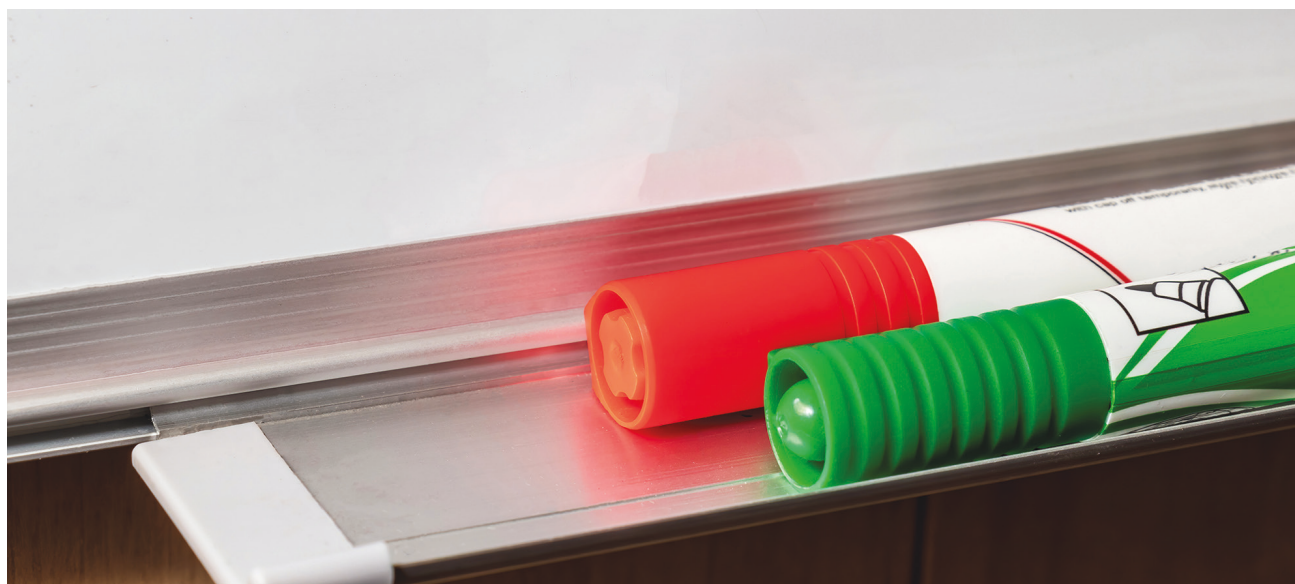




Using Public Records to Scaffold Joint Sense Making

Teachers can more productively use board work to scaffold joint sense making.

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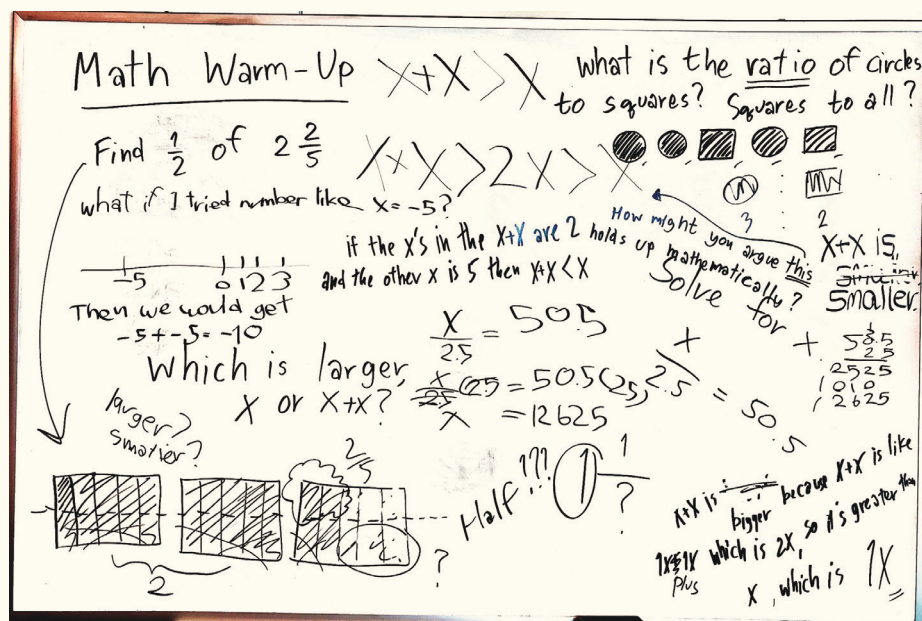
Has your whiteboard ever looked like the one in Figure 1? As teachers, we know effective board work is hard work. What makes board work effective, however, has a lot to do with the purpose of creating that public record in the first place. Others have shared ideas about how public records can support facilitating mathematics discussions in general (Garcia et al., 2021), as well as discussing multiple student solutions (DeLeeuw et al., 2021). In this article, we focus on using board work to scaffold what we call *joint sense making*, because effective

