Journal of Mathematical Behavior 73 (2024) 101140

Contents lists available at ScienceDirect

Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Exploring middle school teachers' views about problem-posing tasks

Jaepil Han, Stephen Hwang, Faith Muirhead, Jinfa Cai *

University of Delaware, USA

ARTICLE INFO

Keywords:
Problem-posing
Problem-posing prompts
P-PBL
Teachers' curricular reasoning
Middle school

ABSTRACT

It is important to understand teachers' views about problem-posing (PP) tasks and the prompts that are used in such tasks to engage students in posing problems. In this study, we explored 15 middle school mathematics teachers' views about PP prompts. We found that the teachers' views were motivated by their curricular reasoning around engaging and challenging their students and addressed five main prompt characteristics: openness, promoting critical thinking, providing scaffolding, more or less intimidating, and allowing for differentiation. The teachers' reasoning suggested they attended to how PP can create opportunities for sensemaking, deepen students' learning of mathematics, and foster students' identities as creative doers of mathematics. However, they did not address connecting students' life experiences to mathematics, another key goal of teaching mathematics through PP. The findings have implications for curriculum developers and researchers regarding the design of PP tasks and the implementation of such tasks in the classroom, and they suggest several directions for future research.

1. Introduction

Problem posing (PP) is a core activity that drives scientific and mathematical work (Einstein et al., 1938; Hadamard, 1945). The role of PP in school mathematics has been recognized in both curriculum standards and recommendations (e.g., National Council of Teachers of Mathematics [NCTM], 1989, 2014) and research in mathematics education (Cai, Hwang, Jiang, & Silber, 2015; Silver, 1994). More specifically, formulating and expressing well-constructed and interesting problems draws deeply on the poser's knowledge and understanding of the subject while also engaging the poser's creativity and reasoning (Cai, Hwang, Jiang, & Silber, 2015; Ellerton, 1986; Silver, 1994, 1997). Traditionally, teachers (often with support from curriculum materials) have been the ones to pose mathematical problems for their students in the classroom (see, e.g., Butts, 1980). In typical mathematics instruction, this has often meant that students have more opportunities to solve problems than to pose them. But, when teachers proactively create opportunities for their students to pose the problems, they open up even more learning opportunities to support students' sensemaking, engage their interest and creativity in mathematics, and shape their identities as doers of mathematics (Cai, 2022). Indeed, research has established that students at various educational levels can successfully pose mathematical problems (Cai, Hwang, Jiang, & Silber, 2015), making PP a low-floor, high-ceiling activity that allows students with different levels of mathematical achievement to participate successfully (Silber & Cai, 2021).

In fact, problem-posing based learning (P-PBL)—the use of PP tasks with students to learn mathematics—aims to achieve several

* Correspondence to: University of Delaware, 437 Ewing Hall, Newark, DE 19716, USA.
E-mail address: jcai@udel.edu (J. Cai).

https://doi.org/10.1016/j.jmathb.2024.101140
Received 17 May 2023; Received in revised form 16 February 2024; Accepted 17 February 2024
0732-3123/

pedagogical goals, including creating rich opportunities for students to engage in mathematical sensemaking (Christou et al., 2005; Cifarelli & Cai, 2005), helping students learn mathematical content and processes more deeply (Toluk-Uçar, 2009), connecting students' life experiences to the mathematics they are learning (Walkington & Bernacki, 2015), and developing students' identities as creative doers of mathematics who have ownership of their learning (Akay & Boz, 2010; Silver, 1994, 1997). For teachers who wish to use PP to help their students make progress towards these goals, PP tasks are needed that are likely to generate the desired kinds of thinking and activity—tasks whose characteristics elicit the engagement of their students to pose good mathematical problems that can help the class make progress toward their learning goals. However, because published curricula generally fail to provide a pool of well-designed PP tasks (Cai & Jiang, 2017; Park et al., 2019), teachers end up generating their own PP tasks or redesigning problem-solving tasks to be PP tasks (Cai & Hwang, 2021). This relies on the teachers' understanding of and views about PP tasks, views that ultimately inform their curricular decisions around P-PBL.

In this paper, we focus on mathematics teachers' views about PP with respect to a key aspect of PP tasks: the prompts that specify what students are supposed to do in response to the PP task (Cai et al., 2022). The characteristics of prompts can influence the PP process and the kinds of problems that students pose (Cai et al., 2023; Silber & Cai, 2017; Yao et al., 2021) and thus play an important role in whether or not a PP task will support the desired student learning. However, the field has not yet characterized how teachers view PP prompts and how they may affect P-PBL. Here, we report findings from a larger PP research project in which middle school mathematics teachers are actively developing their views about PP and PP tasks as they learn how to teach mathematics through PP. A better understanding of teachers' views about PP prompts and their thinking about those prompts can inform efforts to support teachers as they learn to teach through PP (Li et al., 2022; Xu et al., 2020). Through task-based interviews, we examined the teachers' views about two specific PP prompts that have been shown to successfully elicit PP activity and that differ in the level of structure they impose. Moreover, we probed the teachers' pedagogical thinking that underlies those views in order to better understand teachers' pedagogical considerations around PP. Thus, the research questions we addressed were: What are the views about PP prompts held by in-service middle school mathematics teachers who are learning how to teach through PP? What pedagogical considerations are featured in the teachers' explanations of their views about PP prompts?

2. Theoretical background

2.1. Problem posing, P-PBL, and prompts

PP, defined as the act of formulating (or reformulating) and expressing problems in mathematics (Silver, 1994), is a mathematically rich activity. As an overarching definitional framework for PP, Cai and Hwang (2020) characterized PP for students and teachers separately. For students, they defined PP as consisting of two intellectual activities: posing mathematical problems based on given problem situations and posing problems by changing existing problems. For teachers, they also included another three intellectual activities: posing mathematical problems for students to solve, predicting the kinds of problems that students could pose based on given problem situations, and generating PP situations for students to pose problems. Indeed, predicting students' posed problems and generating PP situations are both key activities teachers engage in when using P-PBL (Cai & Hwang, 2020; Zhang & Cai, 2021).

To be more precise, a PP task usually includes a pair of distinguishable parts: a *problem situation* and a *PP prompt* (Cai, 2022; Cai & Hwang, 2023). The *problem situation* provides the poser with the contextual information and data that they may draw from to craft problems. This may be presented in many ways, including words, pictures, graphs, patterns, tables, and mathematical expressions, and the information may come from real-world referents or be purely mathematical or abstract. The *PP prompt*, the focus of the present study, tells the poser what they are expected to do in response to the task. Prompts may be made up of only a few words (e.g., "Pose some problems"), or they may include multiple statements. They may be very general, as with the example in the previous sentence, or they may include explicit guidance and conditions that the posed problems must satisfy (e.g., "Pose three different problems that can be solved using the given information"). Different prompts may be paired with the same problem situation to generate PP tasks with varying characteristics that elicit different PP behavior from students (Cai et al., 2023; Silber & Cai, 2017; Yao et al., 2021). In this paper, we focus on teachers' views about specific PP prompts and the pedagogical considerations that underlie those views.

Some recent research focusing on PP prompts has begun to produce evidence of how differences in prompts can significantly affect students' PP. For example, although Jung and Park (2010) did not clearly distinguish between situations and prompts in their analysis, they did find that variations in terms of text, mathematical content, numerical expression/equation, or visual representation influenced students' PP. With respect to prompts, their findings suggest that fifth graders felt that PP tasks with more specific guidance were more accessible but that the students were less likely to pose more advanced problems in response. However, in a mixed-methods study of 61 preservice teachers, Silber and Cai (2017) found some evidence that more structured PP tasks might result in more complex problems posed, but in a limited way. Silber and Cai used four PP tasks (Doorbell, Road Trip, Food Drive, and Dots) to compare PP under free and structured task conditions. They asked the preservice teachers to pose multiple problems for each task, which previous research has suggested results in a pattern of first posing simpler mathematical problems and then posing more complex mathematical problems (Christou et al., 2005; Silver & Cai, 1996; Stickles, 2011). The effect of the more structured condition on the preservice teachers' PP diverged based on the task. Silber and Cai found no significant effect for the Doorbell task, but for the Road Trip task, the participants in the structured condition posed significantly more complex problems than those in the free condition—but only for their first response. This effect was not found for the second or third responses for this task, meaning that after posing their first problem, participants in the structured condition subsequently wrote problems of about the same complexity as those in the free condition. For the Food Drive and Dots tasks, participants in both the free and structured groups tended to increase the complexity of their posed problems with each subsequent response.

