Going Forward & Backward with Jin Ma

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Jin Ma. A colleague, a thinker, a scholar, a husband and father, but most of all: a friend.

In a previous lifetime, or so it seems, I worked at Purdue University, in West Lafayette, Indiana. During my time at Purdue, I had the good fortune to hire Jin Ma as a postdoc, fresh from his PhD at the University of Minnesota. He was so outstanding we were able to change his status after two years, from a postdoc to tenure track. The rest was inevitable: Associate Professor, and then Professor.

When Jin arrived at Purdue, I was working on Forward-Backward SDEs along with my PhD student, Fabio Antonelli. Fabio and I welcomed Jin, and we discussed the problems connected to FBSDEs with him. At the time, just proving the existence and uniqueness of a solution seemed overwhelming, except for proving it on a special, short time interval, which became part of Fabio's PhD thesis.[1] This intrigued Jin.

Jin had a friend in China, Jiongmin Yong, who is now in the States, at the University of Central Florida, and together they proved a key result showing that solving a FBSDE could be thought of as finding the nodal sets of an HJB equation which had a viscosity solution.[7] I found this awesome, and we worked together to show that we could find a solution, using what we call a "four step scheme."[5]

J. Yong came into the game again when Jin and I developed what we called the four step scheme to prove a general existence and uniqueness result. We managed to show that there was a solution by relating it to a certain PDE, and composing it with a forward solution of an SDE. The problem was, however, that we needed a classical solution of the PDE (and not, for example, a weak solution).

"No problem," I told Jin. "We only have to go up to the 8th floor, where the PDE guys hang out, and ask for their help." We found, of course, that wonder of a man known to the world as Jim Douglas, Jr., who informed us that he was fairly sure such PDEs as the one we needed had never before been studied. (Sadly, Jim Douglas, Jr. passed away in 2016, at the age of 88.) This was depressing news, because we needed such a solution for the final step of the problem on which we were working, and neither of us looked forward to proving that a classical solution of this strange new PDE existed, if indeed it did!

"I'll ask Jiongmin Yong," Jin cheerfully said. And he did. Jiongmin raced through the literature, worldwide, and found an obscure paper by a German, published only two years earlier, that remarkably studied a class of PDEs that included the one we needed! This led to the completion of our Four Step Scheme paper. Later we tackled finding numerical solutions of FBSDEs, and to do this we once again used that mysterious PDE, along with some help from Jim Douglas, Jr. [3].

I told Jin about how I became interested in FBSDEs, it being via a question of Darrell Duffie, who thought they'd be just the ticket to solve a conjecture (the consul rate conjecture) of the renowned economist, Fisher Black (yes, the same Black that gave us the Black-Scholes equation in Mathematical Finance). I told Darrell about our progress, and then he and Jin got together to solve Black's conjecture, and as a reward, Jin's postdoc was converted to a tenure track position [4].

One day I mentioned to both Fabio and Jin that there should be a theory of weak solutions of FBSDEs. Jin ended up writing a lovely paper with Fabio Antonelli on weak solutions of FBSDEs, which avoids the short time horizon trap [2].

Jin retained his interest in FBSDEs, eventually writing a book on the subject together with Jiongmin Yong, published by Springer. It has become the go-to reference on the subject[8].

The world changed when Jianfeng Zhang arrived at Purdue. He was a brilliant student, and a bundle of energy. He worked with Jin, and under Jin's tutelage, Jianfeng wrote what amounted to several PhD theses.The intellectual climate of Purdue, amongst the probabilists, skyrocketed. It was a wonderful time to be there.

Two of Jin's early papers with Jianfeng Zhang involved Backwards SDEs (and not Forward-Backward ones), one involving representation theorems [9], and the other involving the path regularity of the solutions [10]. This latter paper built on the earlier one of Jin, Jianfeng, and myself [6]. Both issues were in the air in the early 2000's, and path regularity was an important issue relating to hedging in Math Finance.

Jin left Purdue in 2007 to join the Mathematics Department at USC, in Los Angeles.

Perhaps my favorite Jin Ma story is the following. My wife, Diane Rubenstein, and I enjoyed watching reruns of the classic TV series, *Perry Mason*. In the television show, there was a private detective, Paul Drake, and the always beautiful actress Barbara Hale, who played the role of Perry Mason's confidential secretary, Della Street. Paul Drake would usually greet

Della Street with the salutation, "Hi, Gorgeous," or "Hi, Beautiful." When my wife Diane called me at the office on occasion, I would answer the phone, "Hi, Gorgeous," since Diane was pretty just like Della Street.

Jin Ma thought that was a nice way to be, and when he went home one day he greeted his wife Yun with the phrase, "Hi, Gorgeous." It was out of context, although certainly not inaccurate, and his wife immediately became suspicious that Jin was up to no good. He wasn't, of course.

Jin survived the Cultural Revolution, and when it was ending and everyone on Earth wanted urgently to go to college, the entrance exams were brutal. Jin passed them at no less a level than one that got him entrance to Fudan University, the top place in China at the time. Showing that kind of fortitude, it's little surprise that his life since has been a good one, with one achievement after the next, not the least of which are his two children. Jin's "gorgeous" family is the highest achievement a man can have, and it is fitting to celebrate Jin on his 65th birthday.

Happy Birthday to my colleague, and my friend, Jin Ma.

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