Thunder and lightning: A revolution in wave-matter interactions

David G. Grier, Ella King, and Mia C. Morrell

Department of Physics and Center for Soft Matter Research, New York University, New York, NY 10003, USA

ABSTRACT

A quarter century of progress in holographic optical trapping has yielded fundamental advances in the science of classical wave-matter interactions. These efforts have drawn attention to the connection between wavefront topology and wave-mediated forces, including the interrelated roles of orbital and spin angular momentum, and the interplay between conservative intensity-gradient forces and non-conservative phase-gradient forces. Holographically structured force landscapes can act as knots, micromachines and even tractor beams and have permeated application areas ranging from biomedical research to quantum computing. Lessons learned from holographic optical trapping recently have been applied to acoustic micromanipulation, with remarkable effect. Beyond an overall leap in the force scales that can be achieved with sound, advances in acoustic trapping are casting new light on the nature of wave-matter interactions, including the role of nonreciprocal wave-mediated interactions in creating novel states of organization.

Keywords: Holographic optical trapping, holographic acoustic trapping, wave-matter interactions

1. INTRODUCTION

The introduction of holographic optical trapping twenty-five years ago[1–4] catalyzed a revolution in the science and applications of wave-matter interactions.[5] The scene for this burst of activity was set by the discovery of optical tweezers, which demonstrated that forces exerted by structured light waves can usefully trap and manipulate small objects.[6] Holographic trapping unlocked the potential of this breakthrough by leveraging computer-generated diffractive elements to sculpt ordinary beams of light into optical force landscapes[7] with tailored properties.[8] The dynamic control afforded by holographically projected arrays of optical tweezers has been used to assemble biological cells and colloidal building blocks into functional structures[9–12] and to organize ultracold atoms into quantum computers.[13–15] Holographic projection also can be used to shape the wavefronts of trap-forming beams,[4,16] thereby affording independent control over forces and torques that are governed by the wave's amplitude, phase and polarization.[17]

Figure 1(a) presents a standard implementation of holographic optical trapping that illustrates both the simplicity and flexibility of the technique and also its fundamental limitation. The wavefronts of a laser beam are imprinted with a computer-generated hologram by a spatial light modulator (SLM). The modified beam then is relayed to the input pupil of a microscope objective lens, which focuses it into the sample. The field in the focal plane of the objective lens at axial coordinate $z = 0$ may be usefully approximated as the Fourier transform of the modified beam in the plane of the SLM.[2,4] This insight provides a basis for computing the holograms needed to project desired volumetric light fields.

The basic implementation in Fig.‘1(a) typically uses a phase-only SLM and linearly polarized light. Elaborations on this theme also can provide independent control over the amplitude[18] and polarization[19,20] of the projected light, allowing for fully vectorial trapping. Even these sophisticated implementations, however, are limited to the single wavelength of light provided by the laser.

Further author information: (Send correspondence to D.G.G.)

D.G.G.: E-mail: david.grier@nyu.edu, Telephone: +1 (212) 647-1471

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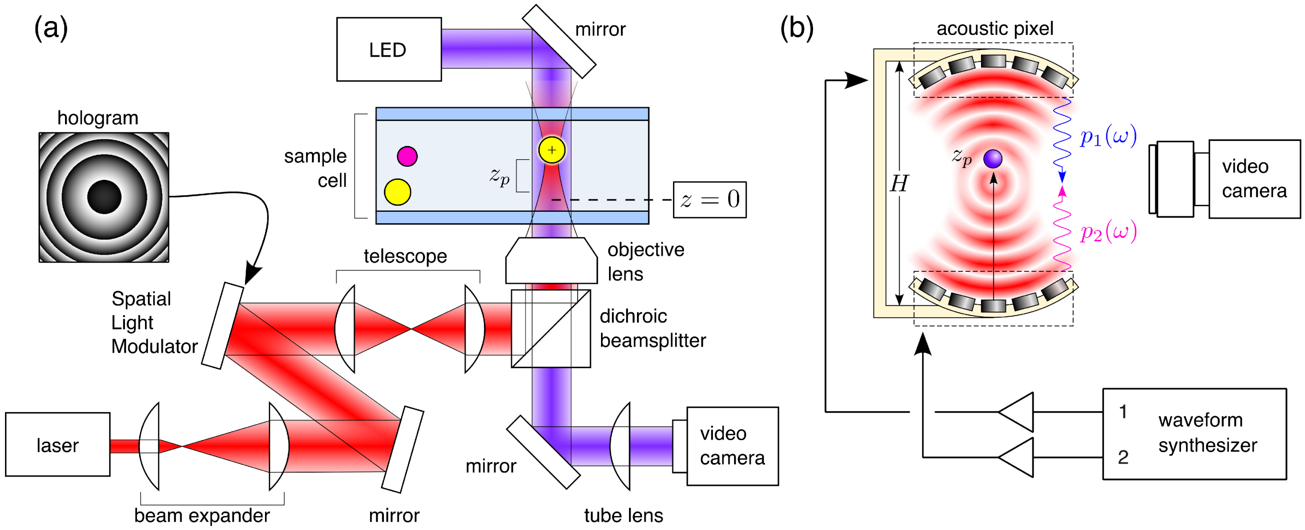

Figure 1. (a) Dynamic holographic optical trapping system integrated with video microscopy. Light from a laser is imprinted with a hologram by a spatial light modulator before being projected into the sample volume by an objective lens. The same lens is used to create images of trapped objects, with the trapping light being separated from imaging light by a dichroic beamsplitter. (b) Acoustic trapping system featuring spectral control of trapped objects' dynamics. Two banks of ultrasonic transducers project spectrally rich sound waves into the cavity. Trapped objects are tracked with a video camera.

Ideas that emerged in the context of holographic optical trapping have inspired initiatives in other branches of inquiry, including the recent introduction of holographic acoustic trapping.[21] Figure 1(b) illustrates a simplified approach to acoustic trapping that has been popularized by the TinyLev project.[22] This instrument comprises two counterpoised acoustic "pixels" that project ultrasound into the air-filled space between them. A similar two-sided design has been realized for holographic optical trapping[23] with significant benefits for trapping strength. Acoustic pixels have the signature benefit of providing independent control over the amplitude, phase and frequency content of the waves they project. While most holographic acoustic trapping systems have mimicked their optical counterparts by projecting sound at a single frequency, recent reports[24,25] have drawn attention to the possibilities of spectral holography implemented with polyphonic projection. Superimposing two counterpropagating harmonic waves at the same frequency creates a standing wave with static acoustic traps along the central vertical axis. Superimposing waves at multiple frequencies can create dynamic trapping patterns.

2. WAVE-MEDIATED FORCES

The forces exerted by waves have been studied since the eighteenth century. A recent reformulation inspired by the goals of holographic optical trapping has also shed new light on the general nature of wave-matter interactions. Most recently, these insights have been brought to bear on the surprisingly complex phenomenology of sound-matter interactions.

The electric field in a monochromatic electromagnetic wave at frequency ω,

$$\mathbf{E}(\mathbf{r}, t) = \sum_{j=1}^{3} u_j(\mathbf{r})\, e^{i\varphi_j(\mathbf{r})}\, e^{-i\omega t}\, \hat{\epsilon}_j \tag{1}$$

has a real-valued amplitude profile, $u_j(\mathbf{r})$, and phase profile, $\varphi_j(\mathbf{r})$, in each of the three Cartesian directions, $\hat{\epsilon}_j$. These components are interrelated through Maxwell's wave equation and contribute to the time-averaged force on an illuminated particle at position $\mathbf{r}$:,[26–28]

$$\mathbf{F}(\mathbf{r}) = \frac{1}{2} \sum_{j=1}^{3} \Re\{\alpha\, E_j \nabla E_j^*\} \tag{2a}$$

$$= \frac{1}{4}\, \alpha' \,\nabla \sum_{j=1}^{3} u_j^2 + \frac{1}{2}\, \alpha'' \sum_{j=1}^{3} u_j^2 \,\nabla \varphi_j, \tag{2b}$$

up to dipole order in the multipole expansion of the incident and scattered fields. Equation (2) is appropriate for an isotropic particle, such as a homogeneous sphere, whose size, a_p, is small enough to satisfy the Rayleigh condition, $ka_p < 0$, where $k = \omega/c_m$ is the wave number of light in a medium with speed of light c_m. The particle's light-scattering properties contribute to the force it experiences through the real and imaginary parts of the dipole scattering coefficient, $\alpha = \alpha' + i\alpha''$. An analogous set of expressions describe contributions to the force due to the light's magnetic field. Magnetic forces tend to be very much weaker than electric forces for dielectric particles, and consequently are less often discussed.

By isolating contributions to the time-averaged force due to the field's amplitudes and phases, Eq. (2b) helps to clarify the nature of the wave-matter interaction and provides guidance for designing trap-forming holograms. The first term in Eq. (2b) describes the manifestly conservative intensity-gradient force responsible for stable trapping in optical tweezers.[6] The second term describes a force that is directed by phase gradients and generally does not conserve the particle's mechanical energy. Phase-gradient forces account for the transfer of orbital angular momentum from helical modes of light to optically trapped particles.[16,29,30] Equation (2b) furthermore reveals that the optical force experienced by an isotropic particle is independent of constant offsets in the relative phases, $\varphi_j(\mathbf{r})$, and therefore is independent of the light's state of polarization, at least at dipole order. This means that any spin-dependent forces arise at higher order in the multipole expansion and therefore are likely to be comparatively weak.

The formulation of acoustic forces follows largely by analogy to the theory of optical forces.[31] Sound travels as a pressure wave,

$$p(\mathbf{r}, t) = u(\mathbf{r})\, e^{i\varphi(\mathbf{r})}\, e^{-i\omega t} \tag{3}$$

through a fluid medium. If the wave's amplitude, $u(\mathbf{r})$, is sufficiently small, the medium may be treated as inviscid, and the pressure serves as a scalar potential for a velocity wave,

$$\mathbf{v}(\mathbf{r}, t) = -\frac{i}{c_m \rho_m} \nabla p, \tag{4}$$

where c_m is the speed of sound in a medium of mass density ρ_m. The time-averaged dipole-order force experienced by a small isotropic sphere in such a sound wave is

$$\mathbf{F}(\mathbf{r}) = \frac{1}{2} \Re\{A\, p \nabla p^*\} + \frac{1}{2} \sum_{j=1}^{3} \Re\{B\, v_j \nabla v_j^*\}, \tag{5}$$

where A and B are the monopole and dipole scattering coefficients, respectively.

There is no equivalent to the monopole scattering term in optical trapping because illuminated objects generally do not have a scalar response to the light's oscillating potential, such as developing an oscillatory charge. The acoustic scattering coefficients for a sphere of radius a_p depend on the density and sound speed of the particle, ρ_p and c_p, respectively.

3. WAVE-MEDIATED INTERACTIONS

Particles moving through wave-generated force landscapes interact with each other through the waves they scatter. A particle at position $\mathbf{r}_1$ in an electromagnetic wave, $\mathbf{E}_0(\mathbf{r})$, scatters the field,

$$\mathbf{E}_s(\mathbf{r}, \mathbf{r}_1) = \alpha_1\, \mathbf{E}(\mathbf{r}_1) \cdot \underline{\mathbf{S}}_1(k(\mathbf{r} - \mathbf{r}_1)), \tag{6}$$

to position $\mathbf{r}$, where $\underline{\mathbf{S}}_1(k\mathbf{r})$ is the tensor-valued Lorenz-Mie scattering function for the particle. The total field incident on a neighboring particle at position $\mathbf{r}_2$ is

$$\mathbf{E}(\mathbf{r}_2, \mathbf{r}_1) = \mathbf{E}_0(\mathbf{r}_2) + \mathbf{E}_s(\mathbf{r}_2, \mathbf{r}_1) \tag{7}$$

Substituting this into Eq. (2a) yields the net force on particle 2:

$$\mathbf{F}_2(\mathbf{r}_2) \approx \mathbf{F}_0(\mathbf{r}_2) + \mathbf{F}_{21}(\mathbf{r}_2, \mathbf{r}_1), \tag{8}$$

where the wave-mediated interaction is dominated by interference between the scattered and incident waves:

$$\mathbf{F}_{21}(\mathbf{r}_2, \mathbf{r}_1) = \frac{1}{2} \sum_{j=1}^{3} \Re\left\{\alpha_2 \left[E_{0j}(\mathbf{r}_2) \nabla_2 E_{sj}^*(\mathbf{r}_2, \mathbf{r}_1) + E_{sj}(\mathbf{r}_1, \mathbf{r}_2) \nabla_2 E_{0j}^*(\mathbf{r}_2)\right]\right\}. \tag{9}$$

This expression is asymmetric under exchange of the labels, 1 and 2, which means that the wave-mediated interaction force is nonreciprocal,

$$\mathbf{F}_{21}(\mathbf{r}_2, \mathbf{r}_1) \neq -\mathbf{F}_{12}(\mathbf{r}_1, \mathbf{r}_2), \tag{10}$$

unless the two particles have identical scattering characteristics. The difference represents a net force acting on the two particles' center of mass. This apparent violation of Newton's third law arises because the two-particle system is not closed; the recoil momentum is carried away by the scattered wave.

The reciprocal part of the wave-mediated interaction is oscillatory in the particles' separation, and generally is attractive at short range. In the context of optical trapping, this wave-mediated attaction is known as optical binding.[32] In the context of acoustic trapping, it is referred to as the secondary Bjerknes interaction.[33]

The possibility of observing dynamical effects arising from nonreciprocal wave-mediated interactions has been proposed in theory[34] and in simulation.[35] Figure 2 presents a straightforward experimental observation of the phenomenon. In this case, millimeter-scale beads of expanded polystyrene are levitated in an acoustic trapping pattern at 40 kHz created with the instrument depicted in Fig. 1(b). Three beads of different diameters are trapped as a cluster in the central node. Additional beads of the same composition are trapped in the neighboring nodes above and below the three-particle cluster and therefore are separated from each other by half the acoustic wavelength. The traps are strong enough that gravity may be neglected. For particles that differ only in their radius, the net nonreciprocal force tends to drive the pair of particles along the line connecting their centers with the larger particle in the lead. Consequently, the cluster in Fig. 2 tends to revolve around the central $\hat{z}$ axis in the right-handed sense. This rapid motion does not result from the topology of the standing wave, but rather is a consequence of the consolidated cluster's heterogeneous wave-scattering properties.

4. DISCUSSION

The nonreciprocal part of a wave-mediated interaction does mechanical work on pairs of particles and, in that limited sense, violates conservation of energy for the system of particles. Nonreciprocal scattering therefore joins engineered phase gradients as a mechanism to transfer energy between the projected wave and collections of particles. More generally, a suitably structured wave can serve as a reservoir of energy that objects can tap into to establish distinct states of organization. Objects propelled by phase-gradient forces are examples of driven matter and can self-organize in non-trivial ways.[36–38] The trajectory of an individual driven particle, however, is programmed into the projected force landscape.

Motion driven by nonreciprocal wave-mediated interactions has a different nature. As the example in Fig. 2 demonstrates, individual particles might not move at all within the force field created by the wave. The ability of collections of particles to harvest energy from the wave depends not just on their individual scattering properties but also on their instantaneous configuration. The ability of such collections of objects to transduce energy from a reservoir into directed motion identifies them as a class of active matter.[35,39] Activity based on non-reciprocal pair interactions appears not to have been previously identified as a distinct mechanism for self-organization and has distinctive features. The transduction rate in such systems, for example, is not an intrinsic property of

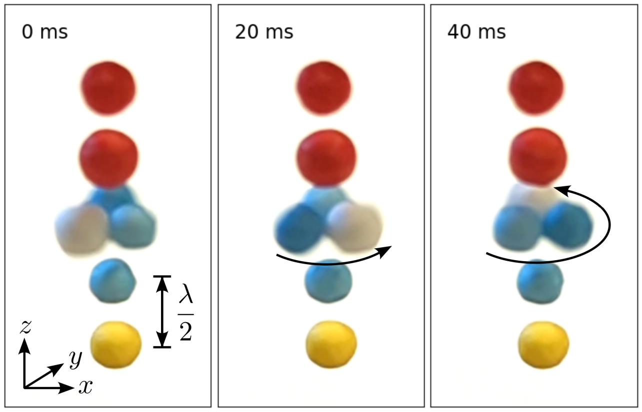

Figure 2. Video 1: Three frames from a video sequence that captures the motions of three expanded polystyrene beads as they revolve rapidly about the axis of a static, non-topological acoustic trap. The particles move because of nonreciprocal wave-mediated interactions. Additional beads are trapped in pressure nodes above and below the central cluster along the central axis of the instrument from Fig. 1(a). http://dx.doi.org/10.1117/12.3009931.1

the constituent particles, but rather depends on their arrangement in space. This form of activity, therefore, is conditioned on the system's organization.

Conditional activity of the kind described here arises naturally in heterogeneous systems whose constituents have different wave-scattering characteristics. It also can arise, however, in collections of nominally identical scatterers if those particles are suitably arranged.[35,40] Conditional activity therefore may be viewed as an emergent property of many-body systems that have access to nonreciprocal wave-mediated interactions. This useful interpretation of the phenomenology is just one product of the ongoing convergence of ideas from the fields of optical and acoustic wave-matter interactions. It signals a larger opportunity to leverage emergent properties to achieve large-scale states of organization that might not be amenable to optical or acoustic manipulation directly.

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REFERENCES

[1] Dufresne, E. R. and Grier, D. G., "Optical tweezer arrays and optical substrates created with diffractive optics," *Rev. Sci. Instr.* **69**(5), 1974–1977 (1998).

[2] Reicherter, M., Haist, T., Wagemann, E., and Tiziani, H. J., "Optical particle trapping with computer-generated holograms written on a liquid-crystal display," *Opt. Lett.* **24**(9), 608–610 (1999).

[3] Mogensen, P. C. and Glückstad, J., "Dynamic array generation and pattern formation for optical tweezers," *Opt. Commun.* **175**(1-3), 75–81 (2000).

[4] Curtis, J. E., Koss, B. A., and Grier, D. G., "Dynamic holographic optical tweezers," *Opt. Commun.* **207**(1-6), 169–175 (2002).

[5] Grier, D. G., "A revolution in optical manipulation," *Nature* **424**(6950), 810–816 (2003).

[6] Ashkin, A., Dziedzic, J. M., Bjorkholm, J. E., and Chu, S., "Observation of a single-beam gradient force optical trap for dielectric particles," *Opt. Lett.* **11**(5), 288–290 (1986).

[7] Korda, P. T., Spalding, G. C., and Grier, D. G., "Evolution of a colloidal critical state in an optical pinning potential landscape," *Phys. Rev. B* **66**(2), 024504 (2002).

[8] Grier, D. G. and Roichman, Y., "Holographic optical trapping," *Appl. Opt.* **45**(5), 880–887 (2006).

[9] Jordan, P., Leach, J., Padgett, M., Blackburn, P., Isaacs, N., Goksör, M., Hanstorp, D., Wright, A., Girkin, J., and Cooper, J., "Creating permanent 3D arrangements of isolated cells using holographic optical tweezers," *Lab Chip* **5**(11), 1224–1228 (2005).

[10] Nedev, S., Urban, A. S., Lutich, A. A., and Feldmann, J., "Optical force stamping lithography," *Nano Lett.* **11**(11), 5066–5070 (2011).

[11] Marago, O. M., Jones, P. H., Gucciardi, P. G., Volpe, G., and Ferrari, A. C., "Optical trapping and manipulation of nanostructures," *Nature Nanotech.* **8**(11), 807–819 (2013).

[12] Kirkham, G. R., Britchford, E., Upton, T., Ware, J., Gibson, G. M., Devaud, Y., Ehrbar, M., Padgett, M., Allen, S., Buttery, L. D., et al., "Precision assembly of complex cellular microenvironments using holographic optical tweezers," *Sci. Rep.* **5**(1), 8577 (2015).

[13] Bergamini, S., Darquié, B., Jones, M., Jacubowiez, L., Browaeys, A., and Grangier, P., "Holographic generation of microtrap arrays for single atoms by use of a programmable phase modulator," *JOSA B* **21**(11), 1889–1894 (2004).

[14] Vala, J., Whaley, K. B., and Weiss, D. S., "Quantum error correction of a qubit loss in an addressable atomic system," *Phys. Rev. A* **72**(5), 052318 (2005).

[15] Bakr, W. S., Gillen, J. I., Peng, A., Fölling, S., and Greiner, M., "A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice," *Nature* **462**(7269), 74–77 (2009).

[16] Simpson, N., Allen, L., and Padgett, M., "Optical tweezers and optical spanners with laguerre-gaussian modes," *J. Mod. Opt.* **43**(12), 2485–2492 (1996).

[17] Ruffner, D. B. and Grier, D. G., "Optical forces and torques in nonuniform beams of light," *Phys. Rev. Lett.* **108**(17), 173602 (2012).

[18] Jesacher, A., Maurer, C., Schwaighofer, A., Bernet, S., and Ritsch-Marte, M., "Full phase and amplitude control of holographic optical tweezers with high efficiency," *Opt. Express* **16**(7), 4479–4486 (2008).

[19] Maurer, C., Jesacher, A., Fürhapter, S., Bernet, S., and Ritsch-Marte, M., "Tailoring of arbitrary optical vector beams," *New J. Phys.* **9**(3), 78 (2007).

[20] Bhebhe, N., Williams, P. A., Rosales-Guzmán, C., Rodriguez-Fajardo, V., and Forbes, A., "A vector holographic optical trap," *Sci. Rep.* **8**(1), 17387 (2018).

[21] Marzo, A. and Drinkwater, B. W., "Holographic acoustic tweezers," *PNAS* **116**(1), 84–89 (2019).

[22] Marzo, A., Barnes, A., and Drinkwater, B. W., "TinyLev: A multi-emitter single-axis acoustic levitator," *Rev. Sci. Instr.* **88**(8) (2017).

[23] Eriksen, R. L., Daria, V. R., and Glückstad, J., "Fully dynamic multiple-beam optical tweezers," *Opt. Express* **10**(14), 597–602 (2002).

[24] Wang, Q., Riaud, A., Zhou, J., Gong, Z., and Baudoin, M., "Acoustic radiation force on small spheres due to transient acoustic fields," *Phys. Rev. Appl.* **15**, 044034 (Apr 2021).

[25] Kandemir, M., Beelen, M., Wagterveld, R., Yntema, D., and Keesman, K., "Dynamic acoustic fields for size selective particle separation on centimeter scale," *J. Sound Vib.* **490**, 115723 (2021).

[26] Chaumet, P. C. and Nieto-Vesperinas, M., "Time-averaged total force on a dipolar sphere in an electromagnetic field," *Opt. Lett.* **25**(15), 1065–1067 (2000).

[27] Ruffner, D. B. and Grier, D. G., "Comment on "scattering forces from the curl of the spin angular momentum of a light field"," *Phys. Rev. Lett.* **111**(5), 059301 (2013).

[28] Yevick, A., Ruffner, D. B., and Grier, D. G., "Tractor beams in the rayleigh limit," *Phys. Rev. A* **93**(4), 043807 (2016).

[29] Gahagan, K. and Swartzlander, G. J., "Optical vortex trapping of particles," *Opt. Lett.* **21**(11), 827–829 (1996).

[30] Friese, M. E., He, H., Heckenberg, N., and Rubinsztein-Dunlop, H., "Transfer of angular momentum to absorbing particles from a laser beam with a phase singularity," in [*Laser Optics '95: Nonlinear Dynamics in Lasers*], **2792**, 190–195, SPIE (1996).

[31] Abdelaziz, M. A. and Grier, D. G., "Acoustokinetics: Crafting force landscapes from sound waves," *Phys. Rev. Res.* **2**(1), 013172 (2020).

[32] Burns, M. M., Fournier, J.-M., and Golovchenko, J. A., "Optical binding," *Phys. Rev. Lett.* **63**(12), 1233 (1989).

[33] Silva, G. T. and Bruus, H., "Acoustic interaction forces between small particles in an ideal fluid," *Phys. Rev. E* **90**(6), 063007 (2014).

[34] Sukhov, S., Shalin, A., Haefner, D., and Dogariu, A., "Actio et reactio in optical binding," *Opt. Express* **23**(1), 247–252 (2015).

[35] Clair, N. S., Davenport, D., Kim, A. D., and Kleckner, D., "Dynamics of acoustically bound particles," *Phys. Rev. Res.* **5**(1), 013051 (2023).

[36] Sokolov, Y., Frydel, D., Grier, D. G., Diamant, H., and Roichman, Y., "Hydrodynamic pair attractions between driven colloidal particles," *Phys. Rev. Lett.* **107**(15), 158302 (2011).

[37] Figliozzi, P., Sule, N., Yan, Z., Bao, Y., Burov, S., Gray, S. K., Rice, S. A., Vaikuntanathan, S., and Scherer, N. F., "Driven optical matter: Dynamics of electrodynamically coupled nanoparticles in an optical ring vortex," *Phys. Rev. E* **95**(2), 022604 (2017).

[38] Sazan, H., Piperno, S., Layani, M., Magdassi, S., and Shpaisman, H., "Directed assembly of nanoparticles into continuous microstructures by standing surface acoustic waves," *J. Colloid Interface Sci.* **536**, 701–709 (2019).

[39] Ramaswamy, S., "The mechanics and statistics of active matter," *Annu. Rev. Condens. Matter Phys.* **1**(1), 323–345 (2010).

[40] Wu, B., VanSaders, B., Lim, M. X., and Jaeger, H. M., "Hydrodynamic coupling melts acoustically levitated crystalline rafts," *PNAS* **120**(29), e2301625120 (2023).