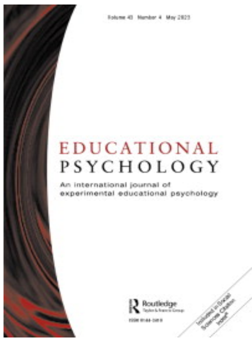

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Predictive relations between early place value understanding and multidigit calculation: approximate versus syntactic measures

Kelly S. Mix[a], Corinne A. Bower[a], Lei Yuan[b], Gregory R. Hancock[a] and Linda B. Smith[c]

[a]Department of Human Development and Quantitative Methodology, University of Maryland, College Park, MD, USA; [b]Department of Psychology and Neuroscience, University of Colorado, Boulder, CO, USA; [c]Department of Psychological and Brain Sciences, Indiana University, Bloomington, IN, USA

ABSTRACT

The current longitudinal study measured 279 kindergartners' (M_{age}=5.76 years; $SD = 0.55$; 135 females) place value understanding using both approximate and syntactic measures based on previous evidence that early variation in performance on approximate measures was associated with subsequent syntactic place value understanding. In the present study, we used the same dataset but traced kindergarten variation on both approximate and syntactic measures to children's multidigit calculation skill in second grade. Path analyses indicated that latent approximate place value knowledge predicted later multidigit calculation performance but more precise understanding of counts and base-ten units did not. However, among the individual task predictors, Base-Ten Counting was the strongest predictor followed by a task asking for specific count of units (e.g. 'Which number has [*two tens*]?') and reading and writing number names (Transcoding). Thus, multidigit calculation skills appear to develop from an emerging, but imprecise understanding of count + unit syntax.

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KEYWORDS
Place value; children; mathematics; longitudinal

Place value notation—the use of spatial position to represent base-ten units—is the backbone of advanced arithmetic operations and thus, a critical gateway skill in elementary mathematics (Booth & Siegler, 2008; Chan et al., 2014; Hiebert & Wearne, 1996). Learning place value is not straightforward, and many children exhibit misunderstandings and errors along the way (e.g. Fuson, 1990; Kamii, 1986). A critical question is how variation in early place value understanding impacts acquisition of subsequent mathematics skills, such as multidigit calculation.

Previous research has demonstrated that early place value understanding takes different forms. On one hand, young children can understand the precise, multiplicative

CONTACT Kelly S. Mix kmix@umd.edu Department of Human Development and Quantitative Methodology, University of Maryland, 3942 Campus Drive, Benjamin Building, Suite 3304, College Park, MD 20742, USA

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structure of counts and units (e.g. 543 represents a count of 5 hundreds, 4 tens, and 3 ones)—what one might call *syntactic* understanding because it focuses on the rules by which place value elements are combined to generate numeric representations (i.e. *count + unit syntax*; Mix et al., 2022). However, place value can also be understood in a less precise way, based on approximations. For example, children can guess that multi-digit numerals stand for greater quantities than single digit numerals without knowing how counts and units are used syntactically. Previous longitudinal research indicates that syntactic place value concepts originate in approximate understandings (Bower et al., 2022; Mix et al., 2022). (Note that this use of the term 'approximate' is not intended to mean approximate number system or ANS (e.g. Barth et al., 2006). Rather, it is used in contrast to syntactic understanding of place value symbols.) The present study was undertaken to evaluate whether these approximate understandings also serve as the entry point for multidigit calculation.

Support for the approximate-syntactic distinction

The approximate-syntactic distinction was proposed and tested in previous research. In one study, Mix et al. (2022), hypothesized that children could compare multidigit numbers (Magnitude Comparison), read and write multidigit numbers (Transcoding), and place multidigit numbers on a number line (Number Line Estimation) without analysing units and counts—that is, without knowing the syntax of place value symbols. In contrast, other tasks directly query count + unit syntax and thus may require syntactic understanding to be correct, including tasks such as showing children two multidigit numbers and directly asking which number has a particular count-unit combination, such as six tens (Which N Has _____?), asking children to match a multidigit number to its expanded notation equivalent (e.g. $56 = 5 \times 10 + 6 \times 1$) (Expanded Notation), or asking children to count large quantities of base-ten blocks to see whether they count by ones or by base-ten units (e.g. tens, hundreds, etc.) (Base-Ten Counting).

One rationale for this distinction was that children showed skill on approximate tasks years before they are taught about place value syntax in school (Byrge et al., 2014; Mix et al., 2014; Yuan et al., 2020). A second rationale was that children's correct responses could, in principle, reflect partial understanding and heuristics, such as paying more attention to the leftmost digit when comparing multidigit numerals (Cheung & Ansari, 2021; Mix et al., 2014), or correctly guessing that the number name 'two hundred forty-five' is written '245' because these number names were heard in that order, rather than knowing that the '2' stands for two hundreds, and so forth. Finally, there is direct empirical support for this distinction. Using confirmatory factor analysis, Mix et al. (2022) determined that, in kindergarten, three measures hypothesised as approximate (Transcoding, Magnitude Comparison, and Number Line Estimation) loaded onto one factor, and three measures hypothesised as syntactic (Which N Has ____?, Expanded Notation, Base-Ten Counting) loaded onto a second factor. By first grade, all six measures loaded onto a single factor in the best-fitting model. Interestingly, kindergarten performance on approximate tasks, but not syntactic tasks, predicted first grade place value performance. The kindergarten two-factor structure was further confirmed in a bottom-up analysis of the same dataset that used network clustering (Bower et al., 2022); however, the two-cluster structure remained evident at first grade, unlike the CFA results, and in the first-grade clusters, Transcoding shifted from clustering with the approximate measures to clustering with the syntactic measures. This pattern indicated that the way children approached Transcoding may have shifted to be more reliant on count + unit syntax and less reliant on heuristics and partial mappings with development. The network analysis also revealed that both Transcoding and Base-Ten Counting acted as central hubs for the rest of the network, consistent with evidence that positional understanding of place value is developmentally linked to reading multidigit numbers (i.e. Transcoding) (Cheung & Ansari, 2021).

