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Tracking Informal Fraction Knowledge and Its Correlates Across First Grade

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Young children have informal knowledge of fractions before learning about fraction symbols in school. In the current study, we followed 103 children in the Mid-Atlantic United States from the fall to the spring of first grade to characterize the development of individual differences in early informal fraction knowledge, as well as its relation to other mathematical and cognitive skills. Most children in our sample showed some early fraction knowledge at the beginning of first grade, especially with nonsymbolic fractions and halving, and this knowledge improved over the school year without explicit instruction in fractions. However, there were large individual differences in early fraction knowledge at the start of first grade, which explained significant variance in math achievement at the end of first grade, even when controlling for whole number knowledge and a variety of cognitive skills. Start-of-year whole number knowledge, but not spatial scaling or proportional reasoning, also predicted early end-of-year fraction knowledge. These data can inform activities for learning in the early years to foster both early fraction and integer knowledge in parallel, which may better prepare students for later formal instruction in fractions.

Public Significance Statement

Most U.S. schools introduce fractions around third grade, but this study shows that many children have early, informal knowledge of fractions when they start first grade. This early knowledge predicts better performance on a standardized math test at the end of first grade. However, there are marked differences in what first graders know about fractions and in how much they learn over the first-grade year.

Keywords: fractions, nonsymbolic, proportional reasoning, numeracy, early mathematics learning

Children's early mathematics competence lays a crucial foundation for their long-term mathematics achievement (Claessens et al., 2009; Jordan et al., 2009; Rittle-Johnson et al., 2017). One important milestone involves the role of understanding of fractions in elementary school to pave the way for later success in algebra and overall mathematics (Bailey et al., 2012; Booth et al., 2014; Resnick et al., 2023; Siegler et al., 2012). At the start of formal fraction instruction (around third grade), there are large individual differences in what children know about fractions (Resnick et al., 2016). Unfortunately, many students who begin fourth grade with a low understanding of fraction magnitudes will show little to no improvement by sixth grade (Resnick et al., 2016). It is therefore essential to chart the roots of these large individual differences. To this aim, we devised materials to document individual differences in young children's early fraction knowledge, examined how this knowledge develops in the first year of formal schooling, and assessed what other numerical and cognitive skills may support its development. These data can inform activities to foster early fraction knowledge alongside whole number knowledge, to better prepare students for later fractions instruction.

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Early Informal Fraction Knowledge

Symbolic Versus Nonsymbolic Representations

Despite widespread difficulties with symbolic fractions (Siegler & Pyke, 2013), young children have informal knowledge and skills with nonsymbolic representations of fractions that may support early formal fraction learning (Siegler et al., 2010). Children frequently use fractions and proportions in everyday life in approximate ways: folding a blanket in half, sharing a cookie equally with a friend, or giving a friend half of a box of six crackers. A growing body of research suggests that humans' neurocognitive architecture is well-suited to process fractions in approximate, nonsymbolic representations (e.g., Jacob et al., 2012; Matthews et al., 2016). Preschoolers and even infants can discriminate which of two nonsymbolic fractions is larger when they are represented visually, such as the fraction ¼ instantiated as a ratio of 25 blue dots to 100 yellow dots or as a ratio of line lengths (e.g., McCrink & Wynn, 2007; Park et al., 2021). Neuroimaging studies suggest that these abilities to perceive nonsymbolic ratio information might be recruited to support the processing of symbolic fraction magnitudes in children (Park, 2021) as well as adults (Mock et al., 2018).

Discrete Versus Continuous Representations

Nonsymbolic representations of ratios or fractions are easier for children to understand when represented as *continuous* relations, such as whole pizzas or a glass of juice, compared with *discrete* representations, such as pieces of chocolate or demarcated area models (e.g., Duffy et al., 2005; Park et al., 2021; Singer-Freeman & Goswami, 2001; Sophian, 2000; Spinillo & Bryant, 1991). For example, preschoolers and kindergartners are more accurate at choosing which nonsymbolic fraction is larger, or matching equivalent nonsymbolic fractions, when the fractions are represented as an undivided portion of a circular spinner compared to when the spinner has divisions shown (e.g., Boyer et al., 2008; Jeong et al., 2007). Much of this evidence comes from studies investigating approximate spatial proportional reasoning in young children, which is closely related to fraction knowledge but can be considered a separate skill (e.g., Möhring et al., 2016). Specifically, children tend to make counting-based errors with *discrete* nonsymbolic fractions (Begolli et al., 2020; Boyer et al., 2008). For example, children may choose a nonsymbolic fraction of four shaded parts out of seven total parts as larger than a nonsymbolic fraction of three shaded parts out of four total parts, because they focus on the absolute number of shaded parts (4 vs. 3) instead of the magnitude of the fraction (4/7 vs. 3/4). Errors with discrete formats decrease as students' formal fraction knowledge improves (Begolli et al., 2020), and one study showed that warming up with continuous proportion problems first may improve students' reasoning with discrete proportion problems, at least for older children (Boyer & Levine, 2015).

Importance of Half

The earliest fraction word that children learn through everyday experiences is "half." Children as young as 3 years of age can solve nonsymbolic fraction analogy problems with "half" (Singer-Freeman & Goswami, 2001) and 6- to 7-year-olds are sensitive to the "half" boundary when matching equivalent nonsymbolic fractions (Spinillo & Bryant, 1991). The special status of half is likely supported by perceptual abilities to transform visual quantities by approximately doubling or halving (H. Barth et al., 2009; McCrink et al., 2013) and by spatial language (Simms & Gentner, 2019).

The importance of "half" in early nonsymbolic fraction reasoning also extends to early symbolic fraction reasoning. For example, most children in a first-grade class could solve equal sharing problems that involved halving or repeated halving, whereas few children could solve sharing problems that did not involve halving (e.g., three children sharing two candy bars; Empson, 1999). First graders also seemed more familiar with the word "half" and "one-half" than other fraction words. In a separate sample, kindergartners and first graders more often drew conventional fraction notations for one-half (i.e., ½) than for other fraction words (Brizuela, 2006). However, children's informal knowledge of one-half can sometimes lead to confusion when learning about other fractions, such as when children think that a half generalizes to any piece of a whole (e.g., Ball, 1993; Brizuela, 2006).

Proportional Reasoning With Whole Numbers

In addition to informal knowledge of nonsymbolic fractions and fraction words, there is evidence that children can reason about proportions and ratios using whole number symbols before they have received formal instruction about fraction symbols. For example, children around ages 6–8 years can use nonsymbolic and symbolic proportions to judge which of two gumball machines will give them a better chance of getting a blue (vs. white) gumball (Szkudlarek & Brannon, 2021). Similarly, children as young as 6 years of age can estimate the position of a whole number by considering its magnitude *relative* to the endpoint of a 0–10 or 0–100 number line (H. C. Barth & Paladino, 2011; Siegler & Booth, 2004) and can spontaneously use the halfway point as a reference for their estimates (Zax et al., 2019). By third grade, around when many children in the United States are formally introduced to symbolic fractions, children can estimate the position of relative magnitudes using whole number notation (e.g., 3 on a 0–8 line) much more accurately than the analogous fraction magnitude (e.g., 3/8 on a 0–1 line; Yu et al., 2022).

In sum, children have some informal knowledge of nonsymbolic and symbolic fraction concepts before formal instruction on symbolic fractions. However, research around informal fraction knowledge is primarily qualitative, with small groups of children, and is two or three decades old (e.g., Empson, 1999). There are a few quantitative studies that have investigated early fraction understanding; however, they have focused on specific aspects of early fraction understanding, such as half (e.g., Simms & Gentner, 2019; Spinillo & Bryant, 1991). In addition, there is not currently a methodology available to measure early fraction knowledge more comprehensively in large samples. A broad quantitative assessment of children's early fraction knowledge would enable systematic measurement of different aspects of early fraction knowledge at the start of formal schooling, which would allow for modeling relations between different mathematical and cognitive skills (e.g., the relation between early fraction knowledge, early whole number knowledge, and overall math competence).

Early Fraction Knowledge May Support Math Achievement

Prior research has established that *symbolic* fraction competence in later elementary school and middle school is strongly related to concurrent and future math achievement (e.g., Bailey et al., 2012; Booth et al., 2014; Siegler et al., 2012; Torbeyns et al., 2015). It is not yet known whether early, *informal* fraction knowledge contributes similarly to mathematical competence, but such a connection is plausible. Previous studies show that children's understanding of fraction magnitude is closely correlated with their math knowledge, both concurrently (Booth & Newton, 2012) and longitudinally (Bailey et al., 2012; Booth et al., 2014; Resnick et al., 2016; Siegler et al., 2012). This relation is also found in cross-national comparisons (Resnick et al., 2023; Torbeyns et al., 2015). Key ideas about fraction magnitudes, such as how increasing the numerator makes the fraction larger when the denominator is held constant and vice versa, may be exactly the kind of insights that children gain from early nonsymbolic fraction knowledge. As children begin to understand that fractions are numbers that have locations on the number line and that fraction magnitudes are determined by the relation between the numerator and denominator, their early fraction knowledge may support overall number competencies.

