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A Non-dimensional Input Excitation Optimization Approach for Battery Health Parameter Estimation ⋆

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Abstract: Model parameter estimation is an important subject in control engineering, including the field of battery management. Input excitation optimization has become an emerging topic lately to improve the accuracy of estimation. Traditional optimization approach suffers from a fundamental issue in parameter uncertainty, as the target parameters for estimation, often needed for computing the optimization objective and constraints, are intrinsically unknown. In this study, we introduce a non-dimensional approach to optimize excitations for estimating the health-related Li-ion battery electrochemical parameters. Guided by the Buckingham π theorem, we derived a control-oriented non-dimensional battery model, excluding uncertain target parameters from the problem formulation. As a result, the optimization problem can be solved without any prior knowledge of target parameters. The applicable control input sequence can be recovered by rescaling the obtained non-dimensional sequence with the best available knowledge of the parameters. Furthermore, the proposed method reveals the fundamental impact of the unknown parameters on the solution of input optimization. In light of this finding, we propose two iterative excitation optimization strategies, which both significantly improve the robustness and reduce the complexity of the optimization problem. The proposed method can be generalized to solve general optimal control problems for a broad class of systems.

Keywords: Li-ion battery, excitation optimization, parameter estimation, non-dimensional.

1. INTRODUCTION

Model-based optimization, control, and diagnostics are ubiquitous in engineering, and the effectiveness of these practice heavily relies on the accuracy of the model and model parameters. For example, in Li-ion battery management, parameter estimation is crucial for ensuring the safety, efficiency, and longevity of the battery systems (Lin et al. (2019)). The importance of parameter estimation has motivated the research on optimal input excitation design for Li-ion batteries (Forman et al. (2012)), as the quality of excitation has major impact on estimation accuracy (Lin et al. (2019)).

Early works in this area focused on excitation design with imposed patterns, such as sinusoidal (Song et al. (2018)), pulse, and other empirical profiles (Park et al. (2018)). Specifically, the parameters of these patterns, e.g. magnitude and frequency, were optimized to maximize the Fisher information (FI) of the target parameters—a metric for quantifying the information content of the output about the parameters (Lehmann and Casella (2006)). Models employed in these studies range from simple equivalent circuit models (Rothenberger et al. (2015)) to electrochemical models (Park et al. (2018)). More recent works have explored the direct optimization of input sequences for estimating battery electrochemical parameters, with the goal of finding the ultimate optimal profile with no imposed pattern (Lai et al. (2020a); Pozzi et al. (2018)), enabled by the efficient analytic sensitivity computation (Lai et al. (2020b)).

These existing studies, while showing the effectiveness of input excitation in improving estimation accuracy, have revealed a fundamental obstacle in excitation design: the optimal excitation often strongly depends on *a priori* unknown system parameters. Since the purpose of excitation is to enable the estimation of parameters, their exact values are intrinsically unknown at the time of design. Uncertainties in parameters can jeopardize the optimality and effectiveness of the generated excitation when applied to parameter estimation. Recent works have attempted to derive a closed-loop control policy using Reinforcement Learning for input generation based on state feedback to improve the robustness of the solution to parameter uncertainty (Huang et al. (2023)).

In this work, we propose a more general non-dimensional approach for battery modeling and control, which can be applied to characterize and mitigate parameter uncertainty in excitation optimization. By applying the Buckingham π theorem (Buckingham (1914)), we reformulate the system model and information objective into dimensionless forms in modulo similarity with respect to the unknown parameters. As a result, the optimal control problem is non-dimensional and can be solved without prior knowledge of uncertain parameters. The

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non-dimensionalization similarity transformation is then inverted by scaling using the best available knowledge of parameters. The non-dimensional approach reveals the essence of optimization under different parameter values, i.e. the parameter values (only) affect the time horizon of the non-dimensional optimization. This insight gives rise to 2 new iterative optimization schemes demonstrated in later sections of the paper, including (1) optimization using a pre-solved library of dimensionless excitations with a range of time horizons, and (2) optimization using a single dimensionless sequence. Both schemes work by first scaling the solution of the non-dimensional optimization problem with the best knowledge of the parameters, applying the obtained input sequence to generate the data for estimation, rescaling based on the estimation results and iterating the process. The advantages of the non-dimensional approach are twofold. First, it renders robustness to parameter uncertainty through the iteration process, where the input sequence can be updated with improved knowledge of parameters and will converge along with the parameter estimates. The updating of the sequence during iteration only involves simple scaling, which can be easily performed and are much more feasible and convenient than re-solving the optimization problem. Second, the proposed approach also facilitates the procedure of solving the optimization problem. This is because the problem is now solved independent of parameter values, which could affect the performance of the algorithm and often requires tuning to obtain the true optimum. Simulation results on estimating key battery health-related parameters are provided to demonstrate the capability of the proposed approach to significantly improve estimation accuracy, even with substantial uncertainty in parameters and measurements. Moreover, the derived non-dimensional approach can potentially be applied to a wide range of general optimization problems, e.g. battery charging control, balancing control, and thermal management, which are usually subject to parameter uncertainty. While non-dimensional analysis has traditionally been used for battery performance analysis and design (Ayerbe et al. (2021); Couto et al. (2023)), to the best of our knowledge, this research is the first to apply a dimensionless similarity approach to battery system estimation and control.