A key remaining question that this dataset can answer is how these early components of place value knowledge—approximate or syntactic—relate to multidigit addition and subtraction. As noted above, only children's approximate performance in kindergarten was predictive of place value understanding in first grade, indicating that children rely on their partial understandings to gain access to the place value system. However, in second grade, multidigit calculation skill was also assessed. Would approximate understandings of place value, syntactic understandings, or both predict this later developing mathematics skill? Although previous longitudinal research has demonstrated that some of the place value measures included here (Base-Ten Counting, Magnitude Comparison, Transcoding, and Number Line Estimation) are significant predictors of later mathematics achievement (Chan et al., 2014; Geary, 2011; Moeller et al., 2011; Siegler & Booth, 2004; Xenidou-Dervou et al., 2017), investigators tended to focus on one or two place value measures rather than including a more complete set of place value skills, and did not compare these skills to see if each predictor makes unique contributions to later achievement. Importantly, none has assessed approximate and syntactic place value separately either.

One possible outcome of such a comparison is that, because multidigit calculation requires a higher degree of syntactic skill, children must exhibit some syntactic skill in kindergarten to master it by second grade. Alternatively, children may need approximate understandings to gain a foothold before they can achieve more precise syntactic understandings, so that early approximate skills may be the stronger predictor of multidigit calculation as well. The present study tests these ideas by analyzing the same dataset used previously to establish the approximate-syntactic distinction in kindergarten (Bower et al., 2022; Mix et al., 2022) but evaluating the longitudinal relations between these earlier concepts and children's multidigit addition and subtraction performance in second grade.

Learning about symbols from symbols

The present study also builds on emerging research showing that children build precise understandings of the place value system from structural regularities in the symbols themselves (Yuan et al., 2019; 2020). The mathematical symbol system, broadly construed, consists of three distinct quantitative representations: written numerical symbols, spoken number names, and physical quantities (Dehaene & Mehler, 1992). Most research has focused on one possible entry point to this system—increasingly accurate nonverbal representations of physical quantities and the link between these representations and symbolic meanings (e.g. Barth et al., 2006). However, the written and verbal conventions for representing large quantities via place value offer a set of predictable relations that could be leveraged by young learners without mapping to physical quantities (e.g. numerals with more digits represent larger quantities). As noted above, the evidence shows that young children recognise such patterns and can use them as an alternative entry point (Byrge et al., 2014; Mix et al., 2014; Yuan et al., 2019; 2020).

Although the present study did not include measures with explicit physical referents (e.g. clouds of dots, collections of objects, etc.), one could argue that placing numerals on a number line or judging which of two written numerals is larger are both indirect measures of magnitude representations. Our other four measures are more squarely focused on written and verbal symbols. Thus, our results contribute to research on the relative contribution of both symbolic and non-symbolic entry points.

Expanding early numeracy assessments

In addition to addressing these theoretical questions, the present study has practical implications for identifying kindergarten children at-risk for later mathematics learning difficulties. Researchers have developed early mathematics assessments for classroom use that successfully identify children at risk for poor mathematics outcomes, but these assessments may not account for as much variance as possible because they tend not to include multidigit items. For example, the *Number Sense Screener* (*NSS,* Jordan & Glutting, 2012) has four items that ask children to read 2- and 3-digit numbers, but it otherwise focuses exclusively on single-digit numbers. Because advanced mathematics relies on strong multidigit number concepts, and because mounting evidence indicates children begin interpreting multidigit numbers in preschool, more extensive assessments of multidigit number understanding in early childhood might explain additional variance in later mathematics outcomes that may be missed by assessments focused on single-digit number understanding.

Furthermore, if research could identify the strongest predictors among specific multidigit number measures, it would expand the toolkit teachers have to rapidly identify kindergarten children who need more exposure to multidigit numbers, as well as highlight skills that might be targeted with instruction. In support of this aim, we examine predictive relations for individual measures, as well as for the same measures grouped into approximate or syntactic classes.

Current study

The current study is the first to longitudinally track both syntactic understanding and approximate understanding of place value in kindergarten, and relate children's early place value performance to their subsequent multidigit calculation skill. Children were tested at three timepoints: spring of kindergarten, spring of first grade, and spring of second grade. We focus on kindergarten because previous analyses of the same dataset indicated there is variation in both approximate knowledge and in (much weaker overall) syntactic knowledge at this age compared to first grade children for whom preliminary analyses indicated ceiling performance on four measures (Bower et al., 2022; Mix et al., 2022). However, we used children's *WISC-IV* Matrix Reasoning scores to estimate general cognitive ability, which were obtained in first grade.

Our study has three specific aims:

1. To determine whether second grade multidigit calculation skill is predicted by kindergarten place value knowledge (approximate, syntactic, or both). We hypothesize that approximate understanding of place value will be a significant predictor as it was for first grade place value (Mix et al., 2022), but due to its syntactic nature, children may also need early syntactic understanding to benefit from multidigit calculation instruction.
2. To determine whether second grade multidigit calculation skill is predicted by specific individual kindergarten place value measures. Recall that Transcoding and Base-Ten Counting acted as central hubs or organizers of children's emerging place value knowledge structure (Bower et al., 2022). These individual skills might similarly predict children's multidigit calculation skill.
3. To determine whether kindergarten multidigit understanding explains additional variance in second grade multidigit calculation skill, above and beyond the variance already explained by early screeners, like the *NSS* (Jordan & Glutting, 2012). The *NSS* successfully identifies young children at-risk for poor mathematics achievement, but it tends to focus on single-digit number concepts. Based on mounting evidence that kindergartners have partial understanding of multidigit numbers, a wider range of place value skills might explain additional variance, as well as providing brief screening tools that could be useful to teachers.

Method

Participants

In year one, the performance levels of 279 kindergartners (135 females; 144 males) (mean age = 5.76 years, SD = 0.55) were assessed. Two years later, in spring of second grade, 197 children remained in the study (97 females; 100 males), but 82 children had moved and could not be located. An *a priori* sensitivity test conducted in G*Power (v.3.1, Faul et al., 2009) indicated that 120 participants would be adequate to reveal a medium effect using a regression model (i.e. Cohen's $f^2 = 0.11$, Cohen, 1988). Thus, our final sample was expected to be adequate to test the hypothesized relations. An attrition analysis indicated no significant differences in kindergarten performance between children who left the study and those who remained (p's $> .122$).

