



Understanding the cognitive processes of mathematical problem posing: evidence from eye movements

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Abstract

This study concerns the cognitive process of mathematical problem posing, conceptualized in three stages: understanding the task, constructing the problem, and expressing the problem. We used the eye tracker and think-aloud methods to deeply explore students' behavior in these three stages of problem posing, especially focusing on investigating the influence of task situation format and mathematical maturity on students' thinking. The study was conducted using a 2×2 mixed design: task situation format (with or without specific numerical information) $\times$ subject category (master's students or sixth graders). Regarding the task situation format, students' performance on tasks with numbers was found to be significantly better than that on tasks without numbers, which was reflected in the metrics of how well they understood the task and the complexity and clarity of the posed problems. In particular, students spent more fixation duration on understanding and processing the information in tasks without numbers; they had a longer fixation duration on parts involving presenting uncertain numerical information; in addition, the task situation format with or without numbers had an effect on students' selection and processing of information related to the numbers, elements, and relationships rather than information regarding the context presented in the task. Regarding the subject category, we found that mathematical maturity did not predict the quantity of problems posed on either type of task. There was no significant main group difference found in the eye-movement metrics.

Keywords Mathematical problem posing · Cognitive process · Eye movements

1 Introduction

Problem posing has long been seen as a vital intellectual activity in scientific investigation. As Einstein pointed out, the formulation of an interesting problem is often more important than its solution (Einstein & Infeld, 1938). Compared to problem solving, however,

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problem posing is a relatively new research area (Brown & Walter, 1993; Cai et al., 2015; Ellerton, 1986; Kilpatrick, 1987; Silver, 1994). Nonetheless, there have been increased efforts aimed at understanding the cognitive and affective aspects of problem posing in mathematics. The state-of-the-art research on problem posing can be summarized into three strands: problem posing as a cognitive activity, problem posing as an instructional approach, and problem posing as a goal (Cai & Leikin, 2020; Liljedahl & Cai, 2021). Despite a growing body of empirical evidence showing that mathematical problem posing is valuable as a goal in itself and as a means to accomplish multiple additional mathematical goals, such as students' mathematical understanding, problem-solving ability, creativity, and attitudes towards mathematics (Bonotto & Santo, 2015; Cai, 2022; Cai et al., 2015; Van Harpen & Sriraman, 2013), our knowledge remains relatively limited when it comes to problem posing as a cognitive activity itself, particularly the cognitive processes involved when participants generate their own problems. A better understanding of the cognitive processes of problem posing can not only help us better understand the nature of problem posing itself but also inform instruction from the view of problem posing as a goal and an instructional approach (Cai et al., 2022).

So far, what we know about problem-posing processes is largely based on analyses of students' posed problems (Cai & Hwang, 2002; Silver & Cai, 1996; Yuan & Sriraman, 2011). More recent studies have discussed the generation processes of problem posing from various angles, such as task types (Christou et al., 2005; Pittalis et al., 2004), problem-posing strategies (Cai & Cifarelli, 2005), and descriptive processes of problem posing (Baumanns & Rott, 2022). However, much more research is needed to develop a broadly applicable understanding of the fundamental processes of problem posing (Cai et al., 2022).

Prior research has shown that task format and mathematical maturity play important roles in understanding problem-posing processes. Several researchers have predicted that task format (Cai et al., 2022; English, 1998; Leung & Silver, 1997) and mathematical knowledge (Voica & Pelczar, 2010; H. Zhang et al., 2020) influence problem posers' behavior. The purpose of this study is to examine students' thinking during different stages of problem posing, especially focusing on the effect of task-situation format (with or without numbers) and subject categories (master's students or sixth graders). In particular, this study employed eye tracking to explore students' thinking involved in problem posing.

2 Theoretical considerations

2.1 The cognitive process of problem posing

A wide variety of problem-posing activities have been discussed in mathematics education (Kontorovich et al., 2012)—for example, structured, semi-structured, and open-ended problem-posing activities (Kilpatrick, 1987). Cai and Cifarelli (Cai & Cifarelli, 2005; Cifarelli & Cai, 2005) explored how two college students formulated and solved their own mathematical problems in an open-ended computer simulation task. They proposed and tested the hypothesis that individual processes in solving and posing problems are recursive, including the process of sense-making (initial reflections), formulating goals (problem posing), and achieving goals (problem solving). Pittalis et al.

(2004) proposed a model of four cognitive processes involved in problem posing: filtering and translating, comprehending, organizing, and editing quantitative information. Christou et al. (2005) built on this model to develop a taxonomy of problem-posing processes related to different types of tasks. Furthermore, Koichu and Kontorovich (2013) analyzed two success stories of two prospective mathematics teachers in the context of problem-posing tasks and identified four similar stages involved in the problem posing. Recently, Baumanns and Rott (2022) developed a descriptive phase model to understand problem-posing processes, which identified five types of activities (situation analysis, variation, generation, problem-solving, and evaluation) based on structured situation.

Despite some evidence on the cognitive processes of problem posing focused on analyzing particular types of tasks or individual processes, we have a considerably less fine-grained understanding of how students generate their problems in a more general sense. In our prior study, we conceptualized a framework of problem posing and verified it by analyzing two problem posers' thinking using think-aloud protocols on the same problem-posing task (L. Zhang et al., 2021). This framework divides the cognitive processes of problem posing into three stages: understanding the task, constructing the problem, and expressing the problem. We elaborate on each of these stages below.

The first stage, *understanding* the task, involves the cognitive process of making sense of the problem-posing situation and its prompt (Cai, 2022; Cai & Hwang, 2023). There is ample evidence that the problem comprehension process plays an important role in problem posing (Cai & Cifarelli, 2005; Christou et al., 2005; Cifarelli & Cai, 2005; Crespo & Sinclair, 2008). Analogous to the process of understanding the problem-solving task (Pólya, 1945), the stage of understanding the task involves making sense of a problem-posing situation and its prompt, including understanding the elements, given relations between elements, and the prompt that lets posers know what they are expected to do (Cai & Rott, 2023). In this study, we focused on understanding the impact of problem-posing situations on students' problem posing.

The second stage, *constructing* the problem, refers to selecting and determining which elements of the problem situation will be used and recognizing the relationships between the selected elements. Mumford et al. (1994) proposed a mental model of constructing problems that includes searching for elements from the problem-posing situation. According to Milinkovic (2015), the mental representation of the problem encompasses the problem space, within which any problem can be defined based on both the given and unknown elements and their relationships. The problem poser selects which elements will be used to construct new problems and then recognizes the relationships between the elements that have been selected (Pittalis et al., 2004). The selection of elements includes determining the specific known condition elements and specifying the target element, which may involve retaining or filtering some elements from the original problem situation (Mumford et al., 1994) or adding elements from other resources such as the poser's existing knowledge (Pittalis et al., 2004).

The final stage is *expressing* the problem, which refers to organizing the language (i.e., making use of syntax and tenses) to express the problem that was constructed in the previous stage. In psychology, it is widely accepted that language generation proceeds through a stage of constructing meaning to clarify "what to say" and a stage of organizing language to clarify "how to say" (Carroll, 1986). Expressing the problem corresponds to "how to say" the new problem based on the "what to say" that has been mentally constructed in the previous stage.

2.2 Eye-tracking methodology in mathematical problem posing

Eye tracking is an increasingly popular methodological tool in mathematics education (Barmby et al., 2014; Hartmann, 2015; Lilienthal & Schindler, 2019). In particular, eye tracking has been shown to be potentially beneficial for studying processes, revealing mental representations, and assessing subconscious aspects of mathematical thinking (Strohmaier et al., 2020). Most eye-tracking studies in mathematics education have claimed that the method allows for the assessment of cognitive processes that would otherwise not be observable. Some cognitive psychology researchers have found that subjects' eye-tracking data, such as fixations, duration of fixation, and saccades, reflect their mental representations, cognitive patterns of their attention, information selection (Chen & Zheng, 2014), the difficulty of tasks, and cognitive load (Chou & Zhou, 2011). In particular, the metrics related to fixation are undoubtedly associated with different types of cognitive processing (Holmqvist et al., 2011).

2.2.1 Eye-tracking methodology employed to track the process of problem posing

Some researchers have used paper-and-pencil tests (Cai & Hwang, 2002; Silver & Cai, 1996), interviews (Nicolaou & Philippou, 2007), or the think-aloud method (Cai & Cifarelli, 2005; Kontorovich et al., 2012) to track problem-posing processes. These studies help us understand the cognitive processes of problem posing to some extent. Magliano and Graesser (1991) proposed the collection of other kinds of data to increase the reliability of the think-aloud report, such as eye-tracking data. The eye-tracking method has the advantage of monitoring subjects' cognitive processes. It is regarded as an alternative tool to gain deeper insight into individuals' thinking (Pieters et al., 2002). However, to the best of our knowledge, no studies have explored the cognitive process of problem posing using eye tracking. The eye-tracking method allows online objective recording of dynamic eye movements throughout the participants' involvement in performing the entire problem-posing task (Land & Tatler, 2009). In addition, it enables the segmentation of the entire time taken by participants engaged in a given problem-posing task (Land & Tatler, 2009), which allows us to deconstructively explore the possible patterns of participants' problem posing at particular stages.

