

Twenty-five years of the jamming phase diagram

Andrea J. Liu & Sidney R. Nagel



Twenty-five years after the proposal of a jamming phase diagram, Andrea Liu and Sidney Nagel discuss how linking jammed granular materials with glasses helps us understand the physics of many systems.

Walking along a sandy beach is a pleasant pastime that can illustrate how nature repeats similar phenomena on widely different scales. Consider: as you gaze over the terrain, you see mounds of granules in great disarray – an array with an overall structure that is much more akin to a liquid than a crystal; yet as you continue to walk, the sand beneath your feet compresses but does not let you sink into the beach the way you would if you were trying to walk on liquid water. It behaves as a solid. Next, you bend over and examine an individual grain of sand, which is made of amorphous SiO_2 , the same substance as the glass used in windowpanes. You realize that the great aggregate structure of glass particles (the sandy beach) has some of the same properties as the aggregate of atoms that forms the sand grain itself. Just like the beach, the glassy grain is a solid even though the atomic structure is that of a liquid. Disorder and rigidity are inherent in both.

One grain of sand, however, is roughly 10^{18} times heavier than a single silicon or oxygen atom. At this vastly different scale, it is responsive to different sets of forces. For a grain, gravity and friction are important and thermal motion is irrelevant. For the glass, it is covalent bonding between atoms that matters, and temperature is crucial for determining when the glass loses rigidity. Despite the difference in the scale and the forces involved, however, certain properties persist.

Introducing the jamming phase diagram

The jamming phase diagram¹ was our attempt, made about 25 years ago, to reconcile on the one hand the obvious similarities that appear between glasses and granular materials and, on the other hand, how these disordered solids differ from crystals, for which the origin of rigidity has been understood for approximately a century. We proposed that disordered systems should develop rigidity near the origin of a phase diagram in which one axis is temperature, another axis is an inverse density, and a third axis is an applied load such as a shear stress. The main point of this diagram was to show that these different axes (and possibly others that could be added) are all important to a material's ability to resist flow. It provided an organizing principle with which to think about many experimental and computational observations of phenomena that occur on different scales but have similar perplexing properties^{2–5}.

Jamming as a subfield^{6,7} is an attempt to understand what is at the very heart of rigidity in a disordered material – a collection of particles that can range in size from atoms or molecules in a glass, to colloids in dried paint, soap bubbles in shaving cream, or grains of sand

on the beach. The study of jamming starts with the assumption that the formation of disordered or glassy solids is essentially a far-from-equilibrium phenomenon. In crystals, the commonly taught form of solids, great care is often taken to ensure that equilibrium is reached as the temperature is lowered from the liquid into the pristine crystalline state. Glasses are different. No matter how slowly one cools a super-cooled liquid, it never reaches a final equilibrium state because the time required to reach equilibrium grows dramatically with decreasing temperature. Thus the final state of a glass is not only disordered but far from equilibrium.

The thinking goes, if the glass is already far from equilibrium, why not take it as far from equilibrium as possible? That is, let's reach the glass state by the fastest temperature quench or pressure crush possible. This preparation method begins to resemble the creation of a sandpile, where thermal effects are totally negligible and equilibration is almost (but never quite) instantaneous; once the particles reach their first mechanically stable positions, they remain there until pushed away by other intruding grains or external stresses. In this sense, jamming in the thermodynamic limit is conceived of as an opposite pole from a crystal: it has been found to be disordered, the least stable rigid state and far from equilibrium, while the perfect crystal is ordered, the ground state and at equilibrium⁸.

The question then arises: is the rigidity of a glass in any significant way different from that of a crystal? Since the first studies of the jamming transition, a great deal of work has been devoted to this question^{6,7}. Understanding rigidity in jammed solids requires ideas that go back to James Clerk Maxwell, who was interested in finding the minimum criteria necessary for a mechanical system to be rigid. As-formed packings of nearly hard particles are close to this threshold for failure; this property has been exploited in robotics applications⁹. Jammed packings just above this threshold show power-law scaling¹⁰ and possess a unique and unexpected characteristic spectrum of elastic excitations⁶. Indeed, these excitations make the jammed state at the Maxwell limit fundamentally different from elastic solids – it cannot be described by elasticity theory at any length scale.

Using ideas from jamming

Much attention has been paid to creating wonderful and complex crystals with unusual properties. But ideas from studying the jammed state can also produce solids of unusual utility. Jammed solids are formed by quenching temperature (or minimizing energy) rapidly. Instead of merely minimizing the energy, one can simultaneously minimize the deviation of the actual response from a desired response. This enables the design of stable rigid systems that have disorder that is correlated in such a way as to produce desired exotic mechanical properties¹¹. It is reasonable that similar ideas could also be used to tune other properties, such as thermal, optical or electronic properties.