mathematics instruction is, at its heart, characterized by teachers and students engaging collaboratively in making sense of mathematical ideas (National Council of Teachers of Mathematics, 2009, 2014). This sense making involves students grappling with a mathematical situation and responding to their peers' thinking about that situation, and teachers helping to facilitate that activity while avoiding the temptation to do the sense making for the students. From our experiences working together with middle and high school mathematics

teachers to learn how to more productively use student mathematical thinking (see Peterson et al., 2022; Stockero et al., 2014; Van Zoest et al., 2023), we have gained insights into how public records (typically a written representation of the discussion displayed on the

board) have the potential to help teachers overcome some of the challenges of that work. In this article, we share these insights by providing suggestions for how teachers can productively use a public record to scaffold joint sense making.

Figure 1 Whiteboard Unlikely to Support Joint Sense Making



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THINKING ABOUT PUBLIC RECORDS

When we use the term *public record*, we mean a visual representation that captures the ideas that emerge during a discussion and that can be accessed at any time by everyone in the class. Although teachers do sometimes contribute in order to *judiciously tell* (Freeburn & Arbaugh, 2017) information that will further the joint sense making, we focus primarily on how student contributions are represented in the public record.

Public records are valuable tools for scaffolding joint sense making because they provide both permanence and focus. Public records provide some *permanence* for the ideas shared in a discussion, so that those ideas become a physical object that the teacher and students can refer to and operate on. Public records help *focus* students on shared ideas by establishing and sustaining a common ground for the sense-making discussion. These affordances help students navigate the mathematics of shared ideas that can be difficult to keep in their memories as they engage in a joint sense-making discussion. In this way, public records have the potential to help reduce the cognitive load (Sweller, 1988) for students during sense-making activities. Sense-making activities, as with other mathematical activities, impose a cognitive load on students' working memory as they encounter and process a lot of information. Some of the cognitive load is key to these sense-making activities, but some of the cognitive load is extraneous, such as trying to remember what a student said early in a discussion or searching for an idea displayed on the board. A teacher's use of a public record has the potential to help reduce this extraneous load on working memory so that more resources can be devoted to the cognitive load intrinsic to sense making.

In this article, we share suggestions for how teachers can use public records to scaffold joint sense making. Before proceeding, we acknowledge two things we are *not* talking about in this article. First, during sense-making discussions, teachers have to decide which student contributions will ultimately be added to the public record. We will not discuss this decision in this article, but teachers can learn more about decision making regarding student contributions in our earlier work (Peterson et al., 2022; Stockero et al., 2014; Van Zoest et al., 2023). Second, we recognize the value in students coming to the board or in a teacher displaying student work during mathematical discussions. Although our suggestions are relevant to the work a teacher does around these student-created public records, our focus here is on the actions of the

teacher when they are the scribe. (For some suggestions for use of student-created public records, see DeLeeuw et al., 2021).

MS. CLUFF'S PUBLIC RECORD

To illustrate how a public record might be created during joint sense making, and to provide a context for our suggestions, consider the vignette in Figure 2, in which Ms. Cluff and her class are having a whole-class discussion related to the problem, "The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?"

SUGGESTIONS REGARDING QUALITY PUBLIC RECORDS

We now use the vignette in Figure 2 as a context to share three suggestions related to using public records to scaffold joint mathematical sense making:

1. Make the public record precise.
2. Purposefully organize the public record.
3. Take advantage of the public record.

For these suggestions to be of most use, it is important to keep in mind the goal of facilitating joint sense making. That is, although these suggestions are applicable to all public records, they are specifically about using public records to scaffold joint sense making.

