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WHAT CAN TIME-SERIES REGRESSIONS TELL US ABOUT POLICY COUNTERFACTUALS?

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We show that, in a general family of linearized structural macroeconomic models, knowledge of the empirically estimable causal effects of contemporaneous and news shocks to the prevailing policy rule is sufficient to construct counterfactuals under alternative policy rules. If the researcher is willing to postulate a loss function, our results furthermore allow her to recover an optimal policy rule for that loss. Under our assumptions, the derived counterfactuals and optimal policies are robust to the Lucas critique. We then discuss strategies for applying these insights when only a limited amount of empirical causal evidence on policy shock transmission is available.

KEYWORDS: Lucas critique, policy counterfactuals, macroeconomic modeling, business cycles, monetary policy, policy shocks.

1. INTRODUCTION

AN IMPORTANT FUNCTION OF MACROECONOMICS IS TO PREDICT the consequences of changes in policy. In this paper, we revisit the role that evidence on policy *shocks*—that is, surprise deviations from a prevailing rule—can play in helping macroeconomists learn about policy *rule* counterfactuals. Existing work mainly uses such policy shocks in two ways. First, in what Christiano, Eichenbaum, and Evans (1999) call the "Lucas program," researchers begin by estimating the causal effects of a policy shock in the data, then construct a micro-founded structural model that matches these effects, and finally trust the model as a laboratory for predicting the effects of changes in policy rules. By design, this approach yields counterfactuals that are robust to the Lucas (1976) critique; on the other hand, the researcher needs to commit to a particular parametric model, thus introducing concerns about model misspecification. An alternative approach, proposed by Sims and Zha (1995), instead relies *only* on the estimated policy shock: in their procedure, the economy is subjected to a new policy shock at each date t, with the shocks chosen so that, t-by-t, the counterfactual policy rule holds.[1] This strategy does not require the researcher

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[1]See, for example, Ramey (1993), Bernanke, Gertler, Watson, Sims, and Friedman (1997), Leeper and Zha (2003), Hamilton and Herrera (2004), Uribe and Yue (2006), Degasperi, Hong, and Ricco (2020), Eberly, Stock, and Wright (2020), Brunnermeier, Palia, Sastry, and Sims (2021), and Antolin-Diaz, Petrella, and Rubio-Ramírez (2021) for important applications and extensions of this method.

to commit to a particular model, but it is subject to the Lucas critique: a rule change announced at date 0 will in general have different effects on private-sector decisions than a sequence of surprise policy shocks at $t = 0, 1, \ldots$.

The contribution of this paper is to propose a method that constructs policy counterfactuals using empirical evidence on *multiple distinct* policy shocks, rather than just a single one. Like Sims and Zha, the method does not rely on a particular parametric structural model; at the same time, for a family of models that nests many of those popular in the Lucas program, it yields counterfactuals that are robust to the Lucas critique. At the heart of our methodology lies an identification result. We prove that, for a relatively general family of macro models, the causal effects of contemporaneous as well as news shocks to a given policy rule are sufficient to construct Lucas critique-robust counterfactuals for alternative policy rules. The core intuition is that, by subjecting the economy to multiple distinct policy shocks at date 0 (rather than a new value of a single shock at $t = 0, 1, \ldots$, as done in Sims and Zha), we are able to enforce the contemplated counterfactual policy rule not just *ex post* along the equilibrium path, but also *ex ante* in private-sector expectations. Under our assumptions, doing so is enough to fully sidestep the Lucas critique. While our exact identification result requires knowledge of the causal effects of a very large number of policy shocks, our proposed empirical method can be applied in the empirically relevant case of a researcher with access to only a couple of distinct shocks. We demonstrate the usefulness of the proposed approach with several applications to monetary policy rule counterfactuals.

Identification Result

The first part of the paper establishes the identification result. Our analysis builds on a general linear data-generating process, with one key added restriction: policy is allowed to affect private-sector behavior *only* through the current and future expected path of the policy instrument.[2] For example, for monetary policy, the private sector only cares about the expected future path of the nominal rate, and not whether this path is the result of the systematic component of policy—that is, the policy *rule*—or due to *shocks* to a given rule. We consider an econometrician that lives in this economy and observes data generated under some baseline policy rule, where that rule is subject to shocks. Using standard time-series methods, she can estimate the causal effects of these policy shocks (Ramey (2016)). She then wishes to predict how a certain historical episode would have unfolded or how a particular shock would have propagated under some alternative policy rule.

In this setting, we establish the following identification result. Suppose the econometrician is able to estimate how contemporaneous shocks to the prevailing rule as well as news about deviations from that rule *at all future horizons* affect the variables that enter her hypothesized counterfactual rule. Then these estimates contain all the information she needs to construct her desired counterfactual; in particular, she need not know any of the structural equations of the underlying model, including the prevailing policy rule. Key to the proof is our assumption on how policy is allowed to shape private-sector behavior. Since only the expected future path of the policy instrument matters, any given *rule*—characterized by the instrument path that it implies—can equivalently be synthesized by adding *shocks* to the baseline rule. All that is required is that those policy shocks imply the

[2]More precisely, the policy rule is allowed to matter only through (a) the expected path of the instrument and (b) equilibrium selection. Our method will construct *one* valid equilibrium corresponding to the hypothesized counterfactual rule; if this rule induces a unique equilibrium, then our method recovers it.

same expected instrument path from date-0 onwards as the counterfactual rule. Finally, we show that, given a loss function, our econometrician can leverage the same logic to also characterize *optimal* policy.[3]

How general is the setting of this identification result? Our two key model restrictions are (i) linearity and (ii) the way that policy is allowed to shape private-sector behavior. We show that (ii) is a feature shared by many business-cycle models, including those with many frictions (e.g., Christiano, Eichenbaum, and Evans (2005)), shocks (e.g., Smets and Wouters (2007)), and rich micro heterogeneity (e.g., Kaplan, Moll, and Violante (2018)). Perhaps the most popular class of structural models violating the restriction is those with an asymmetry of information between the policymaker and private sector (as in Lucas (1972)). In such models, private-sector agents solve a filtering problem, and so the policy rule affects both the dynamics of the policy instrument as well as the information contained in that policy choice; as a result, the policy instrument itself does not afford a full characterization of the policy stance. The linearity assumption (i), on the other hand, is not a conceptual necessity, but rather a practical one. Linearity implies that the effects of policy changes are invariant to their size, their sign, and the state of the economy. Given certainty equivalence, we can thus focus on expected values. As we will see, these simplifications are crucial to connect our theory to empirical evidence. Linearity does, of course, also impose costs: in practice, the methodology that we propose can be used to compare different cyclical stabilization policies (e.g., Taylor rules), but it is less well-suited to study policies that alter the steady state (e.g., changes in the inflation target).

Empirical Strategy

The main challenge to operationalizing our identification result is that empirical evidence on the causal effects of policy shocks is limited. Our theory says that we need to select a linear combination of policy shocks at date 0 that perturbs the current and expected future path of the policy instrument just like the contemplated counterfactual rule. This is a daunting informational requirement: in general, to synthesize the effects of any possible expected policy instrument path of length T (with T large in practice), we would need access to T distinct policy shocks that each imply differentially shaped impulse response paths of the policy instrument, thus allowing us to span all of $\mathbb{R}^T$. While existing empirical evidence falls short of this ideal, recent research has, however, made progress on identifying the effects of at least *some* distinct policy shocks with rather different implications for future expected policy paths.[4] How much can be done with this available evidence?

The idea of our empirical method is to use the available evidence on policy shock transmission to provide a *best Lucas critique-robust approximation* to the desired counterfactual. Given estimates of the dynamic causal effects of a small number n_s of policy shocks

[3]To be clear, our results are silent on the mapping from observables to welfare, and so on the shape of loss functions. Structural models are one way to arrive at such objectives. However, given that objective functions in practice are often derived from a legislated mandate rather than economic theory (e.g., dual mandate), we believe it is useful to have a method of calculating optimal policy for a given objective.

[4]For monetary policy, many of the different popular shock series (e.g., Romer and Romer (2004), Gertler and Karadi (2015), Antolín-Díaz and Rubio-Ramírez (2018), Bauer and Swanson (2022)) are well known to lead to rather different responses of short-term rates. Other identification strategies explicitly aim to identify shocks at different parts of the yield curve (e.g., Gürkaynak, Sack, and Swanson (2005), Antolin-Diaz, Petrella, and Rubio-Ramírez (2021), Inoue and Rossi (2021)), as required by our theory. For fiscal policy, Ramey (2011) and Ramey and Zubairy (2018) estimated the effects of short-lived as well as more persistent shocks. Mertens and Ravn (2010) and Leeper, Walker, and Yang (2013) are similarly focused on disentangling shocks with different policy instrument dynamics..

and their associated policy instrument paths, we face the challenge that our identification result cannot be applied immediately: the counterfactual policy rule needs to hold in ex post equilibrium and ex ante expectation for a large number T of periods, but we only have access to $n_s \ll T$ shocks—more equations than unknowns. Our proposal is simply to choose the linear combination of date-0 shocks that enforces the desired counterfactual rule as well as possible, in a standard least-squares sense. Crucially, since this approach involves no ex post surprises dated $t = 1, 2, \dots$, it is—under our assumptions—fully robust to Lucas critique concerns. Whether or not this best approximation is then in fact a sufficiently accurate representation of the desired counterfactual policy is invariably an application-dependent question.

Applications

We demonstrate the uses and limitations of our empirical method through several examples. Our object of interest is the propagation of a contractionary investment-specific technology shock under different monetary policy rules. As the inputs to our method, we consider the two most popular monetary policy shock series: those of Romer and Romer (2004) and Gertler and Karadi (2015). Importantly, these two shocks reflect different kinds of monetary news—a relatively transitory innovation for Romer and Romer, and a much more gradual rate change for Gertler and Karadi.

Armed with the causal effects associated with those two distinct nominal interest rate paths, we then apply our empirical method to construct counterfactuals for alternative policy rules that: target the output gap, enforce a Taylor-type rule, peg the nominal rate of interest, target nominal GDP, and minimize a simple dual-mandate loss function. We find that, with the exception of the nominal rate peg, the counterfactual rules can be enforced to quite a high degree of accuracy. The conclusion is that, at least for our investment shock, several rather different monetary policy counterfactuals can already be characterized quite sharply simply by combining existing pieces of empirical evidence on monetary policy shock transmission, without commitment to any particular parametric structural model.

Literature

Our identification result provides a bridge between the micro-founded models of the "Lucas program" (as discussed in Christiano, Eichenbaum, and Evans (1999)) and the empirical strategy proposed by Sims and Zha (1995). Our results reveal that, in the structural models typically used in the Lucas program, the estimand of the econometric strategy of Sims and Zha is not equal to the true policy rule counterfactual *only* because of expectational effects related to the future conduct of policy. In theory, using multiple distinct policy shocks at date 0 (rather than a single one at each $t \geq 0$) circumvents this problem; in practice, doing so is feasible because a growing literature on the semi-structural identification of policy shocks provides us with a fairly rich body of empirical evidence (see the references in Footnote 4).[5]

Our work also relates to other more recent contributions to counterfactual policy analysis. Beraja (2020) similarly formed policy counterfactuals without relying on particular

[5]A different route was taken in Leeper and Zha (2003): these authors argued that, if the policy shocks required to implement Sims and Zha are small enough, then it may be credible to *ignore* expectational effects.

parametric models. His approach relies on stronger exclusion restrictions in the non-policy block of the economy, but given those restrictions requires less evidence on policy news shocks. Barnichon and Mesters (2021) used policy shock impulse responses to evaluate the optimality of and then improve upon a given policy decision. While their focus was on a single policy choice, we instead study systematic changes in the policy *rule*, requiring additional assumptions on the economic environment—our two assumptions discussed above.[6] More broadly, our work relates to the increasing popularity of a "sufficient statistics" logic for counterfactual analysis (e.g., Chetty (2009), Arkolakis, Costinot, and Rodríguez-Clare (2012), Nakamura and Steinsson (2018)). Our identification result reveals that, across a broad class of models, the empirically estimable causal effects of policy shocks are precisely such sufficient statistics.