Numerical and Nonnumerical Cognitive Skills That Support Fraction Learning

Although some studies have investigated children's emerging fraction knowledge, little is known about the mathematical and cognitive correlates of this knowledge. Understanding what other early skills support informal fraction knowledge may offer insights into how parents and educators can strengthen children's foundation for later formal fraction knowledge and math achievement. Most existing research investigating these developmental underpinnings has investigated how they support knowledge of *symbolic* fraction magnitudes or arithmetic. The strongest predictor of students' symbolic fraction knowledge seems to be their knowledge of whole number symbols (e.g., Hansen et al., 2015; Vukovic et al., 2014; Wilkins et al., 2021). Specifically, children's understanding of whole number magnitudes and arithmetic in first grade predicts their knowledge of fraction magnitude and arithmetic, respectively, in middle school, even after controlling for general cognitive skills and demographic characteristics (Bailey et al., 2014). Studies with slightly older children suggest that children's understanding of whole number magnitudes and division are particularly important for fraction knowledge (Hansen et al., 2015; Jordan et al., 2013; Siegler & Pyke, 2013; Stelzer et al., 2021).

The strong relationship between whole number knowledge and fraction knowledge seems inherent on one level, because fraction symbols are composed of two whole number symbols, but whole number knowledge can also interfere with fraction reasoning as seen with the whole number bias (e.g., thinking $1/3 < 1/4$; Ni & Zhou, 2005). Nevertheless, a variety of theoretical accounts align with the empirical findings that better whole number knowledge is strongly associated with better fraction knowledge (Moss & Case, 1999; Vamvakoussi & Vosniadou, 2004). Siegler et al.'s (2011) influential *integrated theory of numerical cognition* suggests that children who have a strong sense of whole number magnitudes organized on a well-developed mental number line more easily see fractions as numbers that can be located on a number line. Another dominant theory in math education research, Steffe's (2001) *reorganization hypothesis*, suggests that children build an understanding of fractions by reorganizing their whole number counting schemes to partition and iterate unit fractions. For example, young children who have a stronger understanding of how whole numbers can be decomposed (e.g., 10 is the same as two fives or five twos) are likely to have an easier time understanding relations between parts and wholes when they start learning about fractions. Collectively, these theories and the empirical evidence reviewed above suggest that early whole number competence may also support early fraction knowledge, but this hypothesis remains largely untested.

There are also several nonnumerical cognitive skills that likely play a role in early fractions development. Spatial proportional reasoning with continuous representations, as mentioned earlier, is correlated with symbolic fraction reasoning, even after controlling for many other cognitive skills (Bhatia et al., 2020; Möhring et al., 2016; Park & Matthews, 2021; Ye et al., 2016). Reasoning about symbolic fraction magnitude fully mediates the relation between spatial proportional reasoning and mathematics achievement (Resnick et al., 2023). Spatial proportional reasoning skills are closely related to nonsymbolic fraction knowledge with continuous representations, but it is not yet known whether these spatial skills support broader early fraction knowledge. Spatial scaling, the skill of mapping distances in one space to distances in another space, may also be important in supporting children's reasoning with nonsymbolic and symbolic representations of fractions (Frick & Newcombe, 2012; Möhring et al., 2016). Children's early number knowledge is strongly associated with executive functions like working memory, cognitive flexibility, and inhibitory control (Blair & Razza, 2007; Clark et al., 2010; Clements et al., 2016; Mazzocco & Kover, 2007). Lastly, vocabulary likely plays a role in early fractions development, given findings that young children's vocabulary supports their early numeracy (e.g., Bezuidenhout, 2022; Purpura et al., 2011; Purpura & Reid, 2016) and proportional reasoning (Vanluydt et al., 2021).

Current Study

The current study offers the first comprehensive quantitative assessment of first graders' early fraction knowledge at the beginning and end of the school year. The design allows us to address four aims: (a) to document individual differences in fraction knowledge at the start of formal schooling, (b) to chart whether and how early fraction knowledge changes over the course of the school year, (c) to investigate which numerical and cognitive skills contribute to growth in early fraction knowledge, and (d) to test the relative contributions of whole number knowledge, early fraction knowledge, and general cognitive skills to math achievement at the end of first grade. The current study is the first, to the best of our knowledge, to quantitatively track how informal fraction knowledge changes longitudinally in first grade and to explore whether this informal fraction understanding supports later mathematics achievement in first grade.

Based on prior studies presented above, we hypothesize that most children will have some informal knowledge of fractions at the start of first grade, but that there will be individual differences

(Hypothesis 1). Even though students are not receiving formal instruction in fractions in first grade, we expect that their early fraction knowledge will improve from fall to spring of first grade (Hypothesis 2), as they build other related math skills and may encounter early fraction ideas in everyday, informal experiences. Regarding whether early fraction knowledge relates to other competencies, we hypothesize that whole number knowledge and other cognitive skills like spatial scaling and spatial proportional reasoning will make significant contributions to early fraction knowledge at the end of first grade (Hypothesis 3), given prior evidence that this constellation of skills supports symbolic fraction knowledge in older children. Lastly, because understanding fractions is crucial to a broad understanding of the number system, we hypothesize that students who start first grade with stronger early fraction knowledge will have stronger end-of-year math achievement, even after controlling for whole number knowledge and other cognitive skills (Hypothesis 4).

Method

Transparency and Openness

We report all data exclusions and all measures in the study, and we follow Journal Article Reporting Standards (JARS; Kazak, 2018). All deidentified data, analysis code, and nonproprietary research materials are available on Open Science Framework (OSF) at this link (https://osf.io/e27fp/; Viegut et al., 2023). Data were analyzed using R, Version 4.1.3 and the *tidyverse* (Wickham et al., 2019), *ggpubr* (Kassambara, 2023), *lm.beta* (Behrendt, 2022), and *psych* (Revelle, 2023) packages. This study's design and its analysis were not preregistered.

Participants

The study included 109 first-grade students from three elementary schools in the same Mid-Atlantic community of the United States. Mean age was 6.69 years ($SD = .34$) at the start of the 2021 school year. According to parent/caregiver report, 55% of the children were female and 45% male; the racial/ethnic breakdown was 57% White, 7% Black, 18% Asian, 1% Hispanic, and 7% multiple races, with 10% unreported. Children were drawn from schools that serve primarily middle-income communities, but we did not have access to socioeconomic status (SES) for individual families. Although the specific math curricula of the three schools varied (McGraw-Hill *Reveal*, McGraw-Hill *Math Connects*, and Sadler *Progress in Math*), fraction-related topics were not specifically addressed in any of the curricula during the study period.

Informed consent letters were distributed to the families of all first graders in each school. Consent for participation was given for 91.5% of children. Attrition during the study was low (5.5%). Only two children left the school after the first session, and four additional children missed one of the three sessions due to illness and were excluded. Therefore, the final sample for analysis included 103 students. The sample size was constrained by access to schools, but a post hoc power analysis using the *pwr* package in R (Champely, 2020), calculated that we had 80% power to detect a medium effect size (Cohen's $f^2 = .17$) in regression with our 10 predictors. This study was approved by the University of Delaware Institutional Review Board.

Measures

Early Fraction Knowledge

We created a 43-item measure to assess early fraction knowledge. Item development was based on level of representation (nonsymbolic vs. symbolic) and fraction size, because prior research on early numeracy suggests that both factors affect students' reasoning about numbers (Jordan et al., 2022). Half of the items used continuous representations (22 items; eight nonsymbolic, 13 symbolic) and half used discrete representations (21 items; 12 nonsymbolic, nine symbolic). Because many previous studies have shown that discrete representations are more difficult for young children (e.g., Boyer et al., 2008), we intentionally designed the continuous problems to be more challenging than the discrete problems, to ensure that we could detect a wide range of scores on both types of problems. The nonsymbolic category ($n = 20$) included items involving equal sharing, equivalence, and iterating fractional parts to create a whole. Most items in the symbolic category ($n = 23$) involved mapping fraction words (halves, thirds, and fourths/quarters) to visual models, with three items involving written fraction notation

$$\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right).$$

Figures 1 and 2 show examples of nonsymbolic and symbolic items.