Most of the studies that applied the eye-tracking method in exploring the cognitive process of mathematical problem solving distinguished the eye-tracking data at the stages of problem comprehension and preparing a solution. For example, Werner and Raab (2014) measured the gaze behavior of participants during the first 3 seconds of solving a problem, which is considered to be the first phase of problem solving, representing the task. Compared to an artificially set time limit of 3 seconds to separate the data of each stage of the cognitive process, the study of Hegarty et al. (1992) recorded the overall time of two stages in mathematical problem solving, the problem translation stage and the problem integration and solution planning stage. However, the stages of problem translation and problem integration were united as the process of understanding the problem, meaning that they did not separate the eye-tracking data between the process of understanding the problem and the process of making the plan to solve the problem. Chen and Zheng's (2014) study of undergraduate students' eye movements during creative science problem posing solved this obstacle via a unique method that was applied in the present study as well. The method involved presenting the same identical visual representation of the problem-posing task twice on the computer screen to participants, asking them to signal the moment at which they had finished the understanding phase and began planning the solution (see Fig. 1). This method was employed with the aim of separating the

eye-tracking data into different stages and controlling for potential confounding factors (like differences in the visual content) that could influence eye movements.

2.2.2 Eye-tracking methodology employed to explore the influence of the problem-posing situation

Research has shown that several different task variables, such as the number and length of words in the problem, can influence problem difficulty and problem comprehension (Goldin & McClintock, 1984; Reusser, 1986) and are highly associated with the stage of understanding problem-posing tasks. Similarly, several researchers have investigated the influence of task situation format on problem posing (English, 1998; Leung & Silver, 1997). For example, Leung and Silver (1997) found that preservice teachers' problem-posing performance was better when the task contained specific numerical information than when it did not. Although they predicted that the task situation format would affect problem posers' behavior, their findings provided scant evidence about how the task situation format influences this behavior. In particular, little is known about how participants process particular areas of information present in a problem-posing task situation.

The eye-tracking method has the advantage of recording participants' eye movements on defined areas of interest (AOIs) present in the tasks (Kekule et al., 2019), which allows exploration of the influence of task situation format on participants' problem posing as well as participants' tendency to select and process particular areas of information involved in problem posing. The text of a mathematical problem-posing situation can be categorized into AOIs, including elements, relations between the elements, value, and unit of the elements or relations, and context (Littlefield & Rieser, 1993; Milinkovic, 2015). In particular, uncertain numerical information is more difficult to process than specific numerical information (Just & Carpenter, 1980). Hegarty et al. (1995) found that students took more time doing an initial reading and rereading more words for inconsistent information (considered as more difficult information) than for consistent information when they engaged in solving an arithmetic word problem. Therefore, it's reasonable to hypothesize that participants might give more attention to processing a task situation without numbers (presenting uncertain numerical information) than that with numbers (presenting specific numerical information); in particular, the impact of task situation format on problem posing might be reflected in participants' attention towards processing the area of numerical information, which may yield varying levels of cognitive difficulty (Liversedge & Findlay, 2000).

2.2.3 Eye-tracking methodology employed to explore problem posing of two groups of students with different mathematical maturity levels

Several researchers have explored the difference between experts and novices in mathematical problem posing. Voica and Pelczer (2010) compared problems posed by preservice students (considered novices) and in-service teachers (considered experts) and found that teachers' pedagogical knowledge and classroom experience could constrain and shape their view of the problems posed. Singer and Voica (2017) developed a framework for comparing experts and novices in complex situations which combined problem posing, problem solving, and modelling. However, through case analysis, they admitted that in some cases it was impossible to make clear

distinctions between novice-expert or creative-uncreative students. Kontorovich (2020) argued that people who systematically create problems for high-level mathematics competitions could be considered experienced problem posers and the problems posed by them suitable for determining who the experts are and what attributes they need to pose problems. Overall, no clear definition has been generally agreed upon for who problem-posing experts are.

Mathematical maturity is typically defined as consisting of a combination of mathematical knowledge, problem-solving skills, and a deep appreciation of the discipline of mathematics, which generally increases with the level of mathematical education and experience (Steen, 1983). Several studies have suggested that mathematical maturity might influence students' problem-posing expertise (Kwek, 2015; H. Zhang et al., 2020). However, few empirical studies have examined if mathematical maturity is an important component of problem-posing expertise. Therefore, this study examined the community of mathematics masters' students and primary school students to gain more empirical evidence on the impact of mathematical maturity on problem posing and problem-posing expertise.

Exploring underlying cognitive processes behind expert performance is hard with traditional behavior analysis methods (e.g., observation, think aloud). There is a need to use more objective methods to understand expertise dynamics (Dogusoy-Taylan & Cagiltay, 2014). The eye-tracking method can provide information where two or more participant groups are compared, such as in the expert-novice paradigm (Gegenfurtner et al., 2011). Several studies have used the eye-tracking method to investigate possible characteristics of successful mathematical problem solvers. In terms of problem comprehension, Hegarty et al., (1992, 1995) found that unsuccessful problem solvers seem to use a comprehension strategy that emphasizes looking at numbers and relational terms and successful problem solvers devote a greater percentage of their fixations to variable names than do unsuccessful problem solvers. In terms of identifying and selecting relevant information during the stage of constructing problem, Littlefield and Rieser (1993) proposed that expert problem solvers might have more skill at noticing higher-order relations among the key elements in a problem situation, and they may use this knowledge to constrain their searching process in ways that increase the efficiency and accuracy of their performance. Junior et al. (2017) demonstrated that the most proficient chess players fixed their gazes for a longer time in only the important or key region, which seemed to contribute to their high performance. Therefore, it's reasonable to hypothesize that expert and novice problem posers might have different comprehension strategies and information-identifying and selection strategies so as to make the connections to construct a new problem; in particular, they might distribute varied proportions of their attention towards particular areas of information involved in problem posing.

2.3 Research questions

In this study, we aimed to understand the cognitive process of mathematical problem posing by means of eye tracking. In particular, we aimed to explore the influence of task situation format and mathematical maturity on students' cognitive process while problem posing, including how they understand the task, construct the problem, and express the problem. Specifically, we focused on the following two research questions:

RQ1: What is the influence of task situation format (with or without numbers) on students' problem posing as measured by their performance with respect to the posed problems and by eye tracking?

RQ2: Is there any difference in students' problem posing among master's students and sixth graders as measured by their performance with respect to the posed problems and by eye tracking?

3 Method

3.1 Subject and design

Two groups of subjects participated in this study. We recruited 66 master's students majoring in pure mathematics or mathematical education (note: all with a bachelor's degree in pure math) at a university and 60 sixth graders randomly chosen from 11 classes of a primary school. We labeled the former group as the master's student group and the latter group as the sixth grader group. The reason for selecting these two groups was to increase the degree of difference between the mathematical maturity of the groups and reduce the influence of other factors such as teaching experience on the problem of heterogeneity as much as possible (Kontorovich, in press; Weber et al., 2020). Two mixed-design analyses were conducted with a 2 (task situation format: with numbers, without numbers) $\times$ 2 (subject category: master's students, sixth graders) design. All subjects were presented with two types of problem-posing tasks.

3.2 Materials

The materials consisted of four problem-posing tasks containing two training tasks and two target tasks (see Table 1 and Appendix 1, Table 10). The training tasks were meant to allow the subjects to become familiar with the procedures of the eye-tracking study. For examining the influence of the two different task situation formats on students' thinking, two target tasks were chosen from the test of problem-posing tasks with or without numbers (PPTask-number test). The test consisted of four test items which were translated versions of tasks used by Leung and Silver (1997) to detect the impact of task situation format on problem posing. The English version was first translated into Chinese (standard Mandarin) by a research assistant who is literate in both Chinese and English. The

Table 1 Test items and subjects

		PPTask-number			
		House purchase		Pool maintenance	
		1A	1B	2A	2B
Master's Students	$P_1 = 30$	✓			✓
	$P_2 = 36$		✓	✓	
Sixth Graders	$N_1 = 30$	✓			✓
	$N_2 = 30$		✓	✓	

Format A in the PPTask-number test is the task with numbers (number on); Format B in the PPTask-number test is the task without numbers (number off)

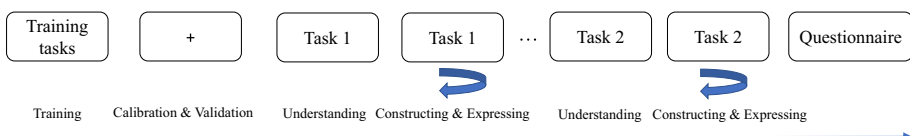
Chinese version of this test was then reviewed by two experts who are literate in both Chinese and English. In addition, a pilot study using these two tests was conducted (L. Zhang et al., 2021).