Make the Public Record Precise

The first suggestion for public record creation and use is to *make the public record precise*. Precise records of student contributions make apparent *what* a student is saying so the class can focus on making sense of the mathematics of that contribution. This precision comes from ensuring that student contributions added to the public record are *clear*, *complete*, and *concise* (Leatham et al., 2021).

First, to make a student contribution *clear*, a teacher may need to go back to the contributing student and ask them to clarify particular aspects of what they have said. This need often arises when students use informal language or pronouns with vague referents. In the vignette in Figure 2, none of the contributions required clarification for the rest of the class to make sense of what was said, and the teacher did not seek any clarification. The teacher's responsibility is to ensure that the contribution is clear, but this may not require asking

Figure 2 Vignette of Classroom Dialogue and Resulting Pieces of Public Record

Liza claimed that the prices would be the same because the price would increase and then decrease by the same amount. Ms. Cluff thought it would be beneficial to use Liza's claim as the focus of a joint sense-making activity. She recorded the claim on the board (as shown below) and asked, "How does Liza's claim hold up mathematically?"

$$\text{final} \stackrel{?}{=} \text{original}$$
 Yes
 The price will
 increase and decrease
 by the same amount

Stuart (another student in the class) stated that he thinks Liza's claim is wrong and that the prices would not increase and decrease by the same amount. Ms. Cluff asked Stuart, "Why not?" to which he responded, "Because, if a necklace is 100 dollars, and it increases by 50%, then it goes up to 150 dollars. And if it decreases by 50%, it goes down to 75 dollars. Cause you're decreasing by half of the amount of the 50% more." After asking Stuart to share how he got both \$150 and \$75, Ms. Cluff recorded Stuart's reasoning.

$$\begin{aligned} \text{necklace} &= \$100 \\ \text{inc by } 50\% &: 50\% \text{ of } 100 = 50 \\ 100 + 50 &= \$150 \\ 50\% \text{ of } 150 &= \$75 \end{aligned}$$

Liza then chimed in, "Well, I used an example too, but I started with \$20." With prompting from Ms. Cluff, Liza shared her example, which Ms. Cluff recorded on the board.

$$\begin{aligned} \text{necklace} &: \$20 + 10 = 30 \\ \$30 - 10 &= 20 \end{aligned}$$

Ms. Cluff then asked the class to think about how Stuart's approach related to Liza's approach. Juanita says, "If you did 50% of 20, you get 10, and then you add that 10 to 20 to get 30. Then 50% of 30 equals 15." Ms. Cluff recorded Juanita's contribution as shown below. Several other students went on to explain that it matters what number you take 50% of, and that the decrease amount would be different because you take 50% of a different number. Liza agreed, as did the rest of the class.

$$\begin{aligned} 50\% \text{ of } \$20 &= 10 \\ \$20 + 10 &= 30 \\ 50\% \text{ of } 30 &= \$15 \\ 30 - 15 &= \$15 \end{aligned}$$

for clarification. (See Van Zoest et al., 2023 for a discussion of productive and counterproductive clarifying.)

A second way for a teacher to make the public record precise is to generate a more *complete* student contribution. This can be done by asking the contributing student for the reasoning or logic behind a claim or for the student to provide an example of what they are claiming. In the vignette in Figure 2, when Stuart asserted that Liza's claim was wrong, the teacher asked him "Why not?" essentially asking him to provide the reasoning behind the assertion. The reasoning Stuart provided made his initial contribution more complete and allowed the teacher the opportunity to add his reasoning to the public record for the class to make sense of.

The third way that a student contribution can be more precise when adding it to the public record is for the teacher to hone the contribution to make it more *concise*. Sometimes a student contribution contains extra verbiage or extraneous information that is unnecessary for—and may even interfere with—making sense of the contribution. To effectively hone, teachers include only the salient points from what a student has said, and they do so in a mathematically efficient manner. Honing helps decrease cognitive load for students, allowing them to focus on the substance of the contribution rather than try to attend to a word-for-word re-presentation of the student's contribution and, in essence, try to carry out this honing themselves. An example of honing can be seen in the teacher's public record of Stuart's contribution in the second paragraph of Figure 2. The teacher honed Stuart's contribution by inserting mathematical notation and by not writing everything Stuart said word-for-word.