We chose not to perfectly balance equal numbers of every item type to avoid floor or ceiling effects and extend the measure to an unwieldy length for first graders. Instead, we carefully selected a set of items to efficiently capture a range of variance in both nonsymbolic and symbolic fraction understanding, based on prior qualitative work with this age group (e.g., Brizuela, 2006; Empson, 1999) and prior studies investigating specific aspects of early fraction knowledge (e.g., Begolli et al., 2020; Spinillo & Bryant, 1991; Szkudlarek & Brannon, 2021).

Items were reviewed by an expert in elementary school math education for clarity and consistency of mathematical language. The early fractions measure was pilot tested during the school year before the study with 29 first graders. Items were printed in a colorful picture booklet, and the experimenter read each question aloud. The raw score was the total number of items answered correctly. On some questions, children also were asked to explain why they gave/chose a particular answer, although these explanations were not used in scoring and are not reported here. The assessment took approximately 15 min to administer. Internal reliability for the full sample at both time points was $\alpha = .82$.

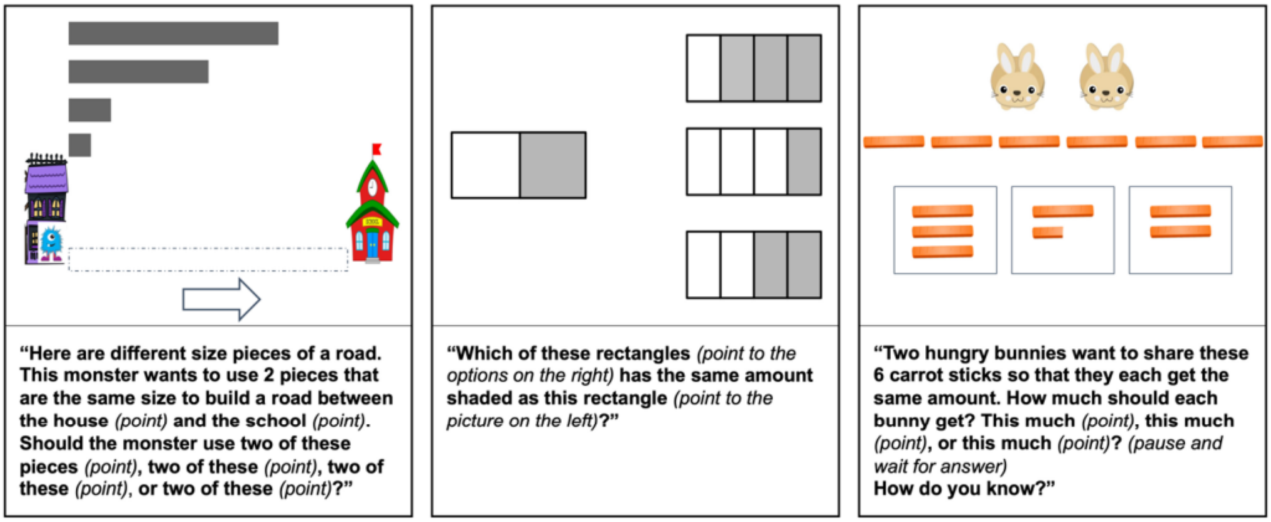
Whole Number Sense

To assess children's whole number knowledge, children were given the number and number relations portions of the Screener for Early Number Sense (SENS), first-grade form (Jordan et al., in press). The SENS is a standardized measure that assesses symbolic number knowledge, including counting, numeral recognition, and comparisons. There are 17 items, and the raw score is the total number of items answered correctly. The SENS took about 9 min to administer for each child. Reliability for the current sample was $\alpha = .85$.

Whole Number Line Estimation

To assess whole number line estimation (WNL) acuity, children were presented with number lines using a 0–100 scale, which is

Figure 1
Examples of Nonsymbolic Fraction Items



Note. See the online article for the color version of this figure.

developmentally appropriate for first graders (Booth & Siegler, 2006). For all 15 items, a 25 cm line appeared in the middle of an iPad screen, with 0 at the left end and 100 at the right end. Target numbers appeared approximately 5 cm above the number line in random order and ranged across the numbers from 0 to 100: 5, 11, 18, 25, 33, 40, 46, 52, 58, 65, 71, 78, 85, 93, and 99. Children were instructed to indicate the location of a given number, one at a time. Following the procedure used by Booth and Siegler (2006), we gave one practice item (50) with feedback, which is common practice with children this age to ensure children understand the task (e.g., Berteletti et al., 2010; Zhu et al., 2017). Student performance was calculated using mean percent absolute error (PAE), which is an average of the absolute difference between the child's estimate and the actual target location divided by the numerical range of the number line (100). Thus, lower PAE indicated higher accuracy. The task took approximately 5 min to administer. Reliability for the current sample at both time points was $\alpha = .87$. To correct for the skewed distribution of children's PAE, we used a log transformation of PAE in the regression analyses, following Bailey et al. (2012).

Executive Functions

To assess inhibitory control and cognitive flexibility, a touch screen adaptation of the hearts and flowers task was used (Davidson et al., 2006). This task has three trial blocks. In the control block (hearts), children were instructed to press a button on the same side of the screen on which the stimulus appears (i.e., left or right); in the flowers block children were instructed to press a button on the opposite side of the screen from which the stimulus appears; in the mixed block, children were required to continue following the rules from the prior two trial blocks and respond appropriately to intermixed heart and flower stimuli. Children were offered up to six practice trials with feedback before beginning each block. Each block contained 20 trials where stimuli were presented on screen for up to 5,000 ms (hearts and flowers blocks) or 6,000 ms (mixed block) or until the child's response. Because our sample had high accuracy even on the challenging mixed block ($M = 95.2\%$), reaction time (RT) on correct trials was a better indicator of their executive functions than accuracy (Camerota et al., 2019). Following Camerota et al. (2019), we calculated a change in response time (on correct trials only) from the control hearts block to the flowers and mixed blocks, to measure inhibitory control ($\Delta RT_{Flowers} = \Delta RT_{Flowers} - \Delta RT_{Heart}$) and cognitive flexibility ($\Delta RT_{Mixed} = \Delta RT_{Mixed} - \Delta RT_{Hearts}$), respectively. We then added the standardized $\Delta RT_{Flowers}$ and standardized ΔRT_{Mixed} scores, which were significantly correlated ($r = .471$, $p < .001$), to create a composite score for executive functions for each child.[1] The task took approximately 6 min to administer. Reliability for the current sample was $\alpha = .74$.