Children were recruited from three states in the Midwestern and Mid-Atlantic regions of the United States (Indiana: $n = 40$ children; Maryland: $n = 186$; Michigan: $n = 53$) with approval from our institutional review boards. Families of 213 children provided written consent. For the remaining 66 children, school administrators requested an opt-out consenting process in which families were notified but only returned their consent forms to indicate exclusion. None of the families opted for exclusion. We obtained demographic data from 46% of the total sample because (a) a demographic questionnaire was not included in the opt-out consents and (b) some families who received the questionnaire did not return it. For the remaining 54%, we used community demographic information—school-wide information for 31% of the families and 2017 neighbourhood census data for the rest. Weighted sample descriptive statistics indicated that the sample was racially diverse (41% Black, 38% White, 12% Latino, 8% Asian), and primarily middle socioeconomic status (average median family income range = \$75,000 to \$99,999).

Procedure and materials

Test sessions took place in a quiet area outside of the classroom and lasted approximately 60 minutes per child. All measures were administered individually by the first author and a team of highly trained research assistants in one of two pseudorandom orders, counterbalanced across children. Reliabilities for each measure were calculated using Cronbach's alpha. Two measures had reliabilities below 0.70, which suggests multidimensionality; however, such measures can be retained if they provide important content coverage (Schmitt, 1996). As an added assurance, we computed an alternative reliability metric to assess replicability of the latent constructs themselves (Coefficient *H*, Hancock & Mueller, 2001), and these metrics indicated adequate construct replicability.

Syntactic place value measures (kindergarten)

The three syntactic place value measures—'Which Number Has __?', Expanded Notation, and Base-Ten Counting—have high reliability as part of a syntactic construct in kindergartners ($H_{syntactic}$ = 0.72, 95% CI [0.66, 0.94], Mix et al., 2022).

'Which N Has __?'. This multiple-choice adaptation of the digit correspondence task (e.g. Kamii, 1989) involved presenting three written numerals in a horizontal line (e.g. 2, 20, and 10) and asking children to select the number that answered a place value question (e.g. 'Which number has two tens?'). There were two practice trials with corrective feedback, followed by six test trials including two trials each probing tens, hundreds, and thousands (stimulus range = 5–4830). The spatial position of the correct response was counterbalanced across trials (maximum possible score = 6, chance = 2; $\alpha = 0.53$).

Expanded notation. In this commonly used multiple choice task (e.g. Mix et al., 2016), children were shown a written numeral (e.g. 73) and asked to select the correct expanded version from among three options (e.g. $70+3$, $700+3$, or $7+3$). The choices were arranged vertically on the right side of the page, and the target number was presented in a larger font on the left side. Before the test trials, the experimenter explained that the plus sign means combining two numbers, and then asked, 'Which of these (pointing to the equations) adds up to be this number (pointing to the target)?' There were two practice trials with corrective feedback, followed by six test trials included two trials each probing two-digit, three-digit, and four-digit numbers (stimulus range = 11–3946) (maximum possible score of 6; chance = 2; $\alpha = 0.57$).

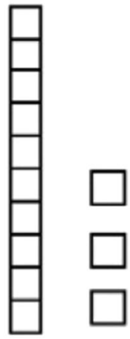

Figure 1. Sample Base-Ten Counting item.

Base-Ten Counting. Children were asked to count the number of squares in a simple line drawing of various quantities represented with base-ten blocks (Figure 1; Chan et al., 2014). The critical question is whether children count the squares by base-ten units (e.g. counting 42: 10–20–30–40–41–42), or make errors such as treating all the units as ones (e.g. counting 42: 1–2–3–4–5–6), treating all the units as tens (e.g. counting 42: 10–20–30–40–50–60) or counting only the ones blocks and ignoring the rest. To encourage counting by base-ten units, Chan et al. (2014) chose quantities that were too high to be counted easily by ones, and presented line drawings that were so small it would be difficult to tag each small square individually. Despite this, we found that some children attempted to count by ones and became fatigued and frustrated. Therefore, we added a five-minute introduction to three-dimensional base-ten blocks to help children understand what objects were depicted in the line drawings and demonstrate how quickly you could count by base-ten units compared to counting by ones. After this short introduction, children were shown the first test trial and told it was a picture of the same blocks. Children were then asked to tell the experimenter how many small squares were in the picture.

The 10 test trials were presented in numerical order from smallest to largest (stimulus range = 13–526). Children were permitted to count by ones on the first trial (target = 13), but if they attempted to do so on the second trial (target = 42), they were allowed to finish, and then reminded that these blocks could also be counted by tens. Children were then allowed to count again, and the better of the two trials was scored. A similar prompt was given if children attempted to count the first hundreds trial by ones. To limit fatigue, children were stopped after the first hundreds trial if they had failed to count by base-ten units on previous tens items and continued to do so on the first hundreds item. Following Chan et al. (2014), each test trial was coded as correct or incorrect based on the child's final response of total number of blocks (maximum possible score = 10; chance = 0; $\alpha = 0.85$).

Approximate place value measures (kindergarten)

The three approximate measures—Number Line Estimation, Transcoding, and Magnitude Comparison—have acceptable reliability as part of an approximate construct in kindergartners ($H_{approximate} = 0.72$, 95% CI [0.67, 0.80], Mix et al., 2022).

Number line estimation. Children were asked to indicate where a number (e.g. 3) would be located on a blank 0–100 number line using a vertical hatch mark (Siegler & Booth, 2004). The target number was printed at the top centre of the page for each trial. There was one practice trial and 15 test trials (stimulus range = 3–96). Test trials were coded for percentage of absolute error (PAE) by measuring the distance from the hatch mark to the correct location and dividing by the scale (in this case, 100). The total score was the PAE averaged across the 15 test trials (range = 0 – 90% PAE, even-odd reliability: $r = 0.76$)

Transcoding. Transcoding (e.g. Byrge et al., 2014) is the abilty to read and write numerals. For the reading assessment, children saw a stimulus number (e.g. '23') and said its name aloud while the experimenter recorded their response (e.g. 'twenty-three'). For the writing assessment, children listened to the experimenter say a multidigit number name and then wrote the numeral they heard. Both assessments had six trials evenly divided among 2-digit, 3-digit and 4-digit numbers (stimulus range = 23–3876). Partially correct responses were not counted correct (e.g. reading 239 as 'two hundred three-nine'); however, full credit was given for written responses that involved numeral reversals (e.g. writing 3 backwards) (maximum possible score = 12; $\alpha = 0.87$).