3.3 Apparatus

The study used the EyeLink-II eye tracker produced by SR Research of Canada. The device consists of two computers connected by Ethernet. One computer presents the experimental materials, and the other computer records the eye-movement data. The experimental materials were presented using a 19-inch display with a refresh rate of 85 Hz and a resolution of 1024*768 pixels. The sampling rate of the eye tracker was 1000 Hz. A forehead and chin rest kept the viewing distance constant and minimized head movements. Participants sat 70 cm from the monitor screen, resulting in a 29° horizontal × 22° vertical visual field. Before each task, a standardized calibration procedure comprising nine white dots randomly appearing on a 3 × 3 black display was undertaken. The presentation of experimental materials and data recording were all run using self-edited Eprime codes. In addition, a voice recorder recorded the think-aloud data generated while the participants posed problems.

3.4 Procedure

The experiment was conducted in a soundproof and uniform light laboratory with voice recordings. The participants were given a description of the eye-tracking apparatus and the experimental procedure along with brief instructions that they were required to report their posed problem facing the screen while completing the problem-posing tasks. Then, participants were asked to adjust the seat height so that they were able to comfortably rest their chin on the chinrest and their forehead against the forehead rest. This would decrease the eye-tracking recording errors involved in self-orally reporting the posed problem. They then underwent a training phase to get familiar with looking at the screen while still, proactively starting or ending each trial, and reporting orally while posing problems via two simple problem-posing tasks. Before presenting the first target task, a calibration and validation procedure was conducted to adjust the eye tracker. Afterwards, each subject was randomly assigned to one of the tests (Test 1: 1A & 2B; Test 2: 1B & 2A) mentioned in Table 1 and tested individually (the instructor then left the subject and entered the eye-tracking data collection room in which the monitor of participants' real-time eye movements was displayed). Each subject independently pressed relevant buttons to begin



Note: The circular arrow at the constructing and expressing stages means that participants can individually decide to pose the next problem or not. If they choose to pose the next problem on this task, the screen will present the content of the same task; if not, the screen will present the content of the next task.

Fig. 1 The procedure of the study

and end each trial according to the training experience. The procedure was repeated for the two tasks. After the test was completed, each subject was required to fill out a questionnaire (presented in Appendix 1, Table 11) about their perception of how well they understood the tasks, which relates to the understanding stage of problem posing. No time limitations were imposed in this experiment. The specific procedure is shown in Fig. 1. The content presented on the computer screen included the text of the problem-posing task as well as different prompts guiding participants to independently end each stage and start the next. The content of the task for the constructing and expressing stage shown in the screen was the same as that for the understanding stage. In addition, to gain a better understanding of the subsequent results, we randomly chose and individually interviewed six master's students after the study. We asked them what their biggest challenge was when posing the problems.

3.5 Data analysis

3.5.1 Responses and post-interview

To examine the subjects' problem-posing performance, the responses of the 126 subjects who completed the problem-posing tasks¹ and the questionnaire were analyzed with respect to an overview and the three stages of problem posing (L. Zhang et al., 2022). We will first present an overview of the subjects' responses including the number of responses, number of mathematical problems, and number of solvable mathematical problems, which to some extent reflects participants' problem-posing fluency (Bicer et al., 2020). Then, the types and sources of data corresponding to the three stages of problem posing are shown in Table 2. According to the theoretical problem-posing framework discussed previously, we examined the following metrics and their corresponding problem-posing stages: The "how well subjects understood the task" metric corresponds to the understanding the task stage; the "largest sum of relations and/or elements" metric, which describes the problem space of the most complicated posed problem, corresponds to the constructing the problem stage; and the "clarity of the posed problem" metric corresponds to the expressing the problem stage.

All problem-posing responses of the 126 subjects were coded by a rater. A second rater coded a stratified subsample of the data (30 subjects: 15 master's students and 15 sixth graders) that was balanced with respect to task situation format (with/without numbers). The double rating of responses on the problem-posing test provided the basis for determining inter-rater reliability, which ranged from 85 to 93% (the inter-rater agreement that was less than 90% (85%) was for coding the index of "problem expressed clearly or not"). In addition, the inter-rater agreement of the data analysis for the same test used in the prior research study (L. Zhang et al., 2022) ranged from 92 to 100%. Regarding the data of the post-interview, we categorized the challenges the interviewees encountered when posing problems.

3.5.2 Areas of interest

To analyze subjects' information processing regarding particular kinds of information involved in problem posing, we considered three areas of interest (AOI). Milinkovic (2015)

¹ An example of the problems posed by a sixth grader and a master's student on the House Purchase Task is shown in Appendix 3.

Table 2 Definition of areas of interest (AOI)

AOI	Definition (Milinkovic, 2015; Littlefield & Rieser, 1993)
Context	- The agents (i.e., persons/things carrying out the action, for example, “Mr. Wang”) - The actions (i.e., “purchase a house”) - The time and place of the actions (i.e., “later” and “the park district”)
Element & relation	- The elements (i.e., the objects acted on, for example, “insurance,” “interest,” “the rest of the payment”) - The relations (i.e., the connection between objects, for example, “reduce”) (Note: because there is little information related to relations in the House Purchase and Pool Maintenance tasks, we combined the elements and relations as one AOI)
Number	- The values and the unit of the elements/relations (i.e., “RMB5,000 per year” in the task situation format with numbers and “a certain amount per year” in the task situation format without numbers, and “by 15%” in the task situation format with numbers and “by a certain percentage” in the task situation format without numbers)

proposed that any problem can be described in terms of its context, givens and unknown elements, and the relationships between the elements. The problem-posing tasks contained specific information relevant to their context, elements, and the relationships between their elements. In this study, we categorized the information presented in the problem-posing tasks into three parts or three AOI: *context*, *element & relation*, and *number*. These are shown in Table 2.

3.5.3 Eye movement measures

Four measures of eye fixations were examined in this study (see Table 3). *Total fixation duration* refers to the sum of the duration for all fixation within all AOI. This metric is very sensitive to longer cognitive processing (Holmqvist et al., 2011). The longer the fixation duration, the more information extraction and more effortful the cognitive processing produced. *The percentage of total fixation duration* refers to the ratio of total fixation duration on each of the AOI to total fixation duration. It indicates the extent to which subjects rely on the information from the target area when executing the task—that is, the subjects’ tendency to proactively select and process information from that area (Tsai et al., 2012). *First fixation duration* is a common early processing measure and refers to the duration of the first fixation on a target area (Yan et al., 2013). This metric typically reflects the time taken for fast processes such as recognition and identification of a certain area (Holmqvist et al., 2011). Finally, the *heatmap* is calculated by averaging the fixation duration on any location in each stage at the pixel level, which is a data-driven approach to investigate which area of information is the most important for the subjects throughout the entire process.

3.5.4 Statistical analysis

A multi-factor analysis was conducted to examine the task situation format and group effect on students’ problem-posing performance. To analyze the eye-movement data, the fixation level recordings were assessed using DataViewer software (SR Research

Table 3 The types and sources of data corresponding to problem posing

Stages of problem posing	Data source	Type of data
Overview	Performance	-The number of responses (Silver & Cai, 1996) -The number of mathematical problems (Silver & Cai, 1996) -The number of solvable mathematical problems (Silver & Cai, 1996)
Understand	Performance <i>Questionnaire</i> Eye movement	How well subjects understood the task situation (Schoenfeld, 1985) -Total fixation duration (i.e., the sum of the duration for all fixation within all AOI) -Percentage of total fixation duration on each of the AOI (i.e., total fixation duration on each of the AOI divided by the total fixation duration) -First fixation duration on each of the AOI (i.e., the duration for which the participant fixated on each of the AOI for the first time) -Heatmap (i.e., fixation duration on any location in the visual stimuli at the pixel level)
Construct	Performance <i>Responses</i> Eye movement	-The sum of the elements and relationships^a in the most complicated problem posed (Note: We selected the problem with the largest sum of the elements and relationships as the most complex problem, which represents the subject's ability to select more elements and connect more relationships between the elements to construct a more complicated problem) -The sum of the relationships in the most complicated problem posed -The sum of the elements in the most complicated problem posed -The total fixation duration on the first problem posed (i.e., the sum of the duration for all fixation within all AOI for the first problem posed) -The percentage of total fixation duration on AOI for the first problem posed (i.e., total fixation duration on each of the AOI divided by the total fixation duration on the first problem posed) -First fixation duration on AOI for the first problem posed (i.e., the duration for which the participant fixated on each of the AOI for the first time on the first problem posed) -Heatmap (i.e., fixation duration on any location in the visual stimuli at the pixel level)
Express	Performance <i>Responses</i> Eye movement	-The number of problems expressed clearly (Note: Ambiguous problems were those that were unclear about what the problem meant or could result in varying interpretations of the same posed problem) -Combined with data from the stage of constructing the problem (Note: Here, we did not divide eye-tracking data between the stages of constructing and expressing the problem because we only paid attention to whether the subjects were able to express clearly in the stage of expressing the problem; in addition, for some subjects and some posed problems, the time between constructing and expressing the problem was too short)

