

Towards Effective Resource Procurement in MEC: a Resource Re-selling Framework

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Abstract—On-demand and resource reservation pricing models, widely used in cloud computing, are currently used in Multi-Access Edge Computing (MEC). Nevertheless the edge's resources are distributed and each server has lower capacity. If too much resources were reserved in advance, on-demand users may not get their jobs served on time, jeopardizing MEC's latency benefits. Concurrently, reservation plan users may possess un-used quota. Therefore, we propose a sharing platform where reservation plan users can re-sell unused resource quota to on-demand users. To investigate the mobile network operator's (MNO's) incentive of allowing re-selling, we formulate a 3-stage non-cooperative Stackelberg Game and characterize the optimal strategies of buyers and re-sellers. We show that users' actions give rise to 4 different outcomes at equilibrium, dependent on the prices and supply levels of the sharing and on-demand pools. Based on the 4 possible outcomes, we characterise the MNO's optimal prices for on-demand users. Numerical results show that having both pools gives the MNO an optimal revenue when the on-demand pool's supply is low, and unexpectedly, when the MNO's commission is low. We develop an interactive prototype, and show that users' decision distributions in studies on our prototype are similar to that of our decision model.

Index Terms—Edge computing, Network economics, Sharing Platforms, Game Theory

1 INTRODUCTION

Edge computing enables a wide variety of low latency and computationally intensive services on mobile and other resource constrained devices (e.g Internet of Things (IoT) devices). These low latency services include video analytics, real-time analytics, virtual and augmented reality (VR/ AR), and connected vehicle decision making. Edge computing brings the power of cloud computing to the network edge, with servers placed at edge access points e.g. base stations or wifi access points [1]. Users and device owners can offload computationally intensive tasks to the nearby edge servers, and receive them within latency requirements [2], due to the close proximity of the edge servers. The wide-area-network (WAN) delay of cloud computing will be avoided [3].

As computing resources are limited at the edge, researchers have been actively investigating the computation offloading and resource allocation problem, [2], [4]–

[7]. Besides the technical challenges of computation offloading, resource pricing solutions to optimize revenue [8]–[16] can serve as complementary mechanisms to control user demands and ensure good quality-of-service despite limited available resources, while maintaining the network operator's incentive. Currently, Edge computing providers have been adopting the traditional cloud computing pricing models [17]–[19]: the *On-Demand pricing* model (OPM) and the *Resource Reservation* model (RRM), e.g. in AWS Local Zones [20] and Juniper [21]. These two models cater to different types of users. In the *On-Demand pricing* model, service users pay for computing resources, as and when they need it, and do not make any long term commitment [22]. In MEC, this is suitable for individual mobile users using, for example, virtual reality or mobile gaming services once, or a couple of times, not across consistent timeslots across the span of months; for firms during special one-off events involving augmented reality displays or machine learning platforms. In contrast, for the *Resource reservation* model, service users reserve computing resource instances in advance, across multiple timeslots, (e.g. for a year), for a discounted price [23]. This caters to users (or firms) who use computing resources in bulk, at periodic and pre-determined timings. These may include IoT vendors with periodic data analytic requirements for their IoT networks or firms behind public augmented reality setups, with a constant need of computing capabilities.

Besides maximizing revenue, will such plans be able to meet user demand? While edge computing is advantageous latency-wise through the proximity to end users, unlike cloud data centers with thousands of servers [24], the distributed nature of MEC involves small-scale data centers placed at the network edge [1], [2]. The server capacity at edge nodes may be limited due to physical constraints [7].

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Therefore, the difficulty from jointly using the *On-Demand pricing* and *Resource Reservation pricing* models in MEC is that after the resource units are reserved in advance by the *Resource Reservation* users, there **might be fewer resource units left for on-demand users**. Consequently, on-demand users may experience a longer service latency, which may jeopardize the utility of MEC since services which use edge computing are often latency sensitive [2], [3]. Cloud providers complement the on-demand and resource reservation models and monetize excess demand through spot instances, leasing the unused capacity at data centers to users at a discounted price [25]. We are unable to consider spot pricing in MEC because unlike the cloud, there is a lack of spare capacity at the edge. Furthermore, raising the price of resource reservations to control the demand, may not suffice as users with long-term usage needs may continue to prefer fixed upfront expenses compared to on-demand expenses that may fluctuate and may be hard to predict.

Therefore, to improve the QoS of users, we propose a sharing platform, titled the *Sharing Quota Model* (SQM). Here, reservation plan users can re-sell their excess unutilized resource quota via the platform to on-demand users, while the MNO collects a commission. This could benefit all parties. The on-demand users can attain a higher QoS (lower delay) and a higher probability of getting their jobs served, the reservation plan users will obtain extra income and the MNO collects a commission. Higher customer satisfaction also benefits the MNO in the long run. It is unclear, however, whether the MNO always has an incentive to offer such a platform, as the SQM may discourage on-demand users from purchasing capacity directly from the MNO. To best integrate the sharing platform with the on-demand pricing and resource reservation models, the following research questions arise:

a) *Given the sharing platform, would on-demand users still buy from the on-demand platform? What would the aggregate equilibrium behavior look like?*

b) *Given the users' decisions, it is not clear if the MNO has an incentive to provide this platform: will it lead to sub-optimal revenue? How should it set prices such that revenue is optimized, whilst minimizing the unmet demand of the on-demand users?*

c) *How can we design the platform? Is it more practical if resource quota re-selling is automatically or manually decided?*

While there have been resource sharing (or trading) platforms in networks, e.g. mobile data market trading platforms [26]–[28], and crowdsourced community wireless networks [29], their market structures are different. Specifically, the mobile data market users buy plans which have the same structure, differentiated by usage levels. In crowdsourced community wireless network model formulations, given prices and membership choices of other users, users decide their network access time. This differs from our market structure, where there are two types of plans (on-demand vs reservation), determined by service application type and usage patterns. Our work also differs from other MEC resource allocation or revenue maximization work, as it uniquely considers how the on demand and resource reservation model can be improved in MEC. Therefore, to answer the above questions and investigate the feasibility of the sharing platform, we perform game theoretic analysis on the three-sided market, formulating a three-stage non-

cooperative Stackelberg Game amongst the MNO, re-sellers (reservation plan users) and buyers (on-demand users). Our **analysis characterises the decision making and conflicting incentives of the three parties**, with the MNO setting the on-demand pool and sharing pool's prices to optimize revenue, the on-demand users choosing the pool to purchase from, aiming to maximize their payoff functions, and the reservation users also aiming to maximize their payoff functions.

Our contributions are summarized as follows:

- With a direct application of on-demand and resource reservation pricing plans in MEC, on-demand users face potentially limited resource availability at the edge, impacting their QoS, while resource reservation plan users might have reserved excess un-utilized resource quota. Therefore we propose a novel model integrating a sharing platform, for the re-selling of unused resource quota from reservation plan users to on-demand users.
- To investigate the MNO's incentive on having a sharing platform and to optimize the MNO's prices for revenue maximization, we will use a **3-stage Stackelberg Game formulation** to characterise how the MNO, buyers and re-sellers interact at equilibrium. In this sequential game, the MNO selects prices to optimize its revenue. Following which, the re-sellers (reservation plan users) choose whether or not to sell, forming the sharing pool. Viewing the prices and relative supplies, the buyers choose which pool (or not) to buy from. We incorporate two platform design options - automated re-selling of unused quota, and manual decision making for re-sellers.
- Using backwards induction, we firstly **characterise the buyer's optimal strategies** for the above two platform design options (Proposition 1). Our analysis shows that the **equilibrium behavior amongst the buyers and re-sellers possesses a 4 region structure** (Proposition 2, with **each region corresponding to whether or not the on-demand and sharing pools coexist**). The likelihood of each of the 4 regions happening depend on the relative magnitudes of prices and supply levels of the two pools. Based on the 4 region structure, we optimize the on-demand and sharing prices (which also impacts the sharing supply), to **optimize the MNO's revenue, and provide insights on the optimal price** (Theorems 1 and 2, Corollary 1).
- We provide numerical results, which show how the system behaves at the optimal point. Our results show that for different willingness to pay and usage distributions, having both the sharing and on-demand pool gives the MNO an optimal revenue when the on-demand pool's supply q_o is low, and when δ , the platform's commission is low, but not below a certain threshold. It would be expected that a high commission would give the platform a higher sharing revenue, nevertheless our results show that it incurred a trade-off in dissuading reservation plan users from re-selling.
- We created a **prototype, an interactive website, to**

conduct preliminary user studies on user decision making in our resource re-selling framework. Our analysis shows that there is no statistically significant evidence that the empirical distribution function of the *data from the user study* and the empirical distribution function of the *data from the model's predictions* (Eq. 1, Prop. 1-2) are distinct.

The rest of this paper is organized as follows. In Section 2, we introduce the related works. In Section 3, we describe the system model. Following which, we formulate the Stackelberg Game in Section 4. In Section 5, we present the game analysis and insights. We present numerical results in Section 6, present our prototype and preliminary users studies in Section 7, discuss future areas of investigation in Section 8, and finally conclude this paper in Section 9.

2 RELATED WORKS

In this work, we propose a novel system model incorporating a resource re-selling platform, to improve the integration of the on-demand and resource reservation pricing models in MEC, given the limited resources at the edge. We also discuss how our model differs from other platforms in networked systems.

Edge Computing resource allocation: Researchers have been actively optimizing resource allocation in MEC, balancing the user's delay requirements with minimizing the energy consumption, under various scenarios [4]–[7], [30]. Another line of work is collaborative edge computing to maintain the QoS in light of user mobility. This involves strategies like VM or service migration across base stations [14], [31]–[33] and user association optimization [34].

Edge Computing revenue maximization: These works involve optimizing revenue while maintaining the quality of service for users (delay, job service rate, etc) [8]–[13]. For example, [8] used the Fisher market model to model the MEC market. [9] jointly optimized over the resource prices and the devices' budget allocation strategies. [10] modelled a dynamic game between the edge resource owners, infrastructure providers, and service users. While our model follows the user payoff model of [10], we model a distinct scenario, involving a novel hybrid sharing, on-demand and resource reservation framework. Thus, unlike [10] we must further account for re-seller incentives, leading to distinct results. In our prior work [35], we proposed a model where users constantly change whether to be an 'owner' or 'renter' of excess resource units, along with dynamic pricing mechanisms. We did not investigate the MNO's incentive of having a sharing platform. To the best of our knowledge, there has not been work on how to improve the integration of the on-demand and the resource reservation pricing models in MEC

Cloud Computing pricing: Maximizing revenue in cloud computing by improving on the on-demand and reserved instance models was studied in [19], [36]–[38]. These works focused on spot instances, which sell the spare capacity at data centers, at discounted prices [25]. For example, [19] proposed a truthful auction for spot instances. [36] proposed a dynamic pricing policy. [37] maximized revenue through capacity segmentation. Unlike in cloud computing,

each edge server has limited resources, making the spot instances model not feasible in edge networks.

