Integrated photonics demonstration of zero-curvature eigenfunctions

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Abstract: We apply the homomorphism between the Schrödinger and Helmholtz-Maxwell wave equations to experimentally demonstrate an integrated photonic analogue of the zero-curvature eigenfunctions predicted in quantum mechanics.

OCIS codes: (130.0130) Integrated optics; (230.0230) Optical Devices; (260.0260) Physical optics.

Zero-curvature (linear) bound-state eigenfunctions that preserve the probability density over an extended region of space have been recently predicted in quantum mechanics [1-4]. These solutions are obtained in idealized potentials, such as 1D infinite potentials wells aided by Dirac delta potentials barriers [1,4], asymmetric infinite square wells [2], symmetric infinite square wells with a potential well [3], and threshold states on single square wells [2]. Under these boundary conditions, Schrödinger's equation reduces to $d^2\psi(x)/dx^2 = 0$, and eigenfunctions with extended zero-curvature solutions of the kind $\psi(x) = ax + b$ are possible, beyond well-known classical turning point.

Here we show an integrated photonics platform to access these zero-curvature states in the optical domain by leveraging the homomorphism between Schrodinger and Helmholtz-Maxwell's wave equations. We show that zero-curvature eigenfunctions can be realized by an optical structure composed of two coupled symmetric dielectric waveguides separated by an intermediary dielectric medium. The refractive index profile of this composite dielectric medium mimics the potential distributions of its quantum-mechanics analogues ($-V(x) \rightarrow n^2(x)$). Figure 1 shows three potential distributions, their corresponding index profiles, and their respective eigenfunctions: (a) a conventional Gaussian distribution in a single symmetric quantum well; (b) a zero-curvature distribution in symmetric double wells with an intermediary potential barrier; (c) and a relatively linear distribution with a controllable inclination in asymmetric wells with an intermediary potential barrier. To obtain the zero-curvature eigenfunctions, we develop a shortcut method where the barrier height matches the energy of a particular level in a single quantum well.

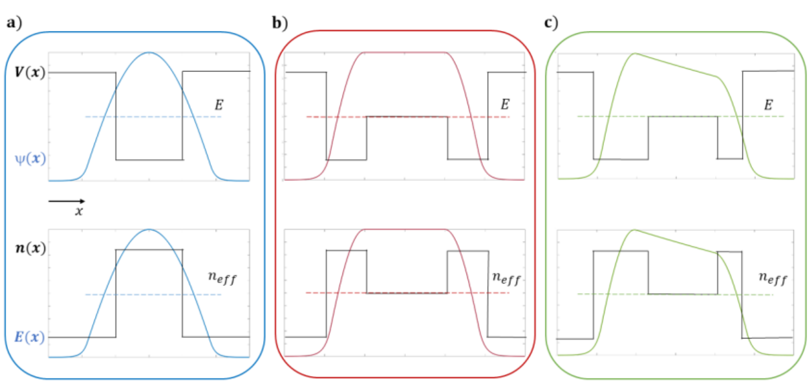

Figure 1. 1D Quantum wells potentials, their corresponding index profiles, and their respective eigenfunctions for the fundamental state. **(a)** Symmetric single well and its spatial distribution showing a conventional Gaussian-like profile. **(b)** Finite symmetric double-well potentials separated by an intermediary potential barrier. The zero-curvature eigenfunction is obtained by matching the height of the intermediary barrier with the eigenenergy of the fundamental state of the symmetric single well. The distribution shows a zero-curvature solution inside the barrier. **(c)** Finite asymmetric wells separated by a barrier. The asymmetry creates a zero-curvature eigenfunction with a controllable inclination.