^aAn example for coding the sum of elements and relationships is shown as follows: Data: "How much can I reduce the monthly heating bill?" (student's posed problem). Coding: the number of elements in this problem (2, the known element "original monthly heating bill, 200RMB" and the unknown element "the reduced monthly heating bill"), the number of relationships in this problem (1, known relationship "reduce 15% heating bill each month"); thus, the sum of elements and relationships of this problem is 3 (2 elements + 1 relationship)

Ltd). The *first fixation duration* was calculated as the sum duration of the first fixation on each word based on AOI segmentation which reflected the initial information processing speed on each of the AOI. The *percentage of total fixation duration on AOI* was derived by: $\frac{\text{Total fixation time on AOI}}{\text{Total fixation time on stimulus}}$. Before counting these two metrics, the fixations falling into regions between words (or larger than two words) were deleted. We then normalized the metric of the first fixation duration using log transform and the metric of the percentage of total fixation duration using Z-score analysis. The samples with abnormalities on these two metrics mentioned above were excluded ($\text{Mean} \pm 2.5 \text{ Sd}$). An ANOVA analysis was conducted to investigate the main group effect, task situation format effect, and the interaction effect between them. The entire data analysis was run on R. Data cleansing was accomplished through the tidyverse package, and the ANOVA model comes from the ezANOVA package. The FDR method was used to control the false positive rate due to multiple comparisons.

The iMap Matlab Toolbox (Caldara & Miellet, 2011) was used to generate the heat-maps for each subject group and difference maps for comparisons between the group and task situation formats. Specifically, the fixation map was separately calculated by summing the fixation duration at the fixation location coordinates of each trial across all valid trials for each group $\times$ task situation format. We then applied a Gaussian kernel function analysis to spatially smooth each fixation map and normalized the data by Z-score. To examine the difference in fixation patterns between groups and task situation formats, a linear mixed model in the iMAP toolbox was used to draw the statistical maps of fixations on any location in the visual stimuli at the pixel level. Finally, a robust statistical FDR-based Threshold-Free Cluster Enhancement (TFCE) approach was applied to calibrate for large multiple comparisons (corrected $p < 0.05$), which is widely used in functional magnetic resonance imaging (fMRI).

4 Results

4.1 Relationship between subjects' problem-posing performance and task situation format

Table 4 shows the mean performance of the sixth graders and master's students on the measures related to the three stages of problem posing. With respect to the expressing the problem stage, the results of a multi-factor analysis of variance indicated a moderately large significant task situation effect for the pool maintenance task ($F = 11.186^{***}$, $\eta_p^2 = 0.084$). In addition, we found that the mean performance on the task situation format with numbers was significantly higher than that on the task situation without numbers for the four measures corresponding to the indices of the constructing the problem stage. The partial eta squared ranged from 0.20 to 0.38 on both tasks (house purchase and pool maintenance, marked in bold text in Table 4), which indicates a large and consistent task situation format effect on students' performance. In understanding the task stage, the results showed that subjects could understand significantly better the task situation format with numbers compared to that without numbers. The group effect on this measure was not significant.

Table 4 Relationship between subjects' problem-posing performance and task situation format

Stages	Indexes	House Purchase			Pool Maintenance		
		Number on	Number off	Group Effect ^a	Number on	Number off	Group Effect
<i>Understand</i>	<i>Understanding</i>						
	Master's students	3.23 (0.73)	3.00 (0.63)	$F=0.853$	3.31 (0.53)	2.70 (0.79)	$F=1.781$
	Sixth graders	3.20 (0.66)	2.80 (0.81)	$\eta_p^2=0.007$	3.50 (0.78)	2.83 (0.65)	$\eta_p^2=0.014$
	<i>Situation format effect^b</i>	$F=6.282^*$, $\eta_p^2=0.049$			$F=26.827^{***}$, $\eta_p^2=0.180$		
<i>Construct</i>	<i>The largest sum of relationships and elements</i>						
	Master's students	4.17 (3.97)	0.81 (2.11)	$F=2.136$	6.22 (3.77)	1.50 (3.65)	$F=0.205$
	Sixth graders	5.27 (3.74)	1.33 (2.40)	$\eta_p^2=0.017$	6.10 (2.82)	1.10 (2.34)	$\eta_p^2=0.002$
	<i>Situation format effect</i>	$F=42.896^{***}$, $\eta_p^2=0.260$			$F=70.887^{***}$, $\eta_p^2=0.368$		
	<i>The largest sum of relationships</i>						
	Master's students	1.37 (1.27)	0.25 (0.65)	$F=1.850$	1.83 (1.34)	0.40 (0.97)	$F=0.307$
	Sixth graders	1.73 (1.26)	0.37 (0.67)	$\eta_p^2=0.015$	1.73 (0.87)	0.30 (0.65)	$\eta_p^2=0.003$
	<i>Situation format effect</i>	$F=48.844^{***}$, $\eta_p^2=0.286$			$F=63.089^{***}$, $\eta_p^2=0.341$		
	<i>The largest sum of elements</i>						
	Master's students	2.80 (2.70)	0.56 (1.48)	$F=2.257$	4.39 (2.50)	1.10(2.68)	$F=0.160$
	Sixth graders	3.53 (2.49)	0.97 (1.73)	$\eta_p^2=0.018$	4.37 (1.96)	0.80 (1.69)	$\eta_p^2=0.001$
	<i>Situation format effect</i>	$F=39.883^{***}$, $\eta_p^2=0.246$			$F=72.257^{***}$, $\eta_p^2=0.372$		
<i>Express</i>	<i>Problems expressed clearly</i>						
	Master's students	2.30 (1.69)	1.92 (1.80)	$F=4.330^*$	2.56 (1.52)	1.57 (1.57)	$F=8.248^{**}$
	Sixth graders	3.30 (1.69)	2.77 (4.06))	$\eta_p^2=0.034$	5.37(6.37)	2.27 (1.86)	$\eta_p^2=0.063$
	<i>Situation format effect</i>	$F=1.063$, $\eta_p^2=0.009$			$F=11.186^{**}$, $\eta_p^2=0.084$		

^aAccording to Cohen (1988)'s partial eta squared, 0.01 is considered a small effect, 0.06 is considered a medium effect, and 0.14 is considered a large effect (*number marked in bold means the effect size was large*); $***p < 0.001$. $**p < 0.01$. $*p < 0.05$

^b"Situation format" refers to "task situation format"

4.2 Relationship between subjects' problem-posing eye movement and task situation format

4.2.1 Total fixation duration

Table 5 shows students' total fixation duration during each problem-posing stage. For the house purchase task, there was a main effect of the task situation format on both stages of students' problem posing. It took longer for students to understand and construct the problems on the tasks without specific numerical information; the mean fixation duration of master's students on the task situation format without numbers and with numbers was 78.60 s and 47.09 s, respectively. In addition, we found that the fixation duration for students' constructing and expressing the problems was shorter than that for their understanding the problems. Meanwhile, the analysis of variance did not reveal a main effect of group on any of the problem-posing stages. For the pool maintenance task, the master's students had greater duration during both problem-posing stages than the sixth graders (understanding stage: $F = 7.294^{**}$, $\eta_p^2 = 0.056$; constructing and expressing stage: $F = 4.901^*$, $\eta_p^2 = 0.039$).

4.2.2 Percentage of total fixation duration on AOI

The panels of Fig. 2 provide graphs of the mean percentages of total fixation duration on each AOI for master's students and sixth graders on the task situation formats with and without numbers during the understanding stage. On the *number* AOI (see Table 6 and Fig. 2 (left)), there was a significant task situation format difference, $F(1, 120) = 23.63$, $p < 0.001^{***}$, $\eta_p^2 = 0.165$. Pairwise comparisons of the task situation format with and without numbers showed that students' percentage of total fixation duration on the number AOI of the task situation format without numbers was significantly higher than that on the task situation format with numbers, $t = 4.86$, $p < 0.001^{***}$, Cohen's $d = 0.88$. On the *element & relation* AOI (see Table 6 and Fig. 2 (middle)), there was a significant main difference in task situation format effect ($F(1, 121) = 41.58$, $p < 0.001^{***}$, $\eta_p^2 = 0.256$) and an interaction effect between group and task situation format ($F(1, 121) = 4.42$, $p < 0.05^*$, $\eta_p^2 = 0.035$). Students' percentage of total fixation duration on element and relation AOI of the task situation format with numbers was significantly higher than that of the task situation format without numbers ($t = 6.45$, $p < 0.001^{***}$, Cohen's $d = 1.16$). In particular, the percentage of total fixation duration for sixth graders was significantly higher than that for master's students on the task situation format with numbers, $t = 2.51$, $p < 0.05^*$, Cohen's $d = 0.65$. On the *context* AOI (see Table 6 and Fig. 2 (right)), there was no significant difference for group, task situation format, or the interaction between group and task situation format effect.