Other Platforms in Networked Systems: The mobile data market has seen the existence of platforms (e.g. China Mobile Hong Kong's 2nd exChange Market) which provide a secondary data market, allowing users to buy and sell data from each other [26], [27]. These works [26]–[28] perform a game theoretic analysis to characterise the equilibrium and optimal decisions under various scenarios, as well as propose alternative platform designs [39]. This platform has a different market structure from the edge computing market - here all the users buy resources under the same type of scheme (monthly data plans with usage quotas) and are differentiated by usage levels. In the edge computing market, users have vastly different kinds of applications and usage patterns, which suit different usage schemes: some users have consistent usage patterns (e.g. IoT vendors) and suit the resource reservation model, and some users have more sporadic, one-off usage patterns (e.g. individual mobile users using applications like virtual reality) and are suited to the on-demand model.

3 THE RESOURCE PROCUREMENT MODEL

We consider an edge MNO that provides edge computing service to N users. The MNO has 2 resource procurement schemes for users to choose from, namely the *Resource Reservation Model* (RRM) and *On-Demand Pricing Model* (OPM). These models, which are commonly used as cloud pricing and procurement schemes [17]–[19], are currently being used in edge networks [20], [21]. Under OPM, the users purchase credits to offload their jobs at the edge servers "on the go", as and when they need it. They pay p_o per unit of resource consumed (CPU cycles), to the MNO. Customers of on-demand pricing could include individual mobile users, e.g. users of applications like AR/VR or mobile gaming, or firms who are using AR or machine learning applications for one-off special events or displays. These users' demand for computing is sporadic, not following a consistent usage pattern across timeslots.

The RRM is catered to customers who have consistent computing requirements across different timeslots. The RRM allows them to reserve (i.e. commit to) computing resource in advance across multiple timeslots, e.g. for a year, at a discounted price per timeslot of p_r . Customers of the reservation plans include users who use computing resources in bulk, at periodic timings, and more frequently in general. For example, these may include IoT vendors with constant sensing and data analysis requirements for their large IoT network, autonomous driving firms, entities behind long-term public augmented reality setups.

The user has chosen between the resource procurement schemes OPM and RRM based on its service application's usage requirements: sporadic, one-off vs consistent, frequent and long-term. The details of the user usage model will be in covered the following subsections. The computing resources at the edge server are first provisioned to the reserved users, with the remaining resources forming the on-demand pool.

Nevertheless, unlike cloud resource allocation where data centers have huge amount of computing resources,

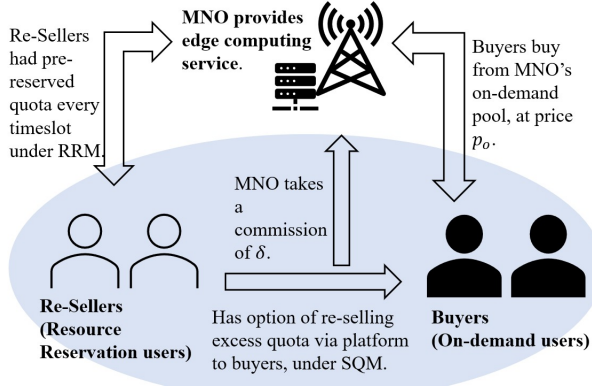


Fig. 1. The integration of the resource-quota sharing platform with the On-Demand and Resource Reservation models.

the server capacity at edge nodes is more limited [1], [2], and the data processing rate is less powerful [40]. Hence, after resources are reserved in advance by RRM users, there might be less resources left for on-demand users. At the same time, RRM users may find that for specific timeslots, they have excess computing resource quota for CPU cycles which they do not require. In this work we propose and explore the impact of adding a resource sharing scheme, the *Sharing Quota Model* (SQM). In this integrated scheme (see Figure 1), RRM users act as *re-sellers*, selling their excess unused resource quota at a price of p_r per unit of resource to on-demand users (*buyers*). The resold resources form the sharing pool and increase the total supply available to on-demand users (*buyers*). The MNO acts as a middleman and takes a commission of δ from the transaction. We investigate the impact that adding SQM has on the profits of the MNO, and hence whether this scheme is feasible.

Re-sellers: The *re-sellers* are users who have previously reserved resources from RRM, and who have excess resources available for sale. Under the proposed sharing scheme SQM, they have 2 choices $s = \{sell, no\}$ in each timeslot. They can choose whether to sell their excess computing resource quota via the MNO's platform to on-demand users (*buyers*), or not. Without loss of generality, we consider the payoff per unit resource quota for seller i , which is expressed as:

$$\pi_i^s(g_i) = \begin{cases} (1 - \delta)p_r - g_i, & \text{if } s = \text{sell} \\ 0, & \text{if } s = \text{no}. \end{cases} \quad (1)$$

Recall that p_r is the per unit price set by the MNO for the transaction, and define δ as the commission level which the MNO takes from the transaction. We let g_i represent the inconvenience cost of selling for seller i (factoring in uncertainty of usage, and the inconvenience of checking the app to decide to share). In this paper, we consider per unit resources in the payoff functions of both buyers and re-sellers. Buyers and re-sellers with multiple units are considered as multiple buyers or re-sellers. Note that if we had considered users buying/re-selling distinct quantities of resources, with the resource usage level of users following $U[0, 1]$, the impact would be that the supply and revenue functions are multiplied by a constant factor.

The inconvenience cost depends on the extent to which each re-seller is willing and able to expend time, energy and resources to periodically check the app, factor in the uncertainty of personal usage, and decide whether or not to sell the excess resource quota. This differs across different vendors and owners, and differs across applications. For example, it would be easier for an IoT vendor to dedicate resources to check the app, as compared to an individual MEC user. Some individual MEC users are more likely to check than others as they are more interested in earning extra income. Besides this, it is more difficult for users of some applications to make the decision to re-sell, as their usage for that particular timeslot is yet to be known, and more unpredictable. g_i is a random variable which follows the uniform $U[0, 1]$ distribution. It captures the varying levels of personal costs across the users. While the distribution has an impact on the way revenue is computed, we perform simulations using both uniform and beta (which has a gaussian-like structure) distributions in Section 6, showing similar results across distributions.

As we can see from Eq. (1), a re-seller's decision is impacted by the price it receives p_r , the MNO's commission level δ , and the inconvenience cost g_i . Re-seller i will share/sell when its profit $(1 - \delta)p_r$ is greater than its cost g_i . If the inconvenience cost is too high, it will not. To obtain the total proportion of re-sellers willing to sell/share, we integrate over the users whose inconvenience cost g_i is less than or equals to $(1 - \delta)p_r$. As g_i follows the uniform $U[0, 1]$ distribution, the proportion of re-sellers willing to share/sell will be:

$$\begin{aligned} prop &= \int_0^{(1-\delta)p_r} dg_i \\ &= (1 - \delta)p_r. \end{aligned} \quad (2)$$

The *quality* of the sharing pool, q_s , indicates the total supply of resource quota available for buyers, and therefore is dependent on the proportion of re-sellers who are willing to share their unused quota, $(1 - \delta)p_r$. It can be expressed as¹:

$$q_s(p_r) = f((1 - \delta)p_r), \quad (3)$$

with $q_s(p_r)$ being a strictly increasing, twice differentiable and concave function. This supply level has an impact on the user experienced delay.

Buyers: The *buyers* are on-demand users who purchase quota to offload their jobs at the edge servers on the go, i.e. not in advance. These buyers send their computing jobs to the edge server, with different willingness to pay (represented by u_i for user i). The willingness to pay is the buyer's utility of job computation at the edge server. For example, some jobs are more urgent and considered more of a necessity than others (e.g. public AR/VR displays, autonomous vehicle computation), and therefore the buyers will still be willing to pay despite the price p_r increasing. On the other hand, some jobs are considered less urgent, or are not viewed as necessities from the buyer's perspective, resulting in a lower willingness to pay: given a high price of

1. Function $q_s(p_r)$ may have various forms in realistic MEC systems, but our analysis permits any functional form. We will discuss a concrete form for numerical results in Section 6.

TABLE 1
Summary of key Notations

Notation	Definition
u_i	Willingness to pay of buyer i .
g_i	The inconvenience cost of re-seller i .
δ	The MNO's commission percentage.
p_r	Price per-unit of re-sold quota at the sharing platform.
p_o	Price per-unit resource at the on-demand pool.
q_o	Supply, and hence quality at the on-demand pool.
$q_s(p_r)$	Supply and hence quality at the sharing platform.
$\pi_i^s(g_i)$	Payoff of re-seller i .
$\pi_i^b(u_i)$	Payoff of buyer i .
$R(p_r, p_o)$	Revenue of the MNO.

resource quota p_r , the buyer is more likely to choose not to purchase the edge computing service. We let u_i follow the uniform $U[0, 1]$ distribution, to represent how the willingness to pay differs across users. Note that the distribution has no impact on the payoff maximizing strategies of the buyers, or the equilibrium outcome of user decisions in light of fixed prices. While it has an impact on the way revenue (Eq. (6)) is computed, we perform simulations using both uniform and beta distributions in Section 6 which show similar trends and effects across distributions.

Under **SQM**, these on-demand users now have 3 options $b \in \{\text{on-demand pool, sharing pool, none}\}$.² Note that, $b = \text{none}$ means not buying from both the on-demand pool and sharing pool. Therefore the payoff of buyer i is:

$$\pi_i^b(u_i) = \begin{cases} u_i q_o - p_o, & \text{if } b = \text{on-demand pool} \\ u_i q_s(p_r) - p_r, & \text{if } b = \text{sharing pool} \\ 0, & \text{if } b = \text{not buying,} \end{cases} \quad (4)$$

where u_i is buyer i 's willingness to pay for a unit resource quota, as described earlier and used in [10]. The payoff when users choose the on-demand and sharing pool respectively is their willingness to pay (satisfaction) u_i multiplied by the quality/supply of the pool (q_o or q_s) minus the payment (p_o or p_r). The quality/supply (q_o or q_s) indicates the relative supply of resources available at the on-demand and sharing pool respectively. The quality at the on-demand pool, q_o , is the amount of supply of computing resources (in CPU cycles) at the on-demand pool, after the resource reservation users have reserved their portion.³ As mentioned earlier, the quality at the sharing pool, $q_s(p_r)$, is a function of the proportion of re-sellers who are willing to re-sell. The higher the supply at each pool, the higher the likelihood the buyer is able to get its job served, hence improving its QoS in terms of delay experienced. Therefore the buyer's payoff $\pi_i^b(u_i)$ is an increasing function of the quality $q_s(p_r)$ or q_o . As different buyers have different willingness to pay u_i , different buyers will make different choices, based on which choice maximizes their individual payoff. In the next few sections, we investigate the equilibrium behavior of the users, and the optimal decisions for the MNO to make based on that. We summarize the key notations used throughout this paper in Table 1.