Silber and Cai (2017) also found that, when operating within more structured PP conditions, their participants were more likely to attend to the mathematical concepts underlying the problem situations for all four of their tasks. This accords with a finding by Yao et al. (2021) that a prompt with an explicit conceptual cue (i.e., a prompt that has a more explicit structure included) tended to lead to participants posing problems that showed evidence of conceptual understanding. Specifically, when generating problems related to a given fraction division (1¾ divided by ½), if the prompt reminded participants of a typical meaningful interpretation of the division operation, the participants were much more likely to pose problems reflecting conceptual understanding. Similarly, in a study of middle school students' PP, Cai et al. (2023) found that a more structured prompt that asked students to pose problems of varying difficulty levels was associated with more in-depth mathematical thinking and more linguistically and semantically complex problems. Moreover, the more structured prompt was associated with posed problems that contained more relationships and required more steps to solve them. Thus, looking across the existing, if still scant, literature on PP prompts, it would appear that providing greater structure in a PP prompt has the potential to strongly impact PP cognition and the nature of the problems that are posed in response. Prompts that include additional specific conditions or guidance tend to support the posing of more complex mathematical problems compared to freer, less structured PP prompts. This finding accords with research on the characteristics of PP situations. For example, Leung and Silver (1997) found that preservice teachers posed higher quality and more complex problems when presented with PP situations that included specific numerical information than when presented with PP situations without such information.

Given the emerging research on the effects of PP prompt characteristics on students' PP thinking and the kinds of problems students pose, and because PP in mathematics classes is currently reliant on mathematics teachers to find or construct PP tasks, it is especially important to understand how teachers view characteristics of PP prompts. Increasing the field's understanding of teachers' thinking in this area would inform attempts to support teachers as they learn to teach mathematics through PP (Li et al., 2022). Moreover, a better understanding of the pedagogical considerations that contribute to teachers' views of PP prompts can shed light on how teachers think about P-PBL, notably the purpose and utility of PP tasks in mathematics instruction. Thus, in the present study, we explored how teachers viewed two kinds of PP prompts—a general prompt and a prompt with more specific conditions. We aimed to better understand how teachers think about these prompts with respect to their students in order to inform efforts to support teachers as they learn to teach mathematics through PP.

2.2. Teachers' views and curricular reasoning about characteristics of mathematical tasks and PP tasks

In their everyday practice, teachers make a variety of pedagogical choices when planning instruction to meet the diverse needs of their students. Planning to teach mathematics through PP is no exception. In particular, as teachers make decisions about incorporating PP tasks into their teaching, they engage in *curricular reasoning*, "a specific form of pedagogical reasoning that teachers employ while working with curriculum materials to plan, implement, and reflect on instruction" (Roth McDuffie and Mather, 2009, p. 302). Through this reasoning, teachers make curricular decisions, such as what instructional tasks to implement and how to implement them in their classrooms, drawing on their views about what a task offers and their anticipation of how their students will respond to it (Dietiker et al., 2018). For P-PBL, in particular, because published mathematics curricula have few PP tasks (Cai & Jiang, 2017; Park et al., 2019), teachers must navigate other curriculum resources, such as the Internet, to find appropriate pre-made PP tasks or transform existing mathematics tasks into new PP tasks that suit their needs (Cai & Hwang, 2021). As part of this process, teachers engage in curricular reasoning around PP tasks, including considering how they think their students will respond to the tasks (Anderson, 2003), the challenges that students may encounter when posing problems (Li et al., 2020), and whether students will enjoy engaging with the tasks (Webel et al., 2015).

Indeed, teachers' proactive and reactive responses to students' challenges before, during, and after instruction are a part of teachers' responsiveness to students' struggles (Warshauer, 2015). Thus, one of the common concerns teachers attend to when creating mathematical tasks or supplementing their curriculum by adapting materials available online is making the mathematical tasks more accessible and appropriately challenging so as to engage students with the underlying mathematics (Hourigan & Leavy, 2023; Wang et al., 2021). For example, Wang et al. (2021) noted that teachers perceived their students as less engaged if their curriculum materials, such as mathematical tasks, were too challenging. In addition, they found that teachers perceived students as more engaged with the mathematics of a lesson when the materials connected to the students' interests and when the teachers granted more autonomy or agency to the students (e.g., allowing students to choose the types of activities). Because teachers' perceptions of different characteristics of mathematical tasks (including the ways in which those tasks may or may not engage their students) influence their curricular reasoning about whether and how to use those tasks, it is particularly important to understand which task characteristics they attend to and how they interpret those characteristics. This is especially true when considering how teachers may adopt a new instructional practice such as teaching mathematics through PP.

However, research on PP and teaching mathematics through PP has not yet explored in detail the specific aspects of PP tasks that teachers consider when engaging in curricular reasoning. Nevertheless, the factors that teachers consider when they reason about instructional tasks and their students' responses to instructional tasks can reveal important aspects of teachers' conceptions of those tasks. For example, to gain insight into teachers' conceptions of mathematics and the teaching of mathematics, Cai (2004) examined the factors they attended to when evaluating student responses to problem-solving tasks (e.g., the use of pictorial vs. symbolic reasoning), highlighting a key difference between Chinese and U.S. teachers' beliefs regarding the appropriateness of generalized strategies. Indeed, teachers' views about mathematics, their students, and mathematical tasks fundamentally influence the implementation of the intended curriculum (Cai, 2004). Thus, in order to better understand how teachers may use P-PBL to teach mathematics and how to support them in doing so, it is important to consider which aspects of PP tasks they attend to in their curricular reasoning.

Given the relationships between PP and problem solving (Cai & Hwang, 2002), it is useful to examine relevant findings in the context of mathematical problem-solving research. Lester (2013) outlined several categories of factors that affect problem-solving instruction, including teachers' beliefs and knowledge (e.g., their views) and the characteristics of problem-solving tasks. Certainly, there has been a great deal of research on the effects of problem-solving task characteristics (see, e.g., Goldin & McClintock, 1984). For example, Kulm (1984) described four categories of task variables in problem-solving research: syntax variables, content and context variables, structure variables, and heuristic behavior variables. *Syntax variables*, the category most parallel to the focus of the present study on PP tasks, involve the particular arrangement of words and symbols in a problem. This includes such variables as length, grammatical structure, and the inclusion of numerals. In research on arithmetic and algebraic word problems, syntax variables have long been recognized as a significant factor in the likelihood of solvers determining the correct solution (Jerman, 1971). This may be related to the fact that syntax variables tend to reflect the mathematical structures or parameters of the task. Moreover, as Barnett (1984) noted, teachers may be able to easily adjust the syntax of problems. This permits teachers to manipulate problem statements based on their views of what characteristics make a problem better for their students.

Ultimately, the field has not yet explored how teachers' views about characteristics of PP tasks may inform their curricular reasoning and potential adaptation, for good or for ill, of P-PBL. Just as teachers can shape students' learning in powerful ways through how they themselves choose to pose problems (Butts, 1980; NCTM, 2014), teachers may also influence students' learning through their curricular reasoning around PP tasks for students. As noted above, curricular reasoning about PP tasks can involve the intellectual activity of envisioning how students will respond to the PP prompt (Cai & Hwang, 2020; Xu et al., 2020). Moreover, previous studies have established that specific task features can greatly influence subsequent PP activity (Cai et al., 2023; Leung & Silver, 1997; Silber & Cai, 2017; Yao et al., 2021). Because teachers' implementation of P-PBL depends on their curricular reasoning about PP and P-PBL, reasoning that is informed by their views about such PP task features, we conducted the present study to better understand these views and what pedagogical considerations teachers make when reasoning about them.

3. Method

3.1. Participants

This study is part of a larger longitudinal research project investigating and supporting the implementation of P-PBL. A total of 15 mathematics teachers (12 female and 3 male) from two middle schools (Grades 6–8) participated in this study. These teachers were first-year participants in the larger research project. Prior to participating in the project, none of the teachers had tried to incorporate PP into their mathematics instruction. However, they were interested in learning about P-PBL and how to implement it in their classrooms, and they had agreed to participate in the larger project with that goal in mind. The teachers attended a 2-day summer workshop and monthly professional-learning (PL) meetings, all held online due to pandemic restrictions. During these meetings, the teachers engaged in activities designed to familiarize them with the concept of teaching mathematics through PP and to work with each other and researchers to integrate PP into their mathematics lessons. Specifically, the teachers engaged with several PP tasks as problem posers themselves. The research team then shared some of the research basis for P-PBL, specifically focusing on characteristics and categories of PP tasks (e.g., tasks based on real-life vs. purely mathematical contexts; structured, semi-structured, and free PP tasks; and kinds of representations used in PP tasks). The teachers also considered how their students might respond to P-PBL by making predictions about the problems their students would pose in response to specific PP tasks (Xu et al., 2020). In addition, the teachers worked with the research team to analyze their curriculum materials with an eye to identifying opportunities to integrate P-PBL. In particular, they attended to existing PP tasks in the form of instances of the "Co-Craft Questions" routine (Zwiers et al., 2017), an instructional routine built into their curriculum that was designed to support English language learners. The Co-Craft Questions routine involves taking an existing problem-solving task in the curriculum, deleting the questions to leave only the problem situation, and then asking students what mathematical problems could be posed based on the situation. Indeed, although positioned in the teachers' curriculum materials as a language-learning routine, Co-Craft Questions is, more generally, a simple routine for teachers to convert problem-solving tasks into PP tasks (Cai & Hwang, 2021). Finally, the teachers discussed potential modifications to their curriculum materials to integrate P-PBL and practiced adapting existing activities to build opportunities for PP. During the monthly PL meetings, the teachers identified focal lessons from their curriculum that they wanted to infuse with PP and collaborated with each other to revise those lessons. Between the monthly meetings, the teachers continued to collaborate on redesigning selected lessons as PP lessons. During the school year, the teachers implemented the lessons that they had designed. Thus, at the time of the interviews, the teachers had already gained some initial experience with teaching mathematics through PP as first-year participants in the project.