2. CONVENTIONAL INPUT OPTIMIZATION METHOD WITH DIMENSIONS

This section examines the conventional excitation optimization methodology for parameter estimation, in the context of battery health-related parameters. We will first introduce the battery model used for control and optimization, followed by the parameter sensitivity dynamics related to the input design objective, and finally the conventional problem formulation and method of solution.

2.1 Battery Model

We employ the widely used single particle model with electrolyte dynamics (SPMe) in this research for its good balance of fidelity and complexity (Moura (2016)). This model simplifies the full-order first-principle Doyle-Fuller-Newman model by assuming a uniform exchange current density (of the intercalation reaction) and molar ion flux in each electrode. As a result, a single spherical particle can be used to capture the Li-ion solid-phase diffusion at each electrode, which is the most important dynamics of the model. Specifically, we can describe the Li-ion diffusion in an electrode particle in spherical coordinates based on Fick's second law, as shown in (1), along with its boundary conditions in (2),

$$\frac{\partial c_{s,i}}{\partial t} = D_{s,i}\left(\frac{\partial^2 c_{s,i}}{\partial r^2} + \frac{2}{r}\frac{\partial c_{s,i}}{\partial r}\right), \tag{1}$$

$$\left.\frac{\partial c_{s,i}}{\partial r}\right|_{r=0} = 0, \quad D_{s,i}\left.\frac{\partial c_{s,i}}{\partial r}\right|_{r=R_{s,i}} = \pm\frac{R_s I}{3A_i\delta_i\varepsilon_{s,i}F}. \tag{2}$$

In these equations, c_s represents the Li-ion concentration in solid particles, D_s denotes the solid-phase diffusion coefficient, R_s is the particle radius, I refers to the battery input current (positive for discharging), ε_s is the active material volume fraction, and A and δ denote the electrode area and thickness. The subscript i indicates the negative electrode (anode) when $i = n$ and positive electrode (cathode) when $i = p$. The $\pm$ sign in (2) takes $+$ for the cathode and $-$ for the anode.

By solving PDE (1) with boundary conditions (2), we can obtain the Li-ion concentration at the surface of the particle, c_{se}, i.e. c_s at $r = R_s$, which is a critical state of the model. Specifically, we apply Laplace transform to derive a transfer function for c_{se}, which can be conveniently used for control purpose subsequently. The obtained transfer function is transcendental, and we further use Padé approximation to simplify it to a rational function by moment matching (Lai et al. (2020b)),

$$c_{se,i}(s) \approx \pm\frac{7R_{s,i}^4 s^2 + 420D_{s,i}R_{s,i}^2 s + 3465D_{s,i}^2}{s(R_{s,i}^4 s^2 + 189D_{s,i}R_{s,i}^2 s + 3465D_{s,i}^2)}\frac{I(s)}{F\varepsilon_{s,i}A_i\delta_i}. \tag{3}$$

The output of the SPMe model is the battery voltage V, which is governed by four terms, as shown in (4). These terms are the open circuit potential (U_i), which is a nonlinear function of $c_{se,i}$, the electrolyte potential $\phi_{e,i}$ associated with the electrolyte Li-ion concentration $c_{e,i}$, the overpotential η_i at the interface of the solid particle and electrolyte, which drives the ionic (de)intercalation reaction, and the lumped Ohmic resistance R_l.

$$\begin{aligned} V = {} & (U_p(c_{se,p}) - U_n(c_{se,n})) + (\phi_{e,p}(c_{e,p}) - \phi_{e,n}(c_{e,n})) \\ & + (\eta_p(c_{se,p}, c_{e,p}) - \eta_n(c_{se,n}, c_{e,n})) - IR_l \end{aligned} \tag{4}$$

Details about the modeling of c_e and η are skipped here due to page limit, and can be found in (Lai et al. (2020b)).