2. For simplification, we use $b = o$, $b = s$, and $b = n$ to represent $b = \text{on-demand pool}$, $b = \text{sharing pool}$, and $b = \text{none}$, respectively.

3. Note that in traditional centralised cloud computing, the supply at data centers (i.e. the q_o equivalent) would be much larger in magnitude, resulting in less of a need for a sharing pool, according to Eq. (4).

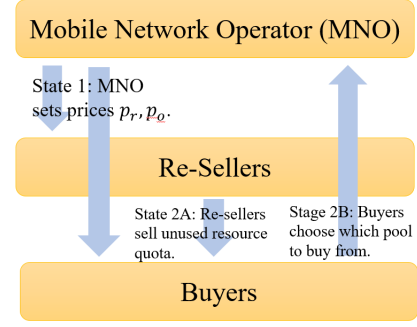


Fig. 2. Game formulation: Non-cooperative game between the MNO, re-sellers and buyers.

4 PROBLEM FORMULATION

The MNO's objective is to optimize its revenue, when integrating the sharing scheme **SQM** with the existing pricing models. The MNO's decisions will be made based on the aggregate behavior of behavior of buyers and re-sellers at equilibrium. Its Revenue Maximization Problem (**RMP**) is:

$$\text{RMP : } \max_{p_o, p_r} R(p_r, p_o) \quad (5)$$

where

$$R(p_r, p_o) = N \int_0^1 p_o \mathbb{1}_{\{\pi_i^{b=o}(u_i) > \max(\pi_i^{b=s}(u_i), 0)\}} du_i + N \int_0^1 p_r \delta \mathbb{1}_{\{\pi_i^{b=s}(u_i) > \max(\pi_i^{b=o}(u_i), 0)\}} du_i. \quad (6)$$

In Eq. (6), N is the number of users, and p_o and p_r are the price per unit resource at the on-demand pool and the sharing pool, respectively. Hence, the first term on the right hand side is the MNO's revenue attained from the on-demand pool, where $\mathbb{1}_{\{\pi_i^{b=o}(u_i) > \max(\pi_i^{b=s}(u_i), 0)\}}$ is the indicator function indicating if buyer i gains a higher payoff from choosing the on-demand pool, over the sharing pool and not buying at all. The second term is the MNO's revenue from the sharing pool, where $\mathbb{1}_{\{\pi_i^{b=s}(u_i) > \max(\pi_i^{b=o}(u_i), 0)\}}$ is the indicator function indicating if buyer i gains a higher payoff from choosing the sharing pool, over the on-demand pool and not buying at all.

To analyse the strategic behaviours and the dynamic interaction amongst the three parties and to obtain equilibrium insights, we formulate a non-cooperative Stackelberg Game [41] (Fig. 2). In Stage 1, the MNO (the leader) sets the on-demand and sharing pool prices p_o and p_r , to maximize its revenue $R(p_r, p_o)$, as in Eq. (7). In Stage 2A, given p_r and δ , the re-sellers (followers) individually decide s , whether they would like to share or not. A re-seller's decision depends on which choice maximizes their individual payoff π_i^s , following Eq. (8). The collective decision of all re-sellers would result in a supply/quality of $q_s(p_r)$ for the sharing pool, which would influence the decisions of the buyers in Stage 2B. In Stage 2B, given the prices p_r and p_o , and the supply/qualities $q_s(p_r)$ and q_o , the buyers (followers) individually decide b , which pool they would buy from. A buyer's decision depends on which choice maximizes their individual payoff π_i^b , as presented in Eq. (9). The decisions of the followers (the re-sellers and buyers) would in turn affect the revenue of the leader MNO.

$$\text{Stage 1 : MNO (leader): } (p_r^*, p_o^*) = \underset{p_r, p_o}{\operatorname{argmax}} R(p_r, p_o) \quad (7)$$

$$\text{Stage 2A : Re-sellers: } s^* = \underset{s \in \{s, n\}}{\operatorname{argmax}} \pi_i^s(g_i) \quad (8)$$

$$\text{Stage 2B : Buyers: } b^* = \underset{b \in \{o, s, n\}}{\operatorname{argmax}} \pi_i^b(u_i) \quad (9)$$

5 GAME ANALYSIS

In this section, we solve the Stackleberg Game (Eqs. (7), (8) and (9)), characterizing and presenting insights on the equilibrium interaction amongst the MNO, buyers, and re-sellers, and characterizing the MNO's optimal prices. We analyze two distinct platform designs which the MNO could implement for the re-selling portion of the sharing platform. Firstly, in Section 5.1, we explore the option where the MNO implements automatic re-selling of unused resource quota on the part of users. While this may not be as feasible in practice as it involves all users knowing upfront whether or not they have excess quota, nevertheless, this portion of the study gives insights on buyer decisions and the aggregate outcome at equilibrium in light of MNO prices, by isolating the MNO-buyer interaction. In Section 5.2 we explore the option where the MNO implements manual decision making for re-sellers: re-sellers weigh their personal inconvenience cost g_i when deciding whether or not to re-sell quota in the sharing pool, according to Eq. (1).

5.1 Case 1: Automatic Re-Selling of Unused Quota

In this subsection we analyze the scenario where the MNO implements automatic re-selling of unused resource quota on the part of re-sellers.

As there is no decision making on the part of re-sellers, we have $q_s(p_r)$ as a constant q_s . The resulting game between the MNO and the buyers is:

$$\text{Stage 1 : MNO decides } (p_r^*, p_o^*) = \underset{p_r, p_o}{\operatorname{argmax}} R(p_r, p_o) \quad (10)$$

$$\text{Stage 2 : Buyers decide } b^* = \underset{b \in \{o, s, n\}}{\operatorname{argmax}} \pi_i^b(u_i) \quad (11)$$

5.1.1 The buyer's optimal strategies

We solve this game by backward induction [41], solving Stage 2 first, and using the equilibrium outcome of Stage 2 to solve Stage 1 and obtain the MNO's optimal prices.

Buyers try to maximize their payoffs π_i^b by choosing whether they will buy computing resource quota from the on-demand pool, the sharing pool, or not buy at all, given the prices p_o , p_r , and the relative supply q_o and q_s . Proposition 1 shows which strategies buyer i will take, given its willingness to pay u_i .

Proposition 1. For given values of prices (p_o, p_r) and quality/supply (q_o, q_s) , the payoff maximizing strategy for buyer i will be:

a) if $q_s > q_o$,

$$b^* = \begin{cases} \text{sharing pool} & \text{if } u_i > \max(\frac{p_r - p_o}{q_s - q_o}, \frac{p_r}{q_s}) \\ \text{on-demand pool} & \text{if } \frac{p_o}{q_o} < u_i \leq \frac{p_o - p_r}{q_o - q_s} \\ \text{none} & \text{otherwise} \end{cases} \quad (12)$$

b) if $q_o > q_s$,

$$b^* = \begin{cases} \text{sharing pool} & \text{if } \frac{p_r}{q_s} < u_i \leq \frac{p_r - p_o}{q_s - q_o} \\ \text{on-demand pool} & \text{if } u_i > \max(\frac{p_o - p_r}{q_o - q_s}, \frac{p_o}{q_o}) \\ \text{none} & \text{otherwise} \end{cases} \quad (13)$$

c) if $q_o = q_s$,

$$b^* = \begin{cases} \text{sharing pool} & \text{if } p_o > p_r \text{ \& } u_i > \frac{p_r}{q_s} \\ \text{on-demand pool} & \text{if } p_r > p_o \text{ \& } u_i > \frac{p_o}{q_o} \\ \text{none} & \text{otherwise} \end{cases} \quad (14)$$

Proof: We compare $\pi_i^o(\mu_i)$, $\pi_i^s(\mu_i)$, and 0, and find the conditions under which each strategy maximizes buyer i 's payoff. When $\pi_i^o(\mu_i)$ is the largest, we have $b = \text{on-demand pool}$. When $\pi_i^s(\mu_i)$ is the largest, we have $b = \text{sharing pool}$. Otherwise, $b = \text{none}$. $\square$

Note that Prop. 1 is independent of the distribution of u_i , users' willingness to pay. Fig. 3 provides a geometric description of Proposition 1 (Prop. 1a in particular). For example, if $q_s > q_o$ (corresponding to Prop. 1a), and when u_i satisfies $\frac{p_o}{q_o} < u_i \leq \frac{p_r - p_o}{q_s - q_o}$, we can see from Fig. 3a that the buyer's payoff from the on-demand pool would be positive and higher than that from the sharing pool. Hence, the buyer with u_i satisfying $\frac{p_o}{q_o} < u_i \leq \frac{p_r - p_o}{q_s - q_o}$ would choose the on-demand pool. Proposition 1 shows that buyers with higher willingness to pay u_i will choose the option with higher quality/supply (q_o or q_s which is the gradient of all the payoff graphs in Fig. 3), as we can see in all three graphs. For an example, we look at the case where $q_s > q_o$ in Fig. 3. Users with higher u_i choose the sharing pool when $p_r - p_o \leq 0$ (Fig. 3c), and also when $0 \leq \frac{p_r - p_o}{q_s - q_o} \leq 1$ (Fig. 3a). Nevertheless, we see that when the price p_r for the sharing pool is too high such that $\frac{p_r - p_o}{q_s - q_o} > 1$ (Fig. 3b, where $\frac{p_r - p_o}{q_s - q_o}$ is the intersection of the two graphs), none of the users will choose the sharing pool even with $q_s > q_o$.