For the constructing and expressing the problem stage, on the *number* AOI (see Table 7 and Fig. 3 (left)), the main effect of the task situation format ($F(1, 120) = 19.26$, $p < 0.001^{***}$, $\eta_p^2 = 0.138$) and the interaction effect between group and task situation format ($F(1, 120) = 7.87$, $p < 0.01^{**}$, $\eta_p^2 = 0.062$) were significant. In particular, the master's students' percentage of total fixation duration on the task situation format without

Table 5 Relationship between subjects' problem-posing *Total fixation duration* and task situation format

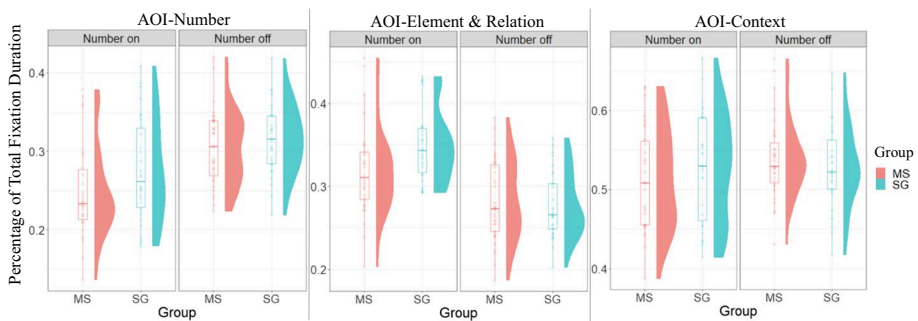
Stage (unit: second)		House purchase			Pool maintenance		
		Number on	Number off	Group effect ^a	Number on	Number off	Group effect
Understand	Master's students	47.09 (17.9)	78.60 (46.2)	$F=0.751$	65.04 (32.0)	85.93 (39.9)	$F=7.294^{**}$
	Sixth graders	49.65 (32.4)	64.46 (43.9)	$\eta_p^2=0.006$	59.37 (27.3)	59.95 (30.9)	$\eta_p^2=0.056$
	<i>Situation format effect^b</i>	$F=12.014^{**}, \eta_p^2=0.09$			$F=3.358, \eta_p^2=0.027$		
Construct & Express	Master's students	27.59 (15.3)	38.7 (33.3)	$F=2.188$	25.69 (11.0)	39.38 (28.3)	$F=4.901^*$
	Sixth graders	17.66 (7.3)	35.70 (28.4)	$\eta_p^2=0.018$	25.9 (16.0)	24.17 (17.7)	$\eta_p^2=0.039$
	<i>Situation format effect</i>	$F=11.350^{**}, \eta_p^2=0.085$			$F=3.119, \eta_p^2=0.025$		

^aAccording to Cohen (1988)'s partial eta squared, 0.01 is considered a small effect, 0.06 is considered a medium effect, and 0.14 is considered a large effect (*number marked in bold means the effect size was large*); *** $p < 0.001$. ** $p < 0.01$. * $p < 0.05$

^b"Situation format" refers to "task situation format"

Table 6 Metrics calculated for *percentage of total fixation duration* and *task situation format* at each of the AOI for the *understanding stage* of problem posing^d

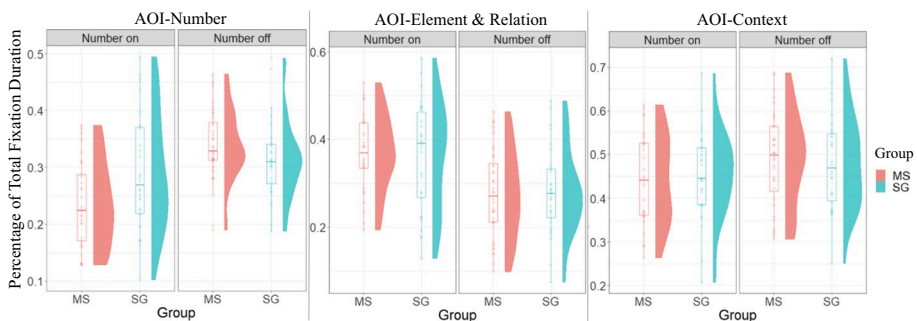
AOI	Effect	MS	MSE	df_1	df_2	F	p	η_p^2	[90% CI]
<i>Number</i> ^a	Group	0.01	0.00	1	120	3.72	0.056	0.030	[0.000, 0.097]
	Situation format ^e	0.07	0.00	1	120	23.63	<0.001***	0.165	[0.075, 0.265]
	Group×situation format	0.00	0.00	1	120	0.68	0.411	0.006	[0.000, 0.048]
<i>Element & relation</i> ^b	Group	0.01	0.00	1	121	2.28	0.134	0.018	[0.000, 0.076]
	Situation format	0.09	0.00	1	121	41.58	<0.001***	0.256	[0.152, 0.358]
	Group×situation format	0.01	0.00	1	121	4.42	0.038*	0.035	[0.001, 0.015]
<i>Context</i> ^c	Group	0.00	0.00	1	120	0.08	0.778	0.001	[0.000, 0.025]
	Situation format	0.00	0.00	1	120	1.34	0.250	0.011	[0.000, 0.061]
	Group×situation format	0.00	0.00	1	120	1.35	0.248	0.011	[0.000, 0.062]

^aThe specific information is shown in Fig. 2 (left)^bThe specific information is shown in Fig. 2 (middle)^cThe specific information is shown in Fig. 2 (right)^dThe number marked in bold means the corresponding effect was significant^e“Situation format” refers to “task situation format”**Fig. 2** Percentage of total fixation duration on each of the AOI for the understanding stage of problem posing

numbers was significantly higher than that on the task situation format with numbers, $t=5.25$, $p<0.001^{***}$, Cohen's $d=1.30$. The *element & relation* AOI (see Table 7 and Fig. 3 (middle)) showed a significant main effect on task situation format ($F(1, 121)=22.50$, $p<0.001^{***}$, $\eta_p^2=0.157$). Students' percentage of total fixation duration on the task situation format with numbers was significantly higher than that on the task situation format without numbers ($t=4.74$, $p<0.001^{***}$, Cohen's $d=0.85$). On the *context* AOI (see Table 7 and Fig. 3 (right)), there was no significant difference for group, task situation format, or the interaction between group and task situation format effect.

Table 7 Metrics calculated for *percentage of total fixation duration* and *task situation format* at each of the AOI for the *constructing & expressing* stage of problem posing^d

AOI	Effect	MS	MSE	df_1	df_2	F	p	η_p^2	[90% CI]
<i>Number</i> ^a	Group	0.01	0.01	1	120	1.27	0.261	0.010	[0.000, 0.060]
	Situation format^e	0.12	0.01	1	120	19.26	<0.001***	0.138	[0.056, 0.236]
	Group×situation format	0.05	0.01	1	120	7.87	0.006**	0.062	[0.010, 0.143]
<i>Element & relation</i> ^b	Group	0.00	0.01	1	121	0.00	0.962	0.000	[0.000, 0.000]
	Situation format	0.24	0.01	1	121	22.50	<0.001***	0.157	[0.070, 0.256]
	Group×situation format	0.00	0.01	1	121	0.14	0.710	0.001	[0.000, 0.030]
<i>Context</i> ^c	Group	0.00	0.00	1	120	0.08	0.778	0.001	[0.000, 0.025]
	Task format	0.00	0.00	1	120	1.34	0.250	0.011	[.000, .061]
	Group×situation format	0.00	0.00	1	120	1.35	0.248	0.011	[.000, .062]

^aThe specific information is shown in Fig. 3 (left)^bThe specific information is shown in Fig. 3 (middle)^cThe specific information is shown in Fig. 3 (right)^dThe number marked in bold means the corresponding effect was significant^e“Situation format” refers to “task situation format”**Fig. 3** Percentage of total fixation duration on each of the AOI for the constructing and expressing stage of problem posing

4.2.3 First fixation duration on AOI

Considering students' first fixation duration on AOI involved in the stage of understanding the problem-posing tasks (see Table 8 and Fig. 4), we found that the main effect of the task situation format on *number* AOI ($F(1, 121) = 6.11, p < 0.05^*, \eta_p^2 = 0.048$) and *context* AOI ($F(1, 120) = 4.18, p < 0.05^*, \eta_p^2 = 0.034$) were both significant. In particular, on the *number* AOI (see Table 8 and Fig. 4 (left)), students' first fixation duration on the task situation format without numbers was significantly higher than that on the task situation format with numbers, $t = 2.47, p < 0.05^*$, Cohen's $d = 0.44$. On *context* AOI (see Table 8 and Fig. 4 (right)) and *element & relation* AOI (see Table 8