2.2 Input Design Problem Formulation

A popular objective for input design is to maximize the Fisher Information (FI) of the data (Lehmann and Casella (2006)), which represents the information content (likelihood function) about the target parameter(s) to be estimated. This objective has been commonly used in battery research community for evaluating and optimizing the current excitation (Park et al. (2018); Lai et al. (2021)).

Suppose the goal is to estimate a target parameter set $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_m]$, using a discrete sequence of system out-

puts $\boldsymbol{y}_{[t_1,\dots,t_N]} = [y(t_1), y(t_2), ..., y(t_N)]$ generated by an input sequence $\boldsymbol{u}_{[t_1,\dots,t_N]} = [u(t_1), u(t_2), ..., u(t_N)]$. Under additive white Gaussian output noises, the (normalized) FI of the target parameter(s) is

$$\boldsymbol{F}_{\text{info}}(\boldsymbol{\theta}) = \frac{1}{\sigma_y^2}\sum_{k=1}^{N}\left(\bar{S}_{\boldsymbol{\theta}}(t_k)\right)^T\left(\bar{S}_{\boldsymbol{\theta}}(t_k)\right) \tag{5}$$

in which σ_y^2 represents the variance of the measurement noises in y, and $\bar{S}_{\boldsymbol{\theta}}(t_k)$ is the $1\times m$ normalized sensitivity (vector) of $y(t_k)$ to $\boldsymbol{\theta}$, with each component

$$\bar{S}_{\theta_j}(t_k) = \theta_j \cdot \frac{\partial y(t_k)}{\partial \theta_j}. \tag{6}$$

For univariate estimation scenarios ($m = 1$), the FI matrix reduces to a scalar. For joint estimation of multiple parameters ($m > 1$), FI is an $m\times m$ matrix. The excitation design problem can be formulated as maximizing the determinant (or other metrics, e.g. trace or smallest eigenvalue) of the FI matrix,

$$\min_{\boldsymbol{u}_{[t_1,\dots,t_N]}} \; -\det \boldsymbol{F}_{\text{info}}(\boldsymbol{\theta}). \tag{7}$$

Evaluation of the FI objective requires computation of the sensitivity $\frac{\partial y_k}{\partial \boldsymbol{\theta}}$, which can be performed based on the underlying system model. In the context of battery parameter estimation, y is the measured terminal voltage V, u is the input current I, and $\boldsymbol{\theta} = [\theta_1, \theta_2]$, where $\theta_1 = \varepsilon_{s,p}$ is the cathode active material volume fraction, and $\theta_2 = D_{s,p}$ is the cathode solid phase lithium diffusion coefficient. These two parameters are selected for estimation here because they are the common targets in literature for two reasons. First, they have important physical significance and are associated with critical battery status. Specifically, ε_s is directly related to battery capacity and hence state of health (SOH), and D_s governs the diffusion dynamics which affects both SOH and state of power (SOP). Second, they are the only parameters in the diffusion equation (1) that are likely to vary significantly from battery to battery and over battery lifetime, and hence prone to uncertainty.

The sensitivity of the output voltage V to ε_s can be calculated by taking the partial derivative of (4) to ε_s as

$$\frac{\partial V}{\partial \varepsilon_s}(t) = \frac{\partial \eta}{\partial \varepsilon_s} + \left(\frac{\partial \eta}{\partial c_{se}} + \frac{\partial U}{\partial c_{se}}\right)\cdot \frac{\partial c_{se}}{\partial \varepsilon_s}(t). \tag{8}$$

In (8), the first three terms on the right side are non-dynamic and can be easily computed based on the functional form of η and U. The final term $\frac{\partial c_{se}}{\partial \varepsilon_s}(t)$ represents the sensitivity of ε_s with respect to the diffusion state c_{se}, which varies dynamically over time and thus complicates the sensitivity calculation. To facilitate computation, we utilize the derived transfer function for c_{se} in (3). Specifically, by taking the partial derivative of (3) to ε_s, we obtain the sensitivity transfer function (STF) for ε_s,

$$\frac{\partial c_{se}}{\partial \varepsilon_s}(s) = \frac{7R_s^4 s^2 + 420 D_s R_s^2 s + 3465 D_s^2}{s(R_s^4 s^2 + 189 D_s R_s^2 s + 3465 D_s^2)}\cdot \frac{I(s)}{F\varepsilon_s^2 A\delta}. \tag{9}$$