Finally, to prove our identification result, we build on recent advances in solution methods for structural macroeconomic models. At the heart of our analysis lies the fact that equilibria in such models can be characterized by matrices of impulse response functions. As in Guren, McKay, Nakamura, and Steinsson (2021) and Wolf (2020), we connect this sequence-space representation to empirically estimable objects. In contemporaneous and independent work, De Groot, Mazelis, Motto, and Ristiniemi (2021) and Hebden and Winkler (2021) showed how to use similar arguments to efficiently *compute* policy counterfactuals by generating impulse responses to policy shocks from a structural model. Our focus is not computational—we aim to calculate policy counterfactuals directly from empirical evidence, forcing us to confront the fact that such evidence is limited.

Outline

Section 2 presents our identification result, mapping the effects of policy *shocks* to counterfactuals for policy *rules*. Section 3 introduces our empirical methodology, and Section 4 provides applications to monetary policy rule counterfactuals. Section 5 concludes.

2. FROM POLICY SHOCKS TO POLICY RULE COUNTERFACTUALS

We begin in Section 2.1 by presenting a stylized version of our identification argument in a simple, illustrative model. We then in Sections 2.2 to 2.5 extend the argument to a general class of infinite-horizon models and discuss its scope and limitations.

The main identification result will be presented for a linearized perfect-foresight economy. Due to certainty equivalence, the equilibrium dynamics of a linear model with uncertainty coincide with the solution to such a linearized perfect-foresight environment. We thus emphasize that all results presented below extend without any change to models with aggregate risk solved using first-order perturbation techniques.[7] In particular, the perfect-foresight transition paths that we characterize will correspond to *expected* transition paths—or impulse response functions—in the analogous linearized economy with aggregate risk.

[6]Building on our insight of the generality of the policy invariance assumption (ii), Barnichon and Mesters (2023) assumed an environment as restrictive as ours as their baseline and then considered the more general case as an extension. Similarly related is Kocherlakota (2019), who presented a dynamic policy game in which the policymaker can select the optimal action via regression analysis. In his setting, the policy action does not cause the private sector to update its beliefs about the future strategy of the policymaker. Policymaker payoffs thus only depend on the current choice and not on the future instrument paths that we emphasize.

[7]For example, see Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016), Boppart, Krusell, and Mitman (2018), or Auclert, Bardóczy, Rognlie, and Straub (2021) for a detailed discussion of this point.

2.1. *A Simple Example*

This section presents our identification result in the context of the three-equation New Keynesian model (Galí (2015), Woodford (2003)). Our broader argument, of course, is that the identification results and empirical method presented in the remainder of the paper actually do *not* require knowledge of the underlying structural model; nevertheless, we find it useful to first explain the logic of our results in a familiar setting before then generalizing it.

Model

The variables of the economy are two private-sector aggregates—output y_t and inflation π_t—and a policy instrument—the nominal rate i_t. They are related through three equations: a Euler equation and a Phillips curve as the private-sector block,

$$y_t = y_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}), \tag{1}$$

$$\pi_t = \kappa y_t + \beta\pi_{t+1} + (\varepsilon_t + \theta\varepsilon_{t-1}), \tag{2}$$

and a simple Taylor rule as the policy rule,

$$i_t = \phi\pi_t + \underbrace{\nu_{0,t}}_{\text{contemp. shock}} + \underbrace{\nu_{1,t-1}}_{\text{1-period news shock}}. \tag{3}$$

In our perfect-foresight setup, the private-sector equations as well as the policy rule hold for $t = 0, 1, 2, \dots$. These equations feature two kinds of disturbances. First, ε_t is a cost-push shock; for the illustrative analysis in this section, we find it useful to assume that it induces a first-order moving average wedge in the Phillips curve (2), implying that the effects of the shock will fully die out after two periods. Second, there are the policy shocks $\nu_{\ell,t-\ell}$; here, $\nu_{0,t}$ is a conventional contemporaneous policy shock, while $\nu_{1,t-1}$ denotes a deviation from the policy rule at time t announced at $t-1$ (a one-period "news" shock). Note that, in principle, (3) could be generalized to feature a full menu of news shocks $\nu_{\ell,t-\ell}$ for all $\ell > 0$; this extension will be important for our general analysis, but is not needed here as we will only construct policy counterfactuals for the MA(1) shock ε_t. As usual, given a vector of the time-0 shocks $\{\varepsilon_0, \nu_{0,0}, \nu_{1,0}\}$, a perfect-foresight transition path—or impulse response function—consists of the paths $\{y_t, \pi_t, i_t\}$ such that (1)–(3) hold at all $t = 0, 1, 2, \dots$.

For the subsequent analysis, the key property of this simple model economy will turn out to be that the coefficients in the two private-sector equations (1)–(2) are independent of the policy rule; that is, γ, κ, and β are unaffected by changes in ϕ. Equivalently, private-sector behavior is affected by policy only through the current and future values of the policy instrument i_t. Our general identification analysis in Sections 2.2 to 2.5 will discuss the generality and limitations of this crucial assumption.

Object of Interest

Under the baseline policy rule, the impulse response of the economy to a cost-push shock is given as the solution of (1)–(3) for some cost-push shock ε_0 together with $\nu_{\ell,0} = 0$ for $\ell = 0, 1$. We wish to instead characterize the behavior of this economy in response to

ε_0 not under the baseline policy rule (3), but instead under some counterfactual policy rule of the form[8]

$$i_t = \tilde{\phi}\pi_t, \tag{4}$$

where $\tilde{\phi} \neq \phi$. Note that this thought experiment supposes that the private sector perfectly understands the change in rule: the new relationship between i and π holds for all $t \geq 0$. Our identification result characterizes the information required to construct this counterfactual.

The Identification Argument

We consider an econometrician living in an economy that satisfies (1)–(3). Using conventional semi-structural time-series methods (Ramey (2016)), and with access to suitable identifying assumptions or instruments, that econometrician can in principle estimate how the macroeconomic aggregates $\{y_t, \pi_t, i_t\}$ respond to the cost-push shock ε_t as well as the policy shocks $\{\nu_{\ell,t-\ell}\}_{\ell=0}^1$ *under the baseline rule (3)*. Our main identification result states that knowledge of these causal effects—and nothing else about the structure of the economy—is sufficient to predict the counterfactual propagation of the shock ε_t under the alternative rule (4). We now describe intuitively why knowledge of these estimable causal effects is sufficient in the simple model (1)–(3), before in Sections 2.2 and 2.3 stating and proving the result for a much more general environment.

The key idea underlying our results is to choose *time-0* policy shocks $\{\nu_{0,0}, \nu_{1,0}\}$ to the baseline rule in order to mimic the desired counterfactual rule. To develop the argument, note first that, because our model has no endogenous state variables, the impulse responses to a time-0 shock will die out after $t = 1$, by our MA(1) assumption. We collect the 2×1 transition paths of $\{y_t, \pi_t, i_t\}$ in response to a cost-push shock ε_0 under the baseline rule as the vectors $\{\boldsymbol{y}_\phi(\varepsilon_0), \boldsymbol{\pi}_\phi(\varepsilon_0), \boldsymbol{i}_\phi(\varepsilon_0)\}$. Similarly, contemporaneous and one-period-ahead policy shocks also have no effects after $t = 1$. For $\ell \in \{0, 1\}$, we collect the corresponding 2×1 impulse responses under the baseline rule to a policy shock $\nu_{\ell,0}$ as the vectors $\{\theta_{y,\nu_\ell,\phi}, \theta_{\pi,\nu_\ell,\phi}, \theta_{i,\nu_\ell,\phi}\} \times \nu_{\ell,0}$; for example, $\theta_{y,\nu_\ell,\phi}$ is the 2×1 impulse response path of y to an ℓ-period-ahead shock to the baseline rule (3). Now consider setting the two monetary policy shocks to values $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ so that, under the baseline rule (3) and in response to the shock tuple $\{\varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$, the counterfactual rule (4) holds at both $t = 0$ and $t = 1$ along the perfect-foresight transition path; that is, we solve the following two equations in the two unknowns $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$:

$$\boldsymbol{i}_\phi(\varepsilon_0) + \theta_{i,\nu_0,\phi}\tilde{\nu}_{0,0} + \theta_{i,\nu_1,\phi}\tilde{\nu}_{1,0} = \tilde{\phi} \times \left[\boldsymbol{\pi}_\phi(\varepsilon_0) + \theta_{\pi,\nu_0,\phi}\tilde{\nu}_{0,0} + \theta_{\pi,\nu_1,\phi}\tilde{\nu}_{1,0}\right]. \tag{5}$$

The left-hand side of this equation gives us the impulse response of the interest rate following our combination of three shocks $\{\varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ under the baseline rule (3), while the right-hand side does the same for inflation, just scaled by $\tilde{\phi}$. By our informational assumptions, the econometrician can evaluate the system of equations (5) given ε_0 and for any candidate set of the two policy shocks $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$. Now suppose a solution $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ to (5) exists, and then compute the responses of $\{y_t, \pi_t, i_t\}$ to the shock tuple $\{\varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ under the baseline policy rule.[9] The content of our identification result is that those impulse responses are in fact identical to the impulse responses to ε_0 alone under the counterfactual rule (4).

[8]For the analysis in this section, we will assume that $\tilde{\phi}$ is such that a unique and determinate equilibrium exists. Our general analysis will cover counterfactual equilibrium non-existence and indeterminacy.

[9]Our general discussion will address the question of when solutions to equations like (5) actually exist.

The intuition underlying the identification result is straightforward. Since the private sector's decisions only depend on the expected path of the policy instrument (here just i_0 and i_1), it follows that it does not matter whether this path comes about due to the systematic conduct of policy or due to policy shocks. Equation (5) leverages this logic, looking for a combination of date-0 policy shocks that results in the counterfactual policy rule (4) holding both at $t = 0$ *and also in expectation* at $t = 1$. In response to these well-chosen shocks, the private sector behaves *as if* the counterfactual rule (4) had been imposed throughout.

Informational Requirements and Relation to Sims and Zha

Our identification result implies that, to predict policy rule counterfactuals, the econometrician does *not* need to know the structural equations of the economy; rather, all she needs are impulse responses to policy shocks. In particular, she needs the causal effects of the policy shocks on the variables that enter her counterfactual rule (here i_t and π_t) and on any other outcome variables she is interested in (e.g., y_t). With those causal effects in hand, she can map outcomes under the baseline rule—that is, impulse responses to some non-policy shock of interest—into counterfactual outcomes by computing impulse responses to $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ that solve (5).