3.2. Interview protocol and data collection

Using an online meeting platform, we conducted semi-structured, task-based interviews (Merriam & Tisdell, 2015) with the teachers during the fall semester. We asked the teachers a set of specific questions about their views about PP and two different PP prompts:

- Prompt A: Pose three different mathematical problems that can be solved using this information.
- Prompt B: Pose one easy mathematical problem, one moderately difficult mathematical problem, and one difficult mathematical problem that can be solved using this information.

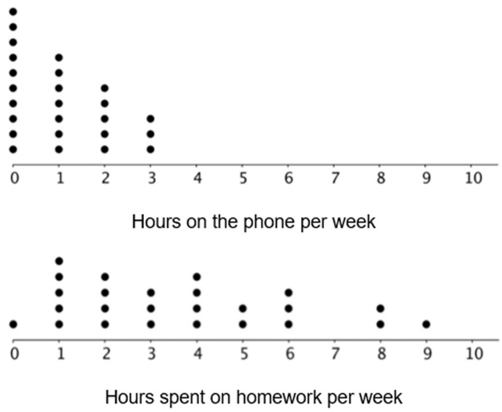
We chose to focus on these two prompts for several reasons. First, these prompts have been used effectively in prior PP studies to elicit PP activity (e.g., Cai & Hwang, 2002; L. Chen et al., 2007; Silber & Cai, 2021; Silver & Cai, 1996), and some characteristics of the problems posed in response to these prompts have been documented. For example, the relatively unstructured "pose three" prompt has produced sequences of posed problems that tend to exhibit increasing complexity (Silver & Cai, 1996). Second, the teachers in this

Lin and Diego both ran for 10 seconds, each at their own constant speed. Lin ran 40 meters and Diego ran 55 meters

Prompt A: Pose three different mathematical problems that can be solved using this information.

Prompt B: Pose one easy mathematical problem, one moderately difficult mathematical problem, and one difficult mathematical problem that can be solved using this information.

(A) Task 1

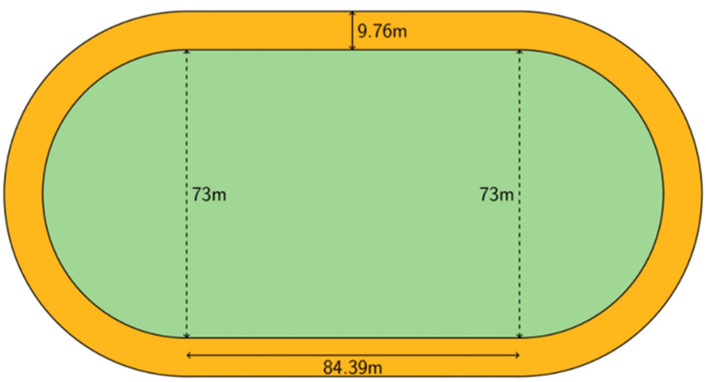


Prompt A: Pose three different mathematical problems that can be solved using the data displayed in these dot plots.

Prompt B: Pose one easy mathematical problem, one moderately difficult problem, and one difficult mathematical problem that can be solved using the data displayed in these dot plots.

(B) Task 2

The field inside a running track is made up of a rectangle that is 84.39 m long and 73 m wide, together with a half-circle at each end.



Prompt A: Pose three different mathematical problems that can be solved using this information.

Prompt B: Pose one easy mathematical problem, one moderately difficult mathematical problem, and one difficult mathematical problem that can be solved using this information.

(C) Task 3

Fig. 1. Three PP tasks consisting of problem situations and PP prompts.

study had previously encountered both of these prompts through participating in the project's PL sessions. Thus, we had reason to believe that the teachers had given some thought to these two prompts and had perhaps even tried using them in PP tasks with their students. Moreover, with an eye to the goals of the larger project, we wanted to better understand how the teachers' views about these prompts were being informed by pedagogical considerations stemming from their early classroom experiences with P-PBL. Third, we anticipated that the teachers might draw on the different levels of structure imposed by the two prompts in discussing how they expected their students would respond to them. Prior research has suggested that PP prompts that provide additional structure may elicit different kinds of posed problems than more open prompts (e.g., Silber & Cai, 2021), and we wanted to capture how the teachers' views were (or were not) related to this kind of phenomenon.

We asked the teachers how they thought their students might respond to these prompts in the context of three different PP situations and to explain why they thought their students would respond in that way. As noted above, PP for teachers includes the intellectual activity of predicting the kinds of problems that students could pose based on given problem situations (Cai & Hwang, 2020). By asking the teachers to make and explain their basis for such predictions, we aimed to gain access to their views about the two different prompts. Moreover, the problem situation for each of the PP tasks was drawn from an actual problem-solving task in the middle school mathematics curriculum the teachers were using (but not all from the same grade level that the teacher taught). The three tasks were chosen to vary in terms of context, specifically the mathematical content area and the use of visual representations (Fig. 1). The first task (Fig. 1A) involved rates (a key topic emphasized throughout their curriculum) and was entirely verbal (no pictorial element). The second task (Fig. 1B) included a set of data represented in a pair of dot plots. The third task (Fig. 1C) included a diagram with included measurements.

We chose to vary the mathematical content area to include content that the teachers frequently covered with their students and content that was less frequently covered (or even skipped in some years). This was intended to provide data on the reasoning and views that the teachers may have already formed with respect to PP with familiar content as well as more on-the-fly reasoning and views the teachers might express when thinking about PP in a more novel content setting. We also chose to vary the use of visual representations because the teachers might anticipate different kinds of student responses if the students had access to a picture (e.g., if the teacher thought their students were more "visual").

Each interview was video recorded and transcribed. The initial transcripts were produced automatically by the online platform. Those transcripts were then manually compared against the video recordings to correct any transcription errors. The length of the interviews ranged from 20 to 50 min.

3.3. Data analysis

Each teacher's responses were coded for two factors: (a) teachers' views about how a prompt would work for their students, including expressions of preference for one prompt over the other, and (b) the reasons the teachers gave for their evaluation or preference. Before initial coding, we read through all the interview transcripts while watching the video recordings or listening to the audio recordings to increase our familiarity with the data. It became clear through this reading that many of the teachers' views included stating a preference for using one prompt over the other with their students for a particular task. Thus, while reading through the transcripts, we also initially recorded the teachers' prompt preferences for each task in addition to leaving marginal notes if there were any similarities or differences across the teachers' justifications for their views. The teachers' justifications were initially coded using an *open coding* technique (Merriam & Tisdell, 2015) that highlighted the teachers' concerns about their students' engagement with the PP tasks and whether the tasks would be especially challenging to their students.

After the first round of coding, we drew on Wang et al.'s (2021) characteristics of engaging and appropriately challenging curriculum materials to generate an analytic framework for interpreting and categorizing the teachers' reasoning and justifications for their prompt evaluations and preferences. The first and second authors independently coded 5 of the 15 transcripts using this analytic framework. The authors compared their coding and discussed any concerns and issues that arose when coding the transcripts until they reached an agreement. Ultimately, the coding process used both deductive and inductive approaches while remaining open to any additional codes. The refined set of final codes used for the analysis is shown in Table 1.

Table 1
Justification codes (adapted from Wang et al., 2021*)*.

Codes	Description
Opens up a variety of possible responses	The teacher refers to how the prompt allows students to pose a variety of different mathematical problems.
Promotes critical thinking	The teacher refers to how the prompt might (or might not) be useful for promoting critical thinking.
Provides scaffolding	The teacher refers to how the prompt provides more (or less) guidance or direction for posing specific problems.
Feels more or less intimidating	The teacher refers to how the language of the prompt (its wording or length) may confuse (or help) students pose problems.
Assesses students' understanding	The teacher refers to the prompt as more useful for assessing students' understanding.
Challenges students appropriately	The teacher claims that the prompt is (or is not) appropriately challenging for their students (without providing specific reasons for this judgment).
Acclimatizes to PP routine	The teacher refers to how the prompt is appropriate for making students familiar with PP routines, such as what is expected of them when they are asked to pose problems.
Differentiates for students	The teacher refers to how the prompt is appropriate for differentiating PP tasks for students with different prior knowledge regarding the context or content of the PP situations.

To code the teachers' reasoning and justifications for their views, the authors first identified the main ideas included in each teachers' justifications. The unit of analysis was a monothematic chunk of sentences (Miles, Huberman, & Saldaña, 2014). The length of monothematic chunks ranged from 13 words to 526 words, and each teacher's interview had between three and fourteen monothematic chunks. For example, the excerpt, "I think it's easy for them to take [Prompt] A and just come up with a pattern and spit out three problems that are very similar to each other," from the interview with Ms. Strickland was categorized as "Opens up a variety of possible responses" because Ms. Strickland argued that Prompt A would give students the freedom to come up with any three mathematical problems, even if the problems were similar to one another. If there was a change of justification idea within a teacher's response to an interview question, then each transition of the ideas was coded separately. Thus, for example, a teacher's justification for their prompt preference for a given task could consist of one or more ideas. Typically a teacher's justification comprised one to three blocks or ideas. To ensure high reliability in the data analysis, two coders independently coded one-third of the teacher responses. The inter-rater agreements were 100% for teachers' prompt preferences (Prompt A, Prompt B, Conditional, or Not mentioned) and 93% for their justifications for the prompt preferences. The coders discussed their differences, which were related to ambiguity in the definition of the "Differentiates for students" code, and resolved the differences by refining the definition to exclude potential overlap with the "Challenges students appropriately" code.