The derived STF can be readily implemented in the time-domain, enabling the efficient computation of sensitivity and FI during optimization, as demonstrated in prior research (Lai et al. (2020a, 2021); Huang et al. (2023)). Similarly, the sensitivity of D_s can be computed as,

$$\frac{\partial V}{\partial D_s}(t) = \left(\frac{\partial \eta}{\partial c_{se}} + \frac{\partial U}{\partial c_{se}}\right)\cdot \frac{\partial c_{se}}{\partial D_s}(t), \tag{10}$$

with the corresponding STF

$$\frac{\partial c_{se}}{\partial D_s}(s) = \frac{903R_s^4 s^2 + 41580 D_s R_s^2 s + 800415 D_s^2}{(R_s^4 s^2 + 189 D_s R_s^2 s + 3465 D_s^2)^2}\frac{R_s^2 I(s)}{F\varepsilon_s A\delta}. \tag{11}$$

Remark: as mentioned previously, this conventional problem formulation of input optimization faces a fundamental limitation, i.e. the unknown target parameters $\boldsymbol{\theta}$ appear in the design objective and present uncertainty. Specifically in the case here, both ε_s and D_s appear in their STFs in (9) and (11), and hence eventually show up in the design objective FI, which is why $\boldsymbol{F}_{\text{info}}$ is denoted as dependent on $\boldsymbol{\theta}$ in (7). Consequently, the exact solution of input optimization requires accurate knowledge of the parameters, which are intrinsically unknown as they are the target of estimation. Examples will be provided in a later section to show that imprecise *a priori* values of the target parameters assumed to perform the optimization could significantly affect the optimality and effectivenss of the solution. This limitation of the conventional problem formulation motivated us to explore a new approach that does not require precise knowledge of parameters.

3. NON-DIMENSIONAL FORMULATION OF INPUT OPTIMIZATION

As a powerful technique for model reduction and simplification, nondimensionalization has been extensively employed in engineering design and development. According to the Buckingham π theorem, systems governed by dimensional physical parameters can be represented with a smaller set of parameters, some (or all) of which are dimensionless. This non-dimensionalization process simplifies the model and reveals the influence of physical parameters on performance. In this study, we apply non-dimensional analysis to model-based control, specifically the input excitation optimization problem. It will be shown that by non-dimensionalizing the input and time, the resulting system dynamics will be free from any physical dimensional parameters. As a result, there is no need to know the values of the target parameters for optimization.

The process of the proposed non-dimensional approach is outlined as follows. We examine the Li-ion diffusion dynamics of the cathode solid particle in (1)-(2), which contains both target parameters of interest. By re-defining

$$\tilde{t} = \frac{D_{s,p}}{R_{s,p}^2}t, \quad \tilde{s} = \frac{R_{s,p}^2}{D_{s,p}}s, \qquad \tilde{r} = \frac{r}{R_{s,p}}, \quad \tilde{c}_{s,p} = \frac{c_{s,p}}{c_{s,p\,\max}},$$
$$\tilde{I} = \frac{R_{s,p}^2}{A_p\delta_p F\varepsilon_{s,p}c_{s,p\,\max}D_{s,p}}I = \frac{\alpha}{\varepsilon_{s,p}D_{s,p}}I \tag{12}$$

a non-dimensional diffusion equation and boundary conditions can be derived as

$$\frac{\partial \tilde{c}_{s,p}}{\partial \tilde{t}} = \frac{\partial^2 \tilde{c}_{s,p}}{\partial \tilde{r}^2} + \frac{2}{\tilde{r}}\frac{\partial \tilde{c}_{s,p}}{\partial \tilde{r}}, \tag{13}$$

$$\left.\frac{\partial \tilde{c}_{s,p}}{\partial \tilde{r}}\right|_{\tilde{r}=0} = 0, \quad \left.\frac{\partial \tilde{c}_{s,p}}{\partial \tilde{r}}\right|_{\tilde{r}=1} = \tilde{I}. \tag{14}$$

Then, by applying Padé approximation, the transfer function of the non-dimensional model can be obtained as

$$\tilde{c}_{se,p}(\tilde{s}) = \tilde{G}(\tilde{s})\tilde{I}(\tilde{s}) = \frac{7\tilde{s}^2 + 420\tilde{s} + 3465}{\tilde{s}(\tilde{s}^2 + 189\tilde{s} + 3465)}\cdot \tilde{I}(\tilde{s}). \tag{15}$$