Whether seeking to make a contribution clear, complete, or concise, teachers should strike a balance. For example, "clarifying" shouldn't be belabored by asking for more clarification than is needed for the class to know what the student contributed. When asking students to provide their reasoning so the contribution is complete, it is important to get just enough reasoning so the class can understand the contribution. Asking for too much from the contributing student could take away the opportunity for the whole class to participate in the sense making. When it comes to finding the balance for creating a concise statement of the student contribution, teachers sometimes worry that they might misrepresent the students' ideas or undermine their sense of ownership. One way to address these concerns is for the teacher to write a concise statement of the thinking that captures what they see as the critical mathematical

One of the big 'ah-ha's' for me is how important that public record really is and keeping it short, sweet, to the point, but not so short, sweet, to the point that you can't make sense out of it.

Trevor, Middle School Math Teacher

elements of the contribution and then ask the contributing student for confirmation that they have accurately represented their thinking. Teachers can do this by saying something like, “Does this [referring to the public record] capture what you were saying?”

Purposefully Organize the Public Record

The second suggestion for public record creation and use is to purposefully organize the shared ideas, both within specific ideas and across the ideas. Considering this series of questions can help a teacher decide how to organize the public record purposely to scaffold joint sense making:

- How do these shared ideas fit into the ongoing argument?
- How might these ideas help the class move forward in their joint sense making?
- How might the recording of these ideas help scaffold the class as they move forward in this joint sense making?

Organizing ideas can help scaffold sense-making activities such as comparing, connecting, and contrasting. Here, we share three key actions teachers can take to purposefully organize public records: (a) distinguish between ideas, (b) consider the placement of ideas, and (c) seek parallelism of ideas (Freeburn et al., 2022).

The public record in Figure 3 is from the vignette in Figure 2. We will modify this public record to illustrate what it might look like had each of the three key teacher actions been used to purposefully organize it.

Distinguish between ideas. As a teacher captures student contributions during the joint sense-making activity, it is important for the teacher to distinguish each contribution in the public record. Distinguishing ideas can help identify where one contribution ends and another begins. Drawing this distinction would be difficult to do with the public record in Figure 3, but teachers can distinguish contributions from each other in a variety of ways. In Figure 4, we can see an adaptation of the initial public record from the vignette, in which a teacher used different colors to differentiate contributions from each other. A teacher at a professional development session made the following observation about the use of color to distinguish ideas:

The colors were huge. We like the fact that it distinguishes the different ideas. You didn't have to put a name with [the ideas]. You could refer to [the ideas]

Figure 3 Public Record from Opening Vignette in Figure 2

The price will increase
then decrease by the
same amount

necklace = \$100
inc. by 50% = 50% of 100 = 50
100 + 50 = \$150
50% of 150 = \$75

necklace = \$20 + 10 = 30
\$30 - 10 = 20

50% of \$20 = 10
\$20 + 10 = \$30
50% of 30 = \$15
30 - 15 = \$15

by color if you wanted to, so, then when there was one [idea] up there that was actually incorrect, it wasn't like, "Oh, you're wrong, Chris."

As the teacher points out, the use of different colors can distinguish ideas with some anonymity for students who may not want to be recognized as contributing an idea with an error. Alternatively, a teacher could draw lines or boxes to delineate contributions in the public record. In each case, distinguishing contributions and important aspects of those contributions can help to scaffold the class in their joint sense making. While distinguishing helps to improve the value of the public record, teachers can do more to scaffold these ideas in the overall argument.

Consider the placement of ideas. The placement of contributions in relation to each other is another way to organize the public record purposefully. For example, in the public records in Figures 3 and 4, it would likely take some effort for students to identify which of the contributions support the initial recorded claim and which do not. Compare this with the public

record in Figure 5, in which the contribution that agreed with the original claim was vertically aligned below that claim in a column on the left, while contradictory contributions were placed together in a column on the right.