We emphasize that this argument inherently relies on knowledge of the dynamic causal effects of both the contemporaneous policy shock $\tilde{\nu}_{0,0}$ as well as the policy news shock $\tilde{\nu}_{1,0}$: it is only with those two that we can actually enforce the counterfactual rule along the entire transition path (which here consists of two time periods). The econometric method of Sims and Zha (1995) instead supposes that the econometrician only has access to the causal effects of one policy shock (e.g., $\nu_{0,t}$). With one shock, it is generally not possible to enforce the counterfactual rule contemporaneously and in expectation; instead, the proposal of Sims and Zha is to subject the economy to an initial shock $\tilde{\nu}_{0,0}$ to enforce the counterfactual policy rule at $t = 0$ and then another *surprise* contemporaneous policy shock $\tilde{\nu}_{0,1}$ to also enforce it at $t = 1$. The key difference relative to our construction is that the private-sector block did not at $t = 0$ *expect* the counterfactual policy rule to hold at $t = 1$; rather, the rule only holds at $t = 1$ because of yet another surprise. In other words, under the approach of Sims and Zha, the counterfactual policy rule only holds ex post along the equilibrium transition path, but not in ex ante expectation. As a result, as long as policy at $t = 1$ matters for $t = 0$ decisions, the constructed counterfactual will differ from the true counterfactual $\{\boldsymbol{y}_{\tilde{\phi}}(\varepsilon_0), \boldsymbol{\pi}_{\tilde{\phi}}(\varepsilon_0), \boldsymbol{i}_{\tilde{\phi}}(\varepsilon_0)\}$. We will further elaborate on this connection between our identification result and the empirical methodology of Sims and Zha in Section 2.4.

Discussion & Outlook

The identification result sketched in this section is special in two respects: first, it is presented within the context of a particular explicit structural model; and second, since impulse responses to ε_0 are non-zero only for two periods, the result required knowledge of the effects of two policy shocks ($\nu_{0,0}$ and $\nu_{1,0}$). The remainder of this section will state and prove our main identification result in the context of a general class of infinite-horizon models. In terms of our informational requirements, the key change will be that the econometrician now needs to know the causal effects of *all* policy shocks $\{\nu_{\ell,0}\}_{\ell=0}^{\infty}$, rather than just the first two. The economic intuition, on the other hand, will be exactly the same: the argument will work as long as the private-sector block of the model depends on the policy rule only through the path of the policy instrument, as was the case here.

2.2. *General Environment and Objects of Interest*

We consider a linear, perfect-foresight, infinite-horizon economy. Throughout, we will use boldface to denote time paths for $t = 0, 1, 2, \dots$, and all variables are expressed in deviations from the deterministic steady state. The economy is summarized by the system

$$\mathcal{H}_w \boldsymbol{w} + \mathcal{H}_x \boldsymbol{x} + \mathcal{H}_z \boldsymbol{z} + \mathcal{H}_\varepsilon \boldsymbol{\varepsilon} = \boldsymbol{0}, \tag{6}$$

$$\mathcal{A}_x \boldsymbol{x} + \mathcal{A}_z \boldsymbol{z} + \boldsymbol{\nu} = \boldsymbol{0}. \tag{7}$$

w_t and x_t are n_w- and n_x-dimensional vectors of endogenous variables, z_t is an n_z-dimensional vector of policy instruments, ε_t is an n_ε-dimensional vector of exogenous structural shocks, and ν_t is an n_z-dimensional vector of policy shocks.[10] The distinction between w and x is that the variables in x are observable while those in w are not; in particular, x contains the outcomes of interest for our econometrician as well as the arguments of the counterfactual policy rule that she contemplates.[11] The linear maps $\{\mathcal{H}_w, \mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ summarize the non-policy block of the economy, yielding $n_w + n_x$ restrictions for each t. Our key assumption—echoing the model of Section 2.1—is that the maps $\{\mathcal{H}_w, \mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ do not depend on the coefficients of the policy rule $\{\mathcal{A}_x, \mathcal{A}_z\}$; instead, policy only matters through the path of the instrument z, with the rule (7) giving n_z restrictions on the policy instruments for each t. As in our simple example, entries of the shock vectors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\nu}$ for $t > 0$ should again be interpreted as news shocks. In particular, the policy shock vector $\boldsymbol{\nu}$ collects the full menu of contemporaneous and news shocks to the prevailing policy rule at all horizons, thus generalizing the two-shock setup that we considered in the simple three-equation model.

Given bounded $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$, an equilibrium is a set of bounded sequences $\{\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}\}$ that solve (6)–(7). We assume that the baseline rule $\{\mathcal{A}_x, \mathcal{A}_z\}$ is such that an equilibrium exists and is unique for any $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$.

ASSUMPTION 1: *The policy rule in* (7) *induces a unique equilibrium.*

Given $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$, we write that unique solution as $\{\boldsymbol{w}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \boldsymbol{\nu}), \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \boldsymbol{\nu}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \boldsymbol{\nu})\}$. As in the simple example, we often focus on impulse responses to exogenous shocks $\boldsymbol{\varepsilon}$ when the policy rule is followed perfectly (i.e., $\boldsymbol{\nu} = \boldsymbol{0}$); with some slight abuse of notation, we will simply write those impulse responses as $\{\boldsymbol{w}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\}$.

Scope

Our identification results in Section 2.3 and the empirical analysis in Section 3 will apply to *any* structural model that can be written in the general form (6)–(7). As emphasized before, in addition to linearity, the key property of this environment for our purposes is that policy enters the non-policy block *only* through the path $\boldsymbol{z}$ of policy variables; equivalently, in the linearized economy with aggregate risk, policy affects private-sector decisions only through current and expected future z. How restrictive are those assumptions?

[10]The boldface vectors $\{\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$ stack the time paths for all variables (e.g., $\boldsymbol{x} = (\boldsymbol{x}_1', \dots, \boldsymbol{x}_{n_x}')'$). The linear maps $\{\mathcal{H}_w, \mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ and $\{\mathcal{A}_x, \mathcal{A}_z\}$ are conformable and are all assumed to map bounded sequences into bounded sequences.

[11]For expositional simplicity, we do not include $\boldsymbol{w}$ as an argument of the *baseline* policy rule (7), though doing so would not pose a problem. The key restriction is that the *counterfactual* policy rule only features variables observable to the econometrician.

Our first observation is that many of the explicit, parametric structural models used for counterfactual and optimal policy analysis in the standard Lucas-program approach fit into our framework (6)–(7). Such models are routinely linearized, and their linear representation features the separation between policy rule and non-policy block that our theoretical results require. For example, the analysis in Section 2.1 has already illustrated that one particular canonical model environment—the textbook three-equation New Keynesian model—fits into our framework.[12] By the exact same line of reasoning, even workhorse estimated business-cycle models (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007)) as well as recent quantitative HANK or heterogeneous-firm models (e.g., Auclert, Rognlie, and Straub (2020), McKay and Wieland (2021), Ottonello and Winberry (2020)) fit into our structure. For example, in HANK-type models, the Euler equation of the representative household is simply replaced by a more general "aggregate consumption function" (e.g., Auclert, Rognlie, and Straub (2018), Wolf (2021)):

$$\boldsymbol{c} = \mathcal{C}_y \boldsymbol{y} + \mathcal{C}_\pi \boldsymbol{\pi} + \mathcal{C}_i \boldsymbol{i} + \mathcal{C}_d \boldsymbol{\varepsilon}^d,$$

where c is consumption, y is income, π is inflation, i is the nominal rate, ε^d is a demand shock, and $\mathcal{C}_\bullet$ are matrices of derivatives of the consumption function. Such models continue to fit into our framework precisely because the derivative matrices $\mathcal{C}_\bullet$ depend only on the model's deterministic steady state, and not on policy rules that influence the economy's fluctuations *around* that steady state (e.g., a Taylor rule for nominal interest rates). We will give a concrete numerical illustration of our identification result in the context of a quantitative HANK-type model in Section 2.4. Finally, as we discuss in Appendix A.1 of the Supplemental Material, several canonical behavioral models (e.g., Gabaix (2020)) are also consistent with our assumptions.

While thus clearly relatively general, our framework also has some important limitations. Recall that our two key restrictions on the model are (i) linearity and (ii) the way the policy instrument is allowed to shape private-sector behavior. The separation between policy and non-policy block embedded in (ii) is violated in some structural models. Important examples are environments that feature an asymmetry of information between the policymaker and the private sector (e.g., Lucas (1972)). In such models, private-sector agents solve a filtering problem, and in general the coefficients of the policy rule will matter for this filtering problem both through the induced movements of the policy instrument *and* through the information contained in those movements. The separation between the private-sector and policy blocks of the model at the heart of our results will thus break down—that is, the coefficients in $\mathcal{H}_x$ depend directly on the policy rule (see Appendix A.2 of the Supplemental Material for a formal derivation).

As we discuss in Appendix A.8, the linearity restriction (i), on the other hand, is not conceptual, but practical. By linearity, the effects of policy are sign-, size-, and state-invariant. Given certainty equivalence, we can focus on expected policy instrument paths, thus substantially reducing the informational requirements of our identification results and facilitating their empirical application.[13] The costs of linearity are twofold. First, our identification results will generally not yield *globally* valid policy counterfactuals. Second,

[12]For reference, we in Appendix A.1 of the Supplemental Material (McKay and Wolf (2023)) write down the model (1)–(3) in the form (6)–(7).

[13]To be clear, what we are requiring is linearity of the non-policy block (6). Non-linearity of the policy (e.g., due to a binding zero lower bound), on the other hand, poses no particular challenge. This point is discussed further in Appendix A.9.

we will be able to construct counterfactuals for alternative policy rules that change the policymaker's response to aggregate perturbations (e.g., different Taylor rules), but our results are unlikely to apply to policies that change the model's steady state (e.g., changes in the inflation target).

Objects of Interest

As in our simple model, we wish to learn about systematic policy rule counterfactuals. Specifically, we consider an alternative policy rule

$$\tilde{\mathcal{A}}_x \boldsymbol{x} + \tilde{\mathcal{A}}_z \boldsymbol{z} = \mathbf{0}. \tag{8}$$

This alternative policy rule is also assumed to induce a unique equilibrium. We will discuss further in Section 2.3 what happens if this assumption is violated.

ASSUMPTION 2: *The policy rule in* (8) *induces a unique equilibrium*.

We emphasize that the arguments of the counterfactual policy rule are macroeconomic *observables* x and z; naturally, our empirical identification result will not allow evaluation of counterfactual rules that directly involve unobservable objects.[14] Given this alternative rule $\tilde{\mathcal{A}}$, we ask: what are the dynamic response paths $\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ and $\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ to some given exogenous non-policy shock path $\boldsymbol{\varepsilon}$?

As a special case of the general counterfactual rule (8), we will also study *optimal* policy rules corresponding to a given loss function. Specifically, we consider a policymaker with a simple exogenously given quadratic loss function of the form

$$\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n_x} \lambda_i \boldsymbol{x}_i' W \boldsymbol{x}_i, \tag{9}$$

where i indexes the n_x distinct (again observable) macro aggregates collected in x, λ_i denotes policy weights, and $W = \text{diag}(1, \beta, \beta^2, \dots)$ allows for discounting.[15] As for our general counterfactual rule, we assume that the optimal policy problem has a unique solution.

ASSUMPTION 3: *Given any vector of exogenous shocks* $\boldsymbol{\varepsilon}$, *the problem of choosing the policy variable* z *to minimize the loss function* (9) *subject to the non-policy constraint* (6) *has a unique solution*.

We are then interested in two questions. First, what rule is optimal for a policymaker with preferences as in (9)? Second, given that optimal rule $(\mathcal{A}_x^*, \mathcal{A}_z^*)$, what are the corresponding dynamic response paths $\boldsymbol{x}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$ and $\boldsymbol{z}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$ for a given non-policy shock path $\boldsymbol{\varepsilon}$?

[14] For example, the counterfactual rule cannot depend on the natural rate of interest, though it could of course depend on an estimate of the natural rate based on observables.

