UFIFT-QG-24-01

Gauge Independent Logarithms from Inflationary Gravitons

D. Glavan[1*], S. P. Miao[2⋆], T. Prokopec[3†] and R. P. Woodard[4‡]

[1] *CEICO, Institute of Physics of the Czech Academy of Sciences (FZU), Na Slovance 1999/2, 182 21 Prague 8, CZECH REPUBLIC*

[2] *Department of Physics, National Cheng Kung University, No. 1 University Road, Tainan City 70101, TAIWAN*

[3] *Institute for Theoretical Physics, Spinoza Institute & EMMEΦ, Utrecht University, Postbus 80.195, 3508 TD Utrecht, THE NETHERLANDS*

[4] *Department of Physics, University of Florida, Gainesville, FL 32611, UNITED STATES*

ABSTRACT

PACS numbers: 04.50.Kd, 95.35.+d, 98.62.-g

Dependence on the graviton gauge enters the conventional effective field equations because they fail to account for quantum gravitational correlations with the source which excites the effective field and with the observer who measures it. Including these correlations has been shown to eliminate gauge dependence in flat space background. We generalize the technique to de Sitter background for the case of the 1-loop graviton corrections to the exchange potential of a massless, minimally coupled scalar.

* e-mail: glavan@fzu.cz
⋆ e-mail: spmiao5@mail.ncku.edu.tw
† e-mail: T.Prokopec@uu.nl
‡ e-mail: woodard@phys.ufl.edu

1 Introduction

The background geometry of cosmology with scale factor $a(t)$ is,

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \qquad \Longrightarrow H(t) \equiv \frac{\dot{a}}{a} \quad , \quad \epsilon(t) \equiv -\frac{\dot{H}}{H^2} \; . \tag{1}$$

The Hubble parameter $H(t)$ measures the expansion rate while the first slow roll parameter $\epsilon(t)$ measures the rate at which $H(t)$ changes. Inflation is characterized by positive $H(t)$ and $0 \leq \epsilon(t) < 1$, which means that both the first and second time derivatives of $a(t)$ are positive. The most highly accelerated geometry is de Sitter, with $\epsilon(t) = 0$ and H constant, which serves as a paradigm for primordial inflation.

The accelerated expansion of inflation can rip massless, not conformally invariant virtual quanta out of the vacuum. That process is responsible for the primordial power spectra of scalars [1] and gravitons [2], but these quanta must, at some level, interact with themselves and with other particles to affect kinematics and long-range forces. One studies these changes on a field by computing its 1PI (one-particle-irreducible) 2-point function and then using that to quantum-correct the linearized field equation. For a scalar field the 1PI 2-point function is known as the self-mass $-iM^2(x;x')$. If the scalar is massless and minimally coupled (MMC) then its quantum-corrected, linearized field equation is,

$$\partial_\mu \Big[\sqrt{-g} \, g^{\mu\nu} \partial_\nu \varphi(x) \Big] - \int d^4x' \, M^2(x;x') \varphi(x') = J(x) \; . \tag{2}$$

Here $g_{\mu\nu}(x)$ is the background metric and $J(x)$ is the source. Zero source solutions tell one about modifications to the kinematics of single particle solutions whereas setting J to a static point source determines the exchange potential.

Effects mediated by inflationary scalars are generally stronger and simpler to compute, but they suffer from being model dependent. In contrast, graviton-mediated effects are weaker, and much more difficult to work out, but are completely generic because general relativity is the unique low energy effective field theory of gravity [3–8]. Despite the complexity of working with gravitons on an inflationary background, six cases have been found of significant effects on de Sitter:

- Growth of the fermion field strength [9];

- Temporal and spatial growth of the electric force [10];
- Growth of the of the photon field strength [11];
- Growth of the graviton field strength [12];
- Spatial suppression of the force carried by an MMC scalar [13]; and
- Temporal suppression of the gravitational force [14].

A standard concern when studying vector-mediated or tensor-mediated corrections to effective field equations is how to distinguish physical effects from gauge artifacts such as the acausality apparently implied by the instantaneous Coulomb potential. Exhibiting the gauge dependence of 1PI n-point functions is relatively simple in flat space [15], but very difficult for inflationary backgrounds. The simplest gauge [16, 17] happens to break de Sitter invariance, which has been a point of contention for decades [18–26]. However, other gauges are so difficult to use that, of the ten graviton loops so far evaluated [13, 27–36],[1] only one was performed using a different gauge. This one was of the graviton contributions to the vacuum polarization [34] in a 1-parameter family of de Sitter invariant gauges [39]. When it was used to check the growth of the photon field strength, the same time dependence was found as in the simplest gauge [11], but with a different multiplicative factor [40]. One can therefore conclude that there is nothing particularly misleading about the simplest gauge, but that 1PI functions on de Sitter harbor the same sort of gauge dependence as they do on flat space.

It was recently shown that the usual effective field equations are gauge dependent because they ignore quantum gravitational correlations with the source which excites the effective field and with the observer who measures it [41]. When these correlations are restored on flat space background, the effective field equations for both an MMC scalar [41] and for electromagnetism [42] become gauge independent. One restores source and observer correlations by writing down the position-space diagrams which contribute to t-channel scattering between two massive source particles. These diagrams consist of 4-point, 3-point and 2-point Green's functions. A series of identities due to Donoghue and collaborators [3, 43–45] permits one to extract the t-channel poles from the 3-point and 4-point diagrams, which reduces them to 2-point form. Then the propagator equation is used to regard these 2-point forms as corrections to the 1PI 2-point function, and hence to the effective field equation. Our goal here is to generalize this technique to de Sitter for graviton corrections to an MMC scalar.

[1] An independent computation of graviton corrections to the self-mass of a massless, conformally coupled scalar [37, 38] also used the simplest gauge, but disagrees with [36].

This paper contains seven sections. In section 2 we review the Feynman rules, including the massive source-observer field, along with gravity and the MMC scalar. We also generalize the Donoghue identities from flat space to de Sitter. It turns out that five classes of diagrams contribute to t-channel scattering, in addition to the self-mass. These diagrams are presented in section 3. Section 4 demonstrates an important cancellation which occurs for any background geometry and in any gauge. The remaining diagrams are reduced in section 5, and incorporated as corrections to the renormalized self-mass. Section 6 gives our conclusions, including the crucial issue of whether or not the 1-loop correction to the scalar exchange potential contains a large logarithm.

2 Feynman Rules and Reduction Strategy

The purpose of this section is to present the basics of our computation. We begin with the full action which describes the massive source-observer field Ψ, in addition to gravity and the massless, minimally coupled scalar whose exchange potential it modifies. The necessary Feynman rules are presented, then the general strategy is described. The section closes with some useful derivative identities.

2.1 Propagators and Vertices

We model the source and observer as a massive, minimally coupled scalar Ψ which has a cubic coupling to the massless, minimally coupled scalar φ. The bare Lagrangian is,

$$\mathcal{L} = \frac{[R-(D-2)\Lambda]\sqrt{-g}}{16\pi G} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} \\ -\frac{1}{2}\partial_\mu\Psi\partial_\nu\Psi g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\Big(m^2+\lambda\varphi\Big)\Psi^2\sqrt{-g} \; . \qquad (3)$$

Here G is Newton's constant, Λ is the positive cosmological constant and D is the dimension of spacetime, left arbitrary to facilitate the use of dimensional regularization. It is convenient to change the time variable from co-moving time t to conformal time η, with $d\eta \equiv dt/a(t)$. This makes the de Sitter background conformal to flat space,

$$ds^2 = a^2\Big[-d\eta^2 + d\vec{x}\cdot d\vec{x}\Big] \qquad , \qquad a = -\frac{1}{H\eta} \quad , \quad H \equiv \sqrt{\frac{\Lambda}{D-1}} \; . \qquad (4)$$

We adopt the convention that the indices of partial derivatives are raised and lowered with the Minkowski metric, $\partial^\mu \equiv \eta^{\mu\nu}\partial_\nu$.

Our analysis employs four scalar propagators. The source and observer (Ψ) propagator $i\Delta_m(x;x')$ has a mass $m \gg H$ which is assumed much larger than the Hubble parameter and spatial momenta. The massless scalar (φ) propagator $i\Delta_A(x;x')$ has zero mass, while the graviton propagator requires three scalar propagators $i\Delta_I(x;x')$ whose masses are,

$$M_A^2 = 0 \qquad , \qquad M_B^2 = (D\!-\!2)H^2 \qquad , \qquad M_C^2 = 2(D\!-\!3)H^2 \; . \tag{5}$$

All four propagators obey equations involving the differential operator $\mathcal{D} \equiv \partial_\mu[\sqrt{-g}\, g^{\mu\nu}\partial_\nu] = \partial^\mu a^{D-2}\partial_\mu$,

$$\Bigl(\mathcal{D} - a^D m^2\Bigr) i\Delta_m(x;x') = i\delta^D(x-x') = \Bigl(\mathcal{D} - a^D M_I^2\Bigr) i\Delta_I(x;x') \; . \tag{6}$$

The three propagators $i\Delta_I(x;x')$ can be usefully expanded as differences of a series in D-dependent powers of $aa'\Delta x^2 \equiv aa'(x-x')^\mu(x-x')^\nu\eta_{\mu\nu} + i\epsilon$ and a series of integer powers of the same quantity,

$$\begin{aligned} i\Delta_A = {}& \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}}\Biggl\{\frac{1}{[aa'\Delta x^2]^{\frac{D}{2}-1}} + \frac{D(D-2)}{8(D-4)}\frac{H^2}{[aa'\Delta x^2]^{\frac{D}{2}-2}} + \dots\Biggr\} \\ & - \frac{(\frac{H}{2})^{D-4}}{4\pi^{\frac{D}{2}}}\frac{\Gamma(D-1)}{2\Gamma(\frac{D}{2})}\Biggl\{0 + \tfrac12 H^2\Bigl[\pi\cot\Bigl(\frac{D\pi}{2}\Bigr) - \ln(aa')\Bigr] + \dots\Biggr\}, \end{aligned} \tag{7}$$

$$\begin{aligned} i\Delta_B = {}& \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}}\Biggl\{\frac{1}{[aa'\Delta x^2]^{\frac{D}{2}-1}} + \Bigl(\frac{D-2}{8}\Bigr)\frac{H^2}{[aa'\Delta x^2]^{\frac{D}{2}-2}} + \dots\Biggr\} \\ & - \frac{(\frac{H}{2})^{D-4}}{4\pi^{\frac{D}{2}}}\frac{\Gamma(D-1)}{2\Gamma(\frac{D}{2})}\Biggl\{0 + \frac{H^2}{2(D-2)} + \dots\Biggr\}, \end{aligned} \tag{8}$$

$$\begin{aligned} i\Delta_C = {}& \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}}\Biggl\{\frac{1}{[aa'\Delta x^2]^{\frac{D}{2}-1}} + \Bigl(\frac{D-6}{8}\Bigr)\frac{H^2}{[aa'\Delta x^2]^{\frac{D}{2}-2}} + \dots\Biggr\} \\ & - \frac{(\frac{H}{2})^{D-4}}{4\pi^{\frac{D}{2}}}\frac{\Gamma(D-1)}{2\Gamma(\frac{D}{2})}\Biggl\{0 - \frac{H^2}{2(D-2)(D-3)} + \dots\Biggr\}. \end{aligned} \tag{9}$$

The neglected terms in expressions (7-9) are not only less singular at coincidence ($x'^\mu = x^\mu \Longrightarrow \Delta x^2 = 0$), they also vanish in $D = 4$ dimensions.