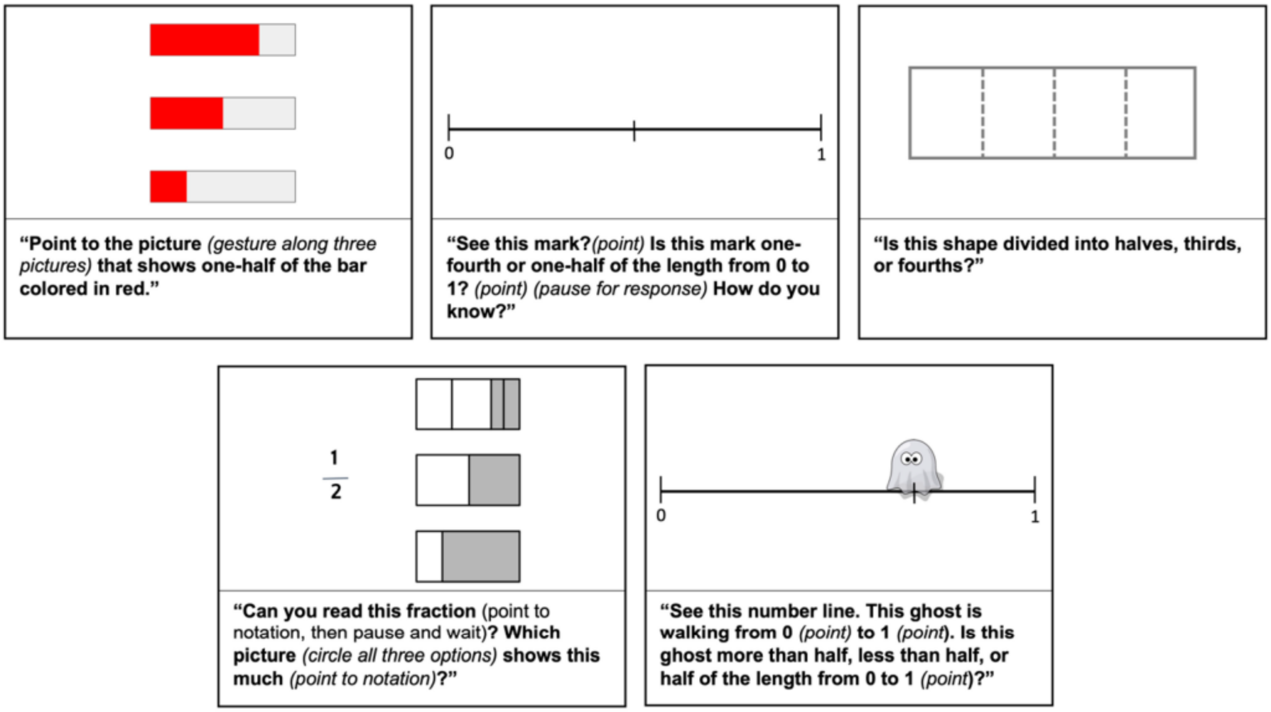
Visual-Spatial Memory Span

To assess children's memory span, we administered the PathSpan for iPad, a touch screen adapted version of the Corsi Block test that is appropriate for first grade (LeFevre et al., 2010). The task assesses both forward and backward memory span in a nonverbal format. The experimenter explained to children that they would be playing a copying game. For *forward span*, children first saw a group of nine green dots on the screen that then flashed in a given sequence. The children were asked to copy the sequence by touching the same dots on the iPad in the same order. For *reverse span*, children followed the same procedure but were instructed to touch the same dots in reverse order. For both the forward and reverse span tasks, the experimenter began by demonstrating a practice sequence before

[1] Two children answered every item of the flowers block incorrectly, but then answered every item of the Mixed block correctly. For these two children, their standardized ΔRT_{Mixed} was duplicated to yield the composite score.

Figure 2

Examples of Symbolic Fraction Items



Note. See the online article for the color version of this figure.

beginning the test trials. Test trials gradually increased in length (up to seven dots). There were two trials for each sequence length, which ended after two consecutive errors at a given length. Performance was calculated based on the total number of correct sequences (out of 14). The task took about 9 min to administer per child. Internal reliability was α = .87 for the current study.

Spatial Scaling

To assess spatial scaling, children completed a spatial scaling task adapted from Frick and Newcombe (2012) on a large iPad screen. Children were asked to help a farmer find eggs in a field hidden by his chickens. They were shown an empty "field" on the screen, which was a green rectangular referent space, and a paper map of the field showing the location of a hidden egg. For each item, the child tapped on the screen using a stylus to indicate where they thought the egg was hidden based on its position in the map. Maps either did not require scaling (i.e., 1:1 correspondence in size) or required scaling at different factors (e.g., 1:2 or 1:4). There were 16 rectangular fields and eight circular fields (24 total items). If children did not respond within 10 s, the trial was repeated once. The task took approximately 5 min to administer. Each trial was scored as 0 (*worst*), 1, 2, or 3 (*best*) points, depending on the absolute deviation (in mm) of the child's estimate from the correct location in the referent space (which was of constant size). Final score was calculated as a percentage out of 72, which was the maximum number of points. Internal reliability in the current study was α = .89.

Spatial Proportional Reasoning

Spatial proportional reasoning was measured with an iPad adaptation of the task developed by Möhring and colleagues (Möhring et al., 2016, 2018). Although this skill is closely related to nonsymbolic fraction knowledge with continuous representations, none of the nonsymbolic items in our early fractions assessment asked children to estimate or judge equivalent proportions with continuous representations. In this task, children mapped stimulus proportions directly onto a continuous rating scale. The experimenter first told children a story about Harry the Bear, who likes his drink to taste a lot like cherry. For each trial, children were presented with a rectangle measuring 2 cm wide but of varying length. The figure was divided into two parts: one part was red, which represented the amount of cherry juice, and the other part was blue, which represented the amount of water. There was a 12-cm horizontal line below the stimulus, where children used a slider to estimate the cherry taste on the horizontal line. On the left part of the line, there was one small cherry to indicate that the juice would taste only a little like cherries, and on the right side, there was a group of cherries to indicate that the juice would taste a lot like cherries. The experimenter began by explaining the anchors on each side of the line and then introduced the children to the task with three practice trials. In the initial trial, the red area of the rectangle covered 28 units of juice out of 30 units of the total amount. The experimenter then positioned the slider in the correct placement of the juice-to-water ratio on the horizontal line. On the second (2/30) and third practice trials (22/30), the child estimated with the slider and received corrective feedback. In all test trials, children completed their judgments and placed the slider in position without additional feedback. There were 16 total test trials, with four levels of juice (3, 4, 5, 6 units) and total amount (6, 12, 18, and 24) combined to make a complete factorial design. Twelve combinations were scaled (i.e., the stimulus proportion was shorter in overall length than the response line) and four were nonscaled. To calculate accuracy, students' PAE was calculated for each trial by dividing the absolute difference between the participant's estimate and the correct target location by the scale of the estimates. The child's average PAE was their total score, with a lower PAE indicating more accurate estimates. The task took approximately 5 min to administer and complete. Reliability for the current study was $\alpha = .72$.

Vocabulary

Children's oral vocabulary comprehension was assessed with the Picture Vocabulary Test of the NIH Toolbox for Assessment of Neurological and Behavioral Function (Slotkin et al., 2012). This standardized assessment measures comprehension of single words. Words varied by difficulty and were measured with an auditory word–picture matching paradigm. Four pictures were displayed on the iPad screen while a word was recited via the iPad. Children were instructed to touch the picture that matched the word. Once a child selected the picture, the next set of images appeared. The task took approximately 4 min to complete. Test–retest reliability based on the test's norms is .84.

Math Achievement

As an indicator of general math achievement, the numeration test of the KeyMath-3 Diagnostic Test (Form A) was administered (Connolly, 2007). The KeyMath-3 is a standardized nationally norm-referenced test for students in kindergarten through 12th grade. The numeration test, which has an internal reliability of .86 for first graders, assesses basic math concepts, including quantity, number order, and place value. Topics gradually get more challenging with items involving rounding and rational numbers. All children started with the first item. Children received one point for each correct response. Following the stopping rule, the test was stopped after four consecutive errors. Thus, time for completion varied for individual children.

Procedure

All children were tested individually in their schools. Tasks were distributed across three sessions during the 2021–2022 school year, with each session taking place over about a 4-week period. Less than 1% of the variance in all measures was explained by the session date, so date of testing was removed from all subsequent analyses. Session 1 occurred in the fall, where children were given the fractions measure, the SENS and the WNL task. Session 2 took place during the winter, where children were assessed on hearts and flowers (executive function), visual-spatial memory span, spatial scaling, spatial proportional reasoning, and vocabulary. Session 3 took place in the spring, where children were reassessed on the fractions and WNL measures. In addition, they were given the KeyMath-3 (numeration) test.

Results

Growth in Early Fraction Knowledge

Mean performance and standard deviations for all measures at each time point are presented in Table 1. As hypothesized, there were substantial individual differences in children's fraction knowledge in first grade (Hypothesis 1).

There was a wide distribution of scores on the early fractions assessment at both fall and spring of first grade (Figure 3). Notably, using a paired samples t test, we found that early fraction knowledge significantly improved from fall ($M = 57.8\%$) to spring ($M = 67.2\%$) as hypothesized (Hypothesis 2), $t(102) = 8.46$, $p < .001$, $d = .56$.