[15] We emphasize that our results are completely silent on the *shape* of the loss function, with structural modeling still the most natural way of obtaining a mapping from observables to welfare. We instead take as given the loss function and ask what kind of empirical evidence would be most useful to figure out how to minimize the loss. We furthermore note that our focus on a separable quadratic loss function is in line with standard optimal policy analysis, but not essential. As shown in Appendix A.3, our results extend to the non-separable quadratic case, where the loss is now given by $\frac{1}{2}\boldsymbol{x}'Q\boldsymbol{x}$ for a weighting matrix Q.

Finally, for both general as well as optimal counterfactual policy rules, we would like to go beyond counterfactuals conditional on particular non-policy shock paths $\boldsymbol{\varepsilon}$, and instead also predict the effects of a rule change on *unconditional* macroeconomic dynamics. In particular, we would like to predict how the change in policy rule would affect the unconditional second-moment properties of the observed macroeconomic aggregates x.

The objective of the remainder of this section is to characterize the information required to recover these desired policy counterfactuals. The key insight is that, exactly as in our simple model, all of the required information can in principle be recovered from data generated under the baseline policy rule.

2.3. *Identification Results*

We begin by defining the dynamic causal effects that lie at the heart of our identification results. By Assumption 1, we can write the solution to the system (6)–(7) as

$$\begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \Theta_{\mathcal{A}} \times \begin{pmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\nu} \end{pmatrix},$$

where the linear map $\Theta_{\mathcal{A}}$ collects the impulse responses of $\boldsymbol{w}$, $\boldsymbol{x}$, and $\boldsymbol{z}$ to the non-policy and policy shocks $(\boldsymbol{\varepsilon}, \boldsymbol{\nu})$ under the prevailing baseline policy rule (7) with parameters $\mathcal{A}$. We will partition it as

$$\Theta_{\mathcal{A}} \equiv \begin{pmatrix} \Theta_{w,\varepsilon,\mathcal{A}} & \Theta_{w,\nu,\mathcal{A}} \\ \Theta_{x,\varepsilon,\mathcal{A}} & \Theta_{x,\nu,\mathcal{A}} \\ \Theta_{z,\varepsilon,\mathcal{A}} & \Theta_{z,\nu,\mathcal{A}} \end{pmatrix}. \tag{10}$$

All of our identification results will require knowledge of $\{\Theta_{x,\nu,\mathcal{A}}, \Theta_{z,\nu,\mathcal{A}}\}$—the impulse responses of the policy instruments z and macroeconomic observables x to contemporaneous as well as all possible future shocks $\boldsymbol{\nu}$ to the prevailing policy rule. Furthermore, to construct counterfactual paths that correspond to a given non-policy shock sequence $\boldsymbol{\varepsilon}$, we also require knowledge of the causal effects of that particular shock sequence under the baseline policy rule, $\{\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) = \Theta_{x,\varepsilon,\mathcal{A}} \times \boldsymbol{\varepsilon}, \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) = \Theta_{z,\varepsilon,\mathcal{A}} \times \boldsymbol{\varepsilon}\}$. We emphasize that, in principle, all of these objects are estimable using data generated under the baseline policy rule: for example, given valid instrumental variables for all the distinct policy shocks $\boldsymbol{\nu}$ as well as a single instrument for the non-policy shock path $\boldsymbol{\varepsilon}$, the required entries of the Θ's can be estimated using semi-structural time-series methods (e.g., see Ramey (2016), for a review).

These informational requirements are the natural generalization of those for the simple model in Section 2.1. First, since we are now considering an infinite-horizon economy, any given shock generates entire *paths* of impulse responses, corresponding to the rows of the Θ's. Second, rather than two policy shocks, we now need to know causal effects corresponding to the full menu of possible contemporaneous and news shocks $\boldsymbol{\nu}$—all columns of the Θ_ν's.

General Counterfactual Rule

We begin with the main object of interest—policy counterfactuals after a non-policy shock sequence $\boldsymbol{\varepsilon}$ under an alternative policy rule.

PROPOSITION 1: *Under Assumptions 1 and 2, we can recover the policy counterfactuals* $\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ *and* $\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ *for a counterfactual rule* $\{\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z\}$ *as*

$$\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) = \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}) \equiv \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{x,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}}, \tag{11}$$

$$\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) = \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}) \equiv \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{z,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}}, \tag{12}$$

where $\tilde{\boldsymbol{\nu}}$ *solves*

$$\tilde{\mathcal{A}}_x\big[\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{x,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}}\big] + \tilde{\mathcal{A}}_z\big[\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{z,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}}\big] = \mathbf{0}. \tag{13}$$

PROOF: The equilibrium system under the new policy rule can be written as

$$\begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ \mathbf{0} & \tilde{\mathcal{A}}_x & \tilde{\mathcal{A}}_z \end{pmatrix} \begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -\mathcal{H}_\varepsilon \\ \mathbf{0} \end{pmatrix} \boldsymbol{\varepsilon}. \tag{14}$$

By Assumption 2, we know that (14) has a unique bounded solution $\{\boldsymbol{w}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$. To characterize $\{\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$ as a function of observables, suppose that we could find a bounded $\tilde{\boldsymbol{\nu}}$ that solves (13). Since (6) also holds under the baseline policy rule, and since (13) imposes the new policy rule, it follows that any $\{\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}})\}$ with $\tilde{\boldsymbol{\nu}}$ solving (13) are also part of a solution of (14), and thus equal $\{\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$.

It now remains to establish that the system (13) actually has a solution. For this, consider the candidate $\tilde{\boldsymbol{\nu}} = (\tilde{\mathcal{A}}_x - \mathcal{A}_x)\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) + (\tilde{\mathcal{A}}_z - \mathcal{A}_z)\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$. Since the paths $\{\boldsymbol{w}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$ solve (14), it follows that they are also a solution to the system

$$\begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ \mathbf{0} & \mathcal{A}_x & \mathcal{A}_z \end{pmatrix} \begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = -\begin{pmatrix} \mathcal{H}_\varepsilon \boldsymbol{\varepsilon} \\ (\tilde{\mathcal{A}}_x - \mathcal{A}_x)\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) + (\tilde{\mathcal{A}}_z - \mathcal{A}_z)\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) \end{pmatrix}. \tag{15}$$

But by Assumption 1, we know that the system (15) has a unique solution, so indeed the paths $\{\boldsymbol{w}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$ are that solution. Finally, it follows from the definition of $\Theta_{\mathcal{A}}$ in (10) that the candidate $\tilde{\boldsymbol{\nu}}$ also solves (13), completing the argument. *Q.E.D.*

Proposition 1 implies that we can recover the desired policy counterfactual as a function of observables alone—our econometrician needs to know the policy shock causal effect matrices $\{\Theta_{x,\nu,\mathcal{A}}, \Theta_{z,\nu,\mathcal{A}}\}$ and the effects of the shock $\boldsymbol{\varepsilon}$ under the baseline rule, $\{\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\}$, but she need *not* know the structural equations of the underlying model. The key equation (13) in Proposition 1 is the infinite-horizon analogue of the bivariate system (5) from our two-period example in Section 2.1. The intuition is as before: since we know how all possible perturbations $\boldsymbol{\nu}$ to the baseline rule affect the variables x and z entering the counterfactual rule, we can always construct a date-0 shock vector $\tilde{\boldsymbol{\nu}}$ that mimics the equilibrium path of z under the new rule. But since the first model block (6) depends on the policy rule *only* via the expected instrument path, the equilibrium allocations under the new counterfactual rule and the perturbed prevailing rule are the same. The only difference relative to the simple two-period model is that, because we now consider an infinite-horizon setting, we in general require evidence on contemporaneous and all possible future news shocks to the baseline rule in order to be able to mimic an arbitrary alternative policy rule.[16]

[16]While Proposition 1 applies to a particular shock path $\boldsymbol{\varepsilon}$, it is immediate that the exact same argument also applies to a particular *historical scenario* (Antolin-Diaz, Petrella, and Rubio-Ramírez (2021)): a historical

What happens if Assumption 2—which maintains that the counterfactual rule delivers a unique equilibrium—is violated? We can distinguish two cases. First, if no equilibrium exists under the contemplated counterfactual policy rule, then the system (13) will simply not have a solution. Second, if multiple equilibria exist, then impulse responses to any $\{\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}\}$ where $\tilde{\boldsymbol{\nu}}$ solves (13) will be *a* valid equilibrium for the counterfactual rule $\{\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z\}$. For example, in the simple New Keynesian model of Section 2.1, applying our identification results for the counterfactual rule $\tilde{\phi} = 0$—that is, a nominal interest rate peg—would deliver the economy's fundamental (minimum state variable, or MSV) equilibrium.

Optimal Policy

A very similar argument applies for optimal policy analysis.

PROPOSITION 2: *Consider a policymaker with loss function* (9). *Under Assumptions* 1 *and* 3, *for any* $\boldsymbol{\varepsilon}$, *the solution to the optimal policy problem is implemented by the rule* $\{\mathcal{A}_x^*, \mathcal{A}_z^*\}$ *with*

$$\mathcal{A}_x^* = \left(\lambda_1 \Theta'_{x_1,\nu,\mathcal{A}} W, \lambda_2 \Theta'_{x_2,\nu,\mathcal{A}} W, \dots, \lambda_{n_x} \Theta'_{x_{n_x},\nu,\mathcal{A}} W\right), \tag{16}$$

$$\mathcal{A}_z^* = \mathbf{0}. \tag{17}$$

Given $\{\mathcal{A}_x^*, \mathcal{A}_z^*\}$, *the corresponding counterfactual paths under the optimal policy rule*, $\boldsymbol{x}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$ *and* $\boldsymbol{z}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$, *are characterized as in Proposition* 1.

PROOF: The solution to the policy problem is characterized by the following conditions:

$$\mathcal{H}_w'(I \otimes W)\boldsymbol{\varphi} = \mathbf{0}, \tag{18}$$

$$(\Lambda \otimes W)\boldsymbol{x} + \mathcal{H}_x'(I \otimes W)\boldsymbol{\varphi} = \mathbf{0}, \tag{19}$$

$$\mathcal{H}_z'(I \otimes W)\boldsymbol{\varphi} = \mathbf{0}, \tag{20}$$

where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \dots)$ and φ is the multiplier on (6). By Assumption 3, we know that the system (18)–(20) together with (6) has a unique solution $\{\boldsymbol{x}^*(\boldsymbol{\varepsilon}), \boldsymbol{z}^*(\boldsymbol{\varepsilon}), \boldsymbol{w}^*(\boldsymbol{\varepsilon}), \boldsymbol{\varphi}^*(\boldsymbol{\varepsilon})\}$.

Now consider the alternative problem of choosing deviations $\boldsymbol{\nu}^*$ from the prevailing rule to minimize (9) subject to (6)–(7). This second problem gives the first-order conditions

$$\mathcal{H}_w'(I \otimes W)\boldsymbol{\varphi} = \mathbf{0}, \tag{21}$$

$$(\Lambda \otimes W)\boldsymbol{x} + \mathcal{H}_x'(I \otimes W)\boldsymbol{\varphi} + \mathcal{A}_x' W \boldsymbol{\varphi}_z = \mathbf{0}, \tag{22}$$

$$\mathcal{H}_z'(I \otimes W)\boldsymbol{\varphi} + \mathcal{A}_z' W \boldsymbol{\varphi}_z = \mathbf{0}, \tag{23}$$

$$W \boldsymbol{\varphi}_z = \mathbf{0}, \tag{24}$$

where φ_z is the multiplier on (7). It follows from (24) that $\boldsymbol{\varphi}_z = \mathbf{0}$. Then (21)–(23) together with (6) determine the same unique solution for $\{\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{w}\}$ as before, and $\boldsymbol{\nu}^*$ adjusts residually to satisfy (7). The original problem and the alternative problem are thus equivalent.

scenario is simply a set of forecast paths $\boldsymbol{x}_{\mathcal{A}}$ and $\boldsymbol{z}_{\mathcal{A}}$ at a given point in time, and we can use the logic of Proposition 1 to recover the analogous counterfactual historical scenario $\boldsymbol{x}_{\tilde{\mathcal{A}}}$ and $\boldsymbol{z}_{\tilde{\mathcal{A}}}$.