As an example, almost all naturally formed materials have a positive Poisson's ratio – that is, when the material is extended along one

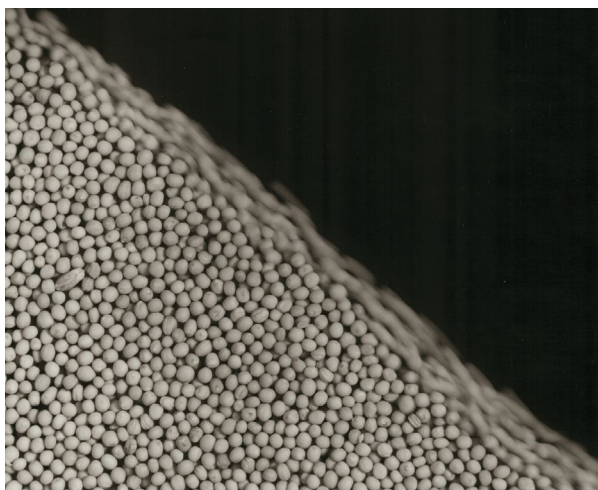
Box 1

That not-so-rare delight

Sidney R. Nagel

There as children once we played
On heaps of granules disarrayed.
It has kept our rapt attention
By flouting typical convention:
If I push it, then it flows,
If I don't, it holds its pose.
It also has this strange propensity
To not possess a constant density:
Gentle shaking will form gaps
That cause the structure to collapse.

This mundane material that lies beneath our feet
Forms a foundation where play and science meet —
Studies, springing from the simple joys of youth,
Lead to profound wonder built on sublime truth:
Just think — when walking on a beach of sand
Wet halos form where carefree footprints land.



axis it compresses along the transverse directions. Negative Poisson's ratios are allowed in elasticity theory but they are scarce in natural or synthesized materials. It turns out to be quite simple to create materials that have a negative Poisson's ratio using minimization or

training protocols; more complex responses¹¹ — including enhanced adaptability¹² — can also be attained. These ideas have led to circuits that can learn how to perform machine-learning tasks adaptively and on their own, without a computer¹³.

We have reviewed here a few of the ideas that have emerged from studying the way particles jam to form a disordered rigid structure. The subject of study emerged by realizing that glasses and granular materials were closely related and could be considered in a more unified way through the concept of the jamming phase diagram. This way of focusing on those aspects that they have in common — especially those associated with being disordered and being far from equilibrium — has led to a fruitful journey. It is astonishing to think that it was inspired simply by taking a walk on a beach (Box 1).

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References

1. Liu, A. J. & Nagel, S. R. Jamming is not just cool anymore. *Nature* **396**, 21–22 (1998).
2. Katgert, G., Tighe, B. P. & van Hecke, M. The jamming perspective on wet foams. *Soft Matter* **9**, 9739–9746 (2013).
3. Brown, E. & Jaeger, H. M. Shear thickening in concentrated suspensions: phenomenology, mechanisms and relations to jamming. *Rep. Prog. Phys.* **77**, 046602 (2014).
4. Behringer, R. P. & Chakraborty, B. The physics of jamming for granular materials: a review. *Rep. Prog. Phys.* **82**, 012601 (2019).
5. Lawson-Keister, E. & Manning, M. L. Jamming and arrest of cell motion in biological tissues. *Curr. Opin. Cell Biol.* **72**, 146–155 (2021).
6. Liu, A. J. & Nagel, S. R. The jamming transition and the marginally jammed solid. *Annu. Rev. Condens. Matter Phys.* **1**, 347–369 (2010).
7. Charbonneau, P. et al. Glass and jamming transitions: from exact results to finite-dimensional descriptions. *Annu. Rev. Condens. Matter Phys.* **8**, 265–288 (2017).
8. Goodrich, C. P., Liu, A. J. & Nagel, S. R. Solids between the mechanical extremes of order and disorder. *Nat. Phys.* **10**, 578–581 (2014).
9. Fitzgerald, S. G., Delaney, G. W. & Howard, D. A review of jamming actuation in soft robotics. *Actuators* **9**, 104 (2020).
10. Durian, D. J. Foam mechanics at the bubble scale. *Phys. Rev. Lett.* **75**, 4780–4783 (1995).
11. Huang, J. C. et al. From jammed solids to mechanical metamaterials: A brief review. *Curr. Opin. Solid State Mater. Sci.* **27**, 101053 (2023).
12. Falk, M. J. et al. Learning to learn by using non-equilibrium training protocols for adaptable materials. *Proc. Natl Acad. Sci. USA* **120**, e2219558120 (2023).
13. Dillavou, S. et al. Demonstration of decentralized physics-driven learning. *Phys. Rev. Appl.* **18**, 014040 (2022).

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Competing interests

The authors declare no competing interests.