The placement of contributions in the public record can also support a class with making comparisons between contributions during the joint sense-making activity. As we saw in the initial public record, a vertical chronological recording of student contributions (see Figure 3) can intertwine contributions that agree or disagree with an initial claim. This intertwining can make it difficult for a class to make comparisons between the contributions. A teacher can organize contributions in a table or other graphic organizer to help students make comparisons and connections that surface at different points in time during a discussion. In addition, labels of "yes" and "no" were added to the columns in Figure 5 to further distinguish the arguments. Grouping the four contributions in columns can help students make connections between the two contributions within the "yes" column or "no" column, as well as between the mathematical ideas in the two columns.

Last, where a teacher places an idea in the public record can be determined by how relevant the idea seems to be to the overall discussion. That is, a teacher may recognize that some of the shared student ideas may not be as essential as others for the sense-making discussion and thus may choose to

Figure 4 Revised Public Record (Using Colors to Distinguish Contributions)

final $\stackrel{?}{=}$ original

Yes

The price will increase and decrease by the same amount

necklace = \$100
inc. by 50% = 50% of 100 = 50
100 + 50 = \$150
50% of 150 = \$75

necklace = \$20 + 10 = 30
\$30 - 10 = 20

50% of \$20 = 10
\$20 + 10 = \$30

50% of 30 = 15
30 - 15 = \$15

Figure 5 Revised Public Record with Similar Contributions in "Yes" and "No" Columns

final $\stackrel{?}{=}$ original

Yes	No
The price will increase and decrease by the same amount. necklace = \$20 + 10 = 30 \$30 - 10 = 20 50% of \$20 = 10 \$20 + 10 = \$30 50% of 30 = 15 30 - 15 = \$15	necklace = \$100 inc. by 50% = 50% of 100 = 50 100 + 50 = \$150 50% of 150 = \$75 50% of \$20 = 10 \$20 + 10 = \$30 50% of 30 = 15 30 - 15 = \$15

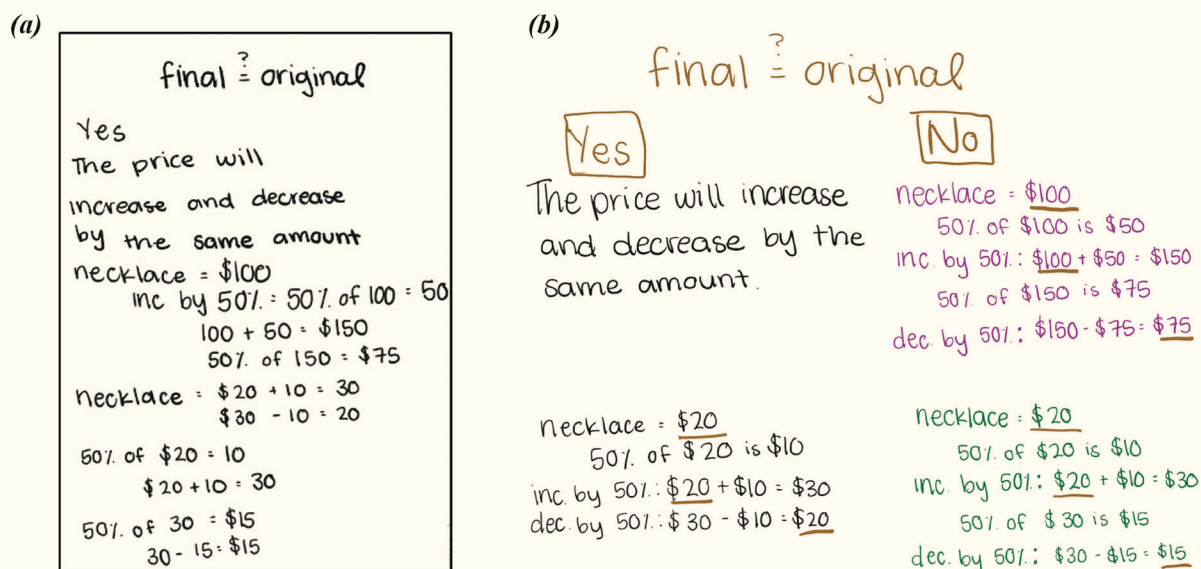
place them peripherally (or not at all), rather than centrally.