Individual Growth

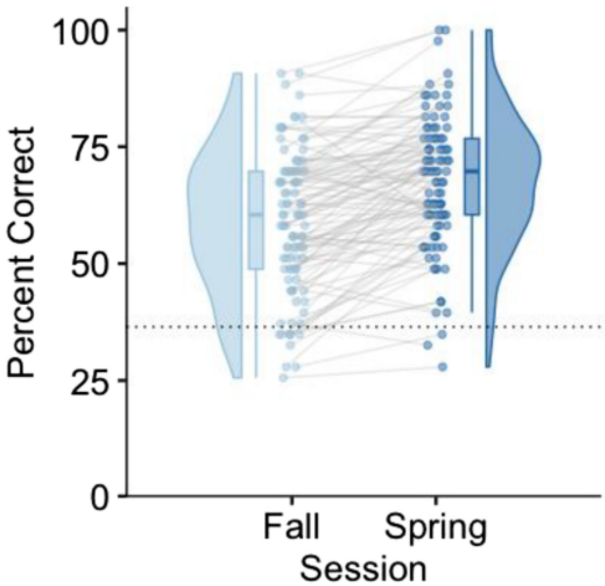
Although we observed improvement at a group level, there was variability in growth in fraction knowledge among individuals. Most children (75%) improved from fall to spring, but informal fraction scores stayed the same from fall to spring for eight children and declined for 18 children. We further examined how

Table 1
Means and Standard Deviations for All Measures by Session

Measure	*M*	*SD*	Range
Session 1			
Early fractions assessment (% correct)	58.8	15.0	25.6–90.7
Whole number line estimation (PAE)	12.3	7.4	4–40.7
Screener for early number sense (SENS; % correct)	74.0	22.8	11.8–100
Session 2			
Hearts and flowers (raw composite score)	694	341	−139–2,104
Spatial scaling (% of total possible points earned)	58.0	11.4	19.4–81.9
Visual-spatial memory span (mean of forward/backward)	4.1	2.2	0–8
Spatial proportional reasoning (PAE)	24.3	5.7	12.5–45.2
Picture vocabulary (standard score)	67.8	27.7	1–99
Session 3			
Early fractions assessment (% correct)	67.3	14.1	27.9–100
Whole number line estimation (PAE)	8.3	6.1	1.9–38.6
KeyMath-3 numeration (scaled score; $M = 10$)	12.1	3.3	3–18

Note. PAE = percent absolute error.

Figure 3
Individual Differences in Fraction Knowledge at Fall Versus Spring of First Grade



Note. Dashed line indicates chance performance (36.5% correct). See the online article for the color version of this figure.

many individuals changed by more than 10% on the early fraction measure, corresponding to a change of 2/3 of a standard deviation, a large effect size. A change of 10% corresponded to change in accuracy on at least four of the 43 items. Fifty-seven children (55.3% of our sample) scored within 10% above or below their fall score in the spring. Only two children declined by more than 10%, and 44 children (42.7%) improved by more than 10%.[2]

To further examine individual differences in early fraction knowledge, we compared children's performance to chance-level performance, which was 36.5% due to a combination of item types, using two-tailed one-sample *t* tests. At Session 1, 69 students (67%) performed significantly better than chance, and 34 students (33%) were performing no better than they would have by guessing. However, by Session 3 at the end of first grade, 90 (87%) out of the 103 students performed significantly better than chance on the early fractions assessment. Of the 34 students who started the fall performing no better than chance, 22 of those students (65%) improved to significantly above chance by the spring. Of the 69 students who were already performing significantly above chance at the fall session, only one student was then performing at chance in the spring. Compared to the rest of the sample, the 13 students who were still performing at chance on the early fractions measure in spring of first grade had lower fall SENS scores, $F(1, 101) = 14.2$, $p < .001$, worse fall WNL, $F(1, 101) = 9.11$, $p = .003$, lower spring KeyMath scores, $F(1, 101) = 20.1$, $p < .001$, worse working memory, $F(1, 101) = 5.78$, $p = .018$, and worse spatial proportional reasoning, $F(1, 101) = 6.49$, $p = .012$. They did not differ from the students performing above chance on vocabulary, executive function, spatial scaling, or age.

Areas of Improvement

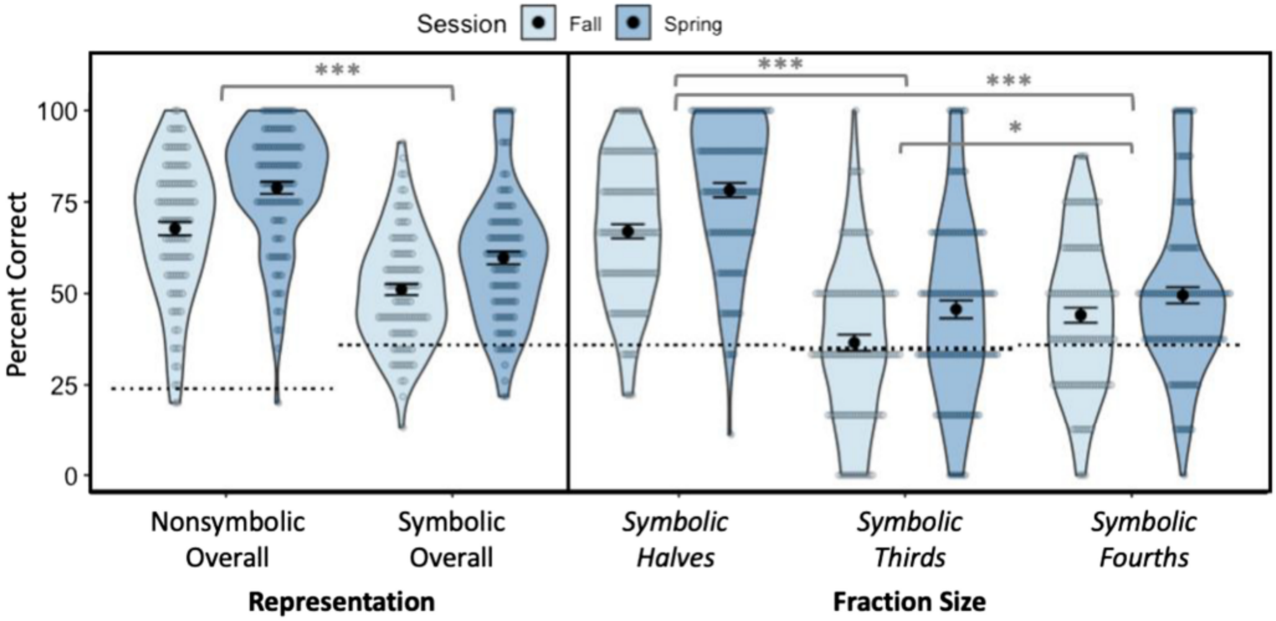
To further understand the growth in children's fraction knowledge, we analyzed children's performance by kind of representation (i.e., nonsymbolic vs. symbolic) and size of fraction (i.e., halves, thirds, and fourths). As shown in Figure 4, there were wide individual differences in all categories. As a group, children were less accurate on symbolic items than nonsymbolic items, and average accuracy on both types of problems increased slightly from fall to spring (Figure 4). These main effects of representation (i.e., nonsymbolic or symbolic) and session (fall or spring) were significant in a linear regression predicting accuracy on the early fractions test ($\beta_{nonsymbolic} = 0.42$, $p < .001$ and $\beta_{spring} = 0.28$, $p < .001$). That is, accuracy was 0.42 *SD*s higher for nonsymbolic items versus symbolic items, and accuracy was 0.28 *SD*s higher in the spring than in the fall. This was true even though symbolic items were designed to have fewer answer choices (and thus a higher probability of answering correctly by chance alone) than nonsymbolic items (see "chance" performance lines in Figure 4). The interaction between representation and session was not significant ($\beta = 0.05$, $p = .48$), showing that children made similar improvements in both nonsymbolic and symbolic fraction knowledge from the fall to the spring. On an individual level, 83% of children in the fall, and 82% of children in the spring, had the same pattern of results found in the group analysis (i.e., were more accurate with nonsymbolic items than with symbolic items) in the fall. About 68% of children improved in symbolic performance and 70% improved in nonsymbolic performance from fall to spring. Children's nonsymbolic and symbolic performance was moderately correlated at both time points (fall $r = .57$, $p < .001$; spring $r = .44$, $p < .001$).

We also examined children's performance on different types of items within nonsymbolic and symbolic categories, as shown in Table 2. Most children already showed some knowledge of verbal fraction words (i.e., "halves," "thirds," "fourths," and "quarter") in fall of first grade; as a group they performed significantly above chance on these items, $t(102) = 12.1$, $p < .001$. However, children were no better than chance at mapping visual models to written fraction symbols (i.e., ½, ⅓, and ¼) at both fall, $t(102) = -2.60$, $p = .99$, and spring assessments, $t(102) = 1.32$, $p = .09$. In addition to these group-level analyses, we examined the proportion of children who performed above chance on each type of item. As shown in Table 2, almost all children were above chance on nonsymbolic items at both fall (98%) and spring (99%), although there was variability across the categories of nonsymbolic items. In line with the group findings, fewer children were above chance on symbolic items at fall (82%) and spring (90%), and less than a third of the sample performed above chance on items with written fraction symbols (fall 19%; spring 31%).

Consistent with prior studies (Brizuela, 2006; Empson, 1999; Singer-Freeman & Goswami, 2001), we found that children were more accurate on items that involved halves than thirds or fourths, although there were wide individual differences, as shown in Table 3 and Figure 3. A linear regression predicting accuracy on the early fractions test from fraction size (i.e., halves, thirds, or

[2] Exploratory analysis of variance (ANOVA) analyses showed that these three groups of students (i.e., no growth, declined, improved) did not significantly differ in whole number knowledge, cognitive skills, or math achievement.