4. Results

Table 2 shows the number of teachers who mentioned each kind of view in relation to the two PP prompts. Looking across the three PP tasks, the teachers' views reflected a consistent pattern of concerns that they attended to. Among the views, the teachers cited five main categories: (a) opens up a variety of possible responses, (b) promotes critical thinking, (c) provides scaffolding, (d) feels more or less intimidating, and (e) differentiates for students. In addition, when the teachers indicated a preference for using a specific prompt with their students for a particular task, it was clear that the teachers considered the nature of the PP situations (e.g., mathematical content, textual vs. visual representations, etc.) and factors external to the tasks related to their students. That is, the teachers' primary concerns were related to the challenges that their students would encounter with the specific PP tasks and how they anticipated the different prompts could modulate those challenges. In the next sections, we examine how the teachers thought about the two different PP prompts and their anticipated effect on students' responses to each PP task.

4.1. Openness

The most common view about the different PP prompts, cited by 11 of the 15 teachers, was that the prompt was particularly open to students posing all kinds of problems. This observation about openness was tied to the teachers' concerns regarding their students' engagement with the task and their desire for there to be an access point in the task where any student could take ownership of the mathematics. Notably, all 11 teachers who cited the openness of the prompt as a reason for their view of the prompt expressed a preference for using Prompt A for Task 1 (8 of 11), Task 2 (4 of 11), or Task 3 (4 of 11).

For Task 1, which was the rate task, 8 of the 15 teachers viewed Prompt A as "lowering the floor" of the task, expecting that Prompt A would provide greater access to the PP task. The teachers considered that Prompt A "keeps it open" (Ms. Khan) and was "more open-ended" (Ms. Hobbs) because it had very few constraints and would allow students to generate any mathematical problem related to the given PP situation that occurred to them. This, in turn, could support the students in achieving the goal of posing multiple problems because there was no constraint against posing similar or repetitive problems. For example, students could ask for the speed of Lin and also ask for the speed of Diego. Indeed, Mr. Mejia anticipated students would easily come up with two problems for this task if Prompt A were used, and thus they would only need to come up with one more problem, which could be similar to or different from the two problems already posed. Similarly, Ms. Strickland shared that "I think it's easy for them to take [Prompt] A and just come up with a pattern and spit out three problems that are very similar to each other." Contrasting the two prompts, Ms. Joyce said, "I think for the most part, they had to pose three [problems], then we'd get more responses out of them or more problems that they would pose out of them than trying to do one in each category." By relaxing the requirement to pose one easy, one moderately difficult, and one difficult problem, Prompt A would allow students to start writing down some mathematical problems more easily. Then, if desired, the activity could be extended by imposing any additional criteria, such as mathematical difficulty.

Table 2
Teachers' views about different PP prompts.

	Prompt A	Prompt B
Opens up a variety of possible responses	11	0
Promotes critical thinking	0	8
Provides scaffolding	0	4
Feels more or less intimidating	4	3
Assesses students' understanding	0	2
Challenges students appropriately	2	1
Acclimatizes to PP routine	1	1
Differentiates for students	7	5

Note. The table shows the number of teachers who expressed each type of view about PP prompts during the interviews.

Similarly, for Task 2, which provided only two dot plots with no accompanying "story," Ms. Garrett argued that the openness of Prompt A would give the students the freedom to respond in ways that could get around a potential lack of familiarity with the mathematical content:

> All kids don't know how to use bar graphs and plots, or anything like that. … But, I feel like if you give them Prompt A on this one and give them a broad spectrum of questions, and not really stresses them out too much about coming out difficult questions, if they don't even know what this means. (Ms. Garrett)

Indeed, in explaining her views on the different prompts for Task 2, Ms. Chen drew on her belief that the more detailed the information that is given in a PP task, the easier it is for students to come up with associated questions:

> Whereas in the first question up above about Lin and Diego, though that was more— they gave me more information, I feel like [Task 1] was kind of more specific. I feel like Prompt B would be better [for Task 1]. But for [Task 2], I kind of feel like they're not, they're literally not asking you anything. You have two diagrams. So I feel like with this one, Prompt A, just because there is no question. It's kind of vaguer. You can ask, 'pose three different mathematical problems' just because I just feel like they would be able to come up with more questions. (Ms. Chen)

Because Task 2 did not have an accompanying narrative, Ms. Chen felt that her students needed a more open prompt so that they could pursue any kind of question that might arise from this "vaguer" problem situation. That is, Ms. Chen interpreted the problem situation itself as more open, and she felt that a more open prompt would give her students a better chance to "come up with more questions."

Along the same lines, Ms. Strickland wanted to use Prompt A for Task 3 because its openness would create more freedom for her students to produce any kind of posed problem. She was concerned that her students did not have much background knowledge of geometry, such as knowing how to find the area of semicircles. Moreover, she expressed that her goal of using PP with her students in a situation like this would be to assess their existing knowledge about an unfamiliar domain. Thus, she did not want the prompt to artificially restrict what the students might reveal:

> I think, for this one, I probably would just use Prompt A just because they don't have as much background information. … I mean, again, some of my students are going to be aware of the fact, like, 'Hey, I can't actually find the area because I don't know semi-circles,' but for the general population, I would probably go with just three problems [Prompt B]. Honestly, I just want to see what you can come up with because I know that you don't even know anything about this. (Ms. Strickland)

Ms. Joyce's views on the prompts for Task 2 also reflected a belief that she might need to dynamically calibrate the PP task to the student's level of understanding:

> I think I would still do Prompt A because it can meet them at whatever level they're thinking for creating problems for this. So, if they can't think of a moderately difficult question or a difficult question in general they can at least think of three easier ones to solve. (Ms. Joyce)

In the openness of Prompt A, Ms. Joyce saw adaptability. The parameters of the prompt would allow her to adjust the PP task to conform to her students' knowledge levels or abilities.

Overall, in their comments about the openness of Prompt A, the teachers revealed their pedagogical considerations about their students' engagement with PP tasks, and in particular, their anticipation that some students might have trouble getting started with or completing a PP task. With these considerations, the teachers tended to view the lack of structure in a prompt such as Prompt A as an advantage for their students, lowering the floor for students to easily complete the PP task. This was particularly notable when paired with anticipation that students would be challenged because they are less familiar with the mathematical content of the problem situation (e.g., dot plots, statistics). By pairing such situations with prompts that have fewer constraints, the teachers anticipated their students would be more likely to complete the tasks, and this, in turn, would allow the teachers to assess their students' mathematical thinking and probe what students could do with the mathematics underlying the given PP tasks.

4.2. Critical thinking

A second view about prompts, cited by 8 of the 15 teachers, had to do with promoting critical thinking. Looking across the three tasks, all 8 teachers who expressed views related to promoting critical thinking reasoned that Prompt B would encourage the students to think more deeply and perhaps engage in critical thinking. On one level, this was simply because the prompt asked for a difficult problem, not just an easy one:

> [For] Prompt A, I feel like they would maybe just give basic things [problems] like I just said. And then, for [Prompt] B, they might come up with some really, really, good, challenging ones [problems]. If you don't ask them to challenge themselves, sometimes they just don't. They go with the easier out. (Ms. Garrett)

Ms. Garrett believed that, at least sometimes, her students needed the explicit nudge of being asked to pose a challenging problem to keep them from just staying at a surface level and posing only easy problems. Similarly, Ms. Love felt that adding specific criteria to the PP prompt, such as posing one easy, one moderately difficult, and one difficult problem, would "push" the students to think more deeply:

> Probably B, just to push them. I mean, I think you're gonna get a little more when you're being specific and have certain criteria for them to meet. And you could even add more criteria, then that was telling them you want them to include a poll or you want them to, whatever it is you're looking for–just give them a few more criteria of what you're looking for in their polling versus just free range to come up with anything. (Ms. Love)

Indeed, Ms. Love saw Prompt B as only the tip of the iceberg for the kinds of additional criteria that could be added to a PP prompt—criteria that would lead to students thinking more deeply about the goals of the lesson.

On another level, some teachers felt that because Prompt B specified that students were to pose problems of different difficulty levels, it could create opportunities for the students to be encouraged to think critically about what makes a problem easy or difficult. For example, Ms. Wise stated:

> Like maybe it wasn't a specific— like so for them easy, medium, hard means, "okay, what does that mean exactly?" And I'm like, "What would be easy to you? What is medium? What does hard look like to you?" (Ms. Wise)

To Ms. Wise, Prompt B looked like it offered students a way to think about how they could make three mathematical problems. Ms. Wise noticed that although the meaning of easy, medium, and hard would differ from one student to another, these criteria gave students some starting points to reference when they thought about other potential problems to pose. Similarly, Ms. Love believed Prompt B's explicit requirement for three different levels of difficulty would force students to actively differentiate their posed problems by thinking about what easy, medium, and difficult meant to them.