Magnitude comparison. Children were shown two choice numerals printed on opposite sides of an 8×11-inch sheet of paper and asked which number represented the larger quantity. The 25 test trials were adapted from Mix et al. (2014) and included one to four-digit numerals (stimulus range = 3–9989) (maximum possible = 25; chance = 13; $\alpha = 0.72$).

Number Sense Screener (kindergarten)

The *NSS* (Jordan & Glutting, 2012) assessed counting skill (3 items), number recognition (4 items), number comparison (7 items), nonverbal calculation (4 items), story problems (5 items), and number combinations of addition and subtraction problems (6 items) (maximum possible = 29). Because the *NSS* is a validated, standardized assessment, we did not code individual responses and assess task reliability in the current sample; however, they reported high internal-consistency reliability from the spring of kindergarten norming sample ($\alpha = 0.86$).

Multidigit calculation (second grade)

The addition and subtraction subtests of the *Comprehensive Mathematical Abilities Test* (*CMAT*; Hresko et al., 2003) were used to assess multidigit calculation. The 48 test items are arranged in order of difficulty moving from whole number calculation (30 items) to problems with decimals, fractions, and mixed numbers (18 items). Testing stopped when children answered three items in a row incorrectly. Age-based standardised *z*-scores were derived and averaged to yield a composite calculation score ($\alpha = 0.86$).

General cognitive ability

Children's general cognitive ability was estimated in first grade using the Matrix Reasoning subtest (*WISC-V*; Wechsler, 2014). The subtest consisted of two practice items and 32 test items in which children choose a figure that completes a repeating pattern or visual analogy. In accordance with *WISC-V* testing procedures, children were stopped after three consecutive incorrect responses. Age-based standardized scores were used as a covariate in all reported analyses. Because this is a validated, standardized assessment, we did not code individual responses and assess task reliability in the current sample, but reliability from the norming sample was high ($\alpha > 0.80$).

Analysis plan

To determine whether kindergartners' place value performance predicted their second-grade CMAT scores, we conducted two path analyses controlling for general cognitive ability. The first used the latent approximate and syntactic factors identified in previous research and the second used the six individual tasks as predictors. We did not include children's *NSS* scores in the latent variable path analysis because we were interested in examining how the two syntactic and approximate place value knowledge predicted second grade *CMAT* performance above-and-beyond general cognitive ability (Matrix Reasoning). However, children's *NSS* scores were included in the path analysis with individual measures to see whether place value performance explained additional variance compared to this standardized test. We chose path analysis rather than multiple regression because the factors and measures were significantly intercorrelated (Table 2). However, we conducted a multiple regression on the individual measures as a check and the results were the same. We used full information maximum likelihood (FIML) estimation (e.g. Enders & Bandalos, 2001) to address missing data due to attrition at the second-grade testing and to reduce potential bias in the parameter estimates. The model was estimated in R (R Development Core Team, 2008) using the psych (Revelle, 2017) and lavaan (Rosseel, 2012) packages.

Results

Performance on all measures was reasonably normally distributed with skewness values between -1 and $+1$ (see Table 1 and Figure S1). Bivariate and partial correlations controlling for Matrix Reasoning performance revealed that all of the measures were significantly intercorrelated (Table 2). This finding was expected given that the kindergarten measures tapped different aspects of place value understanding, which itself has been related to calculation performance in past research.

Latent variable path analysis

We asked whether approximate place value understanding, syntactic place value understanding, or both significantly predicted second grade calculation skills using a latent variable path analysis. Model fit was excellent (RMSEA = 0.05, SRMR = 0.03, CFI = 0.99, Hu & Bentler, 1999) and it accounted for 48% of the variance in second grade

Table 1. Descriptive statistics.

Assessment	Mean	*SD*	Min-Max	Skewness	Kurtosis
Kindergarten					
B-TEN	3.71	2.77	0–10	0.56	−0.59
WN?	4.60	1.17	0–6	−0.65	0.84
EN	3.01	1.78	0–6	0.17	−0.99
TC	6.43	3.08	0–12	0.21	−0.78
NLE	17.91	8.75	3.27–46.47	0.75	0.06
MC	19.07	4.03	1–25	−0.94	1.47
NSS	24.00	3.94	9–29	−0.82	0.23
Second grade					
CMAT SS	0.00	0.92	−2.42–2.77	0.32	0.23

Note: B-TEN: Base-Ten Counting; WN?: Which Number Has __?; EN: Expanded Notation; TC: Transcoding; NLE: Number Line Estimation PAE (higher scores indicate worse performance); MC: magnitude comparison; NSS: *Number Sense Screener*; CMAT SS is a z-score composite variable.

Table 2. Bivariate and partial correlations of measures by grade.

	Kindergarten								
Variable	B-TEN	WN?	EN	TC	NLE	MC	NSS	CMAT	Matrix reasoning
Kindergarten									
B-TEN		0.41	0.58	0.69	−0.51	0.48	0.55	0.55	0.41
WN?	0.30		0.39	0.41	−0.34	0.29	0.38	0.38	0.32
EN	0.57	0.33		0.48	−0.36	0.33	0.43	0.33	0.27
TC	0.61	0.28	0.39		−0.51	0.53	0.59	0.54	0.38
NLE	−0.47	−0.29	−0.35	−0.42		−0.46	−0.54	−0.42	−0.24
MC	0.43	0.20[a]	0.30	0.48	−0.42		0.49	0.37	0.19[a]
NSS	0.43	0.25[a]	0.35	0.49	−0.46	0.42		0.49	0.43
Second grade									
CMAT	0.48	0.32	0.28	0.47	−0.38	0.33	0.40		0.44

Note: All *p's* <.001 except when denoted with [a] (*p*<.05). Partial correlations with matrix reasoning as a covariate are presented below the diagonal and bivariate correlations are presented above the diagonal. B-TEN: Base-Ten Counting; WN?: Which Number Has __?; EN: Expanded Notation; TC: Transcoding; NLE: Number Line Estimation PAE (higher scores indicate worse performance); MC: magnitude comparison; NSS: *Number Sense Screener*.

calculation performance. As shown in Figure 2, only kindergartners' latent approximate place value scores predicted second grade multidigit calculation, suggesting that the approximate partial knowledge acquired informally by many children (Mix et al., 2014; Yuan et al., 2019) has far-reaching developmental effects and may be a critical target for early assessment and remediation.