It is worth noting that the non-dimensional transfer function does not contain any battery parameters, but it is still implicitly related to the parameters through $\tilde{I}$ and $\tilde{s}$ according to (12). Therefore, we can use the chain rule to derive the parameter sensitivities for the non-dimensional system model as

$$\begin{aligned}\frac{\partial \tilde{c}_{se,p}}{\partial \varepsilon_{s,p}}(\tilde{s}) &= \tilde{G}(\tilde{s})\frac{\partial \tilde{I}(\tilde{s})}{\partial \varepsilon_{s,p}} \\ \frac{\partial \tilde{c}_{se,p}}{\partial D_{s,p}}(\tilde{s}) &= \tilde{G}(\tilde{s})\frac{\partial \tilde{I}(\tilde{s})}{\partial D_{s,p}} + \frac{\partial \tilde{G}(\tilde{s})}{\partial \tilde{s}}\frac{\partial \tilde{s}}{\partial D_{s,p}}\tilde{I}(\tilde{s}).\end{aligned} \tag{16}$$

Then, according to the mapping in (12), we can compute

$$\frac{\partial \tilde{I}}{\partial \varepsilon_{s,p}} = -\frac{1}{\varepsilon_{s,p}}\tilde{I}, \quad \frac{\partial \tilde{I}}{\partial D_{s,p}} = -\frac{1}{D_{s,p}}\tilde{I}, \quad \frac{\partial \tilde{s}}{\partial D_{s,p}} = -\frac{1}{D_{s,p}}\tilde{s}. \tag{17}$$

By plugging (17) to (16), we can obtain the (normalized) non-dimensional sensitivities of the two target parameters

$$\varepsilon_{s,p}\cdot\frac{\partial \tilde{c}_{se}}{\partial \varepsilon_{s,p}}(\tilde{s}) = -\frac{7\tilde{s}^2+420\tilde{s}+3465}{\tilde{s}(\tilde{s}^2+189\tilde{s}+3465)}\cdot\tilde{I}(\tilde{s}) \tag{18}$$

$$D_{s,p}\cdot\frac{\partial \tilde{c}_{se}}{\partial D_{s,p}}(\tilde{s}) = -\frac{903\tilde{s}^2+41580\tilde{s}+800415}{(\tilde{s}^2+189\tilde{s}+3465)^2}\cdot\tilde{I}(\tilde{s}). \tag{19}$$

It is interesting to see that the reformulated nondimensional model in (15) and parameter sensitivities in (18) and (19) can now be computed without any original physical parameters. Therefore, the input optimization problem with the nondimensional FI as the objective can be solved independent of parameters, as

$$\min_{\tilde{\boldsymbol{u}}_{[\tilde{t}_1,\dots,\tilde{t}_N]}} -\det \tilde{\boldsymbol{F}}_{\text{info}}. \tag{20}$$

Optimization of battery current under the new formulation will give a non-dimensional sequence $\tilde{\boldsymbol{I}}_{[\tilde{t}_1,\dots,\tilde{t}_N]} = [\tilde{I}(\tilde{t}_1), \tilde{I}(\tilde{t}_2), ..., \tilde{I}(\tilde{t}_N)]$. It is then scaled to obtain the sequence for the original dimensional physical system $\boldsymbol{I}_{[t_1,\dots,t_N]} = [I(t_1), I(t_2), ..., I(t_N)]$ by inverting the mapping between I and $\tilde{I}$, and t and $\tilde{t}$ in (12) as

$$I(t_k) = \frac{\varepsilon_{s,p}D_{s,p}}{\alpha}\tilde{I}(\frac{R_{s,p}^2}{D_{s,p}}\tilde{t}_k), \tag{21}$$

which can be applied to the battery to generate the data for estimation. The *a priori* knowledge of the target parameters $\varepsilon_{s,p}$ and $D_{s,p}$ are needed for scaling the current magnitude and time. The main advantage of the new non-dimensional formulation is that, even though the *a priori* knowledge of parameters is still needed in the final step of scaling, the optimization itself is performed free of parameters and their uncertainty. Therefore, we can simply pre-solve the non-dimensional optimization problem to obtain the dimensionless sequence. And every time when a subject (e.g. battery) needs to be estimated, we do not need to repeat the computationally intensive optimization, but simply scale the pre-designed sequence based on the *a priori* knowledge of parameters. Moreover, the new non-dimensional formulation also reveals the fundamental impact of parameters on the structure of the optimal input sequence. Specifically, if a parameter only affects the scaling of the input magnitude (e.g. $\varepsilon_{s,p}$ in this case), the optimal sequence will just be proportional in magnitude under different parameter values; when the parameter also affects the scaling of time (e.g. $D_{s,p}$), the time horizon $\tilde{t}_N$ of non-dimensional optimization will be proportional with the parameter value if we want to keep the time horizon t_N of the original dimensional sequence the same.