Table 8 Metrics calculated for first fixation duration and task situation format at each of the AOI for the understanding stage of problem posing^c

AOI	Effect	MS	MSE	df_1	df_2	F	p	η_p^2	[90% CI]
Number ^a	Group	0.12	0.07	1	121	1.73	0.191	0.014	[0.000, 0.068]
	Situation format^f	0.43	0.07	1	121	6.11	0.015*	0.048	[0.005, 0.124]
	Group× situation format	0.18	0.07	1	121	2.53	0.115	0.020	[0.000, 0.080]
Element & relation ^b	Group	0.20	0.08	1	119	2.48	0.118	0.020	[0.000, 0.080]
	Situation format	0.13	0.08	1	119	1.64	0.203	0.014	[0.000, 0.067]
	Group× Situation format	0.12	0.08	1	119	1.46	0.229	0.012	[0.000, 0.064]
Context ^c	Group	0.10	0.05	1	120	2.01	0.159	0.016	[0.000, 0.073]
	Situation format	0.21	0.05	1	120	4.18	0.043 ^d	0.034	[0.001, 0.102]
	Group× Situation format	0.05	0.05	1	120	0.98	0.324	0.008	[.000, .055]

^aThe specific information is shown in Fig. 4 (left)

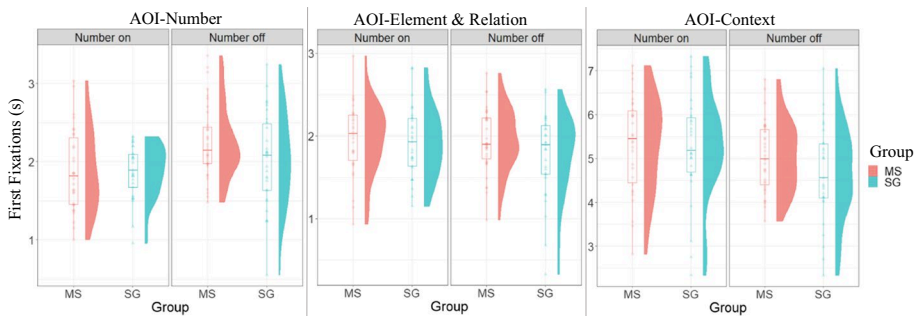
^bThe specific information is shown in Fig. 4 (middle)

^cThe specific information is shown in Fig. 4 (right)

^ddf_r q threshold=0.016, which means the raw value of p lower than 0.016 is deemed as significant

^eThe number marked in bold means the corresponding effect was significant

^f“Situation format” refers to “task situation format”

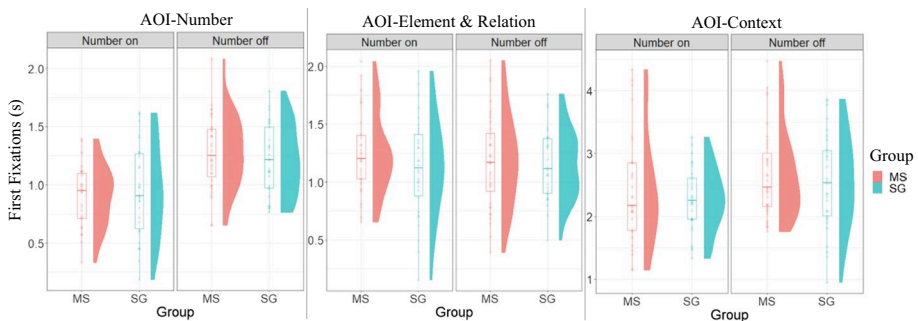
**Fig. 4** First fixation duration on each of the AOI for the understanding stage of problem posing

and Fig. 4 (middle)), there was no significant difference shown for group, task situation format, or the interaction between group and task situation format effect.

Considering students' first fixation duration on AOI involved in the constructing and expressing the problem stage (see Table 9 and Fig. 5), the main effect of the task situation format on *number* AOI ($F(1, 118) = 28.37, p < 0.001^{***}, \eta_p^2 = 0.194$) and *context* AOI ($F(1, 120) = 4.24, p < 0.05^*, \eta_p^2 = 0.034$) were also significant. In particular, on the *number* AOI (see Fig. 5 (left)), students' first fixation duration on the task situation format without numbers was significantly higher than that on the task situation format with numbers, $t = 5.33, p < 0.001$, Cohen's $d = 0.97$. On *context* AOI (see Fig. 5 (right)) and *element &*

Table 9 Metrics calculated for *first fixation duration* and *task situation format* at each of the AOI for the *constructing & expressing stage* of problem posing^c

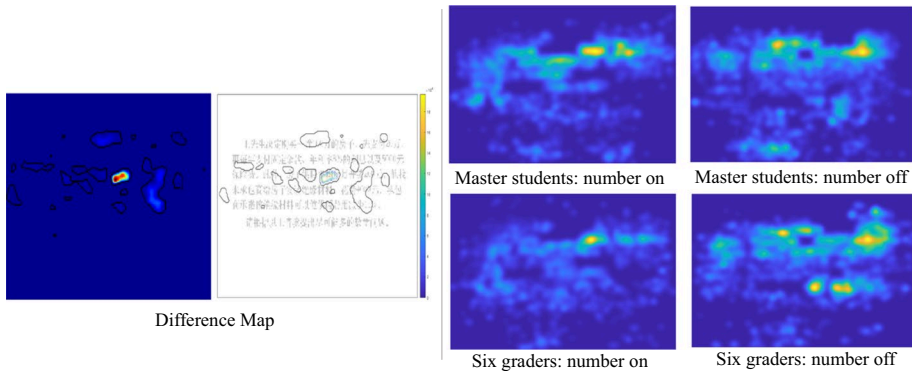
AOI	Effect	MS	MSE	df_1	df_2	F	p	η_p^2	[90% CI]
<i>Number</i> ^a	Group	0.03	0.13	1	118	0.25	0.617	0.002	[0.000, 0.036]
	Situation format ^f	3.55	0.13	1	118	28.37	<0.001 ^{***}	0.194	[0.098, 0.297]
	Group×situation format	0.01	0.13	1	118	0.06	0.805	0.001	[0.000, 0.023]
<i>Element & relation</i> ^b	Group	0.21	0.14	1	122	1.45	0.230	0.012	[0.000, 0.063]
	Situation format	0.00	0.14	1	122	0.00	0.957	0.000	[0.000, 0.000]
	Group×situation format	0.23	0.14	1	122	1.63	0.204	0.013	[0.000, 0.066]
<i>Context</i> ^c	Group	0.06	0.09	1	120	0.69	0.407	0.006	[0.000, 0.048]
	Situation format	0.37	0.09	1	120	4.24	0.042 ^d	0.034	[0.001, 0.103]
	Group×situation format	0.01	0.09	1	120	0.15	0.701	0.001	[0.000, 0.031]

^aThe specific information is shown in Fig. 5 (left)^bThe specific information is shown in Fig. 5 (middle)^cThe specific information is shown in Fig. 5 (right)^dfdr q threshold=0.016, which means the raw value of p lower than .016 is deemed as significant^eThe number marked in bold means the corresponding effect was significant^f“Situation format” refers to “task situation format”**Fig. 5** First fixation duration on each of the AOI for the constructing and expressing stage of problem posing

relation AOI (see Fig. 5 (middle)), there was no significant difference shown for group, task situation format, or the interaction between group and task situation format effect.

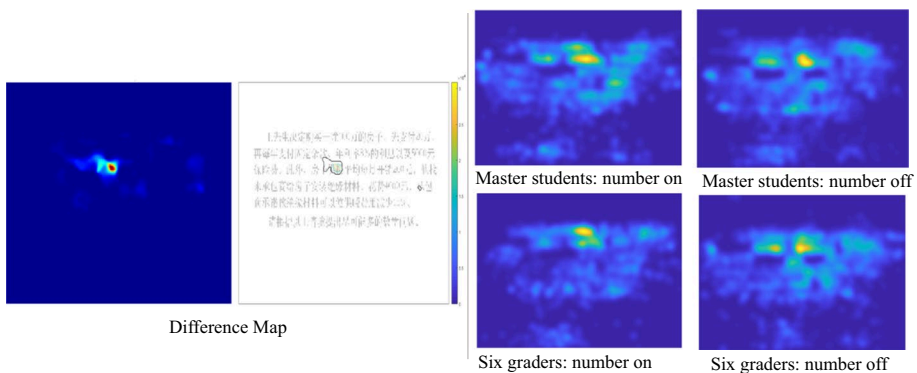
4.2.4 Heat maps

The results of the data-driven analysis are shown in Figs. 6 and 7. Figure 6 illustrates the master's students and sixth graders' fixation distribution while understanding the house purchase task as well as their differences by subtracting the fixation map. Correspondingly, Fig. 7 shows these two groups' fixation distribution while constructing and expressing problems on the house purchase task. There was a significant



Note: The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicate regions of significant difference (at the alpha level of 0.05, two-tailed)

Fig. 6 Difference map (left) and heat maps (right) for two groups of subjects on the two kinds of task situation format (number on and number off) at the stage of *understanding* the House Purchase task. *Note:* The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicate regions of significant difference (at the alpha level of 0.05, two-tailed)



Note: The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicate regions of significant difference (at the alpha level of 0.05, two-tailed)

Fig. 7 Difference map (left) and heat maps (right) for two groups of subjects on the two kinds of task situation format (number on and number off) at the stage of *constructing and expressing the problem* on the house purchase task. *Note:* The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicate regions of significant difference (at the alpha level of 0.05, two-tailed)

main difference in the task situation format involved in both the understanding and the constructing and expressing stages. The significant areas are marked with black borders, $p < 0.05$, corrected (see Fig. 6 (left) and Fig. 7 (left)). The significant differences at different stages of problem posing were associated with the number AOI. Longer fixation for both master's students and sixth graders fell in the area of

numbers on the task situation format without numbers than that on the task situation format with numbers. No significant group differences were found after a multiple comparison correction. These results are consistent with the results on the pool maintenance task. Due to space limitations, that information is shown in Appendix 2.