Individual measures path analysis

This model tested whether the six kindergarten place value measures were equally strong predictors of second grade calculation skill, irrespective of their factor membership, and controlling for *NSS* and general cognitive ability. The fully saturated model accounted for 47% of the variance in second grade calculation performance; however, only three measures were significant individual predictors (Figure 3). Two of these significant predictors (Base-Ten Counting and 'Which N Has __?') loaded onto the syntactic factor in prior work, likely because they both require explicit understanding of base-ten syntax (Mix et al., 2022). The third significant predictor was Transcoding—a measure that has been used to tap syntactic understanding in older participants, but which we conceptualized as approximate because successful reading and writing of multidigit numbers does not require an understanding of base-ten units and could be

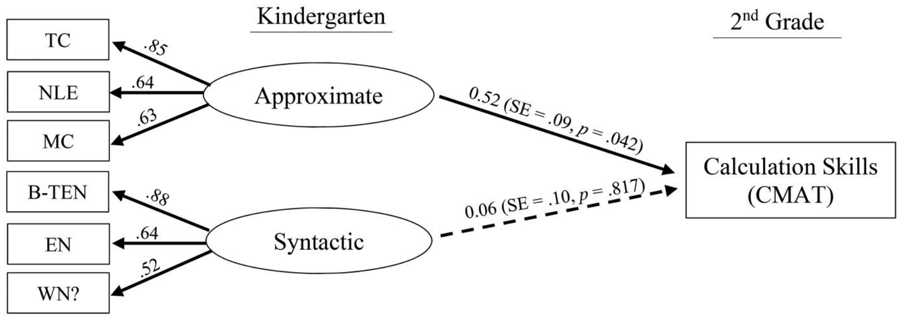

Figure 2. Latent variable path analysis results. *Note:* Kindergarten syntactic and approximate place value understanding predicting second grade multidigit calculation while controlling for matrix reasoning. Standardised coefficients (β) are presented with standard errors and *p*-values in parentheses. Although the covariance between the approximate and syntactic latent variables at kindergarten was included in the model, it is not shown in this figure. B-TEN: Base-Ten Counting task; WN?: Which Number Has __?; EN: expanded notation; TC: transcoding; NLE: Number Line Estimation (PAE; higher scores indicate worse performance); MC: magnitude comparison.

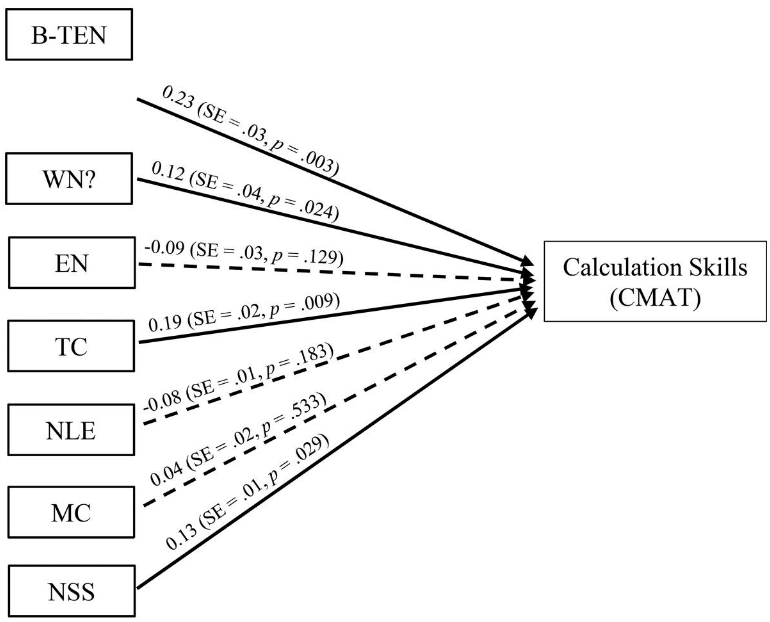

Figure 3. Individual measures path analysis results. *Note:* Kindergarten syntactic and approximate place value measures as predictors of second grade multidigit calculation. Standardised coefficients (β) are presented with standard errors and *p*-values in parentheses. Although the covariances between the tasks at kindergarten were included in the model, it is not shown in this figure. B-TEN: Base-Ten Counting; WN?: Which Number has __?; EN: Expanded Notation; TC: Transcoding; NLE: Number Line Estimation (PAE; higher scores indicate worse performance); MC: Magnitude Comparison; *NSS = Number Sense Screener.*

based on partial understanding and heuristics (Yuan et al., 2019). Thus, although performance on approximate tasks in aggregate are better predictors of second grade calculation skill (as shown in the previous analysis), at the individual task level, an approximate skill that previous work indicates becomes syntactic in first grade

(Transcoding) (Bower et al., 2022) and two syntactic skills were better individual predictors.

Discussion

Place value notation uses the spatial position of base-ten units and the multiplicative relation between these units and their counts to represent multidigit numbers. Building on research showing that children begin extracting partial understanding of this structure without formal instruction (Byrge et al., 2014; Mix et al., 2014; Yuan et al., 2019; 2020), we asked whether this early partial understanding predicts second grade multidigit calculation. Although prior longitudinal research has established such a relation starting in first grade (Chan et al., 2014; Moeller et al., 2011), the current study is the first to trace it back to kindergarten. This study is also the first to include a range of place value measures, specifically targeting both approximate and syntactic understanding. Our results provide clear evidence that, not only does kindergartners' approximate place value performance significantly predict second grade calculation skills, but also the Transcoding, Base-Ten Counting, and Which N Has___? measures are particularly reliable and powerful predictors.

Approximate place value understanding contributes to multidigit calculation

Regarding the first major aim, our findings indicate that only approximate place value understanding in kindergarten significantly predicted multidigit calculation in second grade—kindergartners' syntactic place value understanding did not. Thus, as in previous work indicating that approximate place value skills, but not syntactic place value skills, significantly predicts first grade place value understanding (Mix et al., 2022), the present findings provide evidence that early-emerging heuristics and partial understanding constitute a significant building block for learning multidigit calculation as well.