In the special case where $q_s = q_o$ (Proposition 1c), the qualities/supply of the two options are the same. Geometrically, this means that the gradients of the two payoffs will be the same. If the on-demand pool has a higher price ($p_o > p_r$), the buyers will choose the sharing pool, and vice versa. When the buyers's willingness to pay is low ($u_i < \min(\frac{p_r}{q_s}, \frac{p_o}{q_o})$), the buyer will choose neither.

5.1.2 The MNO's price selection strategy

Following the buyers' optimal selection strategies (Proposition 1), and integrating over the collective behavior of all of the buyers, there will be 4 regions at equilibrium, with boundary conditions as functions of q_o , q_s , p_o and p_r . These 4 regions correspond to the 4 scenarios of whether users will choose the sharing and/or on-demand option, or choose not to buy at all. In Proposition 2, we characterize these 4 regions.

Proposition 2. For given values of prices (p_r, p_o) and quality/supply (q_s, q_o) , the equilibrium will satisfy exactly one of the following 4 conditions:

$$R1 : \{(p_r, p_o) | p_r < \min(\frac{q_s}{q_o}, 1)p_o + (q_s - q_o)^+, p_r > \max(\frac{q_s}{q_o}, 1)p_o - (q_o - q_s)^+\} \quad (15)$$

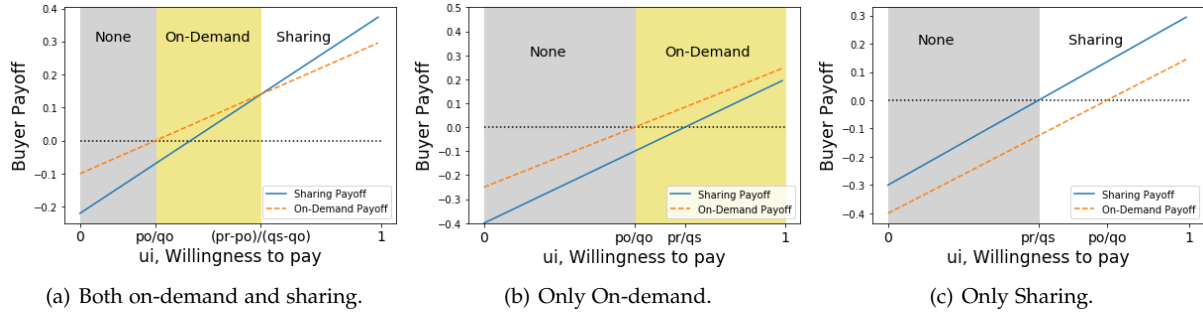


Fig. 3. Illustrations of buyers' payoff, and therefore the decision they make, at different willingness to pay, with $q_s > q_o$.

$$R2 : \{(p_r, p_o) | p_o < q_o, p_r \geq \min(\frac{q_s}{q_o}, 1)p_o + (q_s - q_o)^+\} \quad (16)$$

$$R3 : \{(p_r, p_o) | p_r < q_s, p_r \leq \max(\frac{q_s}{q_o}, 1)p_o - (q_o - q_s)^+\} \quad (17)$$

$$R4 : \{(p_r, p_o) | p_r \geq q_s, p_o \geq q_o\}, \quad (18)$$

where $(a)^+ = a$ if $a \geq 0$, and 0 otherwise. In R1, buyers buy from both pools or neither, like in Fig. 3a. In R2, buyers buy from the on-demand pool or from neither, as in Fig. 3b. In R3, buyers buy from the sharing pool or from neither, as in Fig. 3c. In R4, buyers will buy from neither pool.

Proof: The boundary conditions of the four regions are deduced through Fig. 3. $\square$

Prop. 2 is independent of the distribution of u_i , users' willingness to pay. This proposition indicates how the relative magnitudes of the prices p_r , p_o , and supply q_s and q_o impact the equilibrium outcome. Specifically, when (p_r, p_o, q_s, q_o) satisfies the boundary conditions of one of the regions, the collective behavior of buyers (who behave according to Proposition 1), will result in the equilibrium outcome being in that region. For example, when the prices p_o and p_r satisfy the conditions $p_o < q_o$, $p_r \geq \min(\frac{q_s}{q_o}, 1)p_o + (q_s - q_o)^+$, it will not be incentive compatible for any buyers to choose the sharing pool (regardless of u_i), and the outcome is R2. The special case when $q_s = q_o$ in Proposition 1 also fits this 4 region framework. If $1 < \min(\frac{p_r}{q_s}, \frac{p_o}{q_o})$, the users will choose not to buy and the resulting scenario will be R4. In the alternative where $1 < \min(\frac{p_r}{q_s}, \frac{p_o}{q_o})$ does not hold, if the on-demand price is higher ($p_o > p_r$) the users will choose the sharing pool and we end up in R3, and vice versa for R2. This 4 region structure is unique with respect to the results from work on other sharing platforms in networked systems (mobile data re-selling [26], [28] and crowdsourced wireless community networks [29]).

With the 4 disjoint regions involving different pools being purchased from (Proposition 2), the revenue function $R(p_o, p_r)$ can be simplified as follows:

$$R^{R1}(p_o, p_r) = \begin{cases} Np_o(\frac{p_r - p_o}{q_s - q_o} - \frac{p_o}{q_o}) + Np_r\delta(1 - \frac{p_r - p_o}{q_s - q_o}), & \text{if } q_s \geq q_o \\ Np_r\delta(\frac{p_o - p_r}{q_o - q_s} - \frac{p_r}{q_s}) + Np_o(1 - \frac{p_o - p_r}{q_o - q_s}), & \text{if } q_o > q_s \end{cases} \quad (19)$$

$$R^{R2}(p_o, p_r) = Np_o(1 - \frac{p_o}{q_o}) \quad (20)$$

$$R^{R3}(p_o, p_r) = Np_r\delta(1 - \frac{p_r}{q_s}) \quad (21)$$

$$R^{R4}(p_o, p_r) = 0 \quad (22)$$

In R1, buyers are willing to buy from both the on-demand and sharing pool. The two terms in the objective $R^{R1}(p_o, p_r)$ correspond to the revenue obtained from the two pools. In R1 with $q_s \geq q_o$, $(\frac{p_r - p_o}{q_s - q_o} - \frac{p_o}{q_o})$ and $1 - \frac{p_r - p_o}{q_s - q_o}$ are the proportion of the buyers who choose the on-demand and sharing pool respectively. In R4, buyers find it optimal to not purchase from either pool, and hence the MNO's revenue is 0.

Therefore, to optimize the revenue maximization problem, we will split the problem into four subproblems as follows:

$$\max_{p_o, p_r} R^{R1}(p_o, p_r) \text{ s.t. (15)} \quad (23)$$

$$\max_{p_o, p_r} R^{R2}(p_o, p_r) \text{ s.t. (16)} \quad (24)$$

$$\max_{p_o, p_r} R^{R3}(p_o, p_r) \text{ s.t. (17)} \quad (25)$$

$$\max_{p_o, p_r} R^{R4}(p_o, p_r) \text{ s.t. (18)} \quad (26)$$

For subproblem $R^{R1}(p_o, p_r)$, we further split it into 2 subproblems $R^{R1a}(p_o, p_r)$ and $R^{R1b}(p_o, p_r)$ corresponding to the 2 cases where $q_s \geq q_o$ and $q_o > q_s$. In the following, we give Theorem 1 to characterise the optimal prices p_o and p_r .

Theorem 1. When buyers make decisions according to Proposition 1, given a fixed (q_o, q_s) , and if the condition $(\delta + 1)^2 q_s < 4\delta q_o$ holds, the MNO's optimal prices will be as follows.

- To maximize $R^{R1a}(p_o, p_r)$ in R1, we have:
If Eq. (15) is satisfied

$$(p_{o,1a}^*, p_{r,1a}^*) = \left(\frac{\delta(1 + \delta)q_o(q_s - q_o)}{4q_s\delta - (1 + \delta)^2 q_o}, \frac{q_s - q_o}{2} + \frac{(1 + \delta)^2 q_o(q_s - q_o)}{2(4q_s\delta - (1 + \delta)^2 q_o)} \right),$$

Else,

$$(p_{o,1a}^*, p_{r,1a}^*) = \arg\max_{\{(\frac{q_o}{2}, q_s - \frac{q_o}{2}), (\frac{q_o}{2}, \frac{q_s}{2})\}} R^{R1a}(p_o, p_r).$$

- To maximize $R^{R1b}(p_o, p_r)$ in R1, we have:
If Eq. (15) is satisfied,

$$(p_{o,1b}^*, p_{r,1b}^*) = \left(\frac{2\delta q_o(q_o - q_s)}{4\delta q_o - (\delta + 1)^2 q_s}, \frac{q_s(\delta + 1)(q_o - q_s)}{4\delta q_o - (\delta + 1)^2 q_s} \right),$$

Else,

$$(p_{o,1b}^*, p_{r,1b}^*) = \operatorname{argmax}_{\{(q_o - \frac{q_s}{2}, \frac{q_s}{2}), (\frac{q_o}{2}, \frac{q_s}{2})\}} R^{R1b}(p_o, p_r).$$

- To maximize $R^{R2}(p_o, p_r)$, we have $(p_{o,2}^*, p_{r,2}^*) = (\frac{q_o}{2}, a)$.
- To maximize $R^{R3}(p_o, p_r)$, we have $(p_{o,3}^*, p_{r,3}^*) = (b, \frac{q_s}{2})$.

Note that, in the latter two cases, a is any price p_r which satisfies the boundary conditions of R2 (Eq. (16)), and b is any price p_o which satisfies the boundary conditions of R3 (Eq. (17)).

Proof: See Appendix For each of the subproblems, we obtain the Hessian matrix of the objective function. The Hessians of R1a, R2 and R3 are all negative semidefinite (having eigenvalues which ≤ 0), and therefore these subproblems have concave objectives. As the constraints are linear with respect to p_o and p_r , these subproblems are convex. For the subproblem R1b, the Hessian is negative semidefinite under the condition $(\delta + 1)^2 q_s < 4\delta q_o$. With convexity, we can use the KKT conditions to find their optimal prices. $\square$

In Theorem 1, $(\delta + 1)^2 q_s < 4\delta q_o$ is the condition under which the Hessian of R1b's objective has negative eigenvalues, and hence the condition under which subproblem R1b is convex. We use the KKT conditions to solve the problem. For subproblem R1a, the gradients are 0 at $p_{o,1a}^* = \frac{2\delta q_o(q_o - q_s)}{4\delta q_o - (\delta + 1)^2 q_s}$ and $p_{r,1a}^* = \frac{q_s(\delta + 1)(q_o - q_s)}{4\delta q_o - (\delta + 1)^2 q_s}$. If this point is within the boundary conditions, it is the optimal point. If it is not, the optimal point would be at one of the boundaries (with regions R2 or R3), specifically being $(\frac{q_o}{2}, q_s - \frac{q_o}{2})$ (boundary with R2) or $(\frac{q_o}{2}, \frac{q_s}{2})$ (boundary with R3). Likewise for subproblem R1b. For subproblems R2 and R3, the optimal solutions of $p_{o,2}^* = \frac{q_o}{2}$ and $p_{r,3}^* = \frac{q_s}{2}$ hold at the boundary as well. These optimal prices agree with intuition, that as the quality/supply (likelihood of buyer getting service) q_o (or q_s) increases, the MNO can increase the price to increase its revenue.

