Fitting (LSF) − which puts more emphasis on larger values of the data points − or a weighted version of LSF in which the data points are normalized to provide equal relevance across the entire set of values in the chosen domain using

$$weights = \frac{1}{data\ point\ values}, \tag{3}$$

In this study we make use of weighted LSF to assess the accuracy of a polynomial fit while varying three key parameters: the SG order that must be represented, its cut-off power level, and the degree of polynomial (or number of coefficients). Polynomial fitting is expected to yield a good approximation of lower order SG signal PSD. A higher order SG signal PSD has steep slopes with abrupt transition points that are hard to fit by a polynomial term. In addition, according to Runge's phenomenon, a lower degree of polynomial performs better than a higher degree due to low oscillations at the interval boundaries [4]. However, as the other two parameters along with the weighting factor come into consideration, a higher degree of polynomial may perform better as oscillations of a polynomial fitting with a lower number of coefficients tend to occur in the central part of the spectrum. Section 3 provides numerical results that accurately quantify these expected trends.

2.2 Noise-Tolerant Deconvolution (NTD) Procedure

Deconvolution must be applied to compute the high-resolution PSD of a signal ($|X(f)|^2$) from the data points measured using a low-resolution OSA ($|X_{osa}(f)|^2$). However, noise in the measured data is known to introduce significant artifacts in the deconvolved signal, making this approach unpractical. This section describes a procedure that overcomes this drawback by removing the noise-induced artifacts through two techniques: (i) removing the "out of band" noise and (ii) applying a SG fit to the deconvolved PSD. The NTD procedure requires two input functions: (i) the signal PSD measured by the low-resolution OSA ($|X_{osa}(f)|^2$), and (ii) the OSA transfer function $|H_{osa}(f)|^2$ (an example is reported by the blue plots in Figure 1).

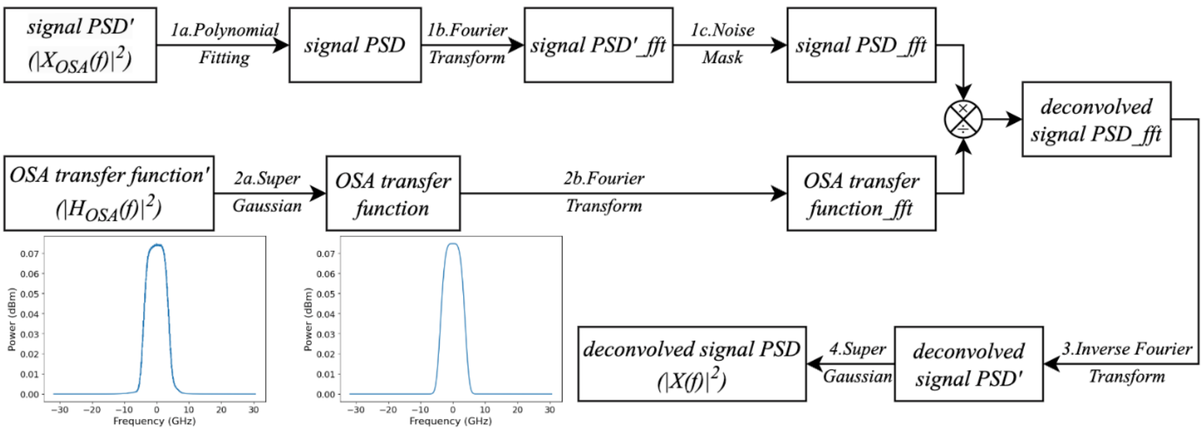


Figure 1. Noise-Tolerant Deconvolution (NTD) procedure

The first step is to ensure that the two input functions are compatible, that is, the two functions must use the same central frequency reference and the same number of data points (length). This means that some level of pre-processing may be necessary. Since a polynomial fitting can capture detailed shapes, such as ripples at the top of the signal PSD, it can be used to model the signal PSD (Step 1a. in Figure 1) to increase the number of data points as necessary. The OSA transfer function $|H_{osa}(f)|^2$, on the other hand, can be approximated using a SG function (Step 2a. in Figure 1), which averages out various noise elements (thermal, shot, etc.) that show up in the experimental data used to characterize of the OSA transfer function.