We experimentally demonstrate these zero-curvature solutions by fabricating and imaging the optical mode of each waveguide with its refractive index profile corresponding to the respective potential distributions. Our dielectric structure is composed of a vertical stack with two silicon nitride (SiN) layers (2.0) separated by a silicon oxynitride (SiON) layer (1.75), drawn as an inset in Fig. 2(b). The width of the waveguide is chosen to be 1000 nm, the thickness of the SiN layers is 226 , and the thickness of the SiON layer is 1000 nm. The waveguide is designed to present the zero-curvature effect near the telecom wavelength ($_0$ = 1.55 μm) for the fundamental mode. In our previous work, we have shown that this structure is scale-invariant with respect to the middle layer thickness under this condition [5]. For comparison, we fabricate a single SiN waveguide (= 1000 and = 1452), as shown in Fig. 2(a). Furthermore, we also fabricate an asymmetric structure with a 40 nm difference between the thickness of the two SiN layers (= 246 and = 206), shown in Fig. 2(c).

We image the mode near-field profiles by using a Z-scan approach, assisted by a high-resolution nanopositioner and tapered nanofiber. The modes profiles are then recovered through the deconvolution algorithm presented in [6]. Figure 2(b) shows the mode profile presenting a zero-curvature wavefunction inside the middle layer region. The effect of the asymmetry on the mode profile is shown in Fig. 2(c). One can see that these zero-curvature solutions can be accessed in the photonic domain opening the door to exploring their properties in optical modes that have spatial distributions significantly different from the traditional ones in waveguides.

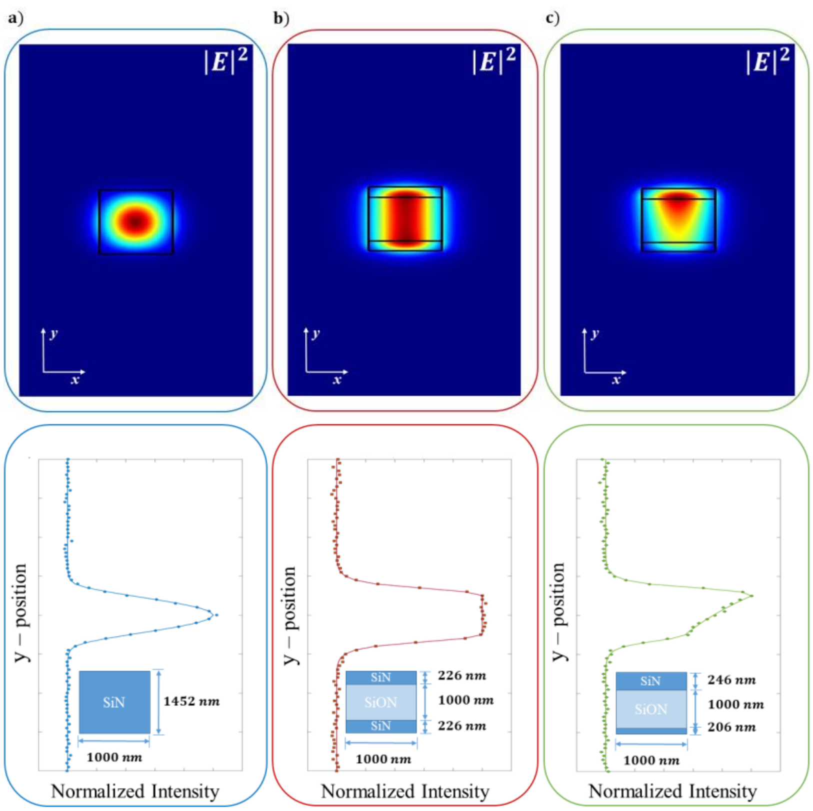

Figure 2. Photonics experimental demonstration of zero-curvature bounded states. **(a)** FDTD simulations of the light intensity ($|\ |^2$) spatial distribution of the single SiN waveguide. Experimentally recovered near-field spatial distribution data (blue circles) and fitting (blue line). **(b)** Waveguide composed of two identical SiN layers separated by a SiON layer designed to present the zero-curvature solution at the telecom wavelength ($_0$=1550 nm). The recovered spatial profile data (red square) and fitting (red line) show a zero-curvature distribution inside the middle layer. **(c)** Waveguide formed by two distinct SiN layers separated by a SiON layer. Spatial profile data (green pyramid) and fitting (green line). The total waveguide thickness was kept the same in all cases (T = 1452 nm). The waveguide's dimensions

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