Based on the above insights, we propose two iterative excitation optimization and parameter estimation schemes to counter the impact of parameter uncertainty, as illustrated in Alg. 1. Scheme 1 applies to the scenario when it is desired to design an input sequence of a fixed time horizon t_N^*. In this case, we can optimize beforehand a library of non-dimensional input sequences $\{\tilde{\boldsymbol{u}}^i_{[\tilde{t}_1,\dots,\tilde{t}_{N_i}]}\}$ under a range of $\tilde{t}_N$ by solving (20). Before performing experiment for a specific subject, we first determine the non-dimensional time horizon $\tilde{t}_N^*$ based on t_N^*, *a priori* knowledge of the parameter $\hat{\boldsymbol{\theta}}^-$, and the time scaling relationship, e.g. $\tilde{t}_N^* = \frac{D_{s,p}}{R_{s,p}^2}t_N^*$ in the case of $D_{s,p}$ estimation. We then select from library a sequence $\tilde{\boldsymbol{u}}_{[\tilde{t}_1,\dots,\tilde{t}_N]}$ with $\tilde{t}_N$ equal (or closest) to $\tilde{t}_N^*$, and convert it to $\boldsymbol{u}_{[t_1,\dots,t_N]}$ based on the scaling relationship, e.g. (21). The obtained dimensional input sequence $\boldsymbol{u}_{[t_1,\dots,t_N]}$ will then be applied to perform experiment and generate the output $\boldsymbol{y}_{[t_1,\dots,t_N]}$, which will be use to estimate (update) $\boldsymbol{\theta}$. The above process is repeated until the estimate converges. Scheme 2 further simplifies the procedure by only using a single non-dimensional sequence, and applies to the scenarios when the requirement on experiment time horizon is not strict. In summary, the key idea behind the proposed new approach is to schedule a series of sequential experiments, during which the input sequence is adapted based on the estimation results (by simple and efficient scaling) after each experiment run. Eventually, the sequence will converge to the true optimum along with the parameter estimates.

Algorithm 1 Non-dimensional approach of iterative excitation optimization and parameter estimation

Input: *a priori* knowledge $\hat{\boldsymbol{\theta}}^-$, a library of sequences $\{\tilde{\boldsymbol{u}}^i_{[\tilde{t}_1,\dots,\tilde{t}_{N_i}]}\}$ pre-optimized under a range of $\tilde{t}_N$ and desired time horizon t_N^* (Scheme 1), a single $\tilde{\boldsymbol{u}}_{[\tilde{t}_1,\dots,\tilde{t}_N]}$ (Scheme 2)

Output: estimate $\hat{\boldsymbol{\theta}}$

$\hat{\boldsymbol{\theta}} \leftarrow \hat{\boldsymbol{\theta}}^-$
repeat
  if Scheme 1 **then**
    $\tilde{t}_N^* \leftarrow t_N^*$ by scaling using $\hat{\boldsymbol{\theta}}$;
    choose $\tilde{\boldsymbol{u}}_{[\tilde{t}_1,\dots,\tilde{t}_N]} \in \{\tilde{\boldsymbol{u}}^i_{[\tilde{t}_1,\dots,\tilde{t}_{N_i}]}\}$ with $\tilde{t}_N = \tilde{t}_N^*$;
    $\boldsymbol{u}_{[t_1,\dots,t_N]} \leftarrow \tilde{\boldsymbol{u}}_{[\tilde{t}_1,\dots,\tilde{t}_N]}$ by scaling using $\hat{\boldsymbol{\theta}}$, e.g. (21).
  else if Scheme 2 **then**
    $\boldsymbol{u}_{[t_1,\dots,t_N]} \leftarrow \tilde{\boldsymbol{u}}_{[\tilde{t}_1,\dots,\tilde{t}_N]}$ by scaling using $\hat{\boldsymbol{\theta}}$, e.g. (21).
  end if
  Apply $\boldsymbol{u}_{[t_1,\dots,t_N]}$ to generate $\boldsymbol{y}_{[t_1,\dots,t_N]}$;
  $\hat{\boldsymbol{\theta}} \leftarrow \boldsymbol{u}_{[t_1,\dots,t_N]}, \boldsymbol{y}_{[t_1,\dots,t_N]}$ (estimation).
until $\hat{\boldsymbol{\theta}}$ converges
return $\hat{\boldsymbol{\theta}}$