Whereas the teachers' views about the openness of Prompt A reflected a pedagogical consideration about broadly engaging students, these views about Prompt B point to a more depth-focused pedagogical concern. The teachers viewed a more structured prompt like Prompt B as a tool to encourage students to think more deeply and even possibly engage in some metacognition or conversations about what makes a problem easy or difficult for them. They viewed more structured PP prompts as potentially increasing the cognitive demand of the PP tasks, suggesting that some teachers were looking at PP tasks as a way to raise the ceiling in their classrooms.

4.3. Scaffolding

A third view about the prompts for the PP tasks was cited by 4 of the 15 teachers. These teachers expressed the view that Prompt B would provide scaffolding in the form of more specific directions (compared to Prompt A) that would actively guide and motivate the students' PP process. More specifically, the teachers argued that Prompt B's specific wording of "one easy," "one moderately difficult," and "one difficult" would actively engage students because they must think beyond simply posing mathematical questions. In particular, Ms. Wise anticipated that the word "easy" would make students pose at least one problem they could think of. The teachers believed the experience of creating the first problem would make students feel more comfortable posing more problems by making small variations. As Ms. Wise pointed out, the word "easy" would encourage students to come up with an easy problem—something fully within their grasp—and then they would feel more comfortable generating other problems.

> I do think they would also respond better to Prompt B than A, mainly because they're gonna go to the first thing they understand the best and then they're going to add to it. I really believe, and even in my own classroom when I tried problem posing, when I changed it to 'easy, medium, hard.' I always got more responses for the easy-medium-hard [Prompt B]. (Ms. Wise)

This reasoning was very similar to some of the reasoning around the teachers' views about Prompt A; in both cases, teachers felt the prompt would help students begin their PP process. With respect to Task 1, Ms. Chen also argued that Prompt B would give students "more guidance" so that students might ask more useful questions like, "How many meters per second?" Such questions would open up space for discussions regarding the given problem situation's topic, such as the unit rates or constant speed. Ms. Chen also thought Prompt A might not make it clear to students what to pose.

> I just feel like Prompt B gives a little more guidance. One that's going to be simple. One that's going to be like 50/50 is medium level, and then a difficult one. So, I just think that B would have more student responses. (Ms. Chen)

Ms. Chen shared her experience of using a prompt similar to Prompt A. When students were asked to pose problems, she found they had "blank stares" because they were not familiar with PP. This experience stood in contrast to those teachers who viewed Prompt B as scaffolding students' efforts so that they could generate both more responses and more mathematically relevant responses, even if sometimes the process might be a bit convoluted: "So I think they were always able to do the easy one and a hard one and then find out what a medium level looks like." (Ms. Wise)

The teachers who viewed Prompt B as offering superior scaffolding may have also been considering that their students might struggle to extract information from some of the problem situations in the tasks, thus making it more difficult to get started with PP. In other words, the need for scaffolding was not necessarily because the students required help with the idea of PP in general (as in the experience Ms. Chen described above) but rather because the problem situation did not provide what the teachers thought would be readily accessible information with which to pose problems. Indirect evidence for this interpretation can be found in the reasoning that three teachers, Ms. Wise, Ms. Khan, and Ms. Reed, gave when they expressed a preference for Prompt A over Prompt B in the case of Task 3. These three teachers viewed the PP situation of Task 3 as providing enough accessible information for students to begin posing questions so the students would not need the extra scaffolding of Prompt B to help them start to parse the situation. In particular, the inclusion of the geometrical shapes (e.g., square, semi-circles) and their dimensions (e.g., length, width) was seen as providing some easy starting points to create mathematical problems. For example, Ms. Wise thought students could create problems related to the circumference or perimeter by accessing related prior knowledge on geometrical shapes, such as the areas or perimeters of shapes:

> So based on my concept, counting, this one I think [Prompt] A would be easier to do. Mainly because I think the first thing they think is about in circumference like they're gonna re-compose that figure and put the two [semi-]circles together and find the radius. Like, 'What's the radius of the semicircle? What's the base? What's the height? How many shapes compose this figure?' Like, there's a lot of different things that they can put in there. (Ms. Wise)

Ms. Khan also mentioned that they considered this PP situation to have "a decent amount of information" for students to pose different questions. In particular, because Task 3 included a detailed visual representation that was familiar to students as well as measurements that included numerals students could work with, Ms. Khan argued that Prompt A was sufficient for this particular task:

> I think Prompt A. That's just because I believe that there's a decent amount of information there that can spiral into different mathematical problems that they could pose. (Ms. Khan)

Similarly, Ms. Reed stated that Task 3 included "tangible information," like the length and width of the track field, with which to pose mathematical problems, even if the content might be harder for her students. Ms. Reed said:

> This one visually is a little bit more difficult for them to say, 'Okay, here's mine.' For problems, I can immediately say that they could probably find the area and perimeter of that square in the center because that's like tangible information. They can reach and grab it immediately. (Ms. Reed)

For these three teachers, the scaffolding provided by Prompt B was not needed by their students for Task 3 because the situation in this task included information that they were familiar with or could perceive immediately once they encountered the task.

Overall, the teachers who expressed pedagogical considerations related to scaffolding when explaining their views about the prompts were split between preferring Prompt A or Prompt B. They considered the accessibility of the problem situation (e.g., their students' familiarity with the mathematical content) to their students when considering which prompt to pair the situation with. Thus, a more structured prompt would not always be the teachers' go-to prompt when they are looking to provide students with more guidance. If teachers evaluated the problem situation to be accessible enough for their students, as Ms. Khan pointed out, then they would be less likely to pair it with a more scaffolded prompt. Thus, this provides evidence that the teachers' curricular reasoning around PP prompts entwined both considerations about the problem situation and the PP prompt.

4.4. More or less intimidating

Another common view, cited by 7 of the 15 teachers, was that the wording and length of a prompt could contribute to it being more or less intimidating. However, unlike the views discussed above, the teachers differed on which of the two prompts this view applied to. Some viewed Prompt A as less intimidating for students, whereas others viewed Prompt B that way. Here, we examine the teachers' reasoning for these views.

On the surface, it would seem clear that Prompt A is quite brief compared to Prompt B simply because it omits the phrase specifying the difficulty levels of the posed problems (the extra condition). Some teachers expected that the simple wording of Prompt A would help students stay focused on the PP task while also not posing too much of a challenge to their students' reading comprehension. For example, Ms. Strickland stated she would like to use Prompt A because it is "less wordy" than Prompt B, and her students might become disengaged from the task if the amount of reading demanded by the task—both the PP situation and the PP prompt—was too taxing. In particular, because the PP situation in Task 1 was presented in multiple sentences without any visual representations, some teachers felt that a relatively short prompt was called for so as not to lose the students' attention. Along the same lines, in the context of Task 2, Ms. Talley viewed Prompt A as the less intimidating prompt and thus the prompt more likely to engage her students. She anticipated they would have a hard time creating more difficult problems for this task: "I would say with my students I'd have to begin with Prompt A because they would have a hard time building their way up to the more difficult." By using Prompt A, Ms. Talley wanted to increase the accessibility of the task, which would allow students to generate problems instead of shutting down completely.

Ms. Reed expressed a similar preference for using Prompt A with her students because she believed the particular words in Prompt B would automatically "shut down" many of her students:

> This is when I'm gonna lose half of the class. They're gonna shut down after they get over there. Prompt B is intimidating because you're already saying that you're going to pose easy, moderate, and difficult, so that word difficult already gonna shut down a lot of kids. I like the fact that when you say just pose three different, that's a little bit better. That way as they're creating questions I can encourage them to make them different without actually having to use that word. (Ms. Reed)

In particular, Ms. Reed viewed the words "easy," "moderate," and "difficult" (especially "difficult") in Prompt B as intimidating for her students, disengaging many of them from PP completely and making it impossible to achieve any of her intended goals for using the PP task. In short, she viewed Prompt A's relatively simple, unthreatening wording as encouraging her students to stay focused on and engaged with the PP process.