Next note that, by Assumption 1, we can rewrite the alternative problem's constraint set as

$$\begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \Theta_{\mathcal{A}} \times \begin{pmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\nu}^* \end{pmatrix}. \tag{25}$$

The problem of minimizing (9) subject to (25) gives the optimality condition

$$\sum_{i=1}^{n_x} \lambda_i \Theta'_{x_i,\nu,\mathcal{A}} W \boldsymbol{x}_i = 0. \tag{26}$$

By the equivalence of the policy problems, it follows that (26) is an optimal policy *rule*, taking the form (16)–(17). Finally, the second part of the result follows from Proposition 1 since (26) is just a special example of a policy rule $\{\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z\}$. *Q.E.D.*

Proposition 2 reveals that, in conjunction with a given policymaker loss function, the information required to construct valid counterfactuals for arbitrary policy rules also suffices to characterize *optimal* policy rules.[17] The intuition is exactly as before: since we know the causal effects of every possible policy perturbation $\boldsymbol{\nu}$ on the policymaker targets $\boldsymbol{x}$, we in particular know the space of those targets that is implementable through policy actions. At an optimum, we must be at the point of this space that minimizes the policymaker loss. As before, it does not matter whether this optimum is attained through some systematic policy rule or through shocks to an alternative rule.

Unconditional Second-Moment Properties

While Propositions 1 and 2 predict counterfactual dynamics *conditional* on particular non-policy shock paths $\boldsymbol{\varepsilon}$, researchers may also be interested in the *unconditional* second-moment properties of macroeconomic aggregates following a change in policy rule. Of course, if researchers have estimated the effects of all distinct non-policy shocks hitting the economy, then such unconditional analysis is simple: apply Propositions 1 and 2 for each such shock and then collect the results in the form of a vector moving average representation.

In practice, however, researchers may not be able to isolate all distinct aggregate non-policy shocks. Our third identification result states that, in some cases, it is nevertheless possible to recover the desired counterfactual second-moment properties. Since the result requires some investment in additional notation, we only state the main idea here and relegate further details to Appendix A.5 of the Supplemental Material. The key assumption allowing us to make progress is "invertibility": we need to assume that the structural vector moving average representation of the observable data x and z under the baseline

[17] By certainty equivalence, the results from our perfect-foresight analysis readily extend to *stochastic* linear-quadratic control problems. We can in that case rewrite the derived optimal policy rule as a forecasting targeting rule (Svensson (1997)):

$$\sum_{i=1}^{n_x} \lambda_i \Theta'_{x_i,\nu,\mathcal{A}} W \mathbb{E}_t[\boldsymbol{x}_i] = \boldsymbol{0}, \tag{27}$$

where now $\boldsymbol{x}_i = (x_{it}, x_{it+1}, \dots)'$. In words, expectations of future targets must always minimize the policymaker loss within the space of (expected) allocations that are implementable via changes in the policy stance. For a timeless perspective, (27) must apply to *revisions* of policymaker expectations at each t.

policy rule is invertible with respect to the structural shocks driving the economy. This assumption, while restrictive (Plagborg-Møller and Wolf (2022)), is routinely imposed in conventional structural vector autoregression analysis (Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007)). Under this assumption, researchers need not be able to separately observe all of the individual structural shocks; instead, it suffices to simply apply our counterfactual prediction results in Propositions 1 and 2 to the Wold innovations and then collect the results in the form of a counterfactual vector moving average. Appendix A.5 also discusses why this argument fails in the non-invertible case.

Role of the Baseline Rule $\{\mathcal{A}_x, \mathcal{A}_z\}$

All identification results in this section were stated using the causal effects of policy shocks $\boldsymbol{\nu}$ *relative to the baseline policy rule* $\{\mathcal{A}_x, \mathcal{A}_z\}$. We would like to emphasize, however, that this baseline rule only plays a limited role in our analysis, and that it in particular does not need to be known by the econometrician.[18]

In our proofs of Propositions 1 and 2, the baseline policy rule $\{\mathcal{A}_x, \mathcal{A}_z\}$ functions as a reference point: we find the sequence of policy shocks relative to that rule that implements the desired counterfactual rule. This choice of reference point, however, is ultimately immaterial: since private-sector behavior in (6) is shaped by policy only through the instrument path $\boldsymbol{z}$, all that matters for our results is knowledge of how macroeconomic outcomes $\boldsymbol{x}$ are related to paths of policy instruments $\boldsymbol{z}$. For example, under the natural assumption that $\Theta_{z,\nu,\mathcal{A}}$ is invertible—that is, the policymaker can implement any sequence of the policy instrument—we could post-multiply all causal effect matrices by $\Theta_{z,\nu,\mathcal{A}}^{-1}$, thus writing policy causal effects not in terms of shocks $\boldsymbol{\nu}$ relative to a given rule, but instead directly in terms of instrument paths $\boldsymbol{z}$. This change in reference point leaves our identification results completely unchanged, but will prove useful when later connecting those theoretical identification results with empirical evidence on policy shock propagation in Section 3.

Discussion

The theoretical identification results in Propositions 1 and 2 offer a bridge between the "Lucas program" (e.g., see Christiano, Eichenbaum, and Evans (1999))—a strategy that relies on micro-founded structural models to form policy counterfactuals—and the purely empirical approach of Sims and Zha (1995). The propositions reveal that, under our assumptions, impulse responses to policy *shocks*—objects that are estimable using semi-structural empirical techniques—suffice to predict the effects of changes in systematic policy *rules*. Key to our argument is the use of *multiple distinct* policy shocks. By using many such shocks (and all realized at date 0), counterfactual rules can be imposed not just ex post but also in ex ante expectation, and this turns out to be enough to circumvent the Lucas critique. We further elaborate on the connection between our results and the approach of Sims and Zha—which uses one policy shock, set to a new level at each date t—in Section 2.4.

Our results can be interpreted as part of the recent effort to bring insights from the "sufficient statistics" approach popular in public finance to macroeconomics (Chetty (2009),

[18] Moreover, our results will continue to hold if the baseline policy rule underwent changes during the sample period. For our purposes, the key requirement is that the private-sector behavioral relationships (6) have remained stable over the observed sample period. We provide further details in Appendix A.4 of the Supplemental Material.

Nakamura and Steinsson (2018)). For a large family of structural models and policy counterfactuals, policy shock impulse responses are sufficient statistics in the sense that we can directly use them to compute the desired counterfactuals, without actually requiring knowledge of the structural equations of the model. To leverage Propositions 1 and 2, an econometrician does not need to make detailed assumptions on the private-sector block, nor does she need to know the policy rule that generated the observed data.

2.4. *Illustration and Relation to Sims and Zha (1995)*

This section provides a visual illustration of our identification results and their relationship to the approach of Sims and Zha (1995). As our laboratory, we use a HANK model as in Wolf (2021), with details of the parameterization relegated to Appendix A.1 of the Supplemental Material. In this environment, we will compute policy counterfactuals in multiple ways: first by using the actual structural equations of the model to simply solve the model with a counterfactual policy rule; and then by using model-implied impulse responses to policy shocks to implement either the approach of Sims and Zha or our identification result in Proposition 1.

We begin by solving the model with a baseline policy rule of

$$i_t = \phi_\pi \pi_t + \sum_{\ell=0}^{\infty} \nu_{\ell,t-\ell} \tag{28}$$

for $\phi_\pi = 1.5$. In particular, we recover (a) the impulse responses $\{\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\}$ to a contractionary cost-push shock ε_t under (28) and (b) the causal effects of contemporaneous and news policy shocks $\boldsymbol{\nu}$ to (28), $\{\Theta_{x,\nu,\mathcal{A}}, \Theta_{z,\nu,\mathcal{A}}\}$. We emphasize that those causal effects would be estimable by an econometrician living in this economy and with access to valid instruments for the cost-push shock ε_t as well as the policy shocks $\{\nu_{0,t}, \nu_{1,t}, \dots\}$.

We entertain the following counterfactual policy rule:

$$i_t = \phi_i i_{t-1} + (1-\phi_i)(\phi_\pi \pi_t + \phi_y y_t) \tag{29}$$

for $\phi_i = 0.9$, $\phi_\pi = 2$, $\phi_y = 0.5$. The dotted and solid lines in all three panels of Figure 1 show the true model-implied impulse responses of output and inflation to a cost-push shock ε_t under the baseline rule (28) (dotted) and the counterfactual rule (29) (solid), where both of these lines are computed from the structural equations of the model.

We now seek to recover the desired counterfactual (solid) *only* through knowledge of the dynamic causal effects of policy shocks, and *without* actually relying on any of the structural equations of the model. The panels of Figure 1 show results for three possible strategies to predict the counterfactual propagation of the cost-push shock.

Estimand of Sims and Zha

The top panel begins with the empirical strategy of Sims and Zha (1995). Here the econometrician was only able to estimate the dynamic causal effects of the first entry of $\boldsymbol{\nu}$ (i.e., the contemporaneous shock $\nu_{0,t}$), and then uses a sequence of such policy shocks—one at each $t = 0, 1, 2, \dots$—to enforce the counterfactual rule (29) ex post along the equilibrium transition path. The right panel shows the sequence of policy shocks that implements this strategy, and the dashed lines in the left and middle panels give the responses of output and inflation to the original cost-push shock *plus* the derived sequence of monetary policy shocks. The main takeaway is that those dashed lines are not equal to