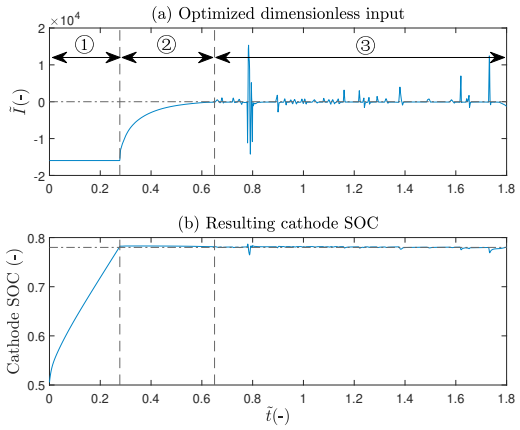

Fig. 1. Current optimization result for $\varepsilon_{s,p}$ estimation using non-dimensional approach: (a) optimized dimensionless sequence, (b) resultant (nondimensional) cathode SOC

4. VALIDATION OF RESULTS

In this section, we demonstrate the effectiveness of the proposed non-dimensional input excitation optimization approach in simulation. The results are presented for two cases of estimation: (1) single-variate parameter estimation of $\varepsilon_{s,p}$, and (2) joint estimation of both $\varepsilon_{s,p}$ and $D_{s,p}$.

During validation, the optimized dimensionless sequence is initially scaled using *a priori* $\hat{\theta}^-$, and applied to an SPMe model to generate the voltage data for estimation. It is noted that θ^- deviates 50% from the true value (used in the simulation model) to emulate the uncertainty in input optimization, and Gaussian noise with both a mean and a standard deviation of 0.01 V is added to the voltage data to emulate the measurement uncertainty (bias and variance). For Scheme 1, i.e. fixed time horizon experiment, the horizon is set as $t_N = 1800\ s$.

4.1 Single-variate Estimation of $\varepsilon_{s,p}$

The optimized non-dimensional sequence $\tilde{I}(\tilde{t})$ for $\varepsilon_{s,p}$ estimation is shown in Fig. 1. It is important to note that according to (12) $\varepsilon_{s,p}$ is not involved in the scaling of time t. Therefore, the update of $\hat{\varepsilon}_{s,p}$ does not require re-scaling the sequence in time, but only in the current magnitude I, and hence the two schemes are identical.

As shown in Fig. 1(a), the dimensionless excitation sequence can be divided into three phases, namely the constant-current charging phase with maximum current, the reduced-current charging phase until the battery state of charge (SOC, which is defined based on c_{se}) reaches 0.78 (where the slope of the open circuit voltage curve takes the peak), and finally an SOC-sustaining phase in which the current drops to zero. The features of the excitation pattern is identical to the previous results using the conventional dimensional approach, which can be explained by correlating to the battery physics governing the $\varepsilon_{s,p}$ sensitivity (Lai et al. (2020a)).

The iterative estimation results are presented in Table 1. It can be observed that the first iteration using the current sequence scaled with 50% initial error in $\varepsilon_{s,p}$ yields an estimation error of 6.6%, showing the significant impact of parameter uncertainty. The errors over subsequent iterations converge to a bound within 3% of the true value, which is attributed to the voltage bias and variance (Fogelquist et al. (2023)). Meanwhile, as the estimates improve, the FI of the re-scaled dimensional sequence also improves substantially from $26.6\,\mathrm{V}^2$ to around $150\,\mathrm{V}^2$.

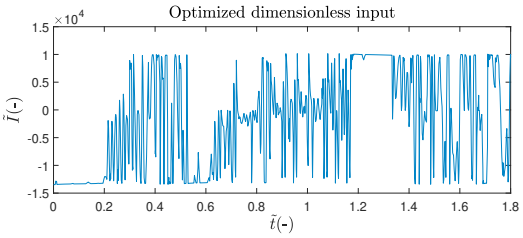

Fig. 2. Optimized dimensionless sequence for $\varepsilon_{s,p}$ and $D_{s,p}$ joint estimation with $\tilde{t}_N = 1.8$

4.2 Joint Estimation of $\varepsilon_{s,p}$ and $D_{s,p}$

Joint estimation of $\varepsilon_{s,p}$ and $D_{s,p}$ presents a more complete and complicated case of estimation, as the goal is to essentially estimate all governing parameters of the lithium diffusion model. One optimized non-dimensional sequence (with $\tilde{t}_N = 1.8$) is shown in Fig. 2. According to (12), $D_{s,p}$ affects the scaling of both time and current magnitude. Therefore, we validate both iterative schemes described in Alg. 1, with the results presented in Table 2 and 3.