> Parallel reasoning about the impact of particular words in the prompt was offered by Ms. Chen, who expressed a preference for using Prompt A with Task 3:
>
> I think for them, it's a more difficult context. Different, more difficult topics than they've encountered so far in the curriculum. And I just feel like if I were to say pose one easy, medium, and hard to them, I think right now it's all hard for them. I do think

> this is tricky for them. … And I just don't think they would be able to differentiate between easy, medium, [and] hard. So I think by just eliminating that vocabulary and just saying pose three different mathematical problems, I feel like that would be a good enough prompt to give them to get them to pose some problems. (Ms. Chen)

Ms. Chen viewed Prompt A as a less intimidating, and thus more suitable prompt for Task 3 specifically because she thought her students would not be able to differentiate what is an easy, moderately difficult, or difficult problem for this task even if they were able to pose problems. Because her students did not have enough background knowledge about the geometry content of Task 3, Ms. Chen saw the less intimidating "good enough" Prompt A as a way to turn this PP task into an opportunity to explore. Indeed, she worried that words like "easy," "medium," and "hard" in Prompt B would inadvertently communicate to the students that there were specific "correct answers" to this PP task:

> So I think kind of just leaving it broad and just [asking to] pose three different mathematical problems would allow them to just explore. I think telling them to do it easy and medium and hard, if I showed them that prompt, they would say I don't even know what this is. So it kind of makes it sound like an easy, medium, hard, that there is a correct answer. Not that I don't like this. I do like Prompt A and B, but I think they look at that [Prompt B] as there are right and wrong answers. (Ms. Chen)

That said, there were some contrasting views among the teachers about whether the wording of Prompt A was indeed less intimidating and more likely to engage students. Notably, the underlying desire was the same for both groups of teachers: to not disengage students with an intimidating prompt. For example, Ms. Brown considered that, from the perspective of her students, "pose three different mathematical problems" was actually a more complex and intimidating instruction. She believed that her students would disengage as soon as they saw those words:

> Once you say, 'pose three different mathematical problems,' they are gone. … these guys are on a second or third-grade reading level. So Prompt B, 'pose one easy math problem and one difficult math problem.' One easy. One hard. And they get that, and you can work with that. (Ms. Brown)

Instead, Ms. Brown viewed the more specific wording of Prompt B as being less intimidating for her students, thus allowing them a better chance to engage with the PP process. This appears because Ms. Brown parsed Prompt B as discrete short prompts rather than as one long prompt with multiple instructions. Thus, she anticipated her students could focus on them one by one instead of thinking about all the problems at the same time. Ultimately, the root of this reasoning in both cases appears to be maintaining student engagement by avoiding stumbling blocks that would derail the students' attention.

4.5. Differentiation

The last common view about the different prompts for the PP tasks, cited by 9 of the 15 teachers, was that the prompts allowed for differentiation of PP tasks for students. Several of the teachers expressed concerns about their students' potential lack of prior knowledge regarding the mathematical content of a PP task. Moreover, the teachers had a conception of PP more generally as a high-cognitive-demand task. For example, this could be seen in Mr. Owen's thoughts about how different prompts might be more suitable to pair with problem situations about which his students might have more or less background knowledge. In essence, Mr. Owen wanted to differentiate the task for different students based on their prior knowledge related to the problem situation (in particular, the geometrical shapes in Task 3). For students with limited prior knowledge of the mathematical concepts in the task, he expressed a preference for using Prompt A.

> With something like this [task], I feel like it's not stuff that we've covered yet. But, if it's something that they've struggled with, then it's difficult getting them [students] to think about questions that they can do. Not really putting that extra challenge on there. [.] Like with something like this, just getting them to think about. (Mr. Owen)

Because his students had not covered the geometric concepts related to the track-and-field figure in Task 3, Mr. Owen viewed Prompt A as more appropriate because it would modulate down the challenge of the task. But at the same time, he hypothesized that he could appropriately differentiate the task by modulating up the challenge with Prompt B for students in higher grades (Mr. Owen taught 6th grade) who had sufficient prior knowledge of the geometric concepts. He felt that it would be necessary to challenge those students, given that they would already know the related concepts well. In a similar vein, Mr. Owen viewed Prompt B as more appropriate for Task 1 with his own students because the mathematical content was repeatedly addressed in their curriculum, and thus, he anticipated his students would be able to pose some challenging questions.

This view of achieving differentiation by using either prompt was also observed in Ms. Strickland's interview. Although Ms. Strickland expressed a preference towards Prompt B for Tasks 1 and 3 (the rate task and the track-and-field task) because of "the amount of thinking it [Prompt B] forces," she saw a need to differentiate Task 2 (the statistics task) for different groups of students. She viewed the prompts as giving her the ability to achieve that differentiation.

> I maybe like to differentiate that based on kids or groups. I do have them sometimes seated like I have two different settings that I use depending on if I'm going to do a small group or if I'm going to have them kind of work more with each other than with me or my special education teacher. I feel like that would be something I'd be intentional about like, "Hey, this group is going to get Prompt B. This group is going to get Prompt A," just to create "These are the kids that need the extension and the extra push. These are the kids who might struggle just to do one question." If everybody was on the level that I would want them at, I would

> definitely go with Prompt B. But just knowing the range of students, Prompt B would be difficult for a lot of them. Prompt A is the more easily accessible. But, I'm partial to Prompt B for some of my students that I'm thinking of who would thrive with something like that. (Ms. Strickland).

Ms. Strickland viewed Prompt A as more accessible to most students, but Prompt B was only appropriate for the students who have the capacity to make an "extra push." Clearly, Ms. Strickland viewed the prompts as offering her the ability to differentiate the task beyond a one-size-fits-all approach.

In the reasoning of Mr. Owen and Ms. Strickland, we can see an echo of the reasoning other teachers used to explain their preferences for using Prompts A and B. Both Mr. Owen and Ms. Strickland viewed Prompt A as more appropriate for the students who often struggle with mathematics or lack prior knowledge related to the mathematical content of the PP situations. However, Prompt B was viewed as more appropriate for the students who had a greater capacity for challenge in mathematics because it called for more cognitive effort and critical thinking.

4.6. Summary

The *most common views* that the teachers expressed about the PP prompts fell into five categories. The most cited view concerned the openness of Prompt A, which was comparatively the more simply stated of the two prompts (i.e., its syntax was simpler, and it did not impose additional conditions on the posed problems). This view was typically cited by teachers in relation to their view of the accessibility of the PP task for their students. So, the openness view was connected to the teachers' concerns regarding their students' engagement with the task and regarding the task providing an access point for any student to take ownership of the mathematics.

The *second most cited* view concerned the capacity of Prompt B to promote critical thinking. Looking across the three tasks, all 8 teachers who expressed this view argued that Prompt B would encourage the students to think more deeply and perhaps engage in critical thinking. *The third most cited view*, also exclusively regarding Prompt B, was about the scaffolding that the prompt could provide for students through the inclusion of more specific directions (compared to Prompt A) that would actively guide and motivate the students' PP process. The teachers who expressed this view found that Prompt B's specific inclusion of the additional conditions of "one easy," "one moderately difficult," and "one difficult" engaged students to think beyond simply posing mathematical questions.

Unlike the above views, which the teachers tended to ascribe to only one of the prompts, the teachers' views about the wording and length of the prompts included both Prompt A and Prompt B. That is, although about half of the teachers believed less intimidating prompts would be more appropriate for keeping their students engaged in the PP process, some of them argued that the brevity of Prompt A made it less intimidating, whereas others saw their students parsing Prompt B into smaller, more approachable bites suitable for their reading comprehension level. The final view expressed by a large number of teachers related to the teachers' desire to differentiate PP tasks to meet the needs of different students. This often involved concerns about students' prior knowledge regarding the PP task context and the teachers' conception of PP as a high-cognitive-demand task.

5. Discussion

This study sheds light both on how teachers think about and understand PP as a newly adopted instructional activity and on how they make sense of specific characteristics of PP tasks—notably, prompts—that could influence their students' engagement with PP. We found that the teachers anticipated challenges that their students would encounter based on variations in task characteristics (e.g., syntax, mathematical content, context, visual representation) as well as on their judgment of their students' prior knowledge related to the tasks and familiarity with PP instruction in general (Cai et al., 2022). Moreover, the views that the teachers expressed about the PP prompts revealed that they attended to the openness of PP tasks, the degree to which they scaffolded and supported students' critical thinking, the ways in which PP tasks might be more or less intimidating (e.g., by changing the syntax), and the opportunities for differentiation offered by PP tasks.

Although the teachers' views varied from task to task, there did appear to be a pair of overarching concerns that were observed across the teachers' responses, concerns that are more generally relevant in teachers' curricular reasoning around any new instructional practice or curriculum material that teachers might evaluate or adopt (Hourigan & Leavy, 2023). As Wang et al. (2021) found, when reasoning through such evaluations, teachers often attend to the accessibility of the practice or material and to the appropriateness of the challenge it offers. Thus, in the present study, we saw on the one hand that the teachers' views about the prompts signaled a strong commitment to increasing their students' participation and their engagement with the mathematics underlying the PP tasks—a concern about accessibility. This was evident in their reasoning about how prompts could affect the accessibility of the PP tasks to lower the floor, how the precise wording of a prompt could make a task less intimidating, and how they could deploy different prompts to differentiate a PP task to allow different subgroups of students to engage with the task. On the other hand, the teachers also expressed views that signaled a concern for maintaining a certain level of cognitive demand in the PP tasks or raising the ceiling to keep an appropriate level of challenge available. This could be seen in their reasoning about how prompts could encourage or scaffold students to engage in critical thinking that would deepen their understanding of mathematics (Cai, Hwang, Jiang, & Silber, 2015). In these ways, the teachers' views and their reasoning around PP prompts and tasks are representative of more general curricular reasoning that teachers engage in around new instructional tools.