5.2 Case 2: Re-sellers weigh inconvenience cost when deciding to re-sell quota

In this subsection, we analyse an alternative platform design in which the MNO implements manual decision making for re-sellers: re-sellers weigh their inconvenience cost g_i when deciding whether or not to re-sell quota in the sharing pool, according to Eq. (1). As mentioned in Section 3, the inconvenience cost g_i factors in the re-seller's uncertainty of usage during the timeslot, and the re-seller's inconvenience, willingness and ability towards checking the app to decide whether or not to re-sell. Here g_i follows the uniform $U[0, 1]$ distribution as it captures the varying levels of inconvenience across users. In Section 6, we perform simulations using both uniform and beta (which has a gaussian-like structure) distributions, showing similar results across distributions.

Under this scenario, we solve the Stackelberg Game (Eq. (7), (8) and (9)). Firstly, in Stage 1 the MNO decides the prices p_r and p_o . Following which, in Stage 2A the re-sellers decide whether they want to sell their excess resource quota, based on what maximizes their payoff (Eq. (1)), resulting in a quality/supply of $q_s(p_r)$. Finally, in Stage 2B, the buyers would make a decision, aiming to maximize their individual payoffs (Eq. (4)), given the values of $(p_r, p_o, q_s(p_r), q_o)$. We

solve the Stackelberg Game by backward induction, solving Stage 2 first, and using the equilibrium outcome of Stage 2 to solve Stage 1 and obtain the MNO's optimal prices.

5.2.1 The re-seller's optimal strategy

The following Proposition shows the re-seller's optimal strategy.

Proposition 3. For given p_r , re-seller i with inconvenience cost g_i 's optimal strategy would be:

$$s^* = \begin{cases} \text{sell,} & \text{if } (1 - \delta)p_r > g_i, \\ \text{no,} & \text{otherwise} \end{cases} \quad (27)$$

Collectively, by Eq. (2) this results in a proportion $(1 - \delta)p_r$ of re-sellers being willing to rent out their unused resources and therefore following Eq. (3), the total quality/supply of the sharing pool will be

$$q_s(p_r) \sim f((1 - \delta)p_r), \quad (28)$$

which is a function of price p_r .

5.2.2 The buyer's optimal strategy

Given $(q_s(p_r), q_o)$ and (p_r, p_o) , a buyer's strategy would still follow Proposition 1 in Section 5.1, with the substitution of q_s by $q_s(p_r)$. Hence, their collective strategies would form distinct regions R1 to R4 at equilibrium, where the four regions indicate whether or not the sharing and/or on-demand pools are feasible. This is according to Proposition 2 in Section 5.1. Once again, the only difference is that q_s is replaced by $q_s(p_r)$.

5.2.3 The MNO's optimal price

Given the strategies of the re-sellers and buyers in Stages 2A and 2B, corresponding to the regions in Proposition 2, the MNO's revenue maximizing problem $R(p_o, p_r)$ can be split into the following subproblems.

$$R_{C2}^{R1a}(p_o, p_r):$$

$$\begin{aligned} \max_{p_r, p_o} & Np_o \left(\frac{p_r - p_o}{q_s(p_r) - q_o} - \frac{p_o}{q_o} \right) + Np_r \delta \left(1 - \frac{p_r - p_o}{q_s(p_r) - q_o} \right) \\ \text{s.t. (15), } & q_s(p_r) > q_o \end{aligned} \quad (29)$$

$$R_{C2}^{R1b}(p_o, p_r):$$

$$\begin{aligned} \max_{p_r, p_o} & Np_r \delta \left(\frac{p_o - p_r}{q_o - q_s(p_r)} - \frac{p_r}{q_s(p_r)} \right) + Np_o \left(1 - \frac{p_o - p_r}{q_o - q_s(p_r)} \right) \\ \text{s.t. (15), } & q_o > q_s(p_r) \end{aligned} \quad (30)$$

$$\begin{aligned} R_{C2}^{R2a}(p_o, p_r) : & \max_{p_r, p_o} Np_o \left(1 - \frac{p_o}{q_o} \right) \\ \text{s.t. (16), } & q_s(p_r) > q_o, \end{aligned} \quad (31)$$

$$\begin{aligned} R_{C2}^{R2b}(p_o, p_r) : & \max_{p_r, p_o} Np_o \left(1 - \frac{p_o}{q_o} \right) \\ \text{s.t. (16), } & q_o > q_s(p_r), \end{aligned} \quad (32)$$

$$\begin{aligned} R_{C2}^{R3a}(p_o, p_r) : & \max_{p_r, p_o} Np_r \delta \left(1 - \frac{p_r}{q_s(p_r)} \right) \\ \text{s.t. (17), } & q_s(p_r) > q_o, \end{aligned} \quad (33)$$

$$\begin{aligned} R_{C2}^{R3b}(p_o, p_r) : & \max_{p_r, p_o} Np_r \delta \left(1 - \frac{p_r}{q_s(p_r)} \right) \\ \text{s.t. (17), } & q_o > q_s(p_r), \end{aligned} \quad (34)$$

and $R_{C2}^{R4}(p_o, p_r)$ is a constant of 0, corresponding to R4 in Proposition 2 (where users choose neither the on-demand nor sharing pool). Other than R4, the other regions are considered with two subproblems which correspond to the disjoint cases $q_s(p_r) \geq q_o$ and $q_o > q_s(p_r)$. This allows us to replace the terms involving $\min()$ and $()^+$. For example, $R_{C2}^{R1a}(p_o, p_r)$ and $R_{C2}^{R1b}(p_o, p_r)$ both correspond to the situation when we are in R1 (where at equilibrium, buyers choose both to buy from the on-demand pool and the sharing pool). They differ depending on whether $q_s(p_r) \geq q_o$ or not.

The subproblems are not jointly convex with respect to p_o and p_r . Hence, with p_r and $q_s(p_r)$ given, we optimize with respect to p_o to attain insights on the MNO's optimal price in R1 and R2. The MNO's revenues in R3 and R4 are not related to p_o (since no users buy from the on-demand pool) and are constant with p_r and $q_s(p_r)$ given.

Theorem 2. *Given $(p_r, q_s(p_r))$, the MNO's optimal on-demand price p_o^* at each region is given by:*

- In R1 with $q_s(p_r) \geq q_o$,

$$p_{o,1a}^* = \frac{q_o p_r (1 + \delta)}{2q_s(p_r, \delta)} \text{ or } \operatorname{argmax}_{\{B_1, B_2\}} R_{C2}^{R1a}(p_o, p_r)$$

- In R1 with $q_s(p_r) < q_o$,

$$p_{o,1b}^* = \frac{p_r(\delta + 1) + q_o - q_s(p_r, \delta)}{2} \\ \text{or } \operatorname{argmax}_{\{B_1, B_2\}} R_{C2}^{R1b}(p_o, p_r)$$

- In R2 with $q_s(p_r) \geq q_o$, $p_{o,2a}^* = \frac{q_o}{2}$ or B_1

- In R2 with $q_s(p_r) < q_o$, $p_{o,2b}^* = \frac{q_o}{2}$ or B_1

where $B_1 = p_r - q_s(p_r) + q_o$ and $B_2 = \frac{p_r q_o}{q_s(p_r)}$ are points on the region boundaries.

Proof: For subproblem R_{C2}^{R1a} , the second derivative of the objective function with respect to p_o is $\frac{-2Nq_s(p_r)}{(q_s(p_r) - q_o)q_o} < 0$ since $q_s(p_r) \geq q_o$ and $q_o > 0$, indicating that the objective function is concave. As the constraints involve linear functions of p_o , the maximization problem is convex with respect to p_o . Therefore, we convert the constraints to non-strict inequalities and use the KKT conditions to solve R_{C2}^{R1a} , with respect to p_o . The Lagrangian is defined as $L(p_o, \lambda_1, \lambda_2) = Np_o(\frac{p_r - p_o}{q_s(p_r) - q_o} - \frac{p_o}{q_o}) + Np_r\delta(1 - \frac{p_r - p_o}{q_s(p_r) - q_o}) + \lambda_1(p_r - p_o - q_s(p_r) + q_o) + \lambda_2(q_o(p_r) - p_r)$, with λ_1 and λ_2 being the dual variables corresponding to the constraints. If the dual variables λ_1 (or λ_2) were greater than 0, the optimal point would lie on the boundary between region R1 and R2 (or R3 respectively). If the dual variables are 0, the optimal point will be the point at which the gradient of the Lagrangian is 0. We solve the rest of the subproblems similarly. $\square$

Theorem 2 gives us insights on how the MNO should set the on-demand price p_o , given that the other parameters p_r and $q_s(p_r)$ are fixed. In R1, when users buy from both the sharing and on-demand pool, the price p_o at which the gradient of the revenue is 0 is $\frac{q_o p_r (1 + \delta)}{2q_s(p_r)}$ for R1a and $\frac{p_r(\delta + 1) + q_o - q_s(p_r)}{2}$ for R1b. We separately analyse the cases when $\frac{q_o p_r (1 + \delta)}{2q_s(p_r)}$ (the point at which the gradient of the revenue is 0) lies within the region's boundaries, and

when it does not. Case 1: If $p_o^* = \frac{q_o p_r (1 + \delta)}{2q_s(p_r)}$ lies within R1a's boundaries ($p_r < p_o + q_s(p_r) - q_o$, $p_r > \frac{q_s(p_r)}{q_o} p_o$), it is the optimal price, as the problem is convex with respect to p_o . Likewise for $\frac{p_r(\delta + 1) + q_o - q_s(p_r)}{2}$ in R1b. These functions indicate that the optimal price p_o^* increases with respect to the on-demand expected quality (supply) q_o and platform commission δ . As the on-demand supply q_o increases, the increase in on-demand price p_o would not deter users away from the on-demand pool. Likewise, when the MNO's commission δ increases, re-sellers will be deterred from selling their unused resource quota, resulting in a smaller supply at the sharing-pool. With a potentially lower payoff at the sharing pool, the increase in the on-demand price p_o would not deter users away from the on-demand pool. At the same time, as the sharing supply $q_s(p_r)$ increases, the sharing pool is seen as more attractive to users (due to the potentially higher individual payoff). In light of this, the optimal on-demand price p_o^* has to decrease. Case 2: If $\frac{q_o p_r (1 + \delta)}{2q_s(p_r)}$ (and $\frac{p_r(\delta + 1) + q_o - q_s(p_r)}{2}$) do not lie within R1a's (respectively R1b's) boundaries, the optimal points $p_{o,1a}^*$ (and $p_{o,1b}^*$) will lie on the region's boundaries, since the problems are convex. Specifically, the optimal prices will be $\operatorname{argmax}_{\{B_1, B_2\}} R_{C2}^{R1a}(p_o, p_r)$ (and $\operatorname{argmax}_{\{B_1, B_2\}} R_{C2}^{R1b}(p_o, p_r)$).