Figure 4
Change in Early Fraction Knowledge by Item Category



Note. Dashed lines indicate chance performance (i.e., estimated % correct if child guessed on every item). Transparent points show individual students' scores. The group mean percent correct and standard error bars are shown in black. See the online article for the color version of this figure.
$^{*}p < .05$. $^{***}p < .001$.

quarters/fourths) and session (fall or spring) showed that children were more accurate with halves than thirds, $\beta = 0.55$, $p < .001$, and fourths, $\beta = 0.42$, $p < .001$. A post hoc comparison using Tukey's adjustment for multiple comparisons showed that children were slightly more accurate on items involving fourths versus thirds, $\beta = 0.22$, $p = .022$. We also found a significant main effect of session, showing that children improved from fall to spring on symbolic items across all fraction sizes, $\beta = 0.22$, $p < .001$. There was no interaction between fraction size and session, $\beta_{thirds} = 0.03$, $p = .61$ and $\beta_{fourths} = 0.08$, $p = .18$. However, on the small set of items that asked children to make judgments about written fraction symbols, there was some observational evidence that children improved more with ½ than with ⅓ or ¼. As shown in Table 3, children showed marked improvement in their ability to map the symbol ½ to the correct visual model, from an average of 31% correct in the fall to 60% correct in spring. In contrast, children's ability to map the symbols ⅓ and ¼ to visual models was very similar in fall and spring.

As an exploratory analysis, we also examined children's attempts to name the fraction symbols ½, ⅓, and ¼ in the fall and spring. Although a few children correctly named these symbols in the fall (½: 6% correct, ⅓: 8% correct, ¼: 11% correct), most children gave an incorrect response that treated the fraction as two separate whole numbers (e.g., "one-two," "one and two," "one plus two," or "one equals two" for ½). Somewhat surprisingly, given students' success with the verbal fraction word "half," children were worse at

Table 2
Early Fractions Accuracy by Item Category

Item category (number of items)	Percent correct		Chance probability[a] (% correct if guessing)	Percent of children descriptively above chance	
	Fall	Spring		Fall	Spring
Nonsymbolic (20)	68	79	24	98	99
Nonsymbolic equal sharing (11)	72	82	46	88	96
Nonsymbolic equivalence (3)	61	75	33	61	74
Nonsymbolic iterating to complete a whole (6)	64	76	12	92	97
Symbolic (23)	51	60	36	82	90
Symbolic fraction word (20)	55	63	36	86	92
Symbolic fraction notation (3)	25	38	33	19	31

[a] Chance probability differs across categories because items had different numbers of answer choices.

Table 3
Accuracy With Symbolic Fractions by Fraction Size

Fraction size (number of items)	Percent correct fall	Percent correct spring	Chance probability[a] (% correct if guessing)
Symbolic halves (9)	67	77	36
Halves word (8)	71	79	36
1/2 notation (1)	31	60	33
Symbolic fourths (8)	44	48	36
Fourths word (4)	44	46	40
Quarters word (3)	53	59	33
1/4 notation (1)	17	20	33
Symbolic thirds (6)	37	44	35
Thirds word (5)	39	47	36
1/3 notation (1)	26	26	33

[a] Chance probability differs across categories because items had different numbers of answer choices.

naming ½ than ⅓ and ¼, especially in the spring (½: 16% correct, ⅓: 27% correct, ¼: 29% correct). One possible explanation may be because the word "half" does not relate closely to the word "two." This pattern is especially noteworthy given that, as mentioned above, children were much better at mapping the notation ½ to the correct visual model than mapping or ¼ in the spring of first grade. Nevertheless, a chi-square test showed that correctly naming the symbol and correctly mapping it to a visual model were associated, $\chi^2(1, N = 618) = 56.71, p < .001$.

Relations Between Early Fraction Knowledge and Other Cognitive Skills

Correlations between all behavioral measures are shown in Table 4. Early fraction knowledge at the start of the year (Session 1) was correlated with working memory, proportional reasoning, and math-specific knowledge, with the strongest correlations being between Session 1 early fraction knowledge and other math-specific knowledge (i.e., SENS, whole number estimation at start and end of year, and end-of-year fraction knowledge). Early fraction knowledge at the end of year (Session 3) was significantly correlated with other math-specific knowledge (i.e., SENS, whole number estimation at start and end of year, and start-of-year fraction knowledge), visual-spatial memory span, and spatial proportional reasoning. Early fractions was not significantly correlated with executive functions, spatial scaling, or picture vocabulary at either time point. Almost all of the measures were significantly correlated with the KeyMath at Session 3 (end of year), although the size of the correlations ranged from small to large (i.e., −.17–.83).

Predicting Fraction Knowledge at the End of First Grade

To determine skills that were uniquely associated with students' end-of-year fraction knowledge, we ran a block-entry regression with end-of-year (Session 3) early fraction knowledge as the dependent variable. Age and gender were entered in the first block, as demographic covariates. In the second block, start-of-year (Session 1) early fraction knowledge and whole number knowledge (SENS, WNL) were entered. The second block allows us to test whether students' whole number knowledge at the beginning of first grade contributes to fraction knowledge at the end of first grade over and above children's baseline fraction knowledge at the beginning of the year. Finally, in the third block, we added the general cognitive variables assessed at Session 2 (picture vocabulary, spatial scaling, spatial proportional reasoning, visual-spatial memory span, and hearts and flowers [executive function]) to test whether these cognitive skills would explain unique variance in fraction knowledge over and above the contributions of the previous predictors. Note that the final step of a block-entry regression model is always statistically identical to the results if all predictors are entered at once. Tests to determine collinearity showed that variance inflation factors for all variables were below 2.5, indicating that multicollinearity was not a major concern in the full model for our math-related measures or cognitive measures.

Results are shown in Table 5. In block 1, we saw that neither age nor gender explained significant variance in end-of-year fraction knowledge, and together they only accounted for 3.4% of the variance. In block 2, we saw that whole number knowledge and start-of-year fraction knowledge explained an additional 58.4% of the variance in end-of-year fraction knowledge. Even when accounting for children's start-of-year fraction knowledge, SENS explained unique variance in end-of-year fraction knowledge. Children who performed one standard deviation above the mean on the SENS at the start of first grade performed an average of .19 *SDs* better on the end-of-first grade fractions assessment. Finally, the addition of all five cognitive predictors in block three only explained an additional 2.8% of the variance, and none of the cognitive predictors were significant when accounting for the other variables. However, SENS still made a significant contribution to end-of-year fraction knowledge even after accounting for the other cognitive skills. Although we hypothesized that both numerical knowledge and general cognitive skills would contribute to first graders' early fraction knowledge, our results highlight that the number sense that students have when they enter first grade is more important for their end-of-year fraction knowledge than other nonnumerical cognitive skills.

Relation Between Start-of-Year Fraction Knowledge and Later KeyMath Achievement

We hypothesized (Hypothesis 4) that children's early fraction knowledge would contribute to their later math achievement. To understand potential contributions to children's end-of-year achievement, we used a block-entry regression with KeyMath as the dependent variable. We again entered age and gender as demographic covariates in the first block and the nonnumerical cognitive

Table 4
Bivariate Correlations Among All Observed Variables

Variable	1	2	3	4	5	6	7	8	9	10
1. Early fractions (fall)	—									
2. SENS	.63**	—								
3. WNL (fall)	−.50**	−.63**	—							
4. Visual-spatial memory span	.43**	.39**	−.36**	—						
5. Proportional reasoning	−.21*	−.24*	.26	−.11	—					
6. Hearts and flowers	−.07	−.07	.11	−.16	−.01	—				
7. Picture vocabulary	.27	.23	−.17	.23	.12	−.17	—			
8. Spatial scaling	.25	.20	−.14*	.27	−.01	−.22*	.26	—		
9. Early fractions (spring)	.76**	.62**	−.49**	.28*	−.28*	−.14	.19	.14	—	
10. WNL (spring)	−.46**	−.60**	.66**	−.26**	.09	.13	−.33*	−.09	−.38**	—
11. KeyMath	.66**	.83**	−.62**	.40**	−.23*	−.17	.24	.25	.66**	−.61**

Note. WNL = whole number line estimation; SENS = screener for early number sense.
* $p < .05$. ** $p < .01$.

predictors in the last block. However, because we were specifically interested in parsing whether fraction knowledge contributed to later math achievement over and above whole number knowledge, we added whole number knowledge in the second block and fraction knowledge in the third block. Results are shown in Table 6. Age and gender explained 4.5% of the variance in end-of-year math achievement, although neither variable was significant. Adding whole number knowledge in the second block explained an additional 66.8% of the variance, with both SENS and WNL making unique contributions. On the next block, early fractions contributed an additional 2.4% of the variance, over and above whole number knowledge. Overall, Model 3 accounted for 73.7% of the variance in KeyMath performance. Finally, the addition of all five cognitive variables only explained an additional 1.2% of the variance, and no cognitive measure significantly predicted KeyMath performance, when considering all other variables.