In addition to these more general issues, the teachers' views about the PP prompts and their reasoning behind those views also showed connections to important pedagogical goals that P-PBL is specifically meant to support. First, the teachers' views clearly showed that they saw PP activities as a venue for creating rich opportunities for mathematical sensemaking for all students (Christou et al., 2005; Cifarelli & Cai, 2005; Silver, 1994). The teachers in this study viewed PP tasks as highly cognitively demanding (Smith & Stein, 2011; Stein & Smith, 1998) and anticipated that their students might face challenges due to a lack of prior content knowledge or familiarity with PP. At the same time, the teachers' views specifically about the PP prompts exhibited their understanding of how the openness or less intimidating nature of some prompts could allow students to get around those challenges and still engage in the desired mathematical sensemaking. Indeed, this accords with research showing that students at various levels can successfully pose mathematical problems (Cai, Hwang, Jiang, & Silber, 2015). Thus, rather than simply considering PP tasks as generically challenging open-ended or nonroutine tasks, perhaps only appropriate for high-achievers or upper-grade students who have prior knowledge (Anderson, 2003), the teachers in this study viewed the PP prompts as tools to adjust the challenge of the task and maximize sensemaking opportunities. By giving students a chance to pose multiple problems, even if the task constraints are loose enough to allow students to pose similar or repetitive problems, the task still creates opportunities to recognize common structures across problems and reason about those common structures (T. Chen & Cai, 2020). Ultimately, thinking about the pedagogical implications of pairing a particular PP prompt with a particular problem situation in this way appears to be a kind of curricular reasoning specific to teaching mathematics through PP (Cai, 2022; Cai & Hwang, 2023).

Second, the teachers' views about the PP prompts in this study reflect a belief in PP as a tool to help students learn mathematical content and processes more deeply (Toluk-Uçar, 2009). Like Ms. Garrett and Ms. Love, several teachers explained how PP tasks where the prompt adds a "push" by imposing additional conditions on the posed problems have more potential to engage students' critical thinking and to develop a deeper understanding of the mathematics in the situation. In this case, the teachers seemed to believe PP prompts could not only lower the floor of the task but also raise the ceiling (Cai, Hwang, Jiang, & Silber, 2015) to make it both appropriately challenging and useful as a space for effective mathematical learning beyond the immediate task (Wang et al., 2021; Webel et al., 2015). For example, by specifying difficulty levels of problems students need to pose (e.g., one easy, one moderately difficult, and one difficult), a PP prompt may encourage students to think both about mathematically rich problems and to critically reflect (individually and with their peers) on what problem difficulty means. This aligns with Silber and Cai's (2017) finding that more structured PP conditions may lead posers to attend to deeper mathematical relationships in problem situations—relationships that are not highlighted by problems that may be answered simply by reading directly from the given information but by problems that require procedures with connections or even nonroutine problems that require students' reasoning skills (Silver, 1994; Smith & Stein, 2011; Stein & Smith, 1998). Furthermore, the teachers' views about how to use the prompts to deepen students' learning from the task were nuanced by the degree to which the teachers felt the mathematical content in a PP task (specifically the problem situation) was already accessible to students (e.g., recognizing that the scaffolding provided by Prompt B may not always be needed if the problem situation involves familiar geometry content). In sum, the teachers' views about this aspect of PP prompts show that their curricular reasoning around PP includes reasoning about how to calibrate and balance task accessibility and robustness (through manipulation of the PP prompt) for cultivating students' mathematical thinking.

Third, the teachers' views and reasoning around the PP prompts highlighted their attention to developing students' identities as creative doers of mathematics who have ownership of their learning (Akay & Boz, 2010; Cai, 2022). As noted above, students' engagement in PP processes can develop their mathematical creativity (Silver, 1997). The teachers in this study recognized that PP tasks can be intimidating to some extent. Thus, their curricular reasoning sometimes focused on how they could make the PP tasks less intimidating so that students could actively engage as creative doers of mathematics, posing their own problems instead of simply following along with the teacher. By increasing the accessibility, the teachers wanted to lower the floor so that students could easily engage in PP processes and thus have an opportunity to direct their own mathematical activity. Moreover, connecting to Wang et al.'s (2021) framework, this concern of the teachers was inextricably bound up with their desire to maintain students' engagement with the task. By choosing a prompt that made students feel less intimidated, teachers aimed to keep them engaged and posing problems. By asking students to actively engage in formulating the mathematical problems they would pursue, the teachers shared agency in the mathematics classroom with the students (Agarwal, 2020).

6. Directions for future research

The findings of the present study suggest multiple avenues for future research. First, there is a need to compare teachers' views of PP tasks and prompts with students' actual reactions to those tasks. Because the teachers in this study reasoned about their prompt preferences based in part on their predictions of how their students would respond, the consistency between those predictions and students' reality is a critical link that connects teachers' instructional decisions to students' existing understanding. However, predicting students' responses is not a trivial task. Indeed, Xu et al. (2020) identified mismatches between teachers' predictions and students' actual PP, suggesting that this kind of pedagogical prediction needs further study. To that end, in a parallel study based on the same longitudinal project as the present study, we are currently investigating the impact of PP prompts on students' PP (Cai et al., 2023). The findings from that study support the hypothesis that students pose more complex problems in response to prompts like Prompt B than they do for prompts like Prompt A. That is, as some of the teachers in this study intuited, PP tasks that use prompts with more direction and guidance (e.g., pose one difficult problem, pose a similar problem) do appear to be more cognitively demanding for students than tasks that use prompts with less direction and guidance because students need to think further with respect to the difficulty levels of posed problems (Cai et al., 2023). Future research in this direction that would move the field forward could investigate actual P-PBL in classrooms, and specifically how students in a classroom environment actually respond to PP tasks with various prompts. This would provide useful data on what different learning opportunities are created by tasks with different prompts.

Second, as discussed above, the views expressed by the teachers in this study about the PP prompts illuminate their reasoning about three intended benefits of teaching mathematics through PP: creating rich opportunities for mathematical sensemaking (Christou et al., 2005; Cifarelli & Cai, 2005), helping students learn mathematical content and processes more deeply (Toluk-Uçar, 2009), and developing students' identities as creative doers of mathematics who have ownership of their learning (Akay & Boz, 2010; Silver, 1997). However, none of the teachers expressed views during the interviews that touched on a fourth key benefit of P-PBL: making connections between students' life experiences and the mathematics they are learning in the classroom (Walkington & Bernacki, 2015). It is possible that Prompts A and B, being fairly generic, did not offer substantial opportunities for the teachers to consider how PP could connect with their students' life experiences. Indeed, one could imagine a prompt that explicitly asks students to draw on their own experiences to pose a problem, effectively making the students' lives the "problem situation." Alternatively, because connecting students' everyday knowledge and mathematical knowledge through PP depends on recognizing and acknowledging certain norms of school mathematics problems (Walkington & Bernacki, 2015), this goal may have been more difficult for the teachers to attend to. However, given the potential to help students connect mathematics with their lives, it seems important to investigate how to support teachers in achieving this goal of P-PBL.

Third, many of the teachers in this study attended to both the level of structure imposed by a PP prompt, whether it be the accessibility of Prompt A or the scaffolded quality of Prompt B, and to the relative openness of the problem situation (cf. Stoyanova and Ellerton, 1996 distinction between "free" and "structured" PP situations). Indeed, Ms. Chen seemed to hypothesize that pairing the relatively open situation of Task 2 with an open, "vaguer" prompt like Prompt A would help her students get started with posing problems. This suggests a line of empirical inquiry into how matching the openness of PP prompts and problem situations influences the overall PP task. Does it make sense to pair more open problem situations with more open PP prompts and more detailed or constrained problem situations with prompts that impose greater structure on the posed problems? How would different pairings of open and structured prompts and situations affect the pedagogical qualities of PP tasks?

Fourth, the teachers who participated in this study had very limited exposure to PP at the time of the interviews (the fall semester of their first year of participation in the project). Because this study is situated within a longitudinal project focused on supporting teachers to teach with PP over multiple years, we believe that further exploration is worthwhile to confirm whether their views are stable over time or whether there are trajectories of teachers' PP use or views about PP and PP prompts (Stickles, 2011) that can be observed. As these teachers continue to learn about P-PBL in subsequent years, they will encounter examples of many other possible PP prompts and gain experience using them in their classrooms. We intend to follow teachers through multiple professional development sessions across several years to look for changes in their perspectives towards teaching with PP (Silver, 1994) as well as the impact of any such changes on their curricular reasoning about PP prompts and other aspects of PP tasks as well as on their implementation of those tasks in their classrooms (Dietiker et al., 2018).

CRediT authorship contribution statement

Faith Muirhead: Writing – review & editing. **Jinfa Cai:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization. **Jaepil Han:** Writing – original draft, Formal analysis. **Stephen Hwang:** Writing – original draft, Formal analysis, Conceptualization.

Declaration of Competing Interest

The authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or nonfinancial interest in the subject matter or materials discussed in this manuscript.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgments

During the preparation of this manuscript, the authors were supported by a grant from the National Science Foundation (DRL-2101552). Any opinions expressed herein are those of the authors and do not necessarily represent the views of the National Science Foundation. We thank Matthew Melville for assistance in conducting interviews and Simon Stewart for assistance with the transcriptions.

References

Agarwal, P. (2020). Organizing for collective agency: negotiating social and disciplinary risks of collaborative learning. In M. Gresalfi, & I. S. Horn (Eds.), *The Interdisciplinarity of the Learning Sciences, 14th International Conference of the Learning Sciences (ICLS) 2020* (Vol. 1, pp. 223–229). International Society of the Learning Sciences.