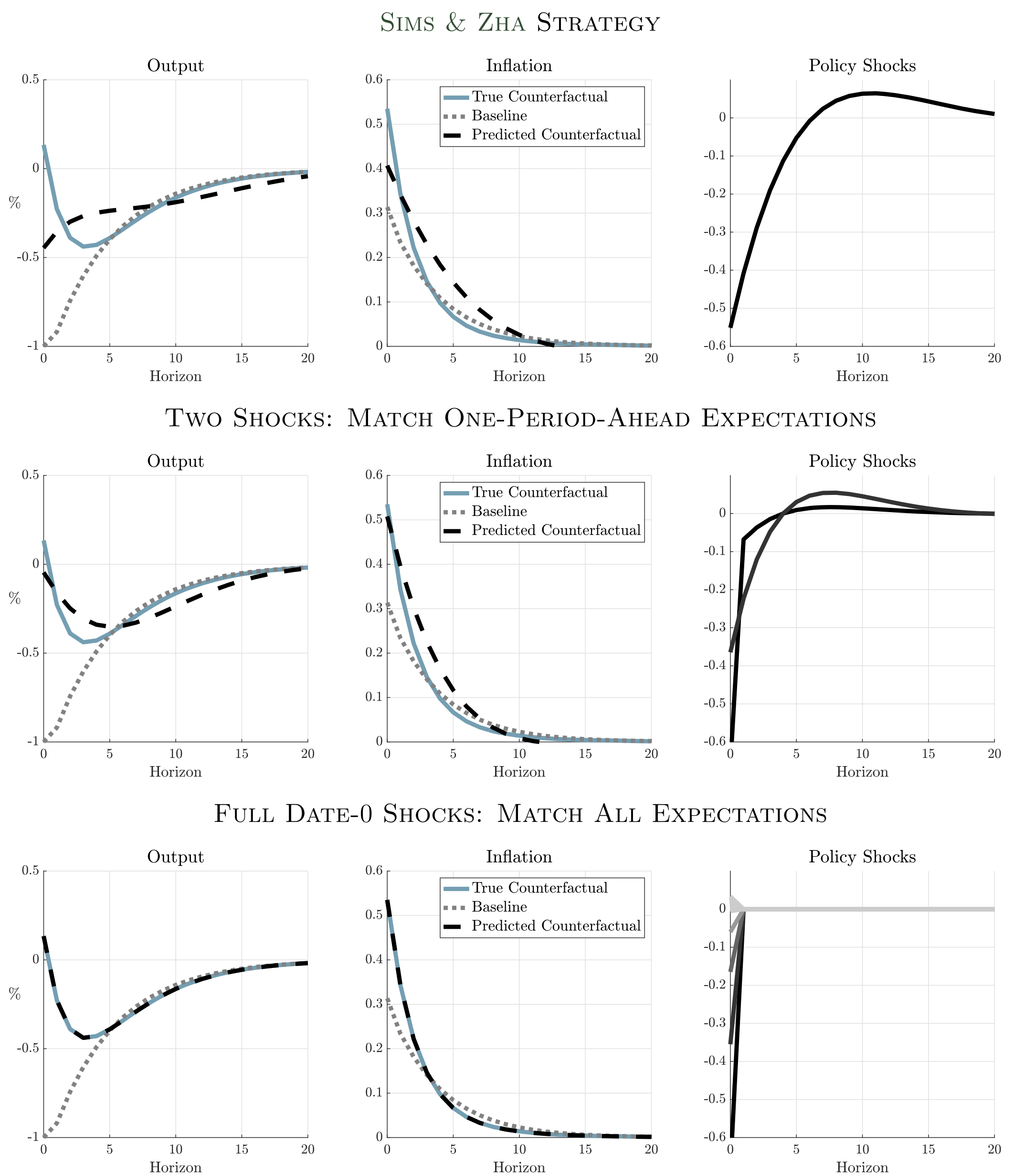

FIGURE 1.—The dotted and solid lines show output and inflation responses to the cost-push shock ε_t under the policy rules (28) and (29) in the HANK model. The dashed lines give counterfactuals constructed through the policy shocks on the right. The top panel uses repeated realizations of a single policy shock to enforce (29) ex post, as in Sims and Zha. The middle panel uses repeated realizations of two policy shocks to enforce (29) ex post and in one-period-ahead expectation. The lower panel shows our method, which uses a single realization of many policy shocks to enforce (29) along the entire expected path. Lighter shades correspond to news about policy at longer horizons.

the true counterfactual (solid). Intuitively, the issue is that the contemplated counterfactual rule is only imposed ex post, but not in ex ante expectation. Since expectations about the future affect the present, enforcing the rule through ex post surprises is not the same as switching and committing to a different rule from time $t = 0$ onwards. Visually, the importance of ex post surprises is evident in the right panel: to map the baseline rule into the counterfactual rule, the econometrician requires a *sequence* of expansionary policy shocks $\nu_{0,t}$, with those shocks remaining large throughout the entire first year after the shock.

Towards Our Identification Result

The middle and bottom panels now illustrate the logic of our identification result—with multiple policy shocks, the econometrician has enough degrees of freedom to impose the counterfactual rule not just ex post, but also in expectation. As a warmup, the middle panel considers a case in which the econometrician is able to estimate the causal effects of the first two entries of $\boldsymbol{\nu}$ (i.e., a contemporaneous and a one-period-forward guidance policy shock). Such access to multiple shocks suggests a natural generalization of Sims and Zha: use the two policy shocks at each $t \geq 0$ to enforce the desired counterfactual rule not only ex post (as Sims and Zha do with one shock), but also in ex ante expectation for the next period.[19] Since the counterfactual policy rule is now imposed both ex post and in ex ante expectation for one period, the predicted counterfactuals (dashed) are closer to the truth (solid); correspondingly, the policy shock sequences in the right panel feature smaller ex post surprises dated $t = 1, 2, \ldots$. The bottom panel—which corresponds to our identification result—simply continues this logic. With access to the causal effects of the full vector of policy shocks $\boldsymbol{\nu}$, the econometrician can rely purely on date-0 shocks (right panel) to enforce the counterfactual rule not just ex post but also in ex ante expectation. Under our assumptions, doing so suffices to circumvent the Lucas critique and recover the correct counterfactual (left and center panels).

To summarize, the top- and bottom-right panels illustrate the core difference between the empirical method of Sims and Zha and our identification result. In the former, the researcher has access to a single policy shock, and uses a sequence of realizations of that shock to enforce the counterfactual rule. In our approach, the researcher has access to many shocks and only uses shocks at date-0 to enforce the counterfactual rule. Our identification result thus clearly has substantially higher informational requirements, but this increase in information brings with it the similarly substantial benefit of robustness to Lucas critique concerns.

2.5. *Discussion*

The central takeaway from the analysis in this section is that—under our maintained structural assumptions—systematic policy rule counterfactuals can, at least in principle, be constructed purely through empirical measurement, and in a way that is robust to Lucas critique concerns. In the remainder of the paper, we discuss how to operationalize our insights. The main challenge is that the informational requirements underlying our identification results are quite high: the researcher needs evidence on the causal effects of a full menu of policy shocks that shift expectations of policy *at all possible horizons*. Section 3 presents an empirical strategy for the relevant case of researchers with access to only a few distinct identified policy shocks. We will then in Section 4 illustrate this empirical strategy through several applications to systematic monetary policy rule counterfactuals.

[19]We present implementation details for this approach in Appendix A.7 of the Supplemental Material.

3. EMPIRICAL METHOD

This section presents our empirical method for constructing policy rule counterfactuals with evidence on multiple, but a limited number of, distinct policy shocks. Section 3.1 illustrates the basic logic of our method with an illustrative example based on the oil shock application of Bernanke et al. (1997). Section 3.2 then introduces the general methodology.

Throughout, the discussion in this section will leverage the following connection between our theoretical identification results in Section 2.3 and empirical evidence on policy shock propagation. For our theoretical analysis, we found it convenient to think of contemporaneous and news shocks $\boldsymbol{\nu}$ that perturb some fixed prevailing policy rule $\{\mathcal{A}_x, \mathcal{A}_z\}$ horizon by horizon. For connecting to data, however, this perspective is less useful—empirical evidence on policy shock causal effects just gives impulse responses and is generally silent on the underlying policy rule. Instead, a more instructive way forward is to realize that the informational requirements underlying our identification results could equivalently be phrased in terms of *policy instrument paths*, as already discussed in Section 2.3: to implement our results, the econometrician needs to know the causal effects associated with all possible time paths of the policy instrument $\boldsymbol{z}$. Empirical work that studies a given identified policy shock simply gives us the dynamic causal effects associated with a *particular* path of the policy instrument, without any reference to the underlying policy rule, to whether a policy shock is contemporaneous or "news," and in fact without even requiring stability of that underlying rule. The basic idea of our empirical method is to combine those instrument time paths to mimic the effects of a switch to a counterfactual policy rule.

3.1. *Illustrative Example*

To illustrate the basic logic of our proposed empirical method as transparently as possible, we begin with a stylized example that emulates the monetary policy counterfactual analysis of Bernanke et al. (1997). Like those authors, we consider an econometrician that wishes to predict the (counterfactual) propagation of oil price shocks in the absence of a monetary policy reaction—that is, the canonical "zeroing-out" policy counterfactual.[20]

Revisiting Bernanke et al. (1997)

Figure 2 provides a stylized representation of how the econometrician could use our identification result to construct her desired oil shock counterfactual. We emphasize that the impulse responses in this figure are purely illustrative; they do not come from any empirical analysis or structural model.

As a first step, the econometrician begins by estimating the effects of an oil price shock under the prevailing monetary reaction function, exactly as in Bernanke et al. (1997). In the stylized example here, the oil shock leads to an increase in prices (top-left panel);

[20]In notation of Section 2, such "zeroing-out" corresponds to a counterfactual policy rule that sets $\boldsymbol{z} = \mathbf{0}$. It is of course well known that rules of this sort—for example, a nominal interest rate peg—often lead to equilibrium indeterminacy, violating Assumption 2 (Sargent and Wallace (1981)). As discussed in Section 2.3, the counterfactuals presented here should thus be interpreted as corresponding to one particular equilibrium associated with this policy rule.

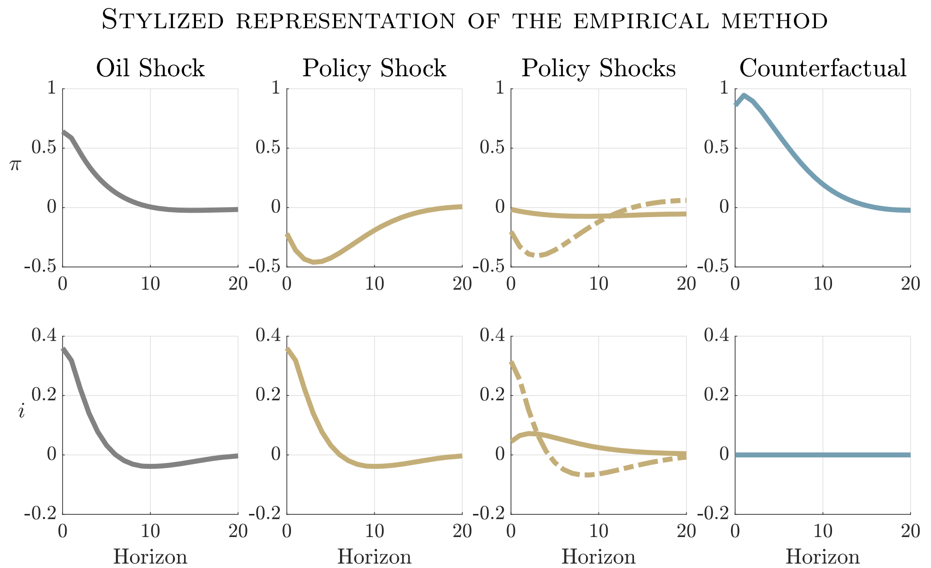

FIGURE 2.—Inflation (π) and nominal interest rate (i) impulse responses to: oil shock under the baseline rule (left panel, solid); monetary policy shocks to the baseline rule (two middle panels, solid and dashed); and oil shock under the counterfactual rule (right panel, solid). All impulse responses are purely illustrative; they do not come from any empirical exercise or structural model.

the monetary authority furthermore leans against this inflationary pressure through an increase in nominal interest rates (bottom-left panel). By our identification result, she next needs to estimate the effects of a monetary policy shock—or a linear combination of such policy shocks—that moves nominal interest rates *from date-*0 *onwards* exactly like the observed endogenous interest rate response to the oil shock. The two middle panels show two possible scenarios. In the left one, the econometrician was able to identify a single monetary policy shock that induces the exact same path of nominal interest rates as the oil shock. In the right one, she estimated two separate policy shocks (one solid, one dashed), with the sum of the two replicating the interest rate path after the oil shock. In both cases, the identified policy shocks decrease inflation (top panels). Given either of these estimates, the econometrician can apply our identification result: she simply needs to subtract the impulse responses shown in the second or third column from those in the first column. The results are then shown in the fourth column: interest rates are now by construction unresponsive, and inflation increases by more than under the baseline policy response. It follows from Proposition 1 that any structural model consistent with (i) our general model framework (6)–(7), (ii) the original propagation of the oil shock (first column), and (iii) either one of the two middle columns on monetary policy shock propagation will necessarily agree with this "zeroing-out" counterfactual displayed in the right panel.