Indeed, even summing the order $[aa'\Delta x^2]^{2-\frac{D}{2}}$ and $[aa'\Delta x^2]^0$ terms of $i\Delta_B$ and $i\Delta_C$ vanishes in $D=4$. This fact has great significance for our calculation.

The great thing about the simplest gauge is that the graviton propagator (in conformal coordinates) consists of a sum of spacetime constant tensor factors multiplying the three scalar propagators $i\Delta_I(x;x')$ [16, 17],

$$i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x') = \Bigl[2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} - \frac{2\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}}{D-3}\Bigr] i\Delta_A(x;x') - 4\delta^0{}_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0{}_{\sigma)} i\Delta_B(x;x') + \frac{2}{(D-3)(D-2)}\Bigl[\overline{\eta}_{\mu\nu} + (D-3)\delta^0{}_\mu\delta^0{}_\nu\Bigr]\Bigl[\overline{\eta}_{\rho\sigma} + (D-3)\delta^0{}_\rho\delta^0{}_\sigma\Bigr] i\Delta_C(x;x') \; . \qquad (10)$$

(Parenthesized indices are symmetrized.) The three tensor factors involve the Minkowski metric $\eta_{\mu\nu}$, the Kronecker delta function $\delta^0{}_\mu$, and a combination $\overline{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0{}_\mu\delta^0{}_\nu$ which gives the purely spatial part of the Minkowski metric. The only contraction we require is,

$$\eta^{\rho\sigma} \times i\Bigl[{}_{\mu\nu}\Delta_{\rho\sigma}\Bigr](x;x') = -\frac{4\eta_{\mu\nu}}{D-2} i\Delta_C - \frac{4\overline{\eta}_{\mu\nu}}{D-3}\Bigl(i\Delta_A - i\Delta_C\Bigr) \; . \qquad (11)$$

Note that summing the three tensor factors in expression (10) produces the tensor factor of the de Donder gauge propagator in flat space,

$$\Bigl[2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} - \frac{2\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}}{D-3}\Bigr] - 4\delta^0{}_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0{}_{\sigma)} + \frac{2}{(D-3)(D-2)}\Bigl[\overline{\eta}_{\mu\nu} + (D-3)\delta^0{}_\mu\delta^0{}_\nu\Bigr] \times \Bigl[\overline{\eta}_{\rho\sigma} + (D-3)\delta^0{}_\rho\delta^0{}_\sigma\Bigr] = 2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2\eta_{\mu\nu}\eta_{\rho\sigma}}{D-2} \; . \qquad (12)$$

This suggests the advantages of singling out a particular scalar propagator — call it $i\Delta_J$ — and then expanding the other two propagators around it,

$$i\Delta_I = i\Delta_J + (i\Delta_I - i\Delta_J) \; . \qquad (13)$$

Substituting (13) into (10) will then produce $i\Delta_J$ times the flat space tensor (12), plus a series of differences of propagators. These differences are less singular at coincidence because expressions (7-9) show that all three propagators have the same leading singularity $\Gamma(\frac{D}{2}-1)/4\pi^{\frac{D}{2}}[aa'\Delta x^2]^{\frac{D}{2}-1}$. The best choice for our calculation is $J=C$, which gives,

$$i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x') = \Bigl[2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2\eta_{\mu\nu}\eta_{\rho\sigma}}{D-2}\Bigr] i\Delta_C + 4\eta_{\mu)(\rho}\overline{\eta}_{\sigma)(\nu}\Bigl(i\Delta_B - i\Delta_C\Bigr) + 2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu}\Bigl(i\Delta_A - 2i\Delta_B + i\Delta_C\Bigr) - \frac{2\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}}{D-3}\Bigl(i\Delta_A - i\Delta_C\Bigr) \; . \qquad (14)$$

This form is especially effective for us because the 4 indices of the graviton propagator are typically contracted into derivatives from vertices and any spatial derivatives are assumed small compared to the source observer mass m. In the 3-point and 4-point contributions these vertex derivatives must supply at least two factors of m to cancel inverse factors from the Donoghue Identities. Hence we can neglect all the terms on the last line of (14) which have only spatial indices. It turns out that even the last term on the first line of (14) drops out because it is proportional to the difference $(i\Delta_B - i\Delta_C)$, which lacks the leading singularity and actually vanishes in $D = 4$ dimensions.

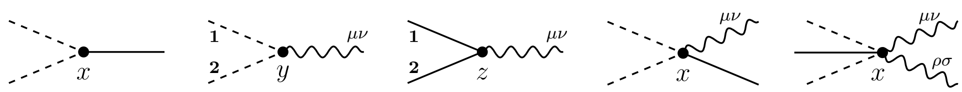

Figure 1: Vertices A-E from left to right, given in expressions (15-19). Solid lines represent the massless, minimally coupled scalar, dashed lines represent the massive source-observer field, and wavy lines represent the graviton.

Figure 1 depicts the vertices we require. From left to right they are,

$$\text{Vertex A} = -i\lambda a_x^D \; , \tag{15}$$

$$\text{Vertex B} = -i\kappa a_y^{D-2}\Big[-\partial_1^\mu \partial_2^\nu + \tfrac{1}{2}\eta^{\mu\nu}(\partial_1\cdot\partial_2 + a_y^2 m^2)\Big] \; , \tag{16}$$

$$\text{Vertex C} = -i\kappa a_z^{D-2}\Big[-\partial_1^\mu \partial_2^\nu + \tfrac{1}{2}\eta^{\mu\nu}\partial_1\cdot\partial_2\Big] \; , \tag{17}$$

$$\text{Vertex D} = -\tfrac{i}{2}\kappa\lambda a_x^D \eta^{\mu\nu} \; , \tag{18}$$

$$\text{Vertex E} = -i\kappa^2 \lambda a_x^D\Big[\tfrac{1}{8}\eta^{\mu\nu}\eta^{\rho\sigma} - \tfrac{1}{4}\eta^{\mu\rho}\eta^{\sigma\nu}\Big] \; . \tag{19}$$

The subscripts on scale factors indicate the coordinate argument, for example, $a_x \equiv -\frac{1}{Hx^0}$ (with $x^0 < 0$). Subscripts on derivatives indicate which leg is differentiated, and we have not bothered to symmetrize indices because they are contracted into the symmetric graviton propagator.

2.2 Reduction Strategy

As discussed in section 1, the gauge dependence of a 1PI 2-point function can be removed by first writing down (in position space) all the diagrams which contribute to the t-channel scattering process $\Psi_1(x) + \Psi_3(x') \longrightarrow$

$\Psi_2(y) + \Psi_4(y')$. This consists of a collection of 2-point, 3-point and 4-point Green's functions,

$$-iV(x;x') \;\; , \;\; -iV(x;x';y) \;\; , \;\; -iV(x;x';y') \;\; , \;\; -iV(x;x';y;y') \; . \qquad (20)$$

The 2-point and 3-point Green's functions must be multiplied by appropriate delta functions to fix y^{μ} and y'^{μ}, for example,

$$-iV(x;x') \longrightarrow \delta^D(x-y)\delta^D(x'-y') \times -iV(x;x') \; . \qquad (21)$$

One then extracts the t-channel poles from 3-point and 4-point contributions using identities derived by Donoghue and collaborators [3, 43–45] and easy to check from general results [46, 47]. In flat position space one can reduce 3-point Green's functions to 2-point form using the relation [41],

$$i\Delta_m(x;y) i\Delta(x;x') i\Delta(y;x') \longrightarrow \frac{i\delta^D(x-y)}{2m^2} \Big[i\Delta(x;x') \Big]^2 \; , \qquad (22)$$

where $i\Delta(x;x')$ denotes the massless scalar propagator in flat space. Reducing the flat 4-point Green's functions to 2-point form requires two such identities [44, 45],

$$i\Delta_m(x;y) i\Delta_m(x';y') i\Delta(x;x') i\Delta(y;y') \longrightarrow -\frac{i}{m^2} \Big[1 - \Big(\frac{\overline{\partial}_x \cdot \overline{\partial}_{x'} - m^2}{3m^2} \Big) \Big] \times \Big\{ \delta^D(x-y) \delta^D(x'-y') \int d^D z \Big[i\Delta(x;z) \Big]^2 i\Delta(z;x') \Big\} \; , \qquad (23)$$

$$i\Delta_m(x;y) i\Delta_m(x';y') i\Delta(x;y') i\Delta(x';y) \longrightarrow +\frac{i}{m^2} \Big[1 + \Big(\frac{\overline{\partial}_x \cdot \overline{\partial}_{y'} + m^2}{3m^2} \Big) \Big] \times \Big\{ \delta^D(x-y) \delta^D(x'-y') \int d^D z \Big[i\Delta(x;z) \Big]^2 i\Delta(z;x') \Big\} \; , \qquad (24)$$

where the over-lined derivatives are understood to act on the external wave functions. Note that the integrals at the end of expressions (23) and (24) is symmetric under interchange of x^{μ} and x'^{μ}.

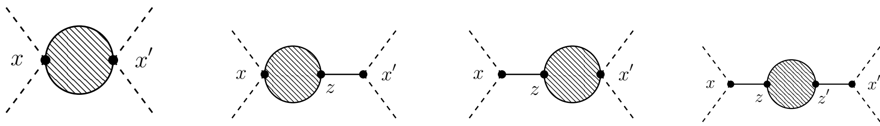

Figure 2: The four classes of diagrams (α through δ, from left to right) produced after reducing diagrams with source and observer correlations to 2-point form. The shaded circles are $-i\lambda^2(a_x a_{x'})^D f_\alpha(x;x')$, $-i\lambda a_x^D f_\beta(x;z')$, $-i\lambda a_{x'}^D f_\gamma(z;x')$ and $-if_\delta(z;z')$, respectively.