In R2, where buyers only choose the on-demand pool, the price p_o at which the gradient of the revenue is 0 is $\frac{q_o}{2}$. We separately analyse the cases when $\frac{q_o}{2}$ (the point at which the gradient of the revenue is 0) lies within the region's boundaries, and when it does not. Case 1: If $p_o^* = \frac{q_o}{2}$ lies within R2's boundary ($p_r \geq p_o + (q_s(p_r) - q_o)$ for $R_{C2}^{R2a}(p_o, p_r)$ and $p_r \geq \frac{q_s}{q_o} p_o$ for $R_{C2}^{R2b}(p_o, p_r)$ respectively), it is the optimal on-demand price because the problem is convex with respect to p_o . Therefore, the optimal on-demand price is an increasing function of the on-demand pool's supply q_o . This indicates that as the supply of the on-demand pool q_o increases, the rise in the on-demand price will not deter users away from the on-demand pool. Case 2: If $p_o^* = \frac{q_o}{2}$ does not lie within region R2's boundaries, the optimal point will lie on the boundary $p_r = B_1$ for $R_{C2}^{R2a}(p_o, p_r)$ and $p_r = B_2$ for $R_{C2}^{R2b}(p_o, p_r)$, respectively. This is because, the optimization problems are convex with respect to p_o .

On this basis, we then focus on the joint optimization of p_o and p_r for the MNO's optimal revenue, which will be the maximum of the revenues across the four regions. According to Corollary 1, the MNO's optimal revenue is calculated as

$$\max_{p_r} R_{\max}(p_r). \quad (35)$$

Corollary 1. *Given $(p_r, q_s(p_r))$, the MNO's optimal revenue will be*

$$R_{\max}(p_r) = \max \left\{ R_{C2}^{R1a}(p_{o,1a}^*, p_r), R_{C2}^{R1b}(p_{o,1b}^*, p_r), R_{C2}^{R2a}(p_{o,2a}^*, p_r), R_{C2}^{R2b}(p_{o,2b}^*, p_r), R_{C2}^{R3}(p_r), 0 \right\}, \quad (36)$$

where $R_{C2}^{R3}(p_r) = Np_r\delta(1 - \frac{p_r}{q_s(p_r)})$ is the revenue in R3 (the region where the buyers only choose the sharing pool), if the boundary conditions of R3 (Eq. (17)) were met. And 0 is the revenue in R4 (the region where the buyers choose neither the

sharing pool nor the on-demand pool), if the boundary conditions of R4 (Eq. (18)) were met.

Since the optimal on-demand price p_o^* in $R_{\max}(p_r)$ is obtained in Theorem 2, the optimization in (35) can be addressed by using a linear search across p_r . In Section 6, we discuss the joint optimization of p_o and p_r and the resulting insights at the optimal and equilibrium point.

6 NUMERICAL SIMULATIONS

In this section we provide numerical results on the optimal and equilibrium outcome of the Stackelberg Game (Eq. (7), (8) and (9)). These results give us insights and show the practical implications of integrating the sharing pool with the On-Demand Pricing and Resource Reservation model. In many of our experiments, for each combination of (on-demand supply q_o , commission δ , and normalization parameter a), we optimize over the prices (p_o, p_r) , obtaining the optimal revenue, and hence optimal region. In Section 7, we perform preliminary user studies on a prototype, to evaluate the extent of which user decisions differ from our decision model.

Firstly, we define a specific form of $q_s(p_r)$, which indicates the relationship between the price p_r and the resulting quality/supply $q_s(p_r)$, as follows:

$$q_s(p_r) = a \log(1 + (1 - \delta)p_r). \quad (37)$$

We use a log function, which models the law of diminishing marginal returns [42]. The relative supply/quality (i.e. likelihood of buyers getting their jobs served) of the sharing pool increases as the proportion of re-sellers willing to sell $(1 - \delta)p_r$ increases. The parameter a is the normalization parameter, normalizing the function of the proportion of re-sellers $\log(1 + (1 - \delta)p_r)$ willing to re-sell to a magnitude such that the supply level $q_s(p_r)$ is comparable with the supply at the on-demand pool q_o . For our numerical simulations, we use Eq. (37) as the specific form of $q_s(p_r)$ and set the number of buyers and the number of re-sellers (N) to be 50. We perform experiments with the willingness to pay u_i and the willingness to share g_i following the Uniform[0,1] as well as the Beta(2,2) distribution. Under the Uniform[0,1] distribution, the users' types u_i and g_i are evenly spread out over the range $[0, 1]$. Under the Beta(2,2) distribution, the users' types follow an inverse parabolic structure, with more users whose types take values towards the center of the range $[0, 1]$ and fewer users taking values towards the edge of the range $[0, 1]$, somewhat similar to a Gaussian distribution. We also model heterogeneous user usage levels, both on the buyer side and the re-seller side, with usage levels following the uniform[0,1] distribution.

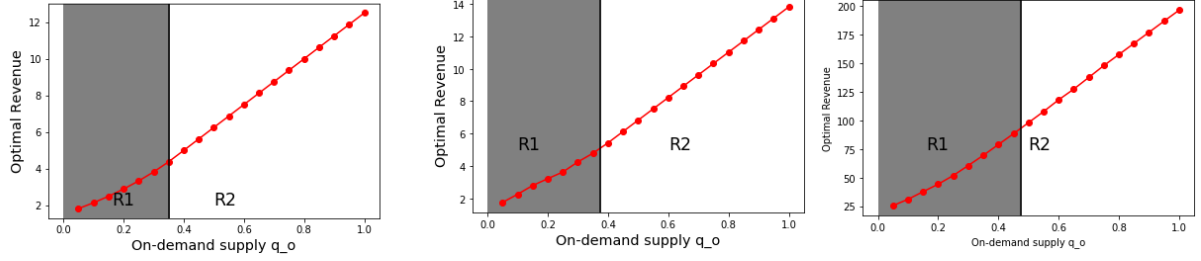
Impact of the on-demand supply q_o on the optimal revenue and region : Firstly, we show how the MNO's optimal revenue and region vary with respect to the on-demand quality/supply q_o in Fig. 4a, 4b and 4c where we set $\delta = 0.2$ and $a = 2$. Given that we do not know what is an appropriate normalization parameter which relates the supply function $q_s(p_r)$ to the supply of the on-demand pool q_o , we set $a = 2$ first, and later decrease the normalization parameter to $a = 1.5$. To obtain the optimal revenue and region, we optimize over the prices (p_o, p_r) , given each

combination of (on-demand supply q_o , commission δ , and normalization parameter a). For both cases where the re-sellers' willingness to share and the buyers' willingness to pay follow uniform (Fig. 4a) and beta distribution (Fig. 4b), and when the users' usage levels are heterogeneous and follow the Uniform[0,1] distribution (Fig. 4c), it can be seen that for smaller values of q_o , i.e., when the on-demand supply is lower, it is more beneficial to supplement buyers with the sharing pool to buy quota for job computation (Region R1). This additional income adds to the revenue of the MNO. Nevertheless, as q_o increases, the additional income from the sharing pool does not outweigh the increase in revenue from the on-demand pool $N_{\text{on-demand}} p_o (1 - \frac{p_o}{q_o})$, hence Region R2 is optimal.

Impact of the commission δ on the optimal revenue and region : In Fig. 5a, we plot the optimal revenue for the MNO against q_o , for different values of commission δ . Conversely, in Fig. 5b, we plot the optimal revenue of the MNO against the commission δ , for different values of q_o . In Figs. 5c, d and e, we show the optimal region at equilibrium for different values of δ , (with g_i and u_i taking values from the Uniform[0,1] and Beta(2,2) distributions, and when the users' usage levels are heterogeneous following Uniform[0,1] respectively). Once again, to obtain the optimal revenue and region, we optimize over the prices (p_o, p_r) , given each combination of (on-demand supply q_o , commission δ , and normalization parameter a). These graphs show that when the commission δ is smallest, it is not profitable to have sharing (hence the optimal region is R2), because the platform does not get much revenue from sharing due to its commission being small. As δ gets larger, it becomes profitable to have the sharing platform (Region R1) due to the increase in commission for the platform and hence revenue $N_{\text{on-demand}} p_o (\frac{p_r - p_o}{q_s(p_r) - q_o} - \frac{p_o}{q_o}) + N_{\text{share}} p_r \delta (1 - \frac{p_r - p_o}{q_s(p_r) - q_o})$. On the other hand, for larger δ , the re-sellers will be less likely to choose to re-sell their unused resource quota. This is seen in Eq. (1), as their payoffs are inversely proportional with respect to δ , and in $q_s(p_r) = a \log(1 + (1 - \delta)p_r)$ (Eq. (37)) where the sharing pool's supply q_s is inversely proportional to δ . This results in a lower quality/supply sharing pool, and therefore a smaller revenue which the MNO can obtain from sharing, making Region R2 (purely on-demand) optimal at high δ . To summarize, it would have been expected that a high commission would give the platform a higher revenue, nevertheless our results show that it incurred a trade-off in dissuading reservation plan users from re-selling, hence decreasing the supply and revenue.