5 Discussion

5.1 The influence of task situation format with or without specific numerical information

Students' performance on tasks with numbers was significantly better than that on tasks without numbers, which is consistent with what Leung and Silver (1997) found. They argued that problem posers tend to use numbers directly provided in the task situation to construct problems rather than assign new numbers to the variables selected for constructing the problem. In the area of mathematical problem solving, researchers (Bai et al., 2015; Daroczy et al., 2015; Goldin & McClintock, 1984) have found that the length of tasks and the form of numbers and symbols in tasks—especially the presence or absence of numbers—affects the difficulty involved in understanding the task. Therefore, problem-posing tasks without numbers might be more difficult for subjects to understand and construct problems than tasks with numbers.

This interpretation is confirmed by subjects' eye-movement behavior. First, our results showed that students spent more total fixation on processing the entire problem on the task situation format without numbers than on the task situation format with numbers. In addition, from the heat maps and difference maps, we found that students looked significantly longer at the task situation format without numbers than at the task situation format with numbers—in particular, the significant area embodied in the *number* AOI. This is supported by the results related to the metric of first fixation duration as well. Specifically, just on the *number* AOI (the number AOI on the task situation format without numbers represented uncertain numerical information, such as “several,” “certain,” and “some”; conversely, the number AOI on the task situation format with numbers represented the specific numbers), students' first fixation duration on the uncertain numerical information was significantly longer than that on the specific numerical information, from which it could be inferred that the students' problem-posing difficulty and cognitive load of early processing of the uncertain numerical information was much higher than that for processing the specific numerical information. These findings echo those of Hegarty et al. (1995), which showed that participants paid more attention when processing more difficult information such as uncertain numerical information compared to specific numerical information. Just and Carpenter (1980) also found that the vocabulary representing unknown values is more complicated than numbers and thus will directly affect processing difficulty.

The influence of including numbers in the tasks manifested not only in the subjects' difficulty in posing problems but also in the tendency of subjects to proactively select and process different types of information. From the results regarding the metric of percentage of total fixation duration on AOI, we found that students proactively paid more attention to the *number* AOI on the task situation format without numbers than on the task situation format with numbers for both problem-posing stages. Conversely, they paid more attention to the *element & relation* AOI on the task situation format with numbers than on the task

situation format without numbers for both problem-posing stages. For the *context* AOI, regardless of whether the setting included numbers, students did not moderate their attention ratio in this area. This also indirectly indicates that the relevance of number and element & relation information is more important to students posing problems compared to contextual information. This result could be connected to Hegarty et al.'s (1992) finding that key information such as numbers and variable names (similar to the “elements” in this study) were fixated on longer and were critical for subjects’ solving of the problem.

5.2 The influence of mathematical maturity (master’s students or sixth graders)

Regardless of task situation format, we found that the sixth graders had significantly more responses and could pose more mathematical problems as well as more clearly expressed problems. However, there was no significant difference in the quality of the problems posed, which is reflected by the largest sum of the relationships and elements used in the problems. Meanwhile, the master’s students had longer fixation than sixth graders while understanding the task (only for the Pool Maintenance task); however, there was no significant difference on other eye-movement metrics. Peters (2010) found that the expert was able to process information very quickly whereas the nonexpert had to rely on explicit semantic processing of the information when parsing the mathematical constructs. Obviously, we cannot yet identify whether the master’s students or the sixth graders were the expert problem posers according to Peters (2010).

It was somewhat surprising that the master’s students did not outperform the sixth graders, and the sixth graders performed better on the quantity of problems posed. To better understand this finding, we randomly chose and individually interviewed six master’s students after the study. We asked them what their biggest challenge was when posing the problems. They indicated that they merely tried to use the mathematical knowledge they had learned in primary school. The cues motivating them to construct new problems were those they had seen in their primary school books. Therefore, master’s students did not attempt to use higher level mathematical knowledge which would reflect their superiority to sixth graders; on the contrary, they recalled problems appearing in materials widely used in primary school. These cues were clearly more familiar to sixth graders. In addition, compared to problem posing, master’s students were more familiar with the activity of problem solving because they rarely engaged in problem-posing activities in class while they were in primary or high school. The sixth graders, meanwhile, had more opportunities to pose problems in their classes. This result makes sense considering the study of Cai and Jiang (2017) which found that the number of problem-posing tasks in Chinese textbooks significantly increased from the 1990s to the 2010s, with hardly any such tasks in textbooks 10 years ago. Finally, we already know that subjects can pose problems which they cannot solve (L. Zhang et al., 2022). Therefore, we cannot directly infer that the subjects with more maturity in mathematics should have better problem-posing performance than primary students.

Moreover, we found some interesting differences between these two groups’ problem-posing performance. For the metric of the percentage of total fixation duration on AOI, compared to the stage of understanding the task, the interaction effect between group and task situation format was significant for the stage of constructing and expressing problems on the *number* AOI. This occurred because the sixth graders proactively reduced their fixation ratio on the uncertain numerical information when starting to construct and express problems, whereas the master’s students proactively increased their fixation ratio on this area. This might be because the uncertain numerical information represents abstract

numbers or unknown quantities: They can be assigned any possible value. Because master's students' understanding of abstract information is more sensitive and deep compared to that of sixth graders (Cummins et al., 1988), they might place more attention on this area. For sixth graders, the uncertain numerical information might simply represent one certain value in their mind (Lenz, 2019). Thus, after they process this information in the first stage, they reduce their attention to processing the same information during the next stage. In addition, on the *element* AOI, we found that the interaction effect between group and task situation format significantly disappeared during the constructing and expressing problems stage compared to the previous stage. This occurred because the master's students increased their fixation ratio on the element & relation area when constructing and expressing the problems on tasks with numbers. This might be because the element and relation information was easily understood, whereas they placed more attention on select elements and organizing the relations between elements to construct problems.

6 Contributions of the eye-tracking method, limitations, and future directions

In this paper, we investigated the influence of task situation format (with or without numbers) and mathematical maturity (master's students or sixth graders) on students' thinking during three stages of problem posing. The current study provides empirical data confirming a framework that conceptualizes the cognitive process of problem posing into three stages (understanding the task, constructing the problem, and expressing the problem; L. Zhang et al., 2021). Although problem posing is an extremely complex and creative activity, we do not yet fully understand the problem-posing process (Cai et al., 2015), and we have provided a preliminary protocol for analyzing subjects' performance during the three problem-posing stages.

It should be indicated that this study represents the first attempt to use the eye-tracking method to record subjects' attention during mathematical problem posing. Although the preliminary protocol mentioned above was informative for us to understand the cognitive process of problem posing, the eye-tracking method helped us transform a conceptual framework of problem posing into an operationalized framework through which we might record and observe subjects' attention during different cognitive processes of problem posing. In addition, we found that the task situation format without numbers was more difficult for students to understand and construct problems than the task situation format with numbers. In particular, the difficulty was mainly embedded in the *number* area, the information from which represents unknown quantities. The task situation format without numbers provides more opportunities for students to understand and represent unknown quantities so as to pose more problems; thus, teachers can better evaluate students' understanding of unknown quantities by designing problem-posing tasks without numbers.

Our future work in this area will focus on four aspects. First, given that the task situation format without numbers was more difficult for students to pose problems from, as was especially reflected in the processing of uncertain numerical information, we will further consider the role of unknown quantities in problem-posing task design. In addition, other characteristics of the task situation itself may have influenced the subjects' performance given that we did not find consistent results across both tasks with respect to partial indices. Therefore, we will further consider whether other characteristics of the task situation (i.e., the familiarity of the task situation) impact the subject's problem-posing performance. In addition, due

to the characteristics of the tasks themselves, most of the information provided by the tasks was related to *context* AOI, and we can hardly compare what kind of information (*context, number, element or relation*) directly influences students' problem posing to a greater extent. Therefore, we will modify the problem-posing tasks to address this concern.