Discussion

We emphasize that the illustrative example displayed in Figure 2 is stylized in two ways. First, using either of the estimated monetary policy shocks, the econometrician was able to *perfectly* enforce the desired policy counterfactual using only date-0 shocks. In actual applications, this will not be possible in general, so approximations will be needed. Second, the counterfactual rule that we considered was particularly simple, taking the form of an exogenous interest rate path rather than a more complicated relationship between endogenous equilibrium outcomes (like, e.g., a Taylor rule). Our empirical method, presented in the next section, is the natural generalization of the stylized example: the researcher considers an arbitrary counterfactual rule of our general form (8), and then enforces it *as well as possible* using the available policy shock evidence.

3.2. *Counterfactuals With a Limited Number of Policy Shocks*

We consider a researcher that has access to estimates of n_s distinct policy shocks associated with n_s distinct response paths of the policy instrument z.[21] We denote the causal effects of these shocks by $\{\Omega_{x,\mathcal{A}}, \Omega_{z,\mathcal{A}}\}$, where each of the n_s columns of the Ω's gives the impulse response to a distinct identified policy shock. Given these lower-dimensional causal effect maps, and given a non-policy shock $\boldsymbol{\varepsilon}$ and a counterfactual rule $\{\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z\}$, the proof strategy of Proposition 1 will fail in general. We would now need to set

$$\tilde{\mathcal{A}}_x\big(\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{x,\mathcal{A}} \times \boldsymbol{s}\big) + \tilde{\mathcal{A}}_z\big(\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{z,\mathcal{A}} \times \boldsymbol{s}\big) = \boldsymbol{0}, \tag{30}$$

where $\boldsymbol{s} \in \mathbb{R}^{n_s}$ denotes weights assigned to the n_s empirically identified policy shocks at date 0. The problem is that this system of T equations (where T is the large maximal transition horizon) in n_s unknowns will generically not have a solution. So how can researchers proceed?

Our proposal is to simply select the weights $\boldsymbol{s}$ on the n_s date-0 shocks to enforce the desired counterfactual rule *as well as possible*. In practice, this means solving the problem

$$\min_{\boldsymbol{s}} \big\| \tilde{\mathcal{A}}_x\big(\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{x,\mathcal{A}} \times \boldsymbol{s}\big) + \tilde{\mathcal{A}}_z\big(\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{z,\mathcal{A}} \times \boldsymbol{s}\big) \big\|. \tag{31}$$

The output of the simple problem (31) is the best approximation to the desired policy counterfactual within the space of empirically identified policy shock paths. By our identification results in Section 2 and because all shocks are dated $t = 0$ (i.e., no ex post surprises), this approach is robust to the Lucas critique. In the illustrative example of Figure 2, the available evidence on policy shocks in the middle panels was sufficient to set the argument of (31) exactly to zero. In actual applications, on the other hand, we will not perfectly enforce the desired policy counterfactual; rather, we will approximate it as closely as possible. The richer the menu of policy shocks we have access to, the better the approximation will become, eventually converging to the truth (as $n_s \to \infty$). The important limitation of our approach is thus that, for small n_s, it will not always be possible to construct an accurate approximation of the desired counterfactual rule—sometimes we will be able to set the implementation error in (31) close to zero, other times it will be large. The practical usefulness of our proposed method is thus an inherently application-dependent question.

By Proposition 2, our identification results also allow researchers to learn about *optimal* counterfactual policy rules, given some exogenously specified loss function. Appendix B.2 of the Supplemental Material shows how to apply our Lucas critique-robust method to

[21]In saying that a researcher has access to policy shocks that induce different instrument paths, we are implicitly assuming that these differences in instrument paths reflect different identification strategies capturing different linear combinations of policy shocks rather than statistical noise or violations of the identifying assumptions. We justify this interpretation in our empirical application in Section 4.

such questions of optimal policy design. Very briefly, the idea is to use date-0 policy shocks to reduce the policymaker loss as much as possible. Our approach thus minimizes the loss function by perturbing the baseline policy response in directions spanned by the set of empirically identified policy shocks.[22] Finally, for both rule counterfactuals and for optimal policy, we in Appendix B also describe how to leverage our results to construct counterfactual average business-cycle statistics.

4. APPLICATION TO MONETARY POLICY COUNTERFACTUALS

This section applies our empirical method to construct monetary policy rule counterfactuals. We proceed in two steps. First, in Section 4.1, we provide a brief review of existing evidence on monetary policy shock transmission—the key input to our empirical method. Second, in Section 4.2, we apply our method to study the propagation of investment-specific technology shocks under various counterfactual monetary rules.

4.1. *A Review of Monetary Policy Shock Evidence*

In order to implement our empirical method, we require evidence on multiple distinct monetary policy shocks that induce different time paths for nominal interest rates. The empirical literature has devised many different strategies to isolate quasi-random variation in the conduct of monetary policy (see Ramey (2016), as well as the discussion below). Since monetary authorities affect current and future expected interest rates, monetary policy is inherently multi-dimensional, and so it is not surprising that distinct identified policy shocks capture different dimensions of policy: some identification schemes will capture transitory impulses, while others reflect more persistent deviations from the policy rule.[23] The empirical evidence that we leverage is consistent with this observation.

Our applications in Section 4.2 will use two of the most canonical monetary policy shock series: those of Romer and Romer (2004) and Gertler and Karadi (2015). Importantly, those two monetary shock series are likely to be informative about very different monetary experiments. While the Romer and Romer shock is rather short-lived (i.e., mostly reflecting contemporaneous shocks $\nu_{0,t}$), the Gertler and Karadi shock is well known to move longer-term nominal interest rates and is thus more likely to have a larger forward guidance component (i.e., in greater part reflecting $\nu_{\ell,t}$ for $\ell > 0$). Our applications in the next section reveal that even this relatively modest amount of evidence is in fact enough to tightly characterize several important monetary policy rule counterfactuals.

While we have chosen to focus on the most well-known and well-understood policy shock series for our main applications, we emphasize that similar arguments about interest rate time profiles apply just as well to several other popular monetary policy shock series. First, as we discuss in detail in Appendix C.4 of the Supplemental Material, the

[22]This part of our empirical method is related to work by Barnichon and Mesters (2021). Those authors argued that, under quite general conditions, evidence on policy shock impulse responses can be used to test the optimality of a policy decision. Our method makes materially stronger assumptions—notably the separation of the policy and non-policy blocks in (6)–(7)—allowing us to explicitly *characterize* optimal policy (and optimal policy rules), as in Proposition 2.

[23]A related argument was made by Sims (1998): there is no need for different identification strategies to yield correlated measures of policy shocks, simply because the identified shocks may capture different sources of variation in policy. We thank our discussant Valerie Ramey for pointing out that connection.

monetary shock series of Miranda-Agrippino and Ricco (2021) and Aruoba and Drechsel (2022)—shock measures that seek to improve on the original series of Romer and Romer and Gertler and Karadi in various ways—induce similar dynamics, with one shock more transitory and the other more persistent. Second, some prior work has explicitly split monetary policy shock series by their effects on different points of the yield curve, leveraging the intuitive idea that no two monetary policy surprises are likely to shift the overall yield curve in exactly the same way. Estimates of this type are, for example, presented in Gürkaynak, Sack, and Swanson (2005), Antolin-Diaz, Petrella, and Rubio-Ramírez (2021), and Inoue and Rossi (2021), and would offer natural alternatives as an input to our empirical method.[24]

4.2. *Counterfactual Policy Rule Exercises*

We apply our empirical method to predict the effects of investment-specific technology shocks under various counterfactual monetary policy rules. In particular, our objects of interest are the counterfactual behavior of the output gap, inflation, and the short-term nominal rate. We choose to focus on investment-specific technology shocks because such shocks are widely argued to be one of the main drivers of aggregate business-cycle fluctuations, at least in the United States (e.g., see Justiniano, Primiceri, and Tambalotti (2010), Ramey (2016)).

We proceed as follows: we estimate the inputs required by our methodology, apply the method and present the main results, and then discuss how to interpret those results in light of our theoretical identification results in Section 2. Appendix C provides further details.

Inputs

The first input to our analysis are the aggregate effects of the non-policy shock of interest $\boldsymbol{\varepsilon}$ under the prevailing baseline policy rule. To recover those effects, we rely on the investment-specific technology news shock series identified by Ben Zeev and Khan (2015)—a shock that induces an anticipated change in the relative price of investment goods. We estimate the propagation of this shock by ordering it first in a recursive vector autoregression (VAR) (as recommended in Plagborg-Møller and Wolf (2021)).

The second input are the causal effects of a menu of different monetary policy shocks. For this, we consider the shock series of Romer and Romer (2004) and Gertler and Karadi (2015), as already discussed in Section 4.1. To correctly account for joint uncertainty in the estimation of the effects of the two policy shocks, we study their propagation through a single VAR. For robustness, we also repeat all of our policy counterfactual applications with the shock series of Miranda-Agrippino and Ricco and Aruoba and Drechsel—two less well-known but arguably somewhat more robust shock series—and find similar results. All results for these alternative shock measures are reported in Appendix C.4.

Counterfactual Policy Results

We use our methodology to construct counterfactuals for several different alternative monetary policy rules: output gap targeting; a standard Taylor (1999) rule; a nominal

[24]We note that this discussion also extends to fiscal shocks. For government spending, Ramey (2011) explicitly distinguished between shocks reflecting gradual military build-ups and more transitory upticks in purchases. For taxes, Mertens and Ravn (2014) separated unanticipated (transitory) and anticipated (gradual) tax shocks. We leave applications of our methodology to fiscal policy counterfactuals to future work.

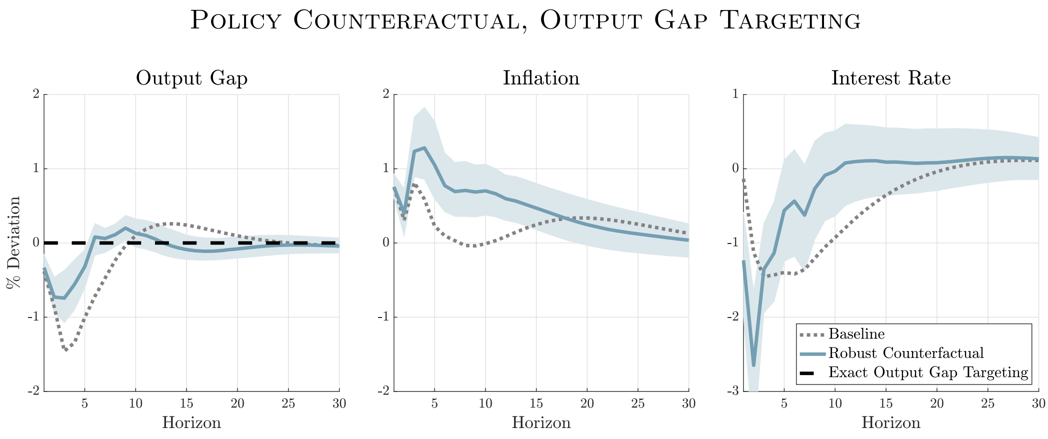

FIGURE 3.—Output gap, inflation, and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted) and the best feasible approximation to output gap targeting (solid), computed following (31). The shaded areas correspond to 16th and 84th percentile confidence bands. Perfect output gap targeting (i.e., $\widehat{y}_t = 0$ for all t) is displayed as the black dashed line.

rate peg; nominal GDP targeting; and the optimal policy rule corresponding to a loss function with equal weight on the output gap and a weighted average of current and lagged inflation (i.e., average inflation targeting).

First, Figure 3 shows our counterfactual results for output gap stabilization. The identified investment technology shock has both a cost-push as well as a negative demand component, consistent with theory (e.g., see Justiniano, Primiceri, and Tambalotti (2010)). Under the baseline policy rule (dotted), nominal interest rates are cut relatively aggressively, though not by enough to stabilize the output gap; furthermore, inflation stays moderately above target.[25] Under our approximation to output gap targeting, nominal interest rates are cut much more aggressively, essentially stabilizing the output gap from around a couple of quarters after the shock, at the cost of persistently higher inflation. Given the well-documented lags in monetary policy transmission, it seems unlikely that *any* nominal interest rate path could actually stabilize the output gap in the immediate aftermath of the investment shock; we thus believe that our empirical analysis yields an accurate approximation to what a strict output gap targeting policy can actually achieve in practice.[26]

Second, Figure 4 shows the results for a Taylor-type rule with strong responses to inflation and the output gap as well as moderate nominal interest rate smoothing. Due to the observed increase in inflation, this policy rule actually dictates a much less aggressive rate cut, resulting in somewhat lower output and inflation at medium horizons. In the right panel, the distance between the dashed and solid lines indicates whether or not our method is able to accurately implement the counterfactual rule. While the solid lines show our counterfactual path of nominal interest rates, the dashed lines instead use the counterfactual Taylor rule to map the output gap and inflation paths shown in the left and

[25]To the extent that our sample saw changes in the systematic conduct of monetary policy, the displayed impulse responses will average over the in-sample observed monetary reactions to the investment shock. See the discussion in Appendix A.4 for further details.

[26]In the notation of Section 2, these statements correspond to the idea that perfect output gap targeting—that is, the rule $\boldsymbol{y} = 0$, with $\boldsymbol{y}$ denoting the output gap—is not implementable (i.e., Assumption 2 is violated).

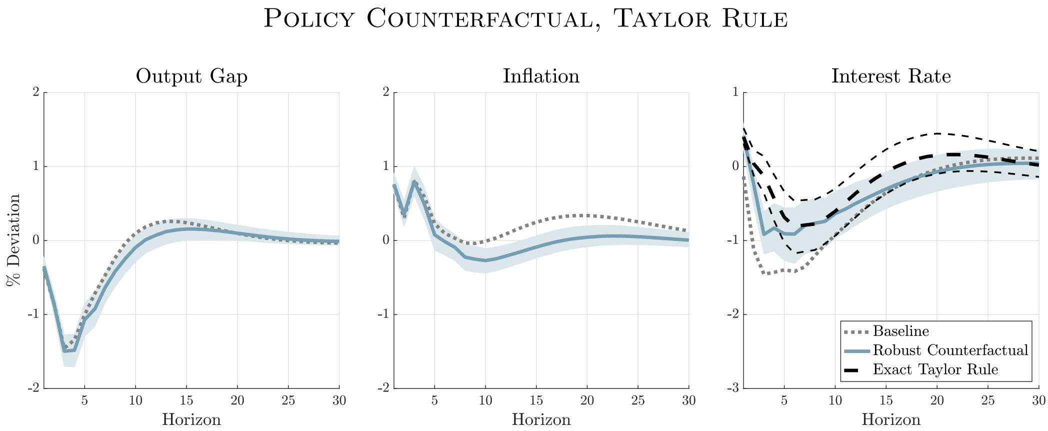

FIGURE 4.—Output gap, inflation, and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing rule (dotted) and the best feasible approximation to a simple Taylor-type rule $\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.5 \times (1.5\widehat{\pi}_t + \widehat{y}_t)$ (solid), computed following (31). The shaded areas correspond to 16th and 84th percentile confidence bands. The distance between dashed and solid lines in the right panel is the implementation error (i.e., the argument of (31)).

middle panels into paths of nominal rates. The distance between the solid and dashed lines is the argument of (31)—that is, the policy rule implementation error. We see that the contemplated counterfactual Taylor rule is imposed relatively well throughout, except at a couple of quarters after the initial shock (where interest rates are still cut by too much relative to the Taylor rule prescription).

Third, we proceed in the spirit of the recent change in the Federal Reserve's policy framework and consider a policymaker with preferences over output and *average* inflation $\bar{\pi}_t$, where $\bar{\pi}_t = \sum_{\ell=0}^{K} \omega_\ell \pi_{t-\ell}$.[27] We then represent the loss function of a dual mandate policymaker with preferences over average inflation as

$$\mathcal{L} = \lambda_\pi \bar{\boldsymbol{\pi}}' W \bar{\boldsymbol{\pi}} + \lambda_y \boldsymbol{y}' W \boldsymbol{y}, \tag{32}$$

with $\lambda_\pi = \lambda_y = 1$, $W = \text{diag}(1, \beta, \beta^2, \ldots)$, and $\beta = 1/1.01$. Results for our optimal policy counterfactual are displayed in Figure 5. The key takeaway here is that this optimal policy counterfactual differs very little from actually observed outcomes. In other words, there is little room to improve upon the observed allocation by changing policy within the space of policy instrument paths spanned by our two identified policy shocks.

Appendices C.3 and C.4 of the Supplemental Material present several further applications. First, we consider the two remaining policy counterfactuals: nominal GDP targeting and a nominal interest rate peg. We find that nominal GDP targeting can be implemented very accurately; interestingly, this counterfactual looks quite similar to our estimated outcomes under the baseline rule, with interest rates cut only slightly less aggressively. Matters look different for a nominal interest rate peg, however. Here, nominal rates in our best Lucas critique-robust counterfactual still fall by quite a bit too much, in particular at short horizons. Our method thus in this case does not allow an accurate characterization

[27]Here K denotes the maximal (lagged) horizon that enters the inflation averaging, and ω_ℓ denotes the weight on the ℓth lag, with $\sum_\ell \omega_\ell = 1$ and $\omega_\ell \geq 0\ \forall \ell$. For our application, we set $K = 20$ and $\omega_\ell \propto \exp(-0.1\ell)$. Suitably stacking the weights $\{\omega_\ell\}$, we can define a linear map $\bar{\Pi}$ such that $\bar{\boldsymbol{\pi}} = \bar{\Pi} \times \boldsymbol{\pi}$.

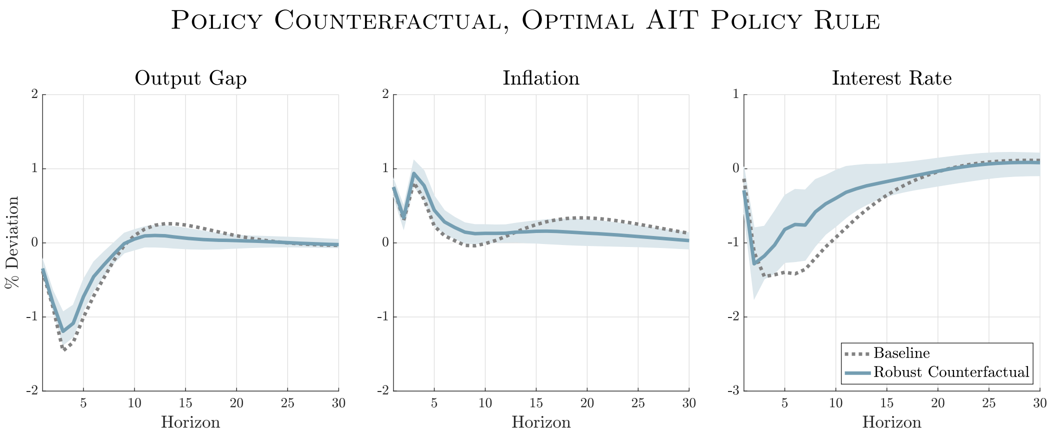

FIGURE 5.—Output gap, inflation, and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted) and the best feasible approximation to an optimal average inflation targeting monetary policy rule (solid), computed as discussed in Appendix B.2. The shaded areas correspond to 16th and 84th percentile confidence bands.

of the desired counterfactual. Second, we repeat our analysis with the alternative shock series of Miranda-Agrippino and Ricco and Aruoba and Drechsel. Those two shocks give similar impulse responses to our baseline shock measures, and so our systematic policy rule counterfactuals are not affected much.

Counterfactual Second-Moment Properties

While our analysis in this section has focused on policy counterfactuals *conditional* on some given non-policy shock, we have also used our identification results to construct counterfactual *unconditional* business-cycle statistics. Specifically, our object of interest is counterfactual aggregate business-cycle statistics under optimal policy for a policymaker with preferences as in (32). As discussed in Sections 2.3 and 3.2, recovering this counterfactual requires us to apply our policy counterfactual mapping separately to the impulse responses for each reduced-form Wold innovation of the observed macroeconomic aggregates, then stacking the resulting impulse responses into a new counterfactual Wold representation, and finally using this Wold representation to derive counterfactual second moments.

Results for this application are presented in Appendix C.5. Consistent with our "conditional shock" results in Figure 5, we find that—at least within the space of identified policy shock causal effects—only moderate policy improvements would have been feasible, with our constructed counterfactual volatilities of the output gap and inflation only somewhat below the actually observed level.

Discussion

The results from our applications in this section reveal that existing empirical evidence on policy shocks is already sufficient to tightly restrict policy rule counterfactuals for several prominent alternative monetary policy strategies. At the same time, we emphasize that our empirical method is clearly not *always* applicable: for some non-policy shocks and some counterfactual rules, it will not be possible to enforce the counterfactual rule accurately. In particular, the counterfactuals that we constructed for the investment shock application were relatively accurate precisely because the investment shock is rather transitory, thus only requiring knowledge of the effects of similarly transitory interest rate changes, along the lines of those implied by the Romer and Romer and Gertler and Karadi monetary policy shocks (see Appendix C.2 for the exact paths). More persistent non-policy shocks $\boldsymbol{\varepsilon}$ necessarily induce more persistent policy instrument movements and thus would correspondingly require empirical evidence on highly persistent policy shocks (e.g., far-ahead forward guidance).

5. CONCLUSIONS

The standard approach to counterfactual analysis for changes in systematic policy rules relies on fully-specified, structural, general-equilibrium models. Our identification results instead point in a very different direction: researchers can estimate the causal effects of distinct policy shocks and combine them to form policy counterfactuals. Importantly, these counterfactuals are valid in a large class of models that encompasses the majority of structural business-cycle models currently used for policy analysis.

An important challenge in implementing this strategy is that its informational requirements are high. We showed how to proceed in the empirically relevant case of evidence on a small number of policy shocks. We illustrated through several examples that empirical evidence is already sufficient to tightly characterize a variety of interesting monetary policy rule counterfactuals, reducing the need for explicit structural modeling. More generally, a key message of this paper is to emphasize the value of empirical strategies that recover the dynamic causal effects associated with different *time paths* of policy instruments. Every additional piece of empirical evidence on a different policy instrument path will expand the space of counterfactual policy rules that can be analyzed with our method.

In closing, we would like to reiterate two important considerations for researchers who contemplate using our approach. First, our method is silent on issues of equilibrium uniqueness. It will construct *one* valid equilibrium for the counterfactual policy rule, but nothing guarantees uniqueness; for that, additional theoretical arguments are needed. Second, our empirical method relies on linearity and thus should only be used when this assumption is appropriate. Structural modelers often use linearization as a means of computing equilibria; in such structural contexts, the uses and limitations of linear methods are well understood. Those same principles apply to the use of our method.

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