This result may seem surprising given that multidigit calculation procedures (e.g. borrowing) require precise understanding of base-ten syntax. However, tasks that permit partial understandings may be the most accessible entry point to this complex system (Cheung & Ansari, 2021; Mix et al., 2014; Yuan et al., 2019). For example, noticing that 3-digit numbers represent larger quantities than 2-digit numbers, and that 3-digit numbers are often present when the word, 'hundred,' is spoken (i.e. Transcoding) may help children discover the hundreds place and its meaning. Thus, separate pieces of structure that can be aligned by common labels or perceptual information might direct attention and guide subsequent learning, similar to what has been demonstrated in learning more generally (Gentner, 2010).

Individual place value skills contribute to multidigit calculation

Regarding our second aim, when we examined the predictive relations between the six individual place value measures taken at kindergarten and children's understanding of multidigit calculation measured in second grade, our results indicated only three significant individual predictors: (1) reading and writing multidigit numbers (Transcoding) (2) identifying the units counted at each place ('Which N Has__?' task); and (3) counting large quantities using base-ten units (Base-Ten Counting task). Interestingly, of these three measures only Transcoding loaded onto the approximate factor in prior work. However, unlike the other approximate measures, Transcoding is a symbolic task that can be performed using rough guesses about the meanings of symbol order, number of digits, and so forth. In contrast, the other two approximate measures—Magnitude Comparison and Number Line Estimation—can be performed based on a rough understanding of the magnitudes represented by multidigit numbers.

It is informative that these latter two measures were not significantly predictive on their own and indicates that the predictive power of the approximate factor, both in the present study and in previous analyses of the same dataset (Bower et al., 2022; Mix et al., 2022), may be rooted in imprecise but foundational insights about number words and written numerals that are tapped by Transcoding rather than a sense of the corresponding physical quantities. These findings lend further support to the claim that children gain access to the place value system through the symbols themselves (Yuan et al., 2019), and underscore the long-reaching effects that variation in these early symbolic understandings appear to have on children's mathematics achievement.

That said, this finding does not contradict previous work demonstrating that Magnitude Comparison and Number Line Estimation are significant predictors of later mathematics outcomes, because these previous studies focused on first grade (e.g. Dietrich et al., 2016; Xenidou-Dervou et al., 2017). Recall that by first grade, children have integrated their understanding of precise base-ten syntax with their understanding of approximate magnitude representations (Mix et al., 2022), so these prior studies likely tapped into this later integrated understanding of base-ten syntax rather than the earlier emerging approximate skills.

The finding that Base-Ten Counting (a syntactic measure) significantly predicts multidigit calculation is also consistent with previous research using first graders (Chan et al., 2014). Our results replicated and extended this finding by demonstrating that Base-Ten Counting performance is one of the strongest predictors compared to a range of other place value measures. Recall that Base-Ten Counting requires children to recognize base-ten units in block representations and count by those units to find the total number of small squares. Clearly, kindergarten children who succeed at Base-Ten Counting are on a strong trajectory towards better mathematics achievement in second grade—a finding that lends additional support to the claim that children gain entry to the place value system through the symbols themselves (e.g. Yuan et al., 2019, 2020).

'Which N Has __?'—also a syntactic task—directly queries the meanings of individual digits, in much the same way as classic interview techniques, such as Kamii's Kamii (1989) Digit Correspondence task, and as such, may have admitted partial understanding less than any of the other measures. Indeed, although Base-Ten Counting also required understanding of spoken number names, it did not require the same level of symbol reading as 'Which N Has __?' Thus, the unique contribution of 'Which N Has ___?' performance to subsequent multidigit calculation may be rooted in its symbol reading components.

The developmental role of Transcoding

Recall that children's ability to read multidigit numerals has been linked with their emerging awareness of the syntax underlying spatial position in multidigit numerals (Cheung & Ansari, 2021), and in fact, that Transcoding is a central hub in children's emerging conceptual structures around which other place value skills are organised, and its influence on the other skills increases across development as children's knowledge networks for place value become more integrated (Bower et al., 2022). Interestingly, in this latter network analysis, Transcoding shifted from clustering with other approximate measures in kindergarten to clustering with syntactic measures in first grade, which is consistent with a shift in children's approach to the task. Specifically, children may generate correct responses on the Transcoding task in kindergarten based on partial or implicit understanding of this syntax, such as guessing that the number name 'eight hundred and sixty-nine' is written '869' simply by recording the number names heard in order from left to right, without knowing that the written '8' means eight hundreds. Even so, early approximate knowledge also carries meaning because it implicitly parses the written form into 'places.'

The potential for correct responses based on implicit understanding of base-ten syntax could explain why Transcoding is one of the most sensitive measures of emergent syntactic understanding: Early in learning, being able to map between number names and written forms in both directions likely aids in understanding classroom instruction about places. Later in learning, through formal instruction, number names and strings of written digits may become linked to the individual places and their counts, which is essential knowledge for calculation. Thus, unlike other measures that directly query count + unit syntax (Base-Ten Counting, Which N Has __?, and Expanded Notation), Transcoding is more likely to invite (and reveal) emergent understanding because it simultaneously requires good guesses at base-ten structure, but does not require full-blown understanding to generate correct responses. Transcoding thus provides a context for children to bootstrap into syntactic understanding through piecemeal, not-quite-right guesses (Bower et al., 2022). The present results underscore this critical role, expanding on previous research indicating how symbols themselves act as an important entry point for children's understanding of place value notation (Yuan et al., 2019) and demonstrating that this role extends to multidigit calculation as well as place value concepts more generally.

Early place value measures as screeners

Regarding our third aim, the results of the present study highlighted three place value measures that were particularly strong early childhood predictors of multidigit calculation skill and could be used to identify children in kindergarten who may be at-risk for subsequent delays in mathematics learning (Transcoding, Base-Ten Counting, 'Which N Has__?'). The 'Which N Has __?' measure exhibited weak reliability but was nonetheless a significant predictor of multidigit calculation skills, and may therefore be useful as a screener, even if its multidimensionality is not fully understood without additional research. However, the other two measures were both strong predictors with higher reliabilities, and thus, may be particularly important tools for screening.