Second, because the master's students surprisingly did not outperform the sixth graders, we will further explore what expert problem posers might look like (Koichu & Kontorovich, 2013; Voica & Pelczer, 2010). Although each student engages in a problem-posing task at a level that is appropriate to their existing mathematical understanding, a task prompt such as "create a problem that would be difficult for you to solve" instead of "pose as many mathematical problems as possible" could result in an increased challenge that motivates subjects to think more deeply (Cai et al., 2022) and the characteristics of subjects with more mathematical problem-posing maturity might be revealed.

Third, regarding the data from the understanding stage, the current study used questionnaire data consisting of subjects' self-perceived understanding of the task rather than the think-aloud data. The shortcoming of the think-aloud data is that it is not very useful for further analysis in the understanding the task stage because 90% of the participants tended to understand the tasks by simply reading them or silently reading them. The think-aloud technique might also distort the thinking process to some extent. Therefore, we will attempt to study how to better use the think-aloud method to record subjects' thinking when understanding problem-posing tasks.

Finally, Koichu and Kontorovich (2013) showed that problem posers may need to generate "warm up" problems before they can construct better-quality problems. Thus, we could pay more attention to problems posed after a "warm up" problem in our eye-tracking design. In addition, considering that the time spent on constructing and expressing the problem could be quite short, it would be difficult to distinguish when participants actually begin to organize their language to express the problem. Also, Meyer and Lethaus (2004) argued that eye tracking would only be a useful tool for language generation research if a speaker's visual inspection of an object and the cognitive processes underlying the production of an utterance about the object are systematically and transparently related. Thus, we combined the stage of constructing the problem and expressing the problem when examining the participants' eye movements. Future research could pay more attention to the EEG/FMRI method in problem posing, which will provide us more windows through which to trace subjects' cognitive thinking while posing problems, such as their language generation and thinking patterns involved in problem-posing activities.

Appendix 1

Table 10 PPT-number test

House Purchase

1A: Mr. Wang decided to purchase a house whose cost was RMB1,000,000. He made a down payment of RMB200,000 and agreed to pay the rest with monthly payments. Each monthly payment included a portion of the principal, an interest charge computed at the rate of 8% per year, plus a charge for insurance which amounts to RMB5,000 per year. Mr. Wang found by talking to the former owner that the average cost to heat the house was RMB200 per month. Later Mr. Wang added insulation to the house which cost him RMB4000, but which the contractor who installed it guaranteed would reduce his heating costs by 15%.

Please pose as many mathematical problems as you can.

1B: Mr. Wang decided to purchase a house. He made a down payment and agreed to pay the rest with monthly payments. Each monthly payment included a portion of the principal, an interest charge, plus a charge for insurance at a certain amount per year. Mr. Wang found by talking to the former owner the monthly cost to heat the house. Later Mr. Wang added insulation to the house which cost him an additional amount, but the contractor who installed it guaranteed would reduce his heating costs by a certain percent.

Please pose as many mathematical problems as you can.

Pool Maintenance

2A: The Park District installs a swimming pool which holds a total capacity of 500 cubic feet. To fill the pool, two inlets with flow rates of 20 and 10 cubic feet per minute respectively are available. A drain will remove water at the rate of 25 cubic feet per minute. A circulating pump is provided which moves the water in the pool through a filtration system at the rate of 5 cubic feet per minute. When the pool is to be cleaned, as it is done once every week, the water is drained and the sides of the pool are scrubbed. The draining and scrubbing together require 2 hours.

Please pose as many mathematical problems as you can.

2B: The Park District installs a swimming pool which holds a fixed capacity of water. To fill the pool, inlets are available and they have different flow rates. A drain will remove water at a rate that is greater than any flow rate of those inlets. A circulating pump is provided which moves the water slowly through a filtration system. When the pool is cleaned and the sides of the pool are scrubbed. The draining and scrubbing together require a specific number of hours.

Please pose as many mathematical problems as you can.

Table 11 Questionnaire test

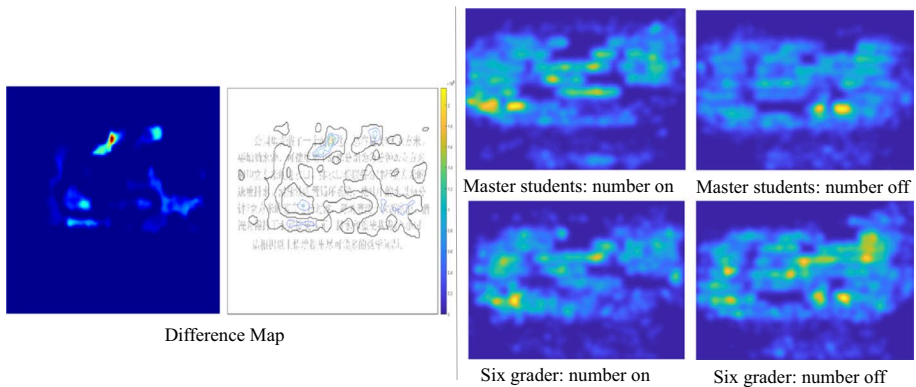
The Questionnaire:

How well do you understand the House Purchase/ Pool Maintenance?

- A. no understanding of the task
- B. understanding a small part of the task
- C. understanding a large part of the task
- D. understanding the whole task

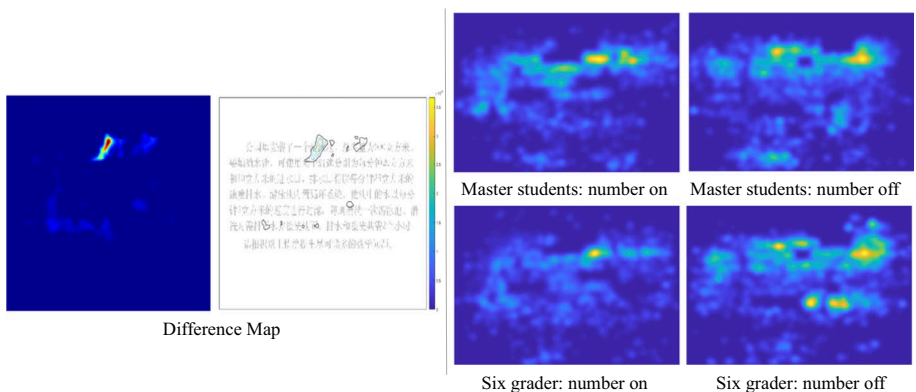
The corresponding problem posing task completed by the participant would be presented under the question, one by one

Appendix 2



Note: The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicated regions of significant difference (at the alpha level of 0.05, two-tailed)

Fig. 8 Difference map (left) and Heat maps (right) for two groups subjects on two kinds of task situation format (number on and number off) on the stage of *understanding* the *Pool Maintenance* task. *Note:* The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicated regions of significant difference (at the alpha level of 0.05, two-tailed)



Note: The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicated regions of significant difference (at the alpha level of 0.05, two-tailed)

Fig. 9 Difference map (left) and Heat maps (right) for two groups subjects on two kinds of task situation format (number on and number off) on the stage of *constructing and expressing problem* on the *Pool Maintenance* task. *Note:* The colors represent Z scores of fixation duration, with warm colors denoting longer fixation duration and cold colors denoting shorter fixation duration. Dark contours in the difference map indicated regions of significant difference (at the alpha level of 0.05, two-tailed)

Appendix 3

Table 12 An Example of the Problems Posed by a Sixth Grader and a Master's Student on the House Purchase task

ID 1117 (sixth grader, test 1: 1A&2B)

1. How much did Mr. Wang spend in total?
2. How much interest did Mr. Wang pay in total?
3. How much did the insulation materials reduce the heating cost?
4. How much less is the heating cost after using insulation material compared to before?
5. How much did it cost to heat the house for 12 months?
6. How much money did Mr. Wang lose compared to paying all at once?
7. A house worth 1 million yuan has paid 200,000 yuan, how much money is left to be paid?
8. What's the ratio of 1 million yuan to 200,000 yuan?
9. How much more does it cost to maintain the house before and after installing insulation material?
10. Can the cost of 4,000 yuan for the insulation material be recouped after using it for 1 year?
11. If 50,000 yuan is paid in one year, what is the interest for that year?

ID 1032 (master's student, test 1: 1A&2B)

1. After installing insulation materials, what is the monthly cost for heating?
2. What is the annual expense after installing insulation material?

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Data availability The dataset generated during the current study is not publicly available as it contains private information of participants that the authors acquired through video recording. Information on how to obtain it and reproduce the analysis is available from the corresponding author on request.

Declarations

Conflict of interest The authors declare no competing interests.

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