Discussion

We followed children from the fall to the spring of first grade to investigate the development of early fraction knowledge and how it relates to other mathematical and cognitive skills. Supporting previous findings from qualitative studies (e.g., Empson, 1999), we showed that most young children in our sample had some informal knowledge of early fraction concepts in first grade, which is years before receiving formal fraction instruction. In particular, children understood nonsymbolic representations and halving. Children's early fraction knowledge improved over the school year, even without explicit instruction on fraction words or symbols. However, our findings showed large individual differences in what children know about early fractions at the start of formal schooling, even with a relatively homogenous sample in terms of race and SES, and these differences explained a small but significant amount of variance in math achievement at the end of first grade, even when controlling for whole number knowledge. These findings have important implications for theories of fraction development and elementary school fraction instruction.

Growth in Early Fraction Knowledge

Children bring considerable fraction knowledge to first grade, and this knowledge improves over the school year. Children in our sample learned about basic fraction concepts without formal instruction on fractions in school. Interestingly, students improved on both nonsymbolic fraction problems, like judging how many equal parts are needed to complete a whole, and symbolic fraction problems, like

Table 5
Regressions Predicting End-of-Year Fractions

Predictor	Model 1 β	Model 1 *SE*	Model 2 β	Model 2 *SE*	Model 3 β	Model 3 *SE*
Age in months	.143	.35	.018	.228	.011	.237
Female	−.108	2.77	−.056	1.86	−.056	1.89
SENS			.193*	.057	.210*	.057
WNL			−.060	5.21	−.039	5.39
Early fractions (fall)			.602***	.078	.638***	.081
Picture vocabulary					−.001	.035
Spatial scaling					−.067	.085
Proportional reasoning					−.094	.167
Visual-spatial memory					−.103	.459
Hearts and flowers					−.102	.537
ΔR^2	3.4%		58.4%		2.8%	

Note. WNL = whole number line estimation; SENS = screener for early number sense. ΔR^2 = change in R^2 from the previous model. All betas are standardized.
* $p < .05$. *** $p < .001$.

Table 6
Regressions Predicting End-of-Year KeyMath in Spring of First Grade

Predictor	Model 1		Model 2		Model 3		Model 4	
	β	*SE*	β	*SE*	β	*SE*	β	*SE*
Age in months	.131	.08	−.012	.046	−.015	.044	−.043	.047
Female	−.161	.643	−.026	.372	−.049	.361	−.056	.371
SENS			.716***	.01	.621***	.011	.622***	.011
WNL			−.175*	1.02	−.121	1.01	−.098	1.06
Early fractions (fall)					.204**	.015	.192**	.016
Picture vocabulary							.007	.007
Spatial scaling							.047	.017
Prop. reasoning							−.016	.033
V-S memory							.012	.090
Hearts and flowers							−.091	.106
ΔR^2	4.5%		66.8%		2.4%		1.2%	

Note. WNL = whole number line estimation; Prop. reasoning = proportional reasoning; V-S memory = visual-spatial memory; SENS = screener for early number sense. All betas are standardized.
* $p < .05$. ** $p < .01$. *** $p < .001$.

judging which of three rectangles shows "one-third" colored in red. This finding shows that children build fraction knowledge through everyday experiences and/or first-grade math instruction that focuses mostly on whole numbers and subsequently raises questions about what kinds of activities, language, or instruction support these developing ideas. Children may be prepared to learn about foundational fraction ideas before the timeline suggested by existing theories (Siegler et al., 2011; Steffe, 2001) and U.S. standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). How did children learn these early fraction concepts without formal instruction?

Our results suggest that strong whole number knowledge is a key correlate and underpinning of children's developing early fraction knowledge. As we hypothesized based on prior research, children's skills with whole numbers, spatial scaling, spatial proportional reasoning, and vocabulary were all correlated with children's initial early fraction knowledge in fall of first grade. However, whole number knowledge as measured by the SENS and number line estimation was most strongly correlated with fall fraction knowledge. SENS performance was also the only significant predictor of later fraction knowledge at the end of the school year, controlling for children's baseline fraction knowledge and other relevant cognitive skills. Previous research using the SENS has shown that children's early number sense is uniquely related to their concurrent and future mathematics achievement, when controlling for reading ability and demographic characteristics (Devlin et al., 2022; Jordan et al., 2007, 2009). Our findings extend this work to show that early whole number knowledge is also strongly associated with future early fraction knowledge.

The strong relation between whole number knowledge and fraction knowledge in first grade is consistent with the integrated theory of numerical development (Siegler et al., 2011) and the reorganization hypothesis (Steffe, 2001). These theories focus on the importance of understanding whole number to support the later learning of symbolic fractions. However, that 66% of children in our sample performed above chance on the early fractions measure at the beginning of first grade also emphasizes the need to incorporate children's early developing fraction knowledge, especially with nonsymbolic representations, in theories of numerical development. For example, early fraction understanding may be supported by neurocognitive architecture equipped to process ratio (e.g., Lewis et al., 2016), which may be in addition to whole number processing systems, or alternatively by an approximate magnitude system from which both integer and fraction knowledge emerge, as well as understanding of weight, length, volume, and so forth (Newcombe et al., 2015). Our results highlight the need to investigate the concurrent development of fraction and whole number learning, which appear to be more intertwined than current theory envisions.

The strong focus on counting and other whole number strategies in early schooling can undermine children's intuitions and informal knowledge of nonsymbolic fractions (Boyer et al., 2008; Rinne et al., 2017). We did see some evidence of this confusion, such as children reasoning that "they each have one" to explain why a nonsymbolic representation of ½ was the same as a nonsymbolic representation of ¼. Nevertheless, we also observed children using whole number knowledge to correctly solve some nonsymbolic problems, such as saying "because three and three makes six" to explain why they chose a picture of three carrots to solve the equal sharing problem with bunnies shown in Figure 1. These seemingly conflicting findings may be explained by the strength of whole number knowledge; children who started the year with stronger whole number knowledge may have been able to learn more from informal everyday experiences with early fraction concepts, such as equal sharing.

Importance of Early Fraction Knowledge to Later Math Achievement

Our findings show for the first time that children's early fraction knowledge in fall of first grade contributes to their math achievement at the end of first grade, over and above their initial whole number knowledge. Although early fractions knowledge only explained a small portion of the variance in math achievement relative to whole number knowledge, the effect size was meaningful. Children with one standard deviation better early fractions knowledge in the fall scored .19 *SD*s better on the KeyMath assessment at the end of first grade, which corresponds to about 18% of the annual gains that occur in first grade on U.S.-based nationally normed tests of mathematics (see Bloom et al., 2008).

Overall, we found that early number knowledge—of both fractions and whole numbers—was more important for first-grade numeracy achievement than spatial proportional reasoning, spatial scaling, vocabulary, or domain-general cognitive skills. These findings are consistent with prior research showing that early whole number competence is crucial for elementary school mathematics achievement (e.g., Aunio & Niemivirta, 2010; Hornung et al., 2014; Jordan et al., 2009), but our findings highlight that early competence with fraction concepts also contributes to later achievement. It would be interesting to examine whether early fraction knowledge uniquely predicts math achievement beyond first grade.

We were surprised by the small contribution of the cognitive predictors, which together explained only 1.2% of the variance in spring KeyMath scores, given that previous studies have shown that spatial and domain-general skills predict math achievement in young children (e.g., Clements et al., 2016; Möhring et al., 2016). However, many past studies that report a relation between spatial or domain-general skills and math achievement in young children have not included a powerful, comprehensive measure of whole number sense like the SENS (e.g., Hawes et al., 2019; Möhring et al., 2016; Ramirez et al., 2013). The SENS was carefully designed to capture many of the most important early math skills, and it is so strongly correlated with KeyMath performance ($r = .83$, see Table 4) that it dominates when pitted against other predictors in a regression model. This explains why most of the individual cognitive predictors are correlated with the KeyMath measure, but fail to add significant variance in the context of SENS. Indeed, some work suggests that when accounting for both prior knowledge and domain-general cognitive skills, prior knowledge strongly tied to the domain investigated frequently cancels out cognitive variables in the same model (Byrnes et al., 2019; Byrnes & Miller, 2007).