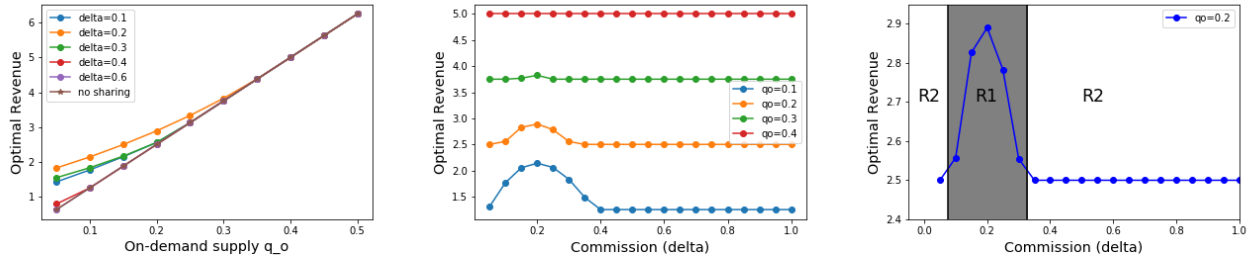
Impact of the on-demand price p_o on the revenue and region : In Fig. 6a, we show how the revenue varies as the on-demand price p_o increases, for given values of $q_o, \delta = 0.2, a = 2, p_r = 0.5$. As the price increases, the equilibrium region shifts from R2 (purely on-demand) to R1 (both on-demand and sharing) as more buyers choose the sharing platform due to the on-demand price increasing. As the on-demand price increases even further, the region transitions to R3 (purely sharing) due to the on-demand price being too high. Nevertheless, R3 is not optimal for the platform because it only collects a proportion δ of the amount transacted.

Impact of the normalization parameter a on the optimal revenue and region : Given that it is not clear what is an ap-

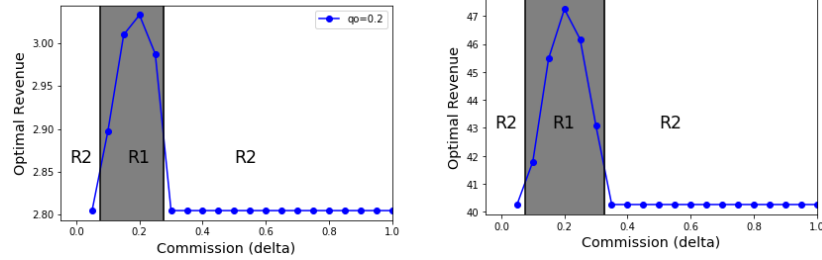


(a) The MNO's optimal revenue and region against q_o . $\delta = 0.2, a = 2$, with u_i, g_i following the uniform $U(0, 1)$ distribution. (b) The MNO's optimal revenue and region against q_o . $\delta = 0.2, a = 2$, with u_i, g_i following beta(2,2) distribution. (c) The MNO's optimal revenue and region against q_o . $\delta = 0.2, a = 2$, with u_i, g_i , and usage levels following the uniform $U(0, 1)$ distribution.

Fig. 4. When on-demand supply is low, the optimal revenue is at Region 1 (offering both a sharing and an on-demand pool). As the on-demand supply increases, the optimal revenue transitions to Region 2 (purely on-demand).



(a) The MNO's optimal revenue against on-demand supply q_o , $a = 2$. (b) The MNO's optimal revenue against commission δ , $a = 2$. (c) The MNO's optimal revenue and region against δ . $q_o = 0.2, a = 2$, with u_i, g_i following the uniform $U(0, 1)$ distribution.



(d) The MNO's optimal revenue and region against δ . $q_o = 0.2, a = 2$, with u_i, g_i following the beta(2,2) distribution. (e) The MNO's optimal revenue and region against δ . $q_o = 0.2, a = 2$, with u_i, g_i , and usage levels following the uniform $(0, 1)$ distribution.

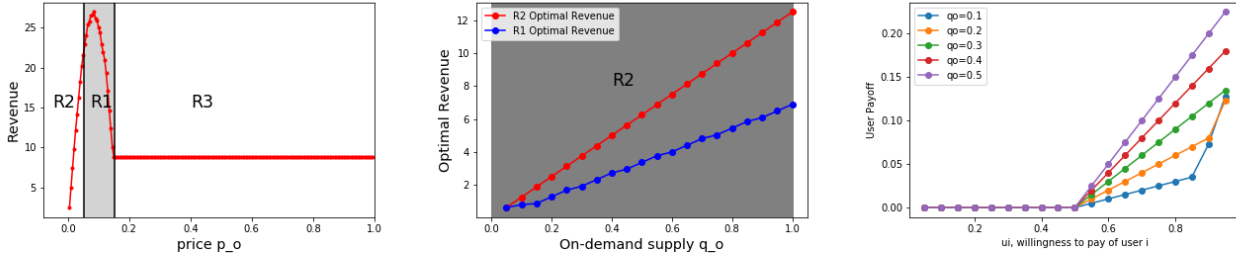
Fig. 5. When the commission level δ is lower, the optimal revenue is at Region 1 (offering both a sharing and an on-demand pool). As the commission level increases, the optimal revenue transitions to Region 2 (purely on-demand).

proprate normalization parameter which relates the supply function $q_s(p_r)$ to the supply of the on-demand pool q_o , we explore different normalization parameters. In contrast to Fig. 5a, we set a smaller normalization parameter $a = 1.5$ for Fig. 6b. Once again, to obtain the optimal revenue and region, we optimize over the prices (p_o, p_r) , given each combination of (on-demand supply q_o , commission δ , and normalization parameter a). With a smaller a , a higher price p_r comes with a lower quality/supply $q_s(p_r)$ relative to q_o and therefore a lower payoff for buyers at the sharing pool. Hence, buyers have lower incentive to buy resources from the sharing pool. That is why region R2 (purely on-demand) gives the optimal revenue for all values of q_o in

Fig. 6b. Nevertheless, although R1's optimal revenue is not the optimal over all regions under this set of parameters, users will still buy from the sharing pool and its revenue is non-zero, as seen in Fig. 6b.

Analyzing individual buyer payoffs : Finally, we plot individual buyer payoffs (Eq. (4)) at equilibrium conditions in Fig. 6c. As we can see, buyers with lower willingness to pay u_i choose neither pool. As their willingness to pay (utility of the job computation) increases, their payoff increases. We can see that for the $q_o = 0.1$ and $q_o = 0.2$ plots, users with high willingness to pay choose the sharing pool, as visualised through the different gradients.

In summary, our simulation results show that under the combination of low δ , low q_o , and large a , having the



(a) The MNO's revenue and region against on-demand price p_o . $q_o, \delta = 0.2, a = 2, p_r = 0.5$, with u_i, g_i following the beta(2,2) distribution. (b) The MNO's optimal revenue and region against on-demand supply q_o , $\delta = 0.2, a = 1.5$. (c) The individual buyer's payoff against willingness to pay u_i , $\delta = 0.2, a = 2$.

Fig. 6. Further Numerical Results and Insights: The impact of the on-demand price and normalization parameter on revenue, and how the individual buyer's payoff varies with the willingness to pay.

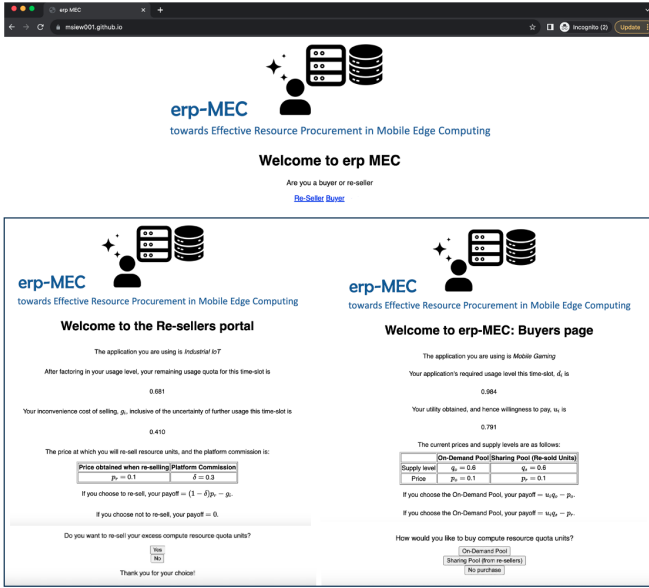


Fig. 7. Screenshots of our prototype - an interactive website, which can capture user decision making given different parameter settings, for user studies. Its url is <https://shikhsh.github.io/erpMEC/>.

sharing platform (Region R1) is optimal. Besides, it is not optimal to have Region R3 (purely sharing), due to the MNO only being able to take a proportion δ of the revenue $N_{\text{share}} p_r \delta (1 - \frac{p_r}{q_s(p_r)})$.

7 PROTOTYPE AND PRELIMINARY USER STUDIES

7.1 Prototype Description

We created a prototype, an interactive website (see Fig. 7 for screenshots), which is able to capture user decisions given the different parameters. The interactive website's url is <https://shikhsh.github.io/erpMEC/>. In Fig. 8 we illustrate how the real-world version of our prototype integrates with the rest of the MEC system.

In our prototype, when users enter the website, they will be given a choice to select whether they are a buyer or re-seller. In the re-sellers portal, the user is shown 1) their specific edge computing application, randomly sampled from the list ['IoT Sensing Analytics', 'Distributed AI', 'Predictive Maintenance', 'Remote Monitoring', 'Industrial

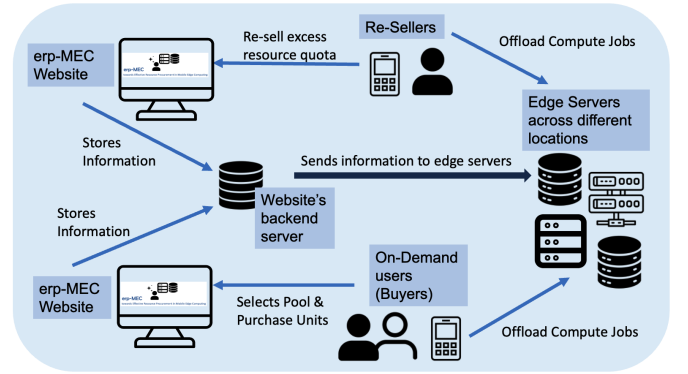


Fig. 8. How the real-world version of our prototype is integrated with the rest of the MEC system.