It is seen that for both schemes, the estimation errors after the first iteration are substantial, namely 7.4% and 7.3% for $\varepsilon_{s,p}$, and -30.2% and -33.7% for $D_{s,p}$ respectively, which are mainly caused by the large uncertainty (50%) in the *a priori* knowledge of the parameters. This again testifies the limitation of the conventional input optimization method, which heavily relies on accurate initial guess that is fundamentally impossible to obtain beforehand. The impact of parameter uncertainty is especially prominent on $D_{s,p}$, as it has a much lower voltage sensitivity than $\varepsilon_{s,p}$ (Lai et al. (2020b)). Over subsequent iterations, the estimates of both parameters converge quickly and eventually settle around 4% of the true values. Even after just 1 more iteration, the error in $D_{s,p}$ estimate drops significantly to 6% in Scheme 2. These results validate the effectiveness of the proposed iterative input optimization and estimation approach in reducing the impact of parameter uncertainty. It is noted that the two schemes perform similarly in this application. The main difference is that the (dimensional) time horizon of Scheme 1 is maintained at $1800s$, while that of Scheme 2 varies over iteration. Since there is no obvious difference in performance, Scheme 2 is preferred in this case since only a single non-dimensional sequence needs to be optimized beforehand.

5. CONCLUSION

In this paper, we proposed a non-dimensional approach for optimizing battery excitation to estimate the health-related parameters. Utilizing the Buckingham π theorem, we derived a non-dimensional battery electrochemical model tailored for control design. The resulting dimensionless model and objective function do not contain any

Table 1. Single-variate iterative estimation result of $\varepsilon_{s,p}$

Iterations	0	1	2	3	4	5	6	7	8
Relative error of $\varepsilon_{s,p}$ (%)	50.00	6.66	4.42	3.24	2.76	2.60	2.70	2.64	2.62
FI of $\varepsilon_{s,p}$ (V^2)	26.63	33.78	78.99	124.09	146.86	154.90	149.85	152.87	-

Table 2. Joint estimation results of $\varepsilon_{s,p}$ and $D_{s,p}$ using Scheme 1

Iterations	0	1	2	3	4	5	6	7	8	9
Relative error of $\varepsilon_{s,p}$ (%)	50.00	7.40	4.60	3.80	3.58	3.36	3.54	3.56	3.64	3.8
Relative error of $D_{s,p}$ (%)	-50.00	-30.28	-9.63	-1.94	-5.34	3.58	-3.13	-2.79	-4.38	-4.13
$\tilde{t}_N$ (−)	2.70	2.35	1.97	1.84	1.90	1.74	1.86	1.85	1.88	-

Table 3. Joint estimation results of $\varepsilon_{s,p}$ and $D_{s,p}$ using Scheme 2

Iterations	0	1	2	3	4	5	6	7	8	9
Relative error of $\varepsilon_{s,p}$ (%)	50.00	7.30	4.14	3.46	3.48	3.38	3.44	3.26	3.46	3.46
Relative error of $D_{s,p}$ (%)	-50.00	-33.68	-6.18	-4.13	-3.50	-4.25	-3.71	-4.25	-3.52	-4.36
t_N (s)	1200	1346	1695	1729	1740	1727	1736	1727	1739	-

unknown target parameters, thus addressing the intrinsic limitation of the traditional approach, which needs good *a priori* knowledge of the parameters beforehand. Specifically, the new problem formulation would allow the optimization to be performed without the parameters, and only requires a simple scaling step to convert the result to the applicable dimensional sequence. Based on this insight, we proposed two iterative excitation optimization and parameter estimation schemes. Simulation validation has been performed for both single-variate and joint estimation of the electrode active material fraction $\varepsilon_{s,p}$ and diffusion coefficient $D_{s,p}$, which are critical parameters related to battery health and governing the key battery diffusion dynamics. It is shown that when subject to 50% initial uncertainty in parameters along with measurement noises, the new approach achieves final estimation errors of around 4%, compared to over 30% for the conventional method. The results demonstrate the effectiveness of the non-dimensional approach in addressing the parameter uncertainty for input design. It is noted that the proposed approach could be easily generalized to the broad class of parameter-affine systems and general control problems, which is a focus of our ongoing work.

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