Akay, H., & Boz, N. (2010). The effect of problem posing oriented Analyses-II Course on the attitudes toward mathematics and mathematics self-efficacy of elementary prospective mathematics teachers. *Australian Journal of Teacher Education, 35*(1), 59–75. https://doi.org/10.14221/ajte.2010v35n1.6

Anderson, J. (2003). Teachers' choice of tasks: a window into beliefs about the role of problem solving in learning mathematics. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Mathematics Education Research: Innovation, networking, opportunity (Proceedings of the 26th Conference of the Mathematics Education Research Group of Australasia* (pp. 72–79). MERGA.

Barnett, J. (1984). The study of syntax variables. In G. A. Goldin, & C. E. McClintock (Eds.), *Task Variables in Mathematical Problem Solving* (pp. 23–68). Franklin Institute Press.

Butts, T. (1980). Posing problems properly. In S. Krulik, & R. Reys (Eds.), *Problem Solving in School Mathematics* (pp. 23–33). National Council of Teachers of Mathematics.
Cai, J. (2004). Why do US and Chinese students think differently in mathematical problem solving? Impact of early algebra learning and teachers' beliefs. *The Journal of Mathematical Behavior, 23*(2), 135–167.
Cai, J. (2022). What research says about teaching mathematics through problem posing. *Éducation et Didactique, 3*(*16*), 31–50.
Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *The Journal of Mathematical Behavior, 21*(4), 401–421.
Cai, J., & Hwang, S. (2020). Learning to teach through mathematical problem posing: Theoretical considerations, methodology, and directions for future research. *International Journal of Educational Research, 102*, Article 101391.
Cai, J., & Hwang, S. (2021). Teachers as redesigners of curriculum to teach mathematics through problem posing: Conceptualization and initial findings of a problem-posing project. *ZDM–Mathematics Education, 53*, 1403–1416.
Cai, J., & Hwang, S. (2023). Making mathematics challenging through problem posing in the classroom. In R. Leikin (Ed.), *Mathematical challenges for all* (pp. 115–145). Springer.
Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-posing research in mathematics education: Some answered and unanswered questions. In F. M. Singer, N. F. Ellerton, & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 3–34). Springer. https://doi.org/10.1007/978-1-4614-6258-3_1.
Cai, J., & Jiang, C. (2017). An analysis of problem-posing tasks in Chinese and US elementary mathematics textbooks. *International Journal of Science and Mathematics Education, 15*(8), 1521–1540.
Cai, J., Koichu, B., Rott, B., Zazkis, R., & Jiang, C. (2022). Mathematical problem posing: task variables, processes, and products. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 119–145). PME.
Cai, J., Ran, H., Hwang, S., Ma, Y., Han, J., & Muirhead, F. (2023). Impact of prompts on students' mathematical problem posing. *The Journal of Mathematical Behavior, 72*, 101087.
Chen, L., Van Dooren, W., Chen, Q., & Verschaffel, L. (2007). The relationship between posing and solving arithmetic word problems among Chinese elementary school children. *Research in Mathematics Education, 11*(1), 1–31.
Chen, T., & Cai, J. (2020). An elementary mathematics teacher learning to teach using problem posing: A case of the distributive property of multiplication over addition. *International Journal of Educational Research, 102*, Article 101420.
Christou, C., Mousoulides, N., Pittalis, M., Pitta-Pantazi, D., & Sriraman, B. (2005). An empirical taxonomy of problem posing processes. *ZDM–Mathematics Education, 37*(3), 149–158.
Cifarelli, V. V., & Cai, J. (2005). The evolution of mathematical explorations in open-ended problem-solving situations. *The Journal of Mathematical Behavior, 24*, 302–324.
Dietiker, L., Males, L. M., Amador, J. M., & Earnest, D. (2018). Curricular noticing: A framework to describe teachers' interactions with curriculum materials. *Journal for Research in Mathematics Education, 49*(5), 521–532.
Einstein, A., Infeld, L., & Hoffmann, B. (1938). The gravitational equations and the problem of motion. *Annals of Mathematics*, 65–100.
Ellerton, N. F. (1986). Children's made-up mathematics problems: A new perspective on talented mathematicians. *Educational Studies in Mathematics, 17*(3), 261–271.
Task variables in mathematical problem solving. In Goldin, G. A., & McClintock, C. E. (Eds.), *Franklin Institute Press*, (1984).
Hadamard, J. (1945). *The Psychology of Invention in the Mathematical Field*. Princeton University Press.
Hourigan, M., & Leavy, A. M. (2023). Elementary teachers' experience of engaging with teaching through problem solving using lesson study. *Mathematics Education Research Journal, 35*, 901–927.
Jerman, M. (1971). Instruction in problem solving and an analysis of structural variables that contribute to problem-solving difficulty (Technical Report No. 180). *Institute for Mathematical Studies in the Social Sciences*. Stanford University.
Jung, S., & Park, M. (2010). The effects of the mathematical problem generation program on problem-solving ability and learning attitude. *Journal of Elementary Mathematics Education in Korea, 14*(2), 315–335.
Kulm, G. (1984). The classification of problem-solving research variables. In G. A. Goldin, & C. E. McClintock (Eds.), *Task Variables in Mathematical Problem Solving* (pp. 1–22). Franklin Institute Press.
Lester, F. K. Jr (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast, 10*(1). https://doi.org/10.54870/1551-3440.1267
Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal, 9*(1), 5–24.
Li, X., Song, N., Hwang, S., & Cai, J. (2020). Learning to teach mathematics through problem posing: Teachers' beliefs and performance on problem posing. *Educational Studies in Mathematics, 105*(3), 325–347. https://doi.org/10.1007/s10649-020-09981-0
Li, X., Zhou, W., Hwang, S., & Cai, J. (2022). Learning to teach mathematics through problem posing: Teachers' beliefs and their instructional practices. *International Journal of Educational Research, 115*, Article 102038. https://doi.org/10.1016/j.ijer.2022.102038
Merriam, S. B., & Tisdell, E. J. (2015). *Qualitative Research: A Guide to Design and Implementation*. John Wiley & Sons.
Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative Data Analysis: A Methods Sourcebook* (3rd ed.). SAGE.
National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Author.
National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.
Park, M., Lee, E. J., & Cho, J. W. (2019). An analysis of problem-posing tasks in 7th grade mathematics textbooks based on 2015 national mathematics curriculum. *Journal of the Korean Society of Mathematical Education Series E: Communications of Mathematical Education, 33*(2), 123–139.
Silber, S., & Cai, J. (2017). Pre-service teachers' free and structured mathematical problem posing. International Journal of Mathematical Education in Science and Technology, *48*(2), 163–184.
Roth McDuffie, A., & Mather, M. (2009). Middle school teachers' use of curricular reasoning in a collaborative professional development project. In J. Remillard, G. Lloyd, & B. Herbel-Eisenmann (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 302–320). Routledge.
Silber, S., & Cai, J. (2021). Exploring underprepared undergraduate students' mathematical problem posing. *ZDM–Mathematics Education, 53*(4), 877–889.
Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.
Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM–Mathematics Education, 29*, 75–80. https://doi.org/10.1007/s11858-997-0003-x
Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521–539.
Smith, M. S., & Stein, M. K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. National Council of Teachers of Mathematics.
Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Reaching in the Middle School, 3*(4), 268–275.
Stickles, P. (2011). An analysis of secondary and middle school teachers' mathematical problem posing. *Investigations in Mathematics Learning, 3*(2), 1–34.
Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. *Technology in Mathematics Education, 4*(7), 518–525.
Toluk-Uçar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and Teacher Education, 25*(1), 166–175.
Walkington, C., & Bernacki, M. (2015). Students authoring personalized "algebra stories": Problem-posing in the context of out-of-school interests. *The Journal of Mathematical Behavior, 40*, 171–191.
Wang, E.L., Tuma, A.P., Doan, S., Henry, D., Lawrence, R.A., Woo, A., & Kaufman, & J.H. (2021). Teachers' perceptions of what makes instructional materials engaging, appropriately challenging, and usable: A survey and interview study of teachers. Retrieved from ⟨https://www.rand.org/content/dam/rand/pubs/research_reports/RRA100/RRA134-2/RAND_RRA134-2.pdf⟩.
Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*, 375–400.

Webel, C., Krupa, E. E., & McManus, J. (2015). Teachers' evaluations and use of web-based curriculum resources in relation to the Common Core State Standards for Mathematics. *Middle Grades Research Journal, 10*(2), 49–64.

Xu, B., Cai, J., Liu, Q., & Hwang, S. (2020). Teachers' predictions of students' mathematical thinking related to problem posing. *International Journal of Educational Research, 102*, Article 101427.

Yao, Y., Hwang, S., & Cai, J. (2021). Preservice teachers' mathematical understanding exhibited in problem posing and problem solving. *ZDM–Mathematics Education, 53*(4), 937–949.

Zhang, H., & Cai, J. (2021). Teaching mathematics through problem posing: Insights from an analysis of teaching cases. *ZDM–Mathematics Education, 53*(4), 961–973.

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., Malamut, J., 2017. Principles for the design of mathematics curricula: Promoting language and content development. Retrieved from Stanford University, UL/SCALE website: ⟨http://ell.stanford.edu/content/mathematics-resources-additional-resources⟩.