Of the three measures, Base-Ten Counting stands out as having the strongest predictive power; however, Base-Ten Counting takes longer to administer and score than Transcoding. More importantly, Transcoding is likely to reveal early partial knowledge of number names and their written forms—partial knowledge that appears to support the next steps in more precise understanding of base-ten principles (Bower et al., 2022). Thus, Transcoding may be useful as a short screener for kindergarten and first-grade children at-risk for mathematics learning difficulties or delays. Note that the four multidigit items on the *NSS* (Jordan & Glutting, 2012) were reading numerals.

We hypothesized that broader assessment of early variation in multidigit number understanding might explain additional variance, above and beyond existing screeners, such as the *NSS*, and our results provided support for this hypothesis. The three individual measures identified under Aim 2 accounted for 0.54 standard deviations in multidigit calculation skill combined when children's performance on the *NSS* was included in the same model. The *NSS* itself was also a strong predictor, accounting for 0.13 standard deviations. These findings suggest that early screening for children at-risk for mathematics difficulties could be enhanced by including more multidigit number items. Alternatively, teachers could use either Transcoding, Base-Ten Counting, or 'Which N Has ___?' as rapid, informal screening tools.

It is noteworthy that the four multidigit items already included on the *NSS* are Transcoding items, yet the Transcoding items we included in our battery provided significant explanatory power even controlling for children's *NSS* scores. This difference may be due to the range of numbers we used. Recall that the *NSS* included three items with 2-digit numbers and one item with 3-digit numbers. In contrast, our Transcoding measure included two items each for 2-digit, 3-digit, and 4-digit numbers, thus permitting children to demonstrate awareness of count-unit syntax for larger quantities—a feat that is entirely possible given that the hundreds and thousands places have more regular number-naming conventions in English than do the decades (e.g. three hundred vs. thirty). Another possible explanation is that our Transcoding task included reading and writing numerals, whereas the items on the *NSS* were reading only. Perhaps writing multidigit numbers has requirements that are particularly predictive of later success with multidigit arithmetic procedures, which are themselves written.

In addition to using the three predictors we identified as screeners, teachers might also use these predictors instructionally, as exercises to help children discover count + unit syntax, to explicitly link number names and written notation to counts and units, thereby promoting the transition from an implicit approximate understanding of this syntax to the explicit understanding required for calculation. For example, kindergarten students could read and write multidigit numbers for a few minutes every day as a warm-up activity. Consistent exposure to the mappings between written multidigit numerals and spoken number names may be crucial for kindergarten students who have not received this exposure informally, and the consistent feedback on accuracy could be a critical component of children's place value acquisition that prepares them to fully understand formal place value instruction in first grade.

Limitations

The present study tested whether various measures of place value skill in kindergarten predicted children's multidigit calculation skill in second grade. One limitation of this design is that prediction, while informative, is not the whole story. There remains much to be learned about how differences in early approximate knowledge moderate children's responses to instruction, or the mechanisms by which early approximate knowledge contributes to the development of syntactic place value concepts. For example, we found that children's ability to read and write multidigit numerals (Transcoding) was a strong predictor of later multidigit calculation skill, but we do not know which instructional activities or home experiences foster this early reading and writing ability. Future research using training designs or home observations will be needed to address such questions.

An additional limitation to the present design is that we included only the calculation subtests from the CMAT as outcome measures. It is possible that early understanding of place value also contributes to children's performance of other mathematics problems. For example, the same understanding of units and counts that supports learning of place value symbols might also support learning of fraction symbols or measurement symbols, as these symbols also involve counts of discrete units (Congdon et al., 2018; Newcombe et al., 2015).

Conclusions

The present results contribute to our understanding of children's struggles to learn place value by highlighting the unique contribution of reading and writing multidigit numerals (i.e. Transcoding) and implicit understanding of base-ten syntax to children's subsequent mathematics performance. Simply understanding multidigit number meanings is not all that contributes to later achievement. Instead, the present findings add to a growing body of research indicating that emergent understanding count + unit syntax and symbol-to-symbol mappings may be the main driver early on (Yuan et al., 2020).

If place value structure is so accessible to kindergarten children, why does it pose such a great obstacle instructionally? Deep understanding of base-ten syntax is likely an incremental process. Though the first steps appearing in kindergarten may be necessary foundations, they are a far cry from mastery. There may also be individual differences such that children who acquire place value easily have greater capacity for recognizing patterns and structures, or have received more caregiver input, or better structured input. Our results show that entering school with even partial knowledge of place value structure is associated with better outcomes, perhaps because such students quickly absorb subsequent mathematics instruction, leading to compounding, positive effects. However, it may be possible to rapidly build these missing understandings in children who do not exhibit partial skills at kindergarten entry if they can be identified early and offered instruction that highlights approximate number meanings and the structure of base-ten symbols.

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ORCID

Kelly S. Mix http://orcid.org/0000-0002-4402-2557
Corinne A. Bower http://orcid.org/0000-0001-7375-2842

Data availability statement

The data that support the findings of this study are publicly available on the University of Maryland's data repository (DRUM) (https://drum.lib.umd.edu/handle/1903/21495).

References

Barth, H., La Mont, K., Lipton, J., Dehaene, S., Kanwisher, N., & Spelke, E. (2006). Non-symbolic arithmetic in adults and young children. *Cognition*, *98*(3), 199–222. https://doi.org/10.1016/j.cognition.2004.09.011

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*(4), 1016–1031. https://doi.org/10.1111/j.1467-8624.2008.01173.x

Bower, C. A., Mix, K. S., Yuan, L., & Smith, L. B. (2022). A network analysis of children's emerging place-value concepts. *Psychological Science*, *33*(7), 1112–1127. 09567976211070242. https://doi.org/10.1177/09567976211070242

Byrge, L., Smith, L. B., & Mix, K. S. (2014). Beginnings of place value: How preschoolers write three-digit numbers. *Child Development*, *85*(2), 437–443. https://doi.org/10.1111/cdev.12162

Chan, W. W. L., Au, T. K., & Tang, J. (2014). Strategic counting: A novel assessment of place-value understanding. *Learning and Instruction*, *29*, 78–94. https://doi.org/10.1016/j.learninstruc.2013.09.001

Cheung, P., & Ansari, D. (2021). Cracking the code of place value: The relationship between place and value takes years to master. *Developmental Psychology*, *57*(2), 227–240. https://doi.org/10.1037/dev0001145

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Lawrence Erlbaum Associates.