We were surprised by the weak correlation (fall $r = .21$, spring $r = .28$) between spatial proportional reasoning and informal fractions knowledge, given that Möhring et al. (2016) showed that 8- to 10-year-old children's scores on the same proportional reasoning measure were strongly correlated with their symbolic fraction performance. The low correlation we observed may have been influenced by some children using nonproportional strategies on the proportional reasoning task, such as focusing on the amount of juice or water alone or focusing on the difference between the juice and water. However, overall performance on the spatial proportional reasoning task in our sample was similar to previous studies with U.S. children of this age (e.g., Tian et al., 2022). Future research is needed to replicate and explain this weak correlation. For example, future studies should examine whether age moderates the relation between spatial proportional reasoning and different kinds of fraction knowledge.

Taken together, our findings raise questions about the codevelopment of whole number knowledge, early fraction knowledge, and overall math achievement in early elementary school. For example, does whole number knowledge necessarily precede early fraction knowledge, or can learning about early fraction concepts like equal sharing or iterating to produce a whole help strengthen whole number knowledge? Longitudinal studies suggest that some math skills codevelop with other math or domain-general skills (e.g., Bailey et al., 2017; Cameron et al., 2019; Kahl et al., 2022; Zhang et al., 2023), including symbolic fraction knowledge and math achievement (Hansen et al., 2017). Future studies with measurements at more frequent intervals using random intercept cross-lagged analyses or similar techniques would be valuable to investigate how early fraction knowledge, whole number knowledge, and overall math performance codevelop over time. If there is also a bidirectional relation between whole number and fraction knowledge, then spending instructional time on early fraction concepts in early elementary school might not only pave the way for future learning in later years but also support ongoing growth in early numeracy.

Limitations

The current study assessed early fraction knowledge prior to formal instruction. Although the schools in the current study did not teach fractions during the study, there is likely variability between other schools on if, when, and how fractions are initially introduced. Teachers may also use words like "half" in everyday language or model nonsymbolic fraction ideas in ways that go beyond the written curricula for first grade. It would therefore be valuable to measure fraction knowledge in combination with measuring additional information about math instruction. Future studies could also expand our investigation of which other skills may support children's early fraction knowledge, such as by measuring children's math-specific vocabulary (Hornburg et al., 2018; Powell & Nelson, 2017; Purpura & Reid, 2016).

We carefully designed our informal fraction measure to capture a broad range of early fraction topics as efficiently as possible, based on previous studies with this age group. However, the uneven distribution of problem types may have influenced some of our conclusions about representation type. For example, whole number knowledge (measured by the SENS) may have predicted performance on our informal fraction measure because both assessments involved symbolic reasoning. However, given the large number of items of nonsymbolic fraction items, and that there were only three items with written fraction notation (i.e., ½, ⅓, and ¼), we do not think this connection is driven by symbolic reasoning alone. To compare specific categories of early fraction knowledge (e.g., nonsymbolic vs. symbolic) with more precision, future studies should balance items along as many dimensions as possible.

Another consideration is that our sample had above average math achievement and was drawn from schools serving primarily white middle-income communities in the same state in the northeastern United States. Future studies should examine whether our results generalize to children from different SES levels, backgrounds, or educational contexts. Mathematical inputs children receive at school and home are likely to vary in other parts of the country (Daucourt et al., 2021) and world (Cankaya & LeFevre, 2016; Miura et al., 1993; Paik et al., 2011). Past research has shown that the home math environment is strongly correlated to family income (Daucourt et al., 2021; Muñez et al., 2021). Children from lower-income backgrounds also tend to enter elementary school with weaker whole number knowledge (e.g., Rittle-Johnson et al., 2017), which may correspond to lower fraction knowledge. However, it is also possible that children who bring different experiences or skills to first grade may develop similar levels of early fraction knowledge through different mechanisms. A broader sample of children and better measurement of the home and school environments would help answer these questions.

Future Directions

Children in our sample of primarily white, middle-class students started first grade with widely different knowledge about early fraction concepts. Some children were unable to answer even simple nonsymbolic questions like recognizing whether a piece of paper was split into three equal parts (12% incorrect), whereas other children could answer questions about symbolic fractions like which bar shows one-fourth (16% correct) or which picture shows the fraction ⅓ (26% correct). These are substantial individual differences, even in a middle-class sample, that warrant further investigation.

Future research should more closely examine these individual differences to test whether there are groups of children with qualitatively different patterns of early fraction knowledge. Prior work shows that older children's fraction knowledge follows distinct growth trajectories from fourth to sixth grade, with some students starting with strong fraction scores and maintaining them, others starting with low fraction scores and improving, and others who maintain consistently low scores (Resnick et al., 2016). It is possible that the development of early fraction knowledge in younger children may follow similar classes of trajectories. It would also be valuable to examine whether the type of early fraction knowledge differs for different groups of students, such as McMullen et al. (2018) showed for symbolic fraction and decimal knowledge in older students. We did not have an appropriate sample size or number of time points to conduct latent growth curve analyses or latent class analyses, but future work could use these methods to further examine individual differences. The growth we observed in children's fraction knowledge from fall to spring also raises questions about the development of that knowledge over the school year. Do children show consistent gradual improvement, or are there spikes where children's fraction knowledge improves more quickly? Are patterns of change related to math content children are learning in school, or does improvement in early fraction knowledge seem to be more closely related to informal or everyday experiences outside of math instruction? Future studies including measurements at more frequent intervals would be able to answer these questions.

The individual differences we observed at the start of first grade, and their growth over the year, suggest that differences in children's early childhood informal education and home experiences likely contribute to their early fraction knowledge. A few students even mentioned recognizing some of the fraction words from previous experiences (e.g., "I learned that in daycare" or "I learned this in Odd Squad [a TV show from PBS]"). Many studies have examined the ways that children's home environment impacts their whole number knowledge or general math achievement (see Daucourt et al., 2021 for a review), but more research is needed that specifically examines environmental supports for early fraction knowledge (e.g., Eason & Ramani, 2020). Future research should seek to characterize the instructional practices and informal experiences before and during first grade that are associated with growth in early fraction knowledge.

Implications for Practice

Formal fraction instruction in the United States typically begins around third grade (although there is variability across curricula and contexts), but our study shows that many first graders have substantial early fraction knowledge that improves over the school year. We also found that early fraction knowledge was significantly associated with later math achievement at the end of first grade, which suggests that children may benefit from more opportunities to build and expand on this informal fraction knowledge in kindergarten and early elementary school. By measuring patterns and individual differences in what children already know about fractions in first grade, our study reveals potential starting points and targets for early fraction instruction.

Our results suggest that early fraction instruction should help children build an understanding of fractions using nonsymbolic representations of fractions and the word "half" or "halves," which many first graders in our sample knew at the start of first grade. A few studies have tested early fraction instruction that builds on children's knowledge of nonsymbolic fractions. In a 5-week study with one first-grade classroom, Empson (1999) used equal sharing situations with nonsymbolic representations to help children build formal fraction knowledge. In a brief experimental lesson with first and second graders, Hurst et al. (2022) combined nonsymbolic and symbolic fraction representations in a version of the card game "War" to teach children about fraction sizes. Our findings suggest that these types of lessons or games might be most effective if they focus on fractions with small denominators, especially one-half and one-fourth. Indeed, a fraction intervention for older students with math difficulties showed that starting with small denominators helped consolidate key fraction concepts (Dyson et al., 2020), and a brief fraction number line lesson that focused on fractions with denominators two, four, and six was effective for teaching second and third graders about fraction sizes (Gunderson et al., 2019).

Our findings also show that some students struggled with even simple nonsymbolic fraction items at the start of first grade, and a subset of these students continued to struggle with such items even at the end of first grade. For these students, it may not be enough to rely upon informal, everyday experiences to help them build understanding of foundational fraction concepts. Instead, these students may need explicit instruction to link nonsymbolic foundations like equal sharing, equivalence, and iterating parts to create a whole with formal fraction ideas, in addition to supports for building whole number concepts.

Overall, this research shows that there is substantial variability in what young children know about early fraction concepts and that these differences predict later achievement. Whole number knowledge is a key underpinning of this informal fraction knowledge. Our results call for theories of numerical development that more fully incorporate concurrent whole number and fraction development in the early grades of schooling. Many children in our sample improved over the school year even without targeted instruction, but future research should develop and test the usefulness of learning tools to build early fraction knowledge that positions all learners for later success, including children at risk.

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