IoT', 'Augmented Reality Display'], 2) their remaining usage quota this time-slot, after factoring in their planned usage level, and 3) their inconvenience cost of re-selling, in light of potential further usage this time-slot. Note that the inconvenience cost also includes the cost of opening the website or app. 4) The re-selling price and platform commission, 5) the payoff equations under the different options, and finally 6) the **option buttons "re-sell" and "no", for the user to choose from**. Parameters 1, 2, and 3 are **user specific parameters**, and are uniformly randomly generated upon each user's entrance for our prototype. When the number of users who use our prototype in user studies is large, we can compare the empirical distribution of user decisions to that of our model. The parameters in 4 are fixed for all users, and can be varied across user studies to view how users make decisions, and the aggregate outcome, under different settings. In practice, the information shown such as the type of application and the inconvenience cost (given potential further usage that time-slot) can be pulled from the user's usage history. The information on the user's remaining resource quota comes from the website's backend calculations.

In the buyers' portal, the user is shown 1) their specific edge computing application, randomly sampled from the list ['IoT Sensing Analytics', 'Mobile Gaming', 'Augmented Reality', 'Data Analytics on Device', 'Smart Home IoT Analytics'] 2) their usage demand this time-slot, 3) their utility received, and hence their willingness to pay, 4) the supply

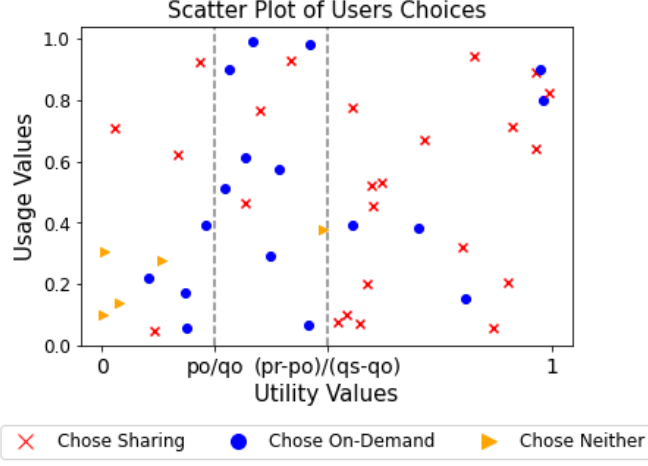


Fig. 9. Results of preliminary user study: The scatter points indicate the user choices given the utility and usage values they were shown on the website. The dotted lines indicate the region boundaries from the analytical model in the paper.

and prices under the sharing pool and re-selling pool, 5) the payoff equations under the different options, and 6) the option buttons “On-Demand Pool”, “Sharing Pool”, “No purchase”, for the user to choose from. As per the re-seller’s portal, parameters 1, 2, and 3 are **user specific parameters**, and are uniformly randomly generated upon each user’s entrance for our prototype. When the number of users who use our prototype in user studies is large, this models the equilibrium and aggregate outcome of heterogeneous users decision making. The parameters in 4 are fixed, and can be varied across user studies to view how users make decisions, and the aggregate outcome, under different settings. In practice, the information shown such as the type of application can be pulled from the user’s usage history. The information on the user’s remaining resource quota comes from the website’s back-end calculations. In practice, the users’ willingness to pay will not be shown on the website, but is inherent in users’ decision making.

Our prototype **stores a string of information upon each user click, on our back-end server** (Fig 8). For re-sellers, the string contains [remaining usage level for the time-slot, inconvenience cost/ uncertainty of usage g_i , re-selling price p_r , commission level δ , type of application, Y or N indicating the **user’s choice of whether or not to re-sell**, the time stamp]. For buyers, the string contains [the user’s usage requirement, the utility/ willingness to pay u_i , the on-demand supply q_o , the sharing supply q_s , the on-demand price p_o , the sharing price p_r , **the user’s choice (Sharing, On-demand, No purchase)**, the time stamp]. Our prototype stores these information, without storing the identity or any other information regarding the user. All clicks are anonymous. In practice, the back-end server will relay the decisions of the users to the edge. Based on this, the resource units at edge servers will be correspondingly allocated to the different users, by the computer at the edge server (Fig 8).

7.2 Preliminary User Study

The goal of the user study is to verify how accurately our decision model (Eq. 1 for re-sellers, and Prop. 1-2, Fig 3a for buyers) models the way users make decisions. We will

evaluate how similar the empirical distribution function of the *data from the user study*, and the empirical distribution function of the *data from the model’s predictions*, are.

For this preliminary user study, we set the sharing supply available to be slightly higher than the on-demand supply ($q_o = 0.6, q_s = 0.7$), and the sharing price to be slightly higher than the on-demand price ($p_o = 0.15, p_r = 0.2$). For our participants, we have had 44 participants be buyers, and 28 participants be re-sellers. Our results are in Fig. 9, Tables 3 and 3. Fig 9 is a scatter plot of buyers clicks, given the usage and utility values they were shown on the website. Under our paper’s math model (Proposition 2/ Fig 3a of paper), buyers will choose the sharing pool when their utility value $\frac{pr-po}{qs-qo} < u_i < 1$, will chose the on-demand pool when $\frac{po}{qo} < u_i < \frac{pr-po}{qs-qo}$, and will choose neither when $u_i < \frac{po}{qo}$. We can see in the scatter plot that the on-demand and sharing region largely agrees with the paper’s model, which is corroborated by the percentage agreement with model in Table 2. The *percentage agreement* is calculated as follows: For each range of utilities u_i corresponding to the regions [sharing, on-demand, neither] (i.e. $[\frac{pr-po}{qs-qo} < u_i < 1, \frac{po}{qo} < u_i < \frac{pr-po}{qs-qo}, u_i < \frac{po}{qo}]$), we will calculate the percentage of data points from our user study which agree with our model. For e.g., in the “sharing” region, there are altogether 22 data points, and 17 agree with our model (Prop 1-2, and Fig. 3a), giving a percentage of 77.3%. We note that there is randomness inherent in user clicks (decisions), as a math model can not fully capture user behavior. Furthermore decisions in a prototype will have differences from user behaviors in the actual system. Next we perform the Kolmogorov-Smirnov (KS) test for both the buyers and re-sellers data. We obtain the **model’s data samples** through using the given utility values, and obtaining the corresponding decisions according to the model (Prop 1-2, Fig 3a for buyers, and Eq 1 for re-sellers). With this and our data samples, we perform a KS test to evaluate whether, for each region (Sharing, On Demand, Neither, for buyers, Re-selling, Not Re-selling, for re-sellers) the two set of data points (data from user studies and model) can be considered to be from the same distribution. We perform the KS test using the scipy library in python. The KS test calculates the mean and standard deviation for each set of data points, and obtains the KS test test statistic,

$$D = \sup_x |F_{data}(x) - F_{model}(x)| \quad (38)$$

which quantifies the distance between the empirical distribution functions of the two data samples (model and user studies data). We choose a confidence level of 95%. As the p-values are above the significance level of 0.05, there is no significant evidence that the data and model distributions are different. Beyond our preliminary study in this work, further studies under different parameter settings, can be made with the help of our interactive prototype.

8 FUTURE WORK

For future work, we will study the long-term scenario where users may switch between reserving resources in advance, and buying on the go through the on-demand and sharing pools. Here, another layer would be added to the Stackelberg game and users will make decisions based on

Region	Sharing	On Demand	Neither
Analytical Model (Prop 1-2)	$\frac{pr-po}{qs-go} < u_i < 1$	$\frac{po}{go} < u_i < \frac{pr-po}{qs-go}$	$u_i < \frac{po}{go}$
Percentage agreement of data and model	77.3%	66.7 %	33.3 %
Test Statistic (of KS test)	0.2917	0.2941	0.2667
P-value (of KS test)	0.2406	0.6487	0.9629
Are the data and model distributions distinct?	No significant evidence	No significant evidence	No significant evidence

TABLE 2

Results for Buyers: Comparing data from user studies with the expected values from the model for each choice: Sharing, On-demand and Neither, through calculating the percentage agreement, and through using the Kolmogorov-Smirnov (KS) test. Our results show that there is no significant evidence of the data following a distinct distribution from our paper's model (at significance level 0.05).

Region	Re-selling	Not re-selling
Analytical Model (Eq 1)	$g_i < (1 - \delta)p_r$	$g_i > (1 - \delta)p_r$
Percentage agreement with model (within region)	90.9%	52.9%
Test Statistic (of KS test)	0.4444	0.1824
P-value (of KS test)	0.0975	0.9532
Are the data and model distributions distinct?	No significant evidence	No significant evidence

TABLE 3

Results for Re-sellers: Comparing data from user studies with the expected values from the model for each choice: Re-selling, and not re-selling, through calculating the percentage agreement, and through using the Kolmogorov-Smirnov (KS) test. Our results show that there is no significant evidence of the data following a distinct distribution from our paper's model (at significance level 0.05).

their expected payoffs in the long run. The expected payoff would depend on the users' application requirements: the probability of them needing job computation at the edge consistently over all timeslots vs one-off and sporadically, the prices set by the MNO and the expected supplies of the different options, which is influenced by the aggregate decisions of other users.

Another line of investigation is to study how the sharing framework can be implemented across multiple edge nodes, with different users being associated with different nodes due to proximity. In light of this, the on-the-go (on-demand and sharing) queues and the sharing supply would differ across nodes. If users are allowed to associate with other nearby nodes, it is thus of interest to investigate how the MNO can jointly price and allocate the resources across nodes to the different users, to maximize revenue and minimize the number of unmet jobs. In this new study on incentives, a Stackelberg game formulation can be used to analyze and characterise the interaction between the different parties, along with a matching algorithm which matches users to resources across edge nodes, with the transmission cost and hence distance between user and node being a factor. In the new Stackelberg formulation, the user decision making functions will be modified to reflect the potential supply levels at different nodes, and the likelihood of the user attaining resources from different nodes, which is a function of the distance.

9 CONCLUSION

In this paper we proposed a novel resource sharing platform to enhance the usage of the on-demand pricing and resource reservation models in edge computing. This helps to optimize the scenario where the on-demand users face a limited supply remaining at the edge server, whilst users under the resource reservation model have reserved excess computing resources, by allowing the re-selling of unused resource quota. We formulated a 3-stage Stackelberg Game to investigate the user's optimal strategies and equilibrium

behavior, as well as whether the MNO has an incentive to provide such a resource sharing platform. By analyzing the formulated game, we characterised the optimal strategies of the re-sellers, buyers and MNO, and showed that there would be 4 regions (both sharing and on-demand, only on-demand, only sharing, none at all) at equilibrium after buyers' decisions. Based on these 4 regions, we optimized the MNO's revenue and analytically characterised the MNO's optimal on-demand prices. Our numerical results show that having a sharing pool gives the MNO an optimal revenue when the on-demand pool's supply is low and when the platform's commission is low.

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