The fundamental principle used here is the multiplication and convolution property of Fourier transform (FT), which states that when two functions need to be convolved, applying a Fourier transform would permit the use of multiplication in place of convolution [4]. Note that in our study convolution is applied in the frequency domain, and multiplication applies to the FT of the frequency domain as shown in eq. (4).

$$|X(f)|^2 * |H_{osa}(f)|^2 \xleftrightarrow{Fourier\ Transform} FT\{|X(f)|^2\} \bullet FT\{|H_{osa}(f)|^2\}, \tag{4}$$

Representing deconvolution with ⊕ we then obtain

$$|X_{osa}(f)|^2 \oplus |H_{osa}(f)|^2 \xleftrightarrow{Fourier\ Transform} \frac{FT\{|X_{osa}(f)|^2\}}{FT\{|H_{osa}(f)|^2\}}. \quad (5)$$

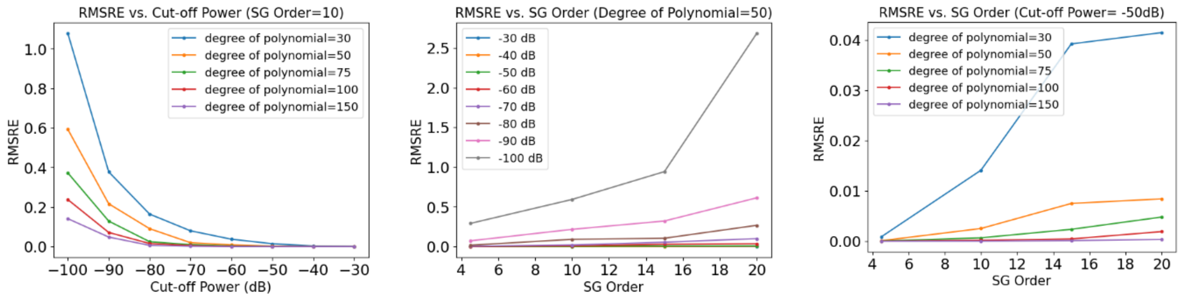
Before applying the division in eq. (5) noise masking is applied to $FT\{|X_{osa}(f)|^2\}$ to remove the ripples that occur in the deconvolved signal (Step 1c. in in Figure 1). The noise mask window must be carefully set because a large window would cause too many ripples in the resulting signal, and a window too small would cause significant distortion. After applying the inverse Fourier transform (IFT) to the term on the right in eq. (5) (Step 3. in Figure 1), some noise artifacts may remain, i.e., some ripples at the top and both ends of the PSD may still be present. To overcome this issue, a SG fitting technique is applied to obtain the final $|X(f)|^2$ (Step 4. in Figure 1). The resulting $|X(f)|^2$ can then be represented using a polynomial fit as described in Section 2.1. The resulting function is the final deconvolved signal PSD.

3. RESULTS

The accuracy of polynomial fitting is quntified numerically using the Root Mean Squared Relative Error (RMSRE) method [6] as follows

$$RMSRE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{polynomial - raw\ data}{polynomial}\right)^2} . \quad (6)$$

Polynomial fitting is tested using different combinations of the three parameters defined in Section 2.1. Figure 2(a) shows RMSRE of polynomial fitting for a 10^{th} order SG signal PSD. Lower cut-off power levels correspond to wider frequency domains to fit, which make the problem harder as illustrated by the increasing RMSRE values reported. Better accuracy is achieved by increasing the degree of polynomial. On the other hand, for a fixed degree of polynomial (e.g., 50), the RMSRE trend is shown in Figure 2(b). Increasing the SG order tends to increase RMSRE, with more pronounced changes as the cut-off power level decreases. Similarly, Figure 2(c) reports an increase in RMSRE when increasing the SG order for a specific cut-off power level (-50dB).



(a)RMSRE vs. Cut-off Power (dB) for varying Degrees of Polynomial *(b)RMSRE vs. SG Order for varying Cut-off Power levels (dB)* *(c)RMSRE vs. SG Order for varying Degrees of Polynomial*

Figure 2. Modelling a super gaussian (SG) function using polynomial fit

Additionally, for a predetermined pair of SG order and degree of polynomial, a decrease in the cut-off power increases the RMSRE (Figure 2(a) and 2(b)). Conversely, as shown in Figure 2(a) and 2(c), an increase in the degree of polynomial causes the RMSRE to drop across a variety of SG order values and cut-off power levels. Lastly, both Figure 2(b) and 2(c) confirm that polynomial fitting performs poorly while trying to fit a higher order SG function across a variety of degrees of polynomial and cut-off power levels.

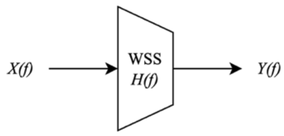
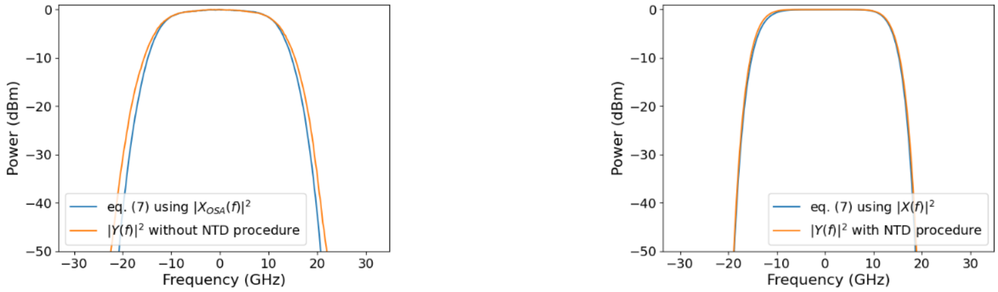


Figure 3. WSS system used to assess the NTD procedure accuracy

The experimental data used to produce the signal PSD to be deconvolved is obtained using an OSA with 0.07nm resolution bandwidth. The OSA transfer function is computed by passing a tuneable narrow laser through the OSA using a light wave measurement system. A weighted polynomial fitting method is applied to increase the number of data points in the signal PSD (from 251pts. to 5001pts.). The WSS device setting in Figure 3 is used to assess the accuracy of the NTD procedure. A 100G signal having a baud rate of 31.6 Gbaud with DP-QPSK modulation is sent through the WSS and the output signal PSD is recorded by the OSA. Theoretically,

$$|Y(f)|^2 = |X(f)|^2 \bullet |H(f)|^2 , \quad (7)$$

where $|X(f)|^2$ and $|Y(f)|^2$ represent the input and output signal PSDs, respectively, and $H(f)$ represents the estimated power transfer function of the WSS. However, due to the OSA 0.07nm low-resolution, the numerical computation and experimental data of $|Y(f)|^2$ have the mismatch reported in Figure 4(a).



(a) Without NTD procedure *(b) With NTD procedure*

Figure 4. $|Y(f)|^2$ *computed through eq. (7) using (a)* $|X_{OSA}(f)|^2$ *and (b)* $|X(f)|^2$ *as defined in Figure 1*

As illustrated in Figure 4(b), the NTD procedure significantly reduces this mismatch, where the computed data through eq. (7) using $|X(f)|^2$ matches the experimental data of $|Y(f)|^2$ more accurately. The improved accuracy can also be quantified numerically by computing the root mean square error (RMSE) in dB, given by

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2} \ , \tag{8}$$

where, $\hat{Y}_i$ (in dB) is the computed output signal, Y_i (in dB) is the experimentally measured data, and n is the length of the signal. The RMSE with and without NTD procedure is found to be 1.1 dB and 5.1 dB, respectively.

4. SUMMARY

The contribution of this paper is twofold. First, the use of polynomial fitting to model signal PSD is investigated while varying critical modelling parameters that are: the SG function order used to model the PSD, the degree of polynomial used to fit the SG function, and the cut-off power level which delimits the domain of the PSD that must be represented. Clear patterns emerge in the test results revealing that a higher degree of polynomial can cope with lower cut-off power levels. It is also evident from these tests that polynomial fitting works comparatively better for lower order SG functions. Second, the proposed Noise-Tolerant Deconvolution (NTD) procedure is shown to be effective in estimating the signal PSD (with virtually infinite resolution) from the measurements collected using a low-resolution OSA. When using the signal PSD estimated through the NTD procedure (as opposed to using the PSD as measured by a low-resolution OSA) it is possible to improve the accuracy of the computed signal PSD at the output of a WSS device, reducing the RMSE from 5.1 dB to 1.1 dB. Possible use cases for the proposed NTD procedure include: (i) integration in the ONE engine so that students are able to use signal PSDs recorded through a low-resolution OSA and have the option to improve their resolution before using them in their optical network emulation efforts; and (ii) integration with low-resolution OSA devices that are deployed in real networks to improve the resolution of the signal PSD that is monitored in real-time. There are scenarios in which the proposed NTD procedure may fail to correctly estimate the signal PSD. For example, when two signals' PSDs are too close to each other spectrally to the point where the OSA transfer function cannot clearly separate their respective power spectral contributions, the proposed NTD procedure may not suffice to recover the two signals' original PSDs. This and other special cases require further investigation.

ACKNOWLEDGMENTS

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