Congdon, E. L., Vasilyeva, M., Mix, K. S., & Levine, S. C. (2018). From intuitive spatial measurement to understanding of units. In K. S. Mix & M. T. Battista (Eds.), *Visualizing mathematics* (pp. 25–46). Springer.

Dehaene, S., & Mehler, J. (1992). Varieties of numerical abilities. *Cognition*, *43*(1), 1–29. https://doi.org/10.1016/0010-0277(92)90030-l

Dietrich, J. F., Huber, S., Dackermann, T., Moeller, K., & Fischer, U. (2016). Place-value understanding in number line estimation predicts future arithmetic performance. *The British Journal of Developmental Psychology*, *34*(4), 502–517. https://doi.org/10.1111/bjdp.12146

Enders, C. K., & Bandalos, D. L. (2001). The relative performance of full information maximum likelihood estimation for missing data in structural equation models. *Structural Equation Modeling*, *8*(3), 430–457. https://doi.org/10.1207/S15328007SEM0803_5

Faul, F., Erdfelder, E., Buchner, A., & Lang, A.-G. (2009). Statistical power analyses using G*Power 3.1: Tests for correlation and regression analyses. *Behavior Research Methods*, *41*(4), 1149–1160. https://doi.org/10.3758/BRM.41.4.1149

Fuson, K. C. (1990). Conceptual structures for multiunit numbers: Implications for learning and teaching multidigit addition, subtraction, and place value. *Cognition and Instruction*, *7*(4), 343–403. https://doi.org/10.1207/s1532690xci0704_4

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A five-year longitudinal study. *Developmental Psychology*, *47*(6), 1539–1552. https://doi.org/10.1037/a0025510

Gentner, D. (2010). Bootstrapping the mind: Analogical processes and symbol systems. *Cognitive Science*, *34*(5), 752–775. https://doi.org/10.1111/j.1551-6709.2010.01114.x

Hancock, G., & Mueller, R. O. (2001). Rethinking construct reliability within latent variable systems. In R. Cudeck, S. du Toit, & D. Sörbom (Eds.), *Structural equation modeling: Present and future* (pp. 195–216). Scientific Software International.

Hiebert, J., & Wearne, D. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and Instruction*, *14*(3), 251–283. https://doi.org/10.1207/s1532690xci1403_1

Hresko, W. P., Schlieve, P. L., Herron, S. R., Swain, C., & Sherbenou, R. J. (2003). *Comprehensive Mathematical Abilities Test (CMAT)*. PRO-ED.

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, *6*(1), 1–55. https://doi.org/10.1080/10705519909540118

Jordan, N. C., & Glutting, J. J. (2012). *Number Sense Screener (NSS) set, K-1*. Research edition. Brookes Publishing.

Kamii, C. (1986). Place value: An explanation of its difficulty and educational implications for the primary grades. *Journal of Research in Childhood Education*, *1*(2), 75–86. https://doi.org/10.1080/02568548609594909

Kamii, C. (1989). *Young children continue to reinvent arithmetic—2nd grade: Implications of Piaget's theory*. Teachers College Press.

Mix, K. S., Bower, C. A., Hancock, G. R., Yuan, L., & Smith, L. B. (2022). The development of place value concepts: Approximation before principles. *Child Development*, *93*(3), 778–793. https://doi.org/10.1111/cdev.13724

Mix, K. S., Levine, S. C., Cheng, Y.-L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology. General*, *145*(9), 1206–1227. https://doi.org/10.1037/xge0000182

Mix, K. S., Prather, R. W., Smith, L. B., & Stockton, J. D. (2014). Young children's interpretation of multidigit number names: From emerging competence to mastery. *Child Development*, *85*(3), 1306–1319. https://doi.org/10.1111/cdev.12197

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H.-C. (2011). Early place-value understanding as a precursor for later arithmetic performance—a longitudinal study on numerical development. *Research in Developmental Disabilities*, *32*(5), 1837–1851. https://doi.org/10.1016/j.ridd.2011.03.012

Newcombe, N. S., Levine, S. C., & Mix, K. S. (2015). Thinking about quantity: The intertwined development of spatial and numerical cognition. Wiley interdisciplinary reviews. *Cognitive Science*, *6*(6), 491–505. https://doi.org/10.1002/wcs.1369

R Development Core Team. (2008). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing

Revelle, W. R. (2017). Psych: Procedures for personality and psychological research (version 1.9.11). (n.p.). https://CRAN.R-project.org/package=psych.

Rosseel, Y. (2012). Lavaan: An R package for structural equation modeling. *Journal of Statistical Software*, *48*(2), 1–36. https://doi.org/10.18637/jss.v048.i02

Schmitt, N. (1996). Uses and abuses of coefficient alpha. *Psychological Assessment*, *8*(4), 350–353. https://doi.org/10.1037/1040-3590.8.4.350

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*(2), 428–444. https://doi.org/10.1111/j.1467-8624.2004.00684.x

Wechsler, D. (2014). *WISC–V: Wechsler Intelligence Scale for Children integrated technical and interpretive manual* (5th ed.). Pearson.

Xenidou-Dervou, I., Molenaar, D., Ansari, D., van der Schoot, M., & van Lieshout, E. C. D. M. (2017). Nonsymbolic and symbolic magnitude comparison skills as longitudinal predictors of mathematical achievement. *Learning and Instruction*, *50*, 1–13. https://doi.org/10.1016/j.learninstruc.2016.11.001

Yuan, L., Prather, R. W., Mix, K. S., & Smith, L. B. (2019). Preschoolers and multi-digit numbers: A path to mathematics through the symbols themselves. *Cognition*, *189*, 89–104. https://doi.org/10.1016/j.cognition.2019.03.013

Yuan, L., Prather, R. W., Mix, K. S., & Smith, L. B. (2020). Number representations drive number-line estimates. *Child Development*, *91*(4), e952–e967. https://doi.org/10.1111/cdev.13333

Yuan, L., Xiang, V., Crandall, D., & Smith, L. (2020). Learning the generative principles of a symbol system from limited examples. *Cognition*, *200*, 104243. https://doi.org/10.1016/j.cognition.2020.104243