Figure 2 shows the generic 2-point forms produced by these reductions. We can regard each of these 2-point contributions as a correction to the 1PI 2-point function by using the propagator equation $i\delta^D(x-z) = \mathcal{D}_z i\Delta_A(x;z)$ and then partially integrating. We will give all the steps for the leftmost diagram,

$$\text{Class}\,\alpha \equiv \lambda^2(a_x a_{x'})^D \times -if_\alpha(x;x') \; , \tag{25}$$

$$= -i\lambda a_x^D \!\int\! d^Dz\, i\delta^D(x-z) \times -i\lambda a_{x'}^D \!\int\! d^Dz'\, i\delta^D(x'-z') \times -if_\alpha(z;z') \; , \tag{26}$$

$$= -i\lambda a_x^D \!\int\! d^Dz\, \mathcal{D}_z i\Delta_A(x;z) \times -i\lambda a_{x'}^D \!\int\! d^Dz'\, \mathcal{D}_{z'} i\Delta_A(x';z') \times -if_\alpha(z;z') \; , \tag{27}$$

$$= -i\lambda a_x^D \!\int\! d^Dz\, i\Delta_A(x;z) \times -i\lambda a_{x'}^D \!\int\! d^Dz'\, i\Delta_A(x';z') \times -i\mathcal{D}_z \mathcal{D}_{z'} f_\alpha(z;z') \; . \tag{28}$$

We merely give the results for the remaining diagrams,

$$\text{Class}\,\beta \equiv -i\lambda a_{x'}^D \!\int\! d^Dz'\, i\Delta_A(x';z') \times -i\lambda a_x^D f_\beta(x;z') \; , \tag{29}$$

$$= -i\lambda a_x^D \!\int\! d^Dz\, i\Delta_A(x;z) \times -i\lambda a_{x'}^D \!\int\! d^Dz'\, i\Delta_A(x';z') \times -i\mathcal{D}_z f_\beta(z;z') \tag{30}$$

$$\text{Class}\,\gamma \equiv -i\lambda a_x^D \!\int\! d^Dz\, i\Delta_A(x;z) \times -i\lambda a_{x'}^D f_\gamma(z;x') \; , \tag{31}$$

$$= -i\lambda a_x^D \!\int\! d^Dz\, i\Delta_A(x;z) \times -i\lambda a_{x'}^D \!\int\! d^Dz'\, i\Delta_A(x';z') \times -i\mathcal{D}_{z'} f_\gamma(z;z') \; , \tag{32}$$

$$\text{Class}\,\delta \equiv -i\lambda a_x^D \!\int\! d^Dz\, i\Delta_A(x;z) \times -i\lambda a_{x'}^D \!\int\! d^Dz'\, i\Delta_A(x';z') \times -if_\delta(z;z') \; . \tag{33}$$

The gauge invariant self-mass is the sum of all four classes,

$$M^2_{\rm inv}(x;x') = \mathcal{D}_x \mathcal{D}_{x'} f_\alpha(x;x') + \mathcal{D}_x f_\beta(x;x') + \mathcal{D}_{x'} f_\gamma(x;x') + f_\delta(x;x') \; . \tag{34}$$

It remains to generalize the Donoghue Identities (22-24) from flat space to de Sitter. This is a matter of preserving general coordinate invariance and taking account of the scalar propagator $i\Delta_A$ possibly differing from the scalar propagators $i\Delta_I$ in the graviton propagator. For the 3-point identity (22) the required generalization is,

$$i\Delta_m(x;y) i\Delta_A(x;x') i\Delta_I(y;x') \longrightarrow \frac{i\delta^D(x-y)}{2m^2 a_x^D}\, i\Delta_A(x;x') i\Delta_I(x;x') \; . \qquad (35)$$

We also need a 3-point identity with a single derivative,

$$i\Delta_m(x;y) i\Delta_A(x;z) \partial_y^\mu i\Delta_C(y;z) \longrightarrow (\overline{\partial}_y^\mu + \overline{\partial}_z^\mu) \frac{i\delta^D(x-y)}{2m^2 a_x^D}\, i\Delta_A(x;z) i\Delta_C(x;z) \; . \qquad (36)$$

The 4-point identities (23-24) have the complication of contracting derivatives at different points. This can be made invariant using the parallel transport matrix $[^\mu g^\nu](x;x')$ whose $3+1$ decomposition in de Sitter conformal coordinates is,

$$[^\mu g^\nu](x;x') \equiv \begin{pmatrix} [^0 g^0] & [^0 g^n] \\ [^m g^0] & [^m g^n] \end{pmatrix} \; , \qquad (37)$$

$$= \frac{\eta^{\mu\nu}}{a_x a_{x'}} + \frac{2}{4-y} \begin{pmatrix} -H^2 \Delta r^2 & -(\frac{1}{a_x} + \frac{1}{a_{x'}}) H \Delta r^n \\ (\frac{1}{a_x} + \frac{1}{a_{x'}}) H \Delta r^m & H^2 \Delta r^m \Delta r^n \end{pmatrix} \; , \qquad (38)$$

where $\Delta r^i \equiv (x-x')^i$ and $y \equiv a_x a_{x'} H^2 \Delta x^2$. Note that $[^\mu g^\nu](x;x') \simeq \eta^{\mu\nu}/a_x a_{x'}$ near spatial coincidence. The appropriate generalizations of the 4-point Donoghue Identities (23-24) are,

$$\begin{aligned} & i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(x;x') i\Delta_C(y;y') \\ & \longrightarrow -\frac{i}{m^2} \Big[1 - \Big(\frac{\overline{\partial}_\mu^x \overline{\partial}_\nu^{x'} [^\mu g^\nu](x;x') - m^2}{3m^2} \Big) \Big] \frac{\delta^D(x-y) \delta^D(x'-y')}{(a_x a_{x'})^D} \\ & \qquad \times \frac{1}{2} \Big\{ \int d^D z\, a_z^D i\Delta_A(x;z) i\Delta_C(x;z) i\Delta_A(z;x') + \Big(x^\mu \longleftrightarrow x'^\mu \Big) \Big\}, \end{aligned} \qquad (39)$$

$$\begin{aligned} & i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(x;y') i\Delta_C(x';y) \\ & \longrightarrow +\frac{i}{m^2} \Big[1 + \Big(\frac{\overline{\partial}_\mu^x \overline{\partial}_\nu^{y'} [^\mu g^\nu](x;y') + m^2}{3m^2} \Big) \Big] \frac{\delta^D(x-y) \delta^D(x'-y')}{(a_x a_{x'})^D} \\ & \qquad \times \frac{1}{2} \Big\{ \int d^D z\, a_z^D i\Delta_A(x;z) i\Delta_C(x;z) i\Delta_A(z;y') + \Big(x^\mu \longleftrightarrow y'^\mu \Big) \Big\}. \end{aligned} \qquad (40)$$

2.3 Derivative Identities

Momentum conservation at a 3-point vertex (15-17) is complicated by the factors of a^{D-2} (for derivative vertices) and a^D (for non-derivative vertices). If we label derivatives of the three propagators (or external wave functions) by "∂_1^μ", "∂_2^μ" and "∂_3^μ" then we might use "∂_4^μ to stand for differentiation of the scale factors. With this understanding, momentum conservation reads,

$$\Bigl(\partial_1 + \partial_2 + \partial_3 + \partial_4\Bigr)^\mu = 0 \; . \tag{41}$$

We often require contractions of two derivatives $\partial_1 \cdot \partial_2 \equiv \eta_{\mu\nu} \partial_1^\mu \partial_2^\nu$. When the vertex contains a factor of a^{D-2} such contractions can be expressed in terms of the scalar covariant d'Alembertian,

$$\mathcal{D}_i \equiv \eta_{\mu\nu} \partial_i^\mu a^{D-2} \partial_i^\nu = a^{D-2} \partial_i \!\cdot\! (\partial_i \!+\! \partial_4) \; . \tag{42}$$

The derivation is,

$$\begin{aligned} \partial_1 \!\cdot\! \partial_2 &= \tfrac12 (\partial_1 \!+\! \partial_2)^2 - \tfrac12 \partial_1^2 - \tfrac12 \partial_2^2 \; , & (43)\\ &= \tfrac12 (\partial_3 \!+\! \partial_4)^2 - \tfrac12 \partial_1^2 - \tfrac12 \partial_2^2 \; , & (44)\\ &= \tfrac12 \partial_3 \!\cdot\! (\partial_3 \!+\! \partial_4) + \tfrac12 \partial_4 \!\cdot\! (\partial_3 \!+\! \partial_4) - \tfrac12 \partial_1^2 - \tfrac12 \partial_2^2 \; , & (45)\\ &= \frac{1}{2 a^{D-2}} \Bigl[\mathcal{D}_3 - \mathcal{D}_1 - \mathcal{D}_2\Bigr] \; . & (46) \end{aligned}$$

A related identity of great utility is,

$$a^{D-2} \partial_1 \!\cdot\! \partial_2 + a^D m^2 = \frac12 \mathcal{D}_3 - \frac12 \Bigl(\mathcal{D}_1 \!-\! a^D m^2\Bigr) - \frac12 \Bigl(\mathcal{D}_2 \!-\! a^D m^2\Bigr) \; . \tag{47}$$

3 The Five New Diagrams

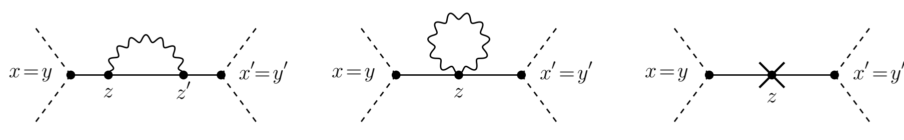

Figure 3: Diagrams $0a$ (left) and $0b$ (right) included in the uncorrected, self-mass.

The scattering process $\Psi_1(x)+\Psi_3(x') \longrightarrow \Psi_2(y)+\Psi_4(y')$ receives a t-channel contribution from the 1-loop self-mass $-iM_0^2(x;x')$ as shown in Figure 3. The analytic expression is,

$$-iV_0(x;x') = -\lambda^2(a_x a_{x'})^D\!\int\! d^Dz\, i\Delta_A(x;z)\!\int\! d^Dz'\, i\Delta_A(x';z')\times -iM_0^2(z;z') \; . \tag{48}$$

The purpose of this section is to give the five other classes of diagrams which make t-channel contributions to the same 1-loop scattering amplitude. We use subscripts to distinguish scale factors at different spacetime points, as in a_x and $a_{x'}$ in expression (48). The same convention applies to derivative operators, whose indices are raised and lowered using the Minkowski metric, as in $\partial_y^\mu \equiv \eta^{\mu\nu}\frac{\partial}{\partial y^\nu}$. Because there are sometimes too many propagators to distinguish how derivatives act by order, we employ a special convention for a $h_{\mu\nu}\Psi_{\rm int}\Psi_{\rm ext}$ vertex: a bar indicates that the derivative acts on the external Ψ leg, a tilde indicates that it acts on the graviton leg, and a derivative without either distinction acts on the internal Ψ leg.