Seek parallelism of ideas. Parallelism (similar structuring) both within and across contributions can scaffold students' sense making of those contributions, helping them to attend to similarities that they may not otherwise attend to. For example, our initial public record (Figure 6a) contained a variety of symbols and structures for the calculations; such variety might hinder students' efforts to see connections. By contrast, each student contribution in Figure 6b has been structured in the same way, potentially scaffolding student attention to their similarities and differences. For example, the original cost of the necklace and parallel computation methods have been included in each of the three examples. In addition, the solutions are positioned in such a way that the common mathematical structure is in parallel placement. Finally, using the same symbols creates parallelism between and within the contributions that students can attend to and make sense of. As with all of these suggestions, attending to parallelism can reduce the cognitive load inherent in comparing and contrasting contributions, allowing that cognitive work to focus primarily on the mathematics within those contributions.

I saw a definite difference when I was more intentional about organizing the record and labeling and how I was recording the student thinking, you know, instead of just writing everything up there in a big mishmash. Having some coherency I thought was really helpful.

Paula, Middle School Math Teacher

Figure 6 (a) Initial Public Record from Vignette, and (b) Reorganized to Highlight Distinguishing, Placement, and Parallelism



Take Advantage of the Public Record

A primary purpose in creating an effective public record is to create a tangible, written representation of ideas that both the teacher and the students can refer to throughout the joint sense-making discussion. References typically entail gesturing (e.g., pointing to the public record), verbal cues (e.g., referring to the public record as “Liza’s claim”), or a combination of gestures and verbal cues (e.g., saying “the red approach” while pointing to a particular part of the public record). Referencing the public record can scaffold joint sense making in at least three ways.

First, frequent referencing of the public record helps to keep the focus of the discussion clear throughout the discussion. Such referencing can also help to refocus the class when tangential student contributions have been put aside (Peterson et al., 2022). Keeping students focused on the discussion at hand is a difficult task, and referencing the public record is a powerful tool to improve that focus. In the vignette in Figure 2, pointing directly to Liza’s claim throughout the discussion, or labeling it as “Liza’s claim” and then referring to it by name, are ways for Ms. Cluff to keep the focus of the discussion clear. In addition to focusing the discussion, such referencing of the public record can also assign competence (Cohen, 1998). Note that assigning competence in this way requires developing classroom norms where both correct and incorrect claims based on sense making are seen as representing smartness and providing opportunities for the class to learn together.

Second, referencing the public record reduces the cognitive load, as it is an efficient way to refer to important ideas from the discussion without the need to restate those ideas. Such referencing is extremely useful when the teacher wants to emphasize *how* they want students to engage with the idea(s) captured in the public record. Ms. Cluff takes advantage of this affordance when she asks, “How does Liza’s claim hold up mathematically?” Reducing the cognitive load in this way can support the teacher in maintaining students’ engagement in sense making and help them to track the big mathematical ideas. The reduction in spoken language can be particularly helpful for students who are learning English.

Third, the public record provides permanence for student contributions so that teachers can use pointing gestures to support the class with attending to the details of multiple contributions. Specific pointing gestures can clarify what pieces of the public record correspond with the teacher’s speech, reducing the cognitive load for students because they do not have to visually search for those pieces on their own. Such gestures can also enhance comprehension of what is being said for students with hearing difficulties by providing visual reinforcement. Referencing different parts of the public record can help scaffold a teacher’s efforts to synthesize current ideas and invite students to make connections among those ideas. A teacher’s referencing supports the synthesis of student contributions by re-presenting the details of those contributions and helping students to track them in the public record. With respect to inviting students to make connections across ideas, referencing allows the teacher to easily identify the ideas they are being invited to connect. Again, such referencing to support connecting gives the desired sense-making action more prominence in the teacher’s speech. Referring explicitly to the clearly recorded and delineated approaches from Liza and Stuart would support Ms. Cluff in asking the class to consider how these approaches are related.

CONCLUSION

As you look ahead to future lessons in your classroom, we invite you to consider the three suggestions discussed in this article as ways to scaffold joint sense making with your students: (1) make public records precise; (2) attend to the organization of ideas within a public record by distinguishing between ideas, considering the placement of ideas, and seeking parallelism of ideas; and (3) reference the public record in meaningful ways. Being mindful of these suggestions can help your public records look more like Figure 6b and less like Figure 1, and taking advantage of these public records can help all students jointly engage in making sense of the mathematics in student contributions. —

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