3.1 Vertex-Vertex

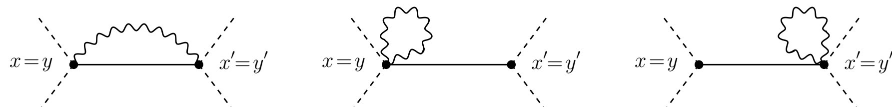

Figure 4: Diagrams $1a$ (left), $1b$ (middle) and $1c$ (right) in which the vertex is corrected.

The first of the new diagrams derive from correlators of gravitons which emerge from the vertex. Figure 4 shows the three possibilities. The first case is where a single graviton emerges from each vertex,

$$-iV_{1a}(x;x') = \Bigl(-\tfrac{i}{2}\kappa\lambda a_x^D \eta^{\mu\nu}\Bigr) i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x') \Bigl(-\tfrac{i}{2}\kappa\lambda a_{x'}^D \eta^{\rho\sigma}\Bigr) i\Delta_A(x;x') \; . \tag{49}$$

The 2nd and 3rd are where two gravitons emerge from the same vertex,

$$-iV_{1b}(x;x') = \Bigl[-i\kappa^2\lambda a_x^D\Bigl(\tfrac18\eta^{\mu\nu}\eta^{\rho\sigma} - \tfrac14\eta^{\mu\rho}\eta^{\nu\sigma}\Bigr)\Bigr] i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x) \Bigl(-i\lambda a_{x'}^D\Bigr) i\Delta_A(x;x') \; , \tag{50}$$

$$-iV_{1c}(x;x') = \Bigl(-i\lambda a_x^D\Bigr) i[{}_{\mu\nu}\Delta_{\rho\sigma}](x';x') \Bigl[-i\kappa^2\lambda a_{x'}^D\Bigl(\tfrac18\eta^{\mu\nu}\eta^{\rho\sigma} - \tfrac14\eta^{\mu\rho}\eta^{\nu\sigma}\Bigr)\Bigr] i\Delta_A(x;x') \; . \tag{51}$$

3.2 Source(Observer)-Vertex

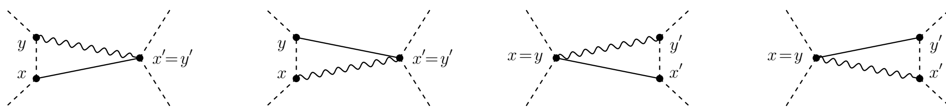

Figure 5: Diagrams $2a$ (left), $2b$ (2nd), $2c$ (3rd) and $2d$ (right) including a correlation from the propagation of the source or observer with the vertex.

Figure 5 shows the second class of new diagrams: those which involve a graviton from the propagation of the source or observer correlated with a graviton from the vertex on the opposite side. (Having the vertex is on the same side produces an irrelevant external leg correction.) The leftmost diagram is,

$$-iV_{2a}(x;x';y) = \Big[-i\kappa a_y^{D-2}\Big(-\overline{\partial}_y^{\mu}\partial_y^{\nu} + \tfrac{1}{2}\eta^{\mu\nu}[\overline{\partial}_y\cdot\partial_y + a_y^2 m^2]\Big)\Big] i[_{\mu\nu}\Delta_{\rho\sigma}](y;x') \\ \times\Big(-\tfrac{i}{2}\kappa\lambda a_{x'}^D\eta^{\rho\sigma}\Big) i\Delta_m(y;x)\Big(-i\lambda a_x^D\Big) i\Delta_A(x;x') \; , \quad (52)$$

$$= \frac{i}{2}\kappa^2\lambda^2(a_x a_{x'})^D a_y^{D-2}\Big[-\overline{\partial}_y^{\mu}\partial_y^{\nu} + \tfrac{1}{2}\eta^{\mu\nu}(\overline{\partial}_y\cdot\partial_y + a_y^2 m^2)\Big] \\ \times i[_{\mu\nu}\Delta^{\rho}{}_{\rho}](y;x') i\Delta_m(x;y) i\Delta_A(x;x') \; . \quad (53)$$

We merely give final results for the other three diagrams,

$$-iV_{2b}(x;x';y) = \frac{i}{2}\kappa^2\lambda^2(a_y a_{x'})^D a_x^{D-2}\Big[-\overline{\partial}_x^{\mu}\partial_x^{\nu} + \tfrac{1}{2}\eta^{\mu\nu}(\overline{\partial}_x\cdot\partial_x + a_x^2 m^2)\Big] \\ \times i[_{\mu\nu}\Delta^{\rho}{}_{\rho}](x;x') i\Delta_m(x;y) i\Delta_A(y;x') \; , \quad (54)$$

$$-iV_{2c}(x;x';y') = \frac{i}{2}\kappa^2\lambda^2(a_x a_{x'})^D a_{y'}^{D-2}\Big[-\overline{\partial}_{y'}^{\rho}\partial_{y'}^{\sigma} + \tfrac{1}{2}\eta^{\rho\sigma}(\overline{\partial}_{y'}\cdot\partial_{y'} + a_{y'}^2 m^2)\Big] \\ \times i[^{\mu}{}_{\mu}\Delta_{\rho\sigma}](x;y') i\Delta_m(x';y') i\Delta_A(x;x') \; , \quad (55)$$

$$-iV_{2d}(x;x';y') = \frac{i}{2}\kappa^2\lambda^2(a_x a_{y'})^D a_{x'}^{D-2}\Big[-\overline{\partial}_{x'}^{\rho}\partial_{x'}^{\sigma} + \tfrac{1}{2}\eta^{\rho\sigma}(\overline{\partial}_{x'}\cdot\partial_{x'} + a_{x'}^2 m^2)\Big] \\ \times i[^{\mu}{}_{\mu}\Delta_{\rho\sigma}](x;x') i\Delta_m(x';y') i\Delta_A(x;y') \; , \quad (56)$$

3.3 Vertex-Exchange

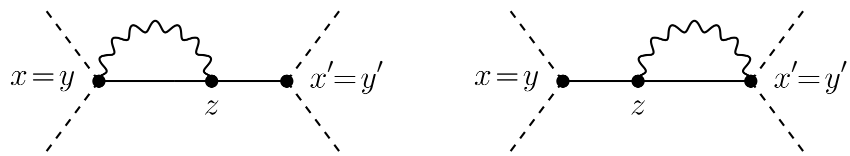

Figure 6: Diagrams $3a$ (left) and $3b$ (right) included a correlation between a vertex and the propagation of the exchange scalar.

Another class of corrections is where a graviton from a vertex correlates with a graviton from the propagation of the exchange scalar. Figure 6 shows these diagrams. Their analytic expressions are,

$$\begin{aligned}
-iV_{3a}(x;x') &= \tfrac{i}{2}\kappa^2\lambda^2(a_x a_{x'})^D \int d^Dz\, a_z^{D-2}\Big[-\overline{\partial}{}_z^{\rho}\partial_z^{\sigma} + \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z\cdot\partial_z\Big] \\
&\qquad \times i[{}^{\mu}{}_{\mu}\Delta_{\rho\sigma}](x;z)\, i\Delta_A(x;z)\, i\Delta_A(z;x') \; , \qquad (57) \\
-iV_{3b}(x;x') &= \tfrac{i}{2}\kappa^2\lambda^2(a_x a_{x'})^D \int d^Dz\, a_z^{D-2}\Big[-\overline{\partial}{}_z^{\mu}\partial_z^{\nu} + \tfrac{1}{2}\eta^{\mu\nu}\overline{\partial}_z\cdot\partial_z\Big] \\
&\qquad \times i[{}_{\mu\nu}\Delta^{\rho}{}_{\rho}](z;x')\, i\Delta_A(x;z)\, i\Delta_A(z;x') \; . \qquad (58)
\end{aligned}$$

Our notation in (57-58) is that $\overline{\partial}_z$ differentiates the scalar propagator external to the loop, while ∂_z acts on the scalar propagator inside the loop.

3.4 Source-Observer

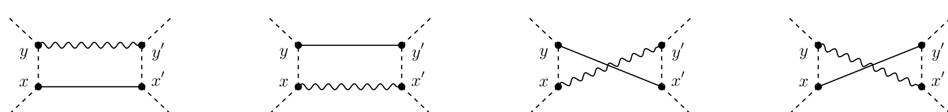

Figure 7: Diagrams $4a$ (left), $4b$ (2nd), $4c$ (3rd) and $4d$ (right) including a correlation between the propagation of the source and the propagation of the observer.

Figure 7 shows diagrams which result from a graviton in the propagation of the source correlating with a graviton from the observer. (Source-source and

Observer-observer correlations either give external state factors or else make vertex corrections which are not enhanced.) The leftmost diagram involves two A vertices (15) and two B vertices (16),

$$
\begin{aligned}
-iV_{4a}(x;x';y;y') = & \Bigl[-i\kappa a_y^{D-2}\Bigl(-\overline{\partial}_y^{\mu}\partial_y^{\nu} + \tfrac{1}{2}\eta^{\mu\nu}[\overline{\partial}_y\cdot\partial_y + a_y^2 m^2]\Bigr)\Bigr] i[{}_{\mu\nu}\Delta_{\rho\sigma}](y;y') \\
& \times\Bigl[-i\kappa a_{y'}^{D-2}\Bigl(-\overline{\partial}_{y'}^{\rho}\partial_{y'}^{\sigma} + \tfrac{1}{2}\eta^{\rho\sigma}[\overline{\partial}_{y'}\cdot\partial_{y'} + a_{y'}^2 m^2]\Bigr)\Bigr] \\
& \times i\Delta_m(x;y)\Bigl(-i\lambda a_x^D\Bigr) i\Delta_A(x;x')\Bigl(-i\lambda a_{x'}^D\Bigr) i\Delta_m(x';y') \; , \qquad (59)
\end{aligned}
$$

$$
\begin{aligned}
= & \; \kappa^2\lambda^2 (a_x a_{x'})^D (a_y a_{y'})^{D-2}\Bigl[\overline{\partial}_y^{\mu}\partial_y^{\nu} - \tfrac{1}{2}\eta^{\mu\nu}(\overline{\partial}_y\cdot\partial_y + a_y^2 m^2)\Bigr]\Bigl[\overline{\partial}_{y'}^{\rho}\partial_{y'}^{\sigma} \\
& - \tfrac{1}{2}\eta^{\rho\sigma}(\overline{\partial}_{y'}\cdot\partial_{y'} + a_{y'}^2 m^2)\Bigr] i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(x;x') i[{}_{\mu\nu}\Delta_{\rho\sigma}](y;y') \; . \qquad (60)
\end{aligned}
$$

For the remaining three diagrams we give only the final forms,

$$
\begin{aligned}
-iV_{4b}(x;x';y;y') = & \; \kappa^2\lambda^2 (a_y a_{y'})^D (a_x a_{x'})^{D-2}\Bigl[\overline{\partial}_x^{\mu}\partial_x^{\nu} - \tfrac{\eta^{\mu\nu}}{2}(\overline{\partial}_x\cdot\partial_x + a_x^2 m^2)\Bigr]\Bigl[\overline{\partial}_{x'}^{\rho}\partial_{x'}^{\sigma} \\
& - \tfrac{\eta^{\rho\sigma}}{2}(\overline{\partial}_{x'}\cdot\partial_{x'} + a_{x'}^2 m^2)\Bigr] i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(y;y') i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x') \; , \qquad (61)
\end{aligned}
$$

$$
\begin{aligned}
-iV_{4c}(x;x';y;y') = & \; \kappa^2\lambda^2 (a_y a_{x'})^D (a_x a_{y'})^{D-2}\Bigl[\overline{\partial}_x^{\mu}\partial_x^{\nu} - \tfrac{\eta^{\mu\nu}}{2}(\overline{\partial}_x\cdot\partial_x + a_x^2 m^2)\Bigr]\Bigl[\overline{\partial}_{y'}^{\rho}\partial_{y'}^{\sigma} \\
& - \tfrac{\eta^{\rho\sigma}}{2}(\overline{\partial}_{y'}\cdot\partial_{y'} + a_{y'}^2 m^2)\Bigr] i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(y;x') i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;y') \; , \qquad (62)
\end{aligned}
$$

$$
\begin{aligned}
-iV_{4d}(x;x';y;y') = & \; \kappa^2\lambda^2 (a_x a_{y'})^D (a_y a_{x'})^{D-2}\Bigl[\overline{\partial}_y^{\mu}\partial_y^{\nu} - \tfrac{\eta^{\mu\nu}}{2}(\overline{\partial}_y\cdot\partial_y + a_y^2 m^2)\Bigr]\Bigl[\overline{\partial}_{x'}^{\rho}\partial_{x'}^{\sigma} \\
& - \tfrac{\eta^{\rho\sigma}}{2}(\overline{\partial}_{x'}\cdot\partial_{x'} + a_{x'}^2 m^2)\Bigr] i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(x;y') i[{}_{\mu\nu}\Delta_{\rho\sigma}](y;x') \; . \qquad (63)
\end{aligned}
$$

3.5 Source(Observer)-Exchange

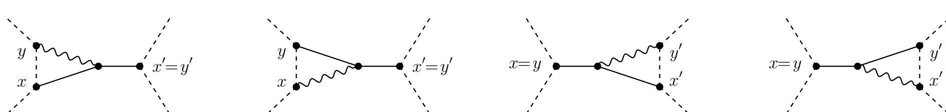

Figure 8: Diagrams $5a$ (left), $5b$ (2nd), $5c$ (3rd) and $5d$ (right) including a correlation between the propagation of the source or observer and the propagation of the exchange scalar.

The final class of contributions consists of gravitons from the propagation of the source or observer correlation with gravitons from the exchange scalar.

Figure 8 shows these diagrams, which each involves two $\varphi\Psi^2$ vertices (15), a single $h_{\mu\nu}\Psi^2$ vertex (16) and a single $h_{\mu\nu}\varphi^2$ vertex (17). The leftmost diagram is,

$$-iV_{5a}(x;x';y) = \Bigl[-i\kappa a_y^{D-2}\Bigl(-\overline{\partial}_y^{\mu}\partial_y^{\nu} + \tfrac{1}{2}\eta^{\mu\nu}[\overline{\partial}_y\!\cdot\partial_y + a_y^2 m^2]\Bigr)\Bigr] i\Delta_m(y;x) \times\!\int\! d^Dz\, i[_{\mu\nu}\Delta_{\rho\sigma}](y;z)\Bigl[-i\kappa a_z^{D-2}\Bigl(-\overline{\partial}_z^{\rho}\partial_z^{\sigma} + \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z\!\cdot\partial_z\Bigr)\Bigr] \times i\Delta_A(z;x)\Bigl(-i\lambda a_x^D\Bigr) i\Delta_A(z;x')\Bigl(-i\lambda a_{x'}^D\Bigr) , \quad (64)$$

$$= \kappa^2\lambda^2(a_x a_{x'})^D\!\int\! d^Dz (a_y a_z)^{D-2}\Bigl[\overline{\partial}_y^{\mu}\partial_y^{\nu} - \tfrac{\eta^{\mu\nu}}{2}(\overline{\partial}_y\!\cdot\partial_y + a_y^2 m^2)\Bigr]\Bigl[\overline{\partial}_z^{\rho}\partial_z^{\sigma} - \tfrac{\eta^{\rho\sigma}}{2}\overline{\partial}_z\!\cdot\partial_z\Bigr] i\Delta_m(x;y) i[_{\mu\nu}\Delta_{\rho\sigma}](y;z) i\Delta_A(x;z) i\Delta_A(z;x') \,. \quad (65)$$

Recall that $\overline{\partial}_z$ indicates differentiation of the scalar propagator external to the loop, where ∂_z acts on the scalar propagator inside the loop. For the remaining three diagrams we give only the final forms,

$$-iV_{5b}(x;x';y) = \kappa^2\lambda^2(a_y a_{x'})^D\!\int\! d^Dz (a_x a_z)^{D-2}\Bigl[\overline{\partial}_x^{\mu}\partial_x^{\nu} - \tfrac{\eta^{\mu\nu}}{2}(\overline{\partial}_x\!\cdot\partial_x + a_x^2 m^2)\Bigr] \times\Bigl[\overline{\partial}_z^{\rho}\partial_z^{\sigma} - \tfrac{\eta^{\rho\sigma}}{2}\overline{\partial}_z\!\cdot\partial_z\Bigr] i\Delta_m(x;y) i[_{\mu\nu}\Delta_{\rho\sigma}](x;z) i\Delta_A(y;z) i\Delta_A(z;x') \,, \quad (66)$$

$$-iV_{5c}(x;x';y') = \kappa^2\lambda^2(a_x a_{x'})^D\!\int\! d^Dz (a_{y'} a_z)^{D-2}\Bigl[\overline{\partial}_{y'}^{\rho}\partial_{y'}^{\sigma} - \tfrac{\eta^{\rho\sigma}}{2}(\overline{\partial}_{y'}\!\cdot\partial_{y'} + a_{y'}^2 m^2)\Bigr] \times\Bigl[\overline{\partial}_z^{\mu}\partial_z^{\nu} - \tfrac{\eta^{\mu\nu}}{2}\overline{\partial}_z\!\cdot\partial_z\Bigr] i\Delta_m(x';y') i[_{\mu\nu}\Delta_{\rho\sigma}](z;y') i\Delta_A(x;z) i\Delta_A(z;x') \,, \quad (67)$$

$$-iV_{5d}(x;x';y') = \kappa^2\lambda^2(a_x a_{y'})^D\!\int\! d^Dz (a_{x'} a_z)^{D-2}\Bigl[\overline{\partial}_{x'}^{\rho}\partial_{x'}^{\sigma} - \tfrac{\eta^{\rho\sigma}}{2}(\overline{\partial}_{x'}\!\cdot\partial_{x'} + a_{x'}^2 m^2)\Bigr] \times\Bigl[\overline{\partial}_z^{\mu}\partial_z^{\nu} - \tfrac{\eta^{\mu\nu}}{2}\overline{\partial}_z\!\cdot\partial_z\Bigr] i\Delta_m(x';y') i[_{\mu\nu}\Delta_{\rho\sigma}](z;x') i\Delta_A(x;z) i\Delta_A(z;y') \,. \quad (68)$$

4 Consolidation

When the derivative identity (47) is used on the $h_{\mu\nu}\Psi^2$ vertex (16) at an external point, and the external state condition $\overline{\mathcal{D}} = a^D m^2$ is employed, the vertex simplies to,

$$-i\kappa a^{D-2}\Bigl(-\overline{\partial}^{\mu}\partial^{\nu} + \tfrac{1}{2}\eta^{\mu\nu}\Bigl[\overline{\partial}\cdot\partial + a^2 m^2\Bigr]\Bigr) = -i\kappa\Bigl(-a^{D-2}\overline{\partial}^{\mu}\partial^{\nu} + \tfrac{1}{4}\eta^{\mu\nu}\Bigl[\widetilde{\mathcal{D}} - (\mathcal{D} - a^D m^2)\Bigr]\Bigr) \,, \quad (69)$$

Recall that $\overline{\partial}^\mu$ acts on the external Ψ leg, ∂^μ acts on the internal Ψ line and $\widetilde{\partial}^\mu$ differentiates the graviton propagator. Of course the factor of $(\mathcal{D}-a^D m^2)$ acts on the internal Ψ propagator to give a delta function, which effectively reduces the number of distinct points of that portion of the diagram. This part of the Source(Observer)-Exchange (V_5) diagrams turns out to cancel the Vertex-Exchange (V_3) corrections. Similarly, these parts of the Source-Observer (V_4) diagrams cancel the main Vertex-Vertex (V_{1a}) correction and all of the Source(Observer)-Vertex (V_2) corrections. The purpose of this section is to demonstrate these cancellations, which are especially powerful because they pertain for any gauge. We close by listing the residual diagrams which remain.

4.1 The 3-5 Complex

It is convenient to label the three terms inside the large parentheses of expression (69) as "i", "ii" and "iii",

$$-a^{D-2}\overline{\partial}^\mu \partial^\nu \iff i \; , \tag{70}$$

$$\tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}} \iff ii \; , \tag{71}$$

$$-\frac{1}{4}\eta^{\mu\nu}(\mathcal{D}-a^D m^2) \iff iii \; . \tag{72}$$

We can decompose $-iV_5(x;x';y)$ accordingly,

$$\begin{aligned} -iV_{5a_i}(x;x';y) \equiv\; & \kappa^2\lambda^2(a_x a_{x'})^D \times a_y^{D-2}\overline{\partial}^\mu \partial_y^\nu \times \int d^D z\, a_z^{D-2}\Big[\overline{\partial}_z^\rho \partial_z^\sigma - \tfrac{1}{2}\eta^{\mu\nu}\overline{\partial}_z\cdot\partial_z\Big] \\ & \times i\Delta_m(x;y) i\big[{}_{\mu\nu}\Delta_{\rho\sigma}\big](y;z) i\Delta_A(x;z) i\Delta_A(z;x') \; , \end{aligned} \tag{73}$$

$$\begin{aligned} -iV_{5a_{ii}}(x;x';y) \equiv\; & \kappa^2\lambda^2(a_x a_{x'})^D \times -\tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}_y} \times \int d^D z\, a_z^{D-2}\Big[\overline{\partial}_z^\rho \partial_z^\sigma - \tfrac{1}{2}\eta^{\mu\nu}\overline{\partial}_z\cdot\partial_z\Big] \\ & \times i\Delta_m(x;y) i\big[{}_{\mu\nu}\Delta_{\rho\sigma}\big](y;z) i\Delta_A(x;z) i\Delta_A(z;x') \; , \end{aligned} \tag{74}$$

$$\begin{aligned} -iV_{5a_{iii}}(x;x';y) \equiv\; & \kappa^2\lambda^2(a_x a_{x'})^D \times \tfrac{1}{4}\eta^{\mu\nu}(\mathcal{D}_y - a^D m^2) \times \int d^D z\, a_z^{D-2}\Big[\overline{\partial}_z^\rho \partial_z^\sigma \\ & - \tfrac{1}{2}\eta^{\mu\nu}\overline{\partial}_z\cdot\partial_z\Big] \times i\Delta_m(x;y) i\big[{}_{\mu\nu}\Delta_{\rho\sigma}\big](y;z) i\Delta_A(x;z) i\Delta_A(z;x') \; . \end{aligned} \tag{75}$$

The same conventions pertain for diagrams $-iV_{5b}(x;x';y)$, $-iV_{5c}(x;x';y')$ and $-iV_{5d}(x;x';y')$. We use a bar to indicate the sum of the "i" and "ii" terms,

$$-iV_{\overline{5}}(x;x';y) \equiv -iV_{5_i}(x;x';y) - iV_{5_{ii}}(x;x';y) \; . \tag{76}$$

Note that $-iV_{5a_{iii}}(x;x';y)$ can be simplified by acting the Ψ kinetic operator $(\mathcal{D}_y - a^D m^2)$ on the Ψ propagator $i\Delta_m(x;y)$,

$$-iV_{5a_{iii}}(x;x';y) \;=\; \delta^D(x-y)\times\tfrac{i}{4}\kappa^2\lambda^2(a_x a_{x'})^D\!\int\! d^Dz\, a_z^{D-2}\Big[\overline{\partial}_z^{\rho}\partial_z^{\sigma} - \tfrac{1}{2}\eta^{\mu\nu}\overline{\partial}_z\!\cdot\!\partial_z\Big] \times i[^{\mu}{}_{\mu}\Delta_{\rho\sigma}](x;z)\, i\Delta_A(x;z)\, i\Delta_A(z;x') \; . \quad (77)$$

One gets the same result for $-iV_{5b_{iii}}(x;x';y)$. Taken together, they cancel $\delta^D(x-y)\times -iV_{3a}(x;x')$,

$$\delta^D(x-y)\times -iV_{3a}(x;x') = \delta^D(x-y)\times -\tfrac{i}{2}\kappa^2\lambda^2(a_x a_{x'})^D\!\int\! d^Dz\, a_z^{D-2}\Big[\overline{\partial}_z^{\rho}\partial_z^{\sigma} - \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z\!\cdot\!\partial_z\Big]\times i[^{\mu}{}_{\mu}\Delta_{\rho\sigma}](x;z)\, i\Delta_A(x;z)\, i\Delta_A(z;x') \; . \quad (78)$$

Hence we can write,

$$-iV_{5a_{iii}}(x;x';y) - iV_{5b_{iii}}(x;x';y) + \delta^D(x-y)\times -iV_{3a}(x;x') = 0 \; . \qquad (79)$$

Similarly, one can show,

$$-iV_{5c_{iii}}(x;x';y') - iV_{5d_{iii}}(x;x';y') + \delta^D(x'-y')\times -iV_{3b}(x;x') = 0 \; . \qquad (80)$$

It follows that the Vertex-Exchange diagrams are completely canceled by the "iii" parts of the Source(Observer)-Exchange diagrams.

4.2 The 1a-2-4 Complex

The Source-Observer diagrams (V_4) each have two $h_{\mu\nu}\Psi^2$ vertices (16), each one of which can be decomposed into terms "i", "ii" and "iii" as in (70-72). For V_{4a} we first expand on the left hand vertex, keeping the right hand vertex together,

$$V_{4a} = V_{\overline{4a}} + V_{4a_{iii}} \; . \qquad (81)$$

We then expand on the right hand vertex,

$$V_{\overline{4a}} = V_{\overline{\overline{4a}}} + V_{\overline{4a}_{iii}} \qquad , \qquad V_{4a_{iii}} = V_{\overline{4a_{iii}}} + V_{(4a_{iii})_{iii}} \; , \qquad (82)$$

where the resulting four terms are,

$$-iV_{\overline{\overline{4a}}} \equiv \kappa^2\lambda^2(a_x a_{x'})^D\Big[a_y^{D-2}\overline{\partial}_y^\mu\partial_y^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big]\Big[a_{y'}^{D-2}\overline{\partial}_{y'}^\rho\partial_{y'}^\sigma - \tfrac{1}{4}\eta^{\rho\sigma}\widetilde{D}_{y'}\Big] \times i\Delta_m(x;y)i\Delta_m(x';y')i\Delta_A(x;x')i[{}_{\mu\nu}\Delta_{\rho\sigma}](y;y') \; , \quad (83)$$

$$-iV_{\overline{4a}_{iii}} \equiv \tfrac{1}{4}\kappa^2\lambda^2(a_x a_{x'})^D\Big[a_y^{D-2}\overline{\partial}_y^\mu\partial_y^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big](\mathcal{D}_{y'} - a_{y'}^D m^2) \times i\Delta_m(x;y)i\Delta_m(x';y')i\Delta_A(x;x')i[{}_{\mu\nu}\Delta^\rho{}_\rho](y;y') \; , \quad (84)$$

$$-iV_{\overline{4a_{iii}}} \equiv \tfrac{1}{4}\kappa^2\lambda^2(a_x a_{x'})^D(\mathcal{D}_y - a_y^D m^2)\Big[a_{y'}^{D-2}\overline{\partial}_{y'}^\rho\partial_{y'}^\sigma - \tfrac{1}{4}\eta^{\rho\sigma}\widetilde{D}_{y'}\Big] \times i\Delta_m(x;y)i\Delta_m(x';y')i\Delta_A(x;x')i[{}^\mu{}_\mu\Delta_{\rho\sigma}](y;y') \; , \quad (85)$$

$$-iV_{(4a_{iii})_{iii}} \equiv \tfrac{1}{16}\kappa^2\lambda^2(a_x a_{x'})^D(\mathcal{D}_y - a_y^D m^2)(\mathcal{D}_{y'} - a_{y'}^D m^2) \times i\Delta_m(x;y)i\Delta_m(x';y')i\Delta_A(x;x')i[{}^\mu{}_\mu\Delta^\rho{}_\rho](y;y') \; . \quad (86)$$

For V_{4b} we also expand first on the left hand vertex and then on the right hand vertex. However, it is best to expand V_{4c} and V_{4d} first on the right hand vertex and then on the left hand one. This convention makes for the simplest correspondence with the Source(Observer)-Vertex diagrams (V_2).

The Source(Observer)-Vertex diagrams (53-56) contain only one $h_{\mu\nu}\Psi^2$ vertex. We expand this as usual, for example,

$$V_{2a} = V_{\overline{2a}} + V_{2a_{iii}} \; . \quad (87)$$

To be explicit, the two terms are,

$$-iV_{\overline{2a}} \equiv -\tfrac{i}{2}\kappa^2\lambda^2(a_x a_{x'})^D\Big[a_y^{D-2}\overline{\partial}_y^\mu\partial_y^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big] \times i\Delta_m(x;y)i\Delta_A(x;x')i[{}_{\mu\nu}\Delta^\rho{}_\rho](y;x') \; , \quad (88)$$

$$-iV_{2a_{iii}} \equiv -\tfrac{i}{8}\kappa^2\lambda^2(a_x a_{x'})^D(\mathcal{D}_y - a_y^D m^2) \times i\Delta_m(x;y)i\Delta_A(x;x')i[{}^\mu_\mu\Delta^\rho{}_\rho](y;x') \; . \quad (89)$$

Let us first act the Ψ kinetic operators on the Ψ propagators in expressions (86) and (89) to conclude,

$$-iV_{(4a_{iii})_{iii}} = \delta^D(x-y)\delta^D(x'-y') \times -\tfrac{1}{16}\kappa^2\lambda^2(a_x a_{x'})^D i\Delta_A(x;x')i[{}^\mu{}_\mu\Delta^\rho{}_\rho](x;x') \; , \quad (90)$$

$$-iV_{2a_{iii}} = \delta^D(x-y)\times\tfrac{1}{8}\kappa^2\lambda^2(a_x a_{x'})^D i\Delta_A(x;x')i[{}^\mu{}_\mu\Delta^\rho{}_\rho](x;x') \; . \quad (91)$$

Adding the two contributions (with appropriate delta functions) gives $-\frac{1}{4}$ times the $-iV_{1a}$ contribution (49),

$$\delta^D(x'-y')\times -iV_{2a_{iii}} - iV_{(4a_{iii})_{iii}} = -\frac{1}{4}\delta^D(x-y)\delta^D(x'-y')\times -iV_{1a} \; . \quad (92)$$

The same relation pertains as well for the "b", "c" and "d" contributions, with the result that $-iV_{1a}$ is completely canceled.

Now act the Ψ kinetic operator of expression (84),

$$-iV_{\overline{4a}_{iii}} = \delta^D(x'-y')\times\frac{i}{4}\kappa^2\lambda^2(a_x a_{x'})^D\Big[a_y^{D-2}\overline{\partial}_y^{\mu}\partial_y^{\nu}-\frac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big] \times i\Delta_m(x;y)i\Delta_A(x;x')i[_{\mu\nu}\Delta^{\rho}{}_{\rho}](y;y') \; . \qquad (93)$$

One gets exactly the same thing from $-iV_{\overline{4c_{iii}}}$, and their sum cancels $-iV_{\overline{2a}}$,

$$-iV_{\overline{4a}_{iii}} - iV_{\overline{4d_{iii}}} = -\delta^D(x'-y')\times -iV_{\overline{2a}} \; . \qquad (94)$$

A number of similar relations pertain,

$$-iV_{\overline{4b}_{iii}} - iV_{\overline{4c_{iii}}} = -\delta^D(x'-y')\times -iV_{\overline{2b}} \; , \qquad (95)$$

$$-iV_{\overline{4c}_{iii}} - iV_{\overline{4a_{iii}}} = -\delta^D(x-y)\times -iV_{\overline{2c}} \; , \qquad (96)$$

$$-iV_{\overline{4d}_{iii}} - iV_{\overline{4b_{iii}}} = -\delta^D(x-y)\times -iV_{\overline{2d}} \; . \qquad (97)$$

Hence the $-iV_2$ diagrams are completely canceled.

4.3 Surviving Residual Diagrams

After all these consolidations there are just three classes of "new" diagrams which remain. The simplest are the two surviving $i = 1$ diagrams (50-51),

$$-iV_{1b}(x;x') = -\kappa^2\lambda^2(a_x a_{x'})^D \times\Big\{\frac{1}{8}i[^{\alpha}{}_{\alpha}\Delta^{\beta}{}_{\beta}](x;x) - \frac{1}{4}i[^{\alpha\beta}\Delta_{\alpha\beta}](x;x)\Big\}i\Delta_A(x;x') \; , \qquad (98)$$

$$-iV_{1c}(x;x') = -\kappa^2\lambda^2(a_x a_{x'})^D \times i\Delta_A(x;x')\Big\{\frac{1}{8}i[^{\alpha}{}_{\alpha}\Delta^{\beta}{}_{\beta}](x';x') - \frac{1}{4}i[^{\alpha\beta}\Delta_{\alpha\beta}](x';x')\Big\} \; . \qquad (99)$$

Next come the four residual $i = 4$ diagrams,

$$-iV_{\overline{\overline{4a}}}(x;x';y;y') = \kappa^2\lambda^2(a_x a_{x'})^D\Big[a_y^{D-2}\overline{\partial}_y^{\mu}\partial_y^{\nu}-\frac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big]\Big[a_{y'}^{D-2}\overline{\partial}_{y'}^{\rho}\partial_{y'}^{\sigma}-\frac{1}{4}\eta^{\rho\sigma}\widetilde{\mathcal{D}}_{y'}\Big] \times i\Delta_m(x;y)i\Delta_m(x';y')i\Delta_A(x;x')i[_{\mu\nu}\Delta_{\rho\sigma}](y;y') \; , \qquad (100)$$

$$-iV_{\overline{\overline{4b}}}(x;x';y;y') = \kappa^2\lambda^2(a_y a_{y'})^D\Big[a_x^{D-2}\overline{\partial}_x^{\mu}\partial_x^{\nu}-\frac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_x\Big]\Big[a_{x'}^{D-2}\overline{\partial}_{x'}^{\rho}\partial_{x'}^{\sigma}-\frac{1}{4}\eta^{\rho\sigma}\widetilde{\mathcal{D}}_{x'}\Big] \times i\Delta_m(x;y)i\Delta_m(x';y')i\Delta_A(y;y')i[_{\mu\nu}\Delta_{\rho\sigma}](x;x') \; , \qquad (101)$$

$$-iV_{\overline{\overline{4c}}}(x;x';y;y') = \kappa^2\lambda^2(a_y a_{x'})^D \Big[a_x^{D-2}\overline{\partial}_x^\mu \partial_x^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_x\Big]\Big[a_{y'}^{D-2}\overline{\partial}_{y'}^\rho \partial_{y'}^\sigma - \tfrac{1}{4}\eta^{\rho\sigma}\widetilde{\mathcal{D}}_{y'}\Big] \times i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(y;x') i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;y') \; , \quad (102)$$

$$-iV_{\overline{\overline{4d}}}(x;x';y;y') = \kappa^2\lambda^2(a_x a_{y'})^D \Big[a_y^{D-2}\overline{\partial}_y^\mu \partial_y^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big]\Big[a_{x'}^{D-2}\overline{\partial}_{x'}^\rho \partial_{x'}^\sigma - \tfrac{1}{4}\eta^{\rho\sigma}\widetilde{\mathcal{D}}_{x'}\Big] \times i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(x;y') i[{}_{\mu\nu}\Delta_{\rho\sigma}](y;x') \; . \quad (103)$$

Finally, there are the four $i=5$ residual diagrams,

$$-iV_{\overline{5a}}(x;x';y) = \kappa^2\lambda^2(a_x a_{x'})^D \Big[a_y^{D-2}\overline{\partial}_y^\mu \partial_y^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big] \int d^Dz \, a_z^{D-2} \Big[\overline{\partial}_z^\rho \partial_z^\sigma - \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z \cdot \partial_z\Big] \times i\Delta_m(x;y) i[{}_{\mu\nu}\Delta_{\rho\sigma}](y;z) i\Delta_A(x;z) i\Delta_A(z;x') \; , \quad (104)$$

$$-iV_{\overline{5b}}(x;x';y) = \kappa^2\lambda^2(a_y a_{x'})^D \Big[a_x^{D-2}\overline{\partial}_x^\mu \partial_x^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_x\Big] \int d^Dz \, a_z^{D-2} \Big[\overline{\partial}_z^\rho \partial_z^\sigma - \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z \cdot \partial_z\Big] \times i\Delta_m(x;y) i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;z) i\Delta_A(y;z) i\Delta_A(z;x') \; , \quad (105)$$

$$-iV_{\overline{5c}}(x;x';y) = \kappa^2\lambda^2(a_x a_{x'})^D \Big[a_{y'}^{D-2}\overline{\partial}_{y'}^\mu \partial_{y'}^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_{y'}\Big] \int d^Dz \, a_z^{D-2} \Big[\overline{\partial}_z^\rho \partial_z^\sigma - \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z \cdot \partial_z\Big] \times i\Delta_m(x';y') i[{}_{\mu\nu}\Delta_{\rho\sigma}](y';z) i\Delta_A(x;z) i\Delta_A(z;x') \; , \quad (106)$$

$$-iV_{\overline{5d}}(x;x';y) = \kappa^2\lambda^2(a_x a_{y'})^D \Big[a_{x'}^{D-2}\overline{\partial}_{x'}^\mu \partial_{x'}^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_{x'}\Big] \int d^Dz \, a_z^{D-2} \Big[\overline{\partial}_z^\rho \partial_z^\sigma - \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z \cdot \partial_z\Big] \times i\Delta_m(x';y') i[{}_{\mu\nu}\Delta_{\rho\sigma}](x';z) i\Delta_A(x;z) i\Delta_A(z;y') \; . \quad (107)$$

It is important to note that the reductions to these residual forms are exact, and independent of the graviton gauge.

5 Gauge Independent Self-Mass

The purpose of this section is to combine the t-channel contributions from the five new diagrams with $-iM_0(x;x')$ to produce a gauge independent self-mass. We begin by discussing the general reduction strategy for the $\overline{5}$ and $\overline{\overline{4}}$ contributions. Then each of these is worked out. The section closes by adding the $1b$ and $1c$ contributions and renormalizing the final result.

5.1 General Reduction Strategy

A number of simplifications concern the graviton propagator. First, because the propagator's indices are typically contracted into derivatives, only the temporal components of which can be significant for the infrared, t-channel exchange contributions of interest to us, we need retain just the first term in expression (14),

$$i[_{\mu\nu}\Delta_{\rho\sigma}] \longrightarrow \Bigl[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}\Bigr] \times i\Delta_C \; . \tag{108}$$

Second, in acting $\widetilde{D}$ on $i\Delta_C$ in 4-point contributions we can ignore the delta function because that term will have one fewer massless propagator,

$$\widetilde{\mathcal{D}} i\Delta_C(x;x') = i\delta^D(x-x') + 2(D-3)H^2 i\Delta_C(x;x') \longrightarrow 2(D-3)H^2 i\Delta_C(x;x') \; . \tag{109}$$

Finally, single derivatives of $i\Delta_C$ can be approximately reflected from one coordinate to the other using the relation,[2]

$$\Bigl(\widetilde{\partial}^\mu_x + \frac{1}{2}\partial^\mu_{a_x}\Bigr) i\Delta_C(x;x') \simeq -\Bigl(\widetilde{\partial}^\mu_{x'} + \frac{1}{2}\partial^\mu_{a_{x'}}\Bigr) i\Delta_C(x;x') \; . \tag{110}$$

This approximation is exact on the first term of the $i\Delta_C(x;x')$ expansion; and the higher terms should not matter because they vanish for $D = 4$.

Our basic strategy is to follow the same reductions as in flat space [41], collecting de Sitter obstructions as they occur. However, we need not retain terms with more than two factors of H because any additional factors of H will necessarily come in the dimensionless form of H/m which vanishes when the source and observer masses become large.

One major change from the flat space reduction concerns the external state factors for the source and observer. In flat space these are simple phase factors; in de Sitter they are spatial plane waves times Hankel functions with imaginary arguments. Their series expansions in powers of k^2/a^2 are more useful for our purposes,

$$u_m(x;\vec{k}) = \frac{e^{-i\mu t + i\vec{k}\cdot\vec{x}}}{\sqrt{2\mu a^{D-1}}} \sum_{n=0}^{\infty} \frac{\Gamma(1+\frac{i\mu}{H})}{n!\,\Gamma(1+n+\frac{i\mu}{H})} \Bigl(\frac{-k^2}{4H^2a^2}\Bigr)^n \; , \; \mu^2 \equiv m^2 - (\tfrac{D-1}{2})^2 H^2 \; . \tag{111}$$

[2]Recall that $\partial^\mu_{a_x} = -(D-2)Ha_x\delta^\mu_{\ 0}$ indicates the derivative acting on the factor of a_x^{D-2} which accompanies vertices from the kinetic energy.

Note that this expression employs the co-moving time $t = \ln(a)/H$, rather than the conformal time. We can expand external first derivatives as,

$$\partial_0 u_m(x;\vec{k}) = -ia\Big[\mu - i(\tfrac{D-1}{2})H + \tfrac{k^2}{2ma^2} + \dots\Big]u_m \quad , \quad \partial_i u_m(x;\vec{k}) = ik_i u_m \; . \tag{112}$$

We sometimes also need second time derivatives,

$$\partial_0^2 u_m(x;\vec{k}) = -a^2\Big[m^2 - i(D-2)\mu H - \tfrac{1}{2}(D-1)(D-2)H^2 + \tfrac{k^2}{a^2} + \dots\Big]u_m(x;\vec{k}) \; . \tag{113}$$

We regard the two in-coming wave functions as,

$$u_m(x;\vec{k}_1) \qquad , \qquad u_m(x';\vec{k}_3) \; . \tag{114}$$

The corresponding out-going wave functions are,

$$u_m^*(y;\vec{k}_2) \qquad , \qquad u_m^*(y';\vec{k}_4) \; . \tag{115}$$

And spatial momentum conservation implies,

$$\vec{k}_1 + \vec{k}_3 = \vec{k}_2 + \vec{k}_4 \qquad \Longrightarrow \qquad \vec{k}_1 - \vec{k}_2 = \vec{k}_4 - \vec{k}_3 \equiv \vec{q} \; . \tag{116}$$

5.2 $V_{\overline{5}}(x;x';y;y')$ Diagrams

Here we give the reduction for the residual diagrams of type $\overline{5}$ listed in (104-107). The details are given for part $\overline{5a}$, and the remaining three parts are inferred from symmetry.

The initial step of the reduction is contracting the simplified graviton propagator (108) into the tensor structures and derivatives of the two 3-vertices in (104),

$$\Big[a_y^{D-2}\overline{\partial}_y^{\mu}\partial_y^{\nu} - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Big]a_z^{D-2}\Big[\overline{\partial}_z^{\rho}\partial_z^{\sigma} - \tfrac{1}{2}\eta^{\rho\sigma}\overline{\partial}_z\cdot\partial_z\Big]\Big[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2\eta_{\mu\nu}\eta_{\rho\sigma}}{D-2}\Big]$$
$$= a_z^{D-2}\Big[a_y^{D-2}\overline{\partial}_y\cdot\overline{\partial}_z\partial_y\cdot\partial_z + a_y^{D-2}\overline{\partial}_y\cdot\partial_z\partial_y\cdot\overline{\partial}_z - \tfrac{1}{2}\overline{\partial}_z\cdot\partial_z\widetilde{\mathcal{D}}_y\Big] . \tag{117}$$

The derivatives in the resulting three terms above are then partially integrated using (41), and reflected over the graviton propagator using (110) so as to extract as many of them from the triangular loop. Not all derivatives can be extracted this way. The ones remaining inside we arrange when possible to be contracted with another derivative at the same vertex. This procedure gives the following for the first term in (117),

$$(a_y a_z)^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\partial_y\!\cdot\!\partial_z \longrightarrow -\tfrac{1}{2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\big(a_z^{D-2}\mathcal{D}_y + a_y^{D-2}\mathcal{D}_z\big) \tag{118}$$

$$+\tfrac{1}{2}(a_y a_z)^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big[\overline{\partial}_y\!\cdot\!(\overline{\partial}_y+\partial_{a_y}) + \overline{\partial}_z\!\cdot\!(\overline{\partial}_z+\partial_{a_z}) + 2\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big] \tag{119}$$

$$+\tfrac{1}{4}(D-2)(a_y a_z)^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big\{H(a_y-a_z)\Big[2\big(\overline{\partial}_{y^0} - \overline{\partial}_{z^0}\big) + \widetilde{\partial}_{y^0} - \widetilde{\partial}_{z^0} + (D-2)H(a_y-a_z)\Big] + H^2(a_y^2+a_z^2)\Big\}\,. \tag{120}$$

Note that we recognized scalar d'Alembertians (42) where that was not impeded by other derivatives or powers of the scale factor. They are defined to act directly onto the corresponding propagator or external leg. The first two lines in the expression above contain terms that are de Sitter generalizations of flat space contributions, that we refer to as flat space descendants, while the remaining lines contain only pure de Sitter terms.

The second term in (117), upon reorganizing the derivatives, doubles the contribution of the first term, and provides additional contributions,

$$(a_y a_z)^{D-2}\overline{\partial}_y\!\cdot\!\partial_z\partial_y\!\cdot\!\overline{\partial}_z \longrightarrow (a_y a_z)^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\partial_y\!\cdot\!\partial_z \tag{121}$$

$$+\tfrac{1}{2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\big(a_z^{D-2}\widetilde{\mathcal{D}}_y + a_y^{D-2}\widetilde{\mathcal{D}}_z\big) + \tfrac{1}{4}\big(\mathcal{D}_y - \overline{\mathcal{D}}_y - \widetilde{\mathcal{D}}_y\big)\big(\mathcal{D}_z - \overline{\mathcal{D}}_z - \widetilde{\mathcal{D}}_z\big) \tag{122}$$

$$+\tfrac{1}{4}(D-2)(a_y a_z)^{D-2}\Big\{H(a_y-a_z)\Big[2\overline{\partial}_y\!\cdot\!\widetilde{\partial}_y\overline{\partial}_{z^0} - 2\overline{\partial}_z\!\cdot\!\widetilde{\partial}_z\overline{\partial}_{y^0} \tag{123}$$

$$-\overline{\partial}_y\!\cdot\!\overline{\partial}_z\big(\widetilde{\partial}_{y^0}-\widetilde{\partial}_{z^0}\big) - (D-2)H(a_y-a_z)\overline{\nabla}_y\!\cdot\!\overline{\nabla}_z\Big] - H^2(a_y^2+a_z^2)\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big\}. \tag{124}$$

The latter can again be recognized as flat space descendants in line (122), and as pure de Sitter contributions in lines (123) and (124). Finally, the third term in (117) produces only flat space descendants,

$$-\tfrac{1}{2}a_z^{D-2}\overline{\partial}_z\!\cdot\!\partial_z\widetilde{\mathcal{D}}_y \longrightarrow -\tfrac{1}{4}\widetilde{\mathcal{D}}_y\big(\widetilde{\mathcal{D}}_z - \overline{\mathcal{D}}_z - \mathcal{D}_z\big)\,. \tag{125}$$

Further reduction of terms in (118-125) is accomplished by acting the scalar kinetic operators where possible, applying 3-point Donoghue identities (35) and (36), and finally acting the remaining derivatives onto external legs. We consider these reductions separately for flat space descendants and for pure de Sitter contributions.

5.2.1 Reduction of flat space descendants

Flat space descendant contributions comprises twice the terms in lines (118) and (119), and terms in lines (122) and (125), that added together read,

$$(a_y a_z)^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big[2\overline{\partial}_y\!\cdot\!\overline{\partial}_z+\overline{\partial}_y\!\cdot\!(\overline{\partial}_y\!+\!\partial_{a_y})+\overline{\partial}_z\!\cdot\!(\overline{\partial}_z\!+\!\partial_{a_z})\Big] \tag{126}$$

$$-\tfrac{1}{2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big[2a_z^{D-2}\mathcal{D}_y+2a_y^{D-2}\mathcal{D}_z-a_z^{D-2}\widetilde{\mathcal{D}}_y-a_y^{D-2}\widetilde{\mathcal{D}}_z\Big] \tag{127}$$

$$+\tfrac{1}{4}\big(\mathcal{D}_y-\overline{\mathcal{D}}_y\big)\big(\mathcal{D}_z-\overline{\mathcal{D}}_z-\widetilde{\mathcal{D}}_z\big)+\tfrac{1}{2}\widetilde{\mathcal{D}}_y\overline{\mathcal{D}}_z\,. \tag{128}$$

Simplest to reduce are terms in the last line (128), whose contribution we label by "(A)", that consist of products of scalar d'Alembertians. Only terms containing combination $(\mathcal{D}_y\!-\!\overline{\mathcal{D}}_y)$ that contracts the massive propagator contribute in the end,

$$(A):\quad \tfrac{1}{4}\big(\mathcal{D}_y\!-\!\overline{\mathcal{D}}_y\big)\big(\mathcal{D}_z\!-\!\overline{\mathcal{D}}_z\!-\!\widetilde{\mathcal{D}}_z\big)i\Delta_m(x;y)\longrightarrow \tfrac{1}{4}i\delta^D(x\!-\!y)\big(\mathcal{D}_z\!-\!\overline{\mathcal{D}}_z\!-\!\widetilde{\mathcal{D}}_z\big)\,, \tag{129}$$

while the remaining ones vanish in the large mass limit.

The terms in line (127) that have two external derivatives are best considered together with the latter two terms in first line (126) that contain two external would-be scalar d'Alembertians. We label this contribution by "(B)". It is only the middle term in (126) and the first term in (127) that contain sufficient number of derivatives acting on either external mode functions or on the massive propagator to generate a nonvanishing contribution. This contribution is obtained by first acting the scalar d'Alembertian, then applying the 3-point Donoghue identity (35), followed by acting derivatives on external mode functions using (112),

$$(B):\qquad a_z^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big[a_y^{D-2}\overline{\partial}_y\!\cdot\!(\overline{\partial}_y\!+\!\partial_{a_y})-\mathcal{D}_y\Big]i\Delta_m(x;y) \tag{130}$$

$$\longrightarrow a_z^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big\{a_y^{D-2}\Big[\overline{\partial}_y\!\cdot\!(\overline{\partial}_y\!+\!\partial_{a_y})-m^2a_y^2\Big]i\Delta_m(x;y)-i\delta^D(x\!-\!y)\Big\} \tag{131}$$

$$\longrightarrow a_z^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big[\tfrac{1}{2m^2a_y^2}\overline{\partial}_y\!\cdot\!(\overline{\partial}_y\!+\!\partial_{a_y})-\tfrac{3}{2}\Big]i\delta^D(x\!-\!y) \tag{132}$$

$$\longrightarrow a_z^{D-2}\Big[-\overline{\partial}_y\!\cdot\!\overline{\partial}_z-Ha_y\overline{\partial}_{z^0}\Big]i\delta^D(x\!-\!y)\,. \tag{133}$$

In the last step above one needs to account for all the terms generated by multiple derivatives acting on external mode functions,

$$\overline{\partial}_z\!\cdot\!\overline{\partial}_y\,\overline{\partial}_y\!\cdot\!(\overline{\partial}_y\!+\!\partial_{a_y})\longrightarrow m^2a_y^2\Big[\overline{\partial}_y\!\cdot\!\overline{\partial}_z-2Ha_y\overline{\partial}_{z^0}-\tfrac{(D-2)H^2}{m^2}\overline{\partial}_{y^0}\overline{\partial}_{z^0}\Big]\,, \tag{134}$$

and keep only ones contributing in the large mass limit.

The last flat space descendant contribution, that we label by "(C)", is the first term in line (126) with all four derivatives extracted. Reducing it is straightforward and again follows from applying the 3-point Donoghue identity, followed by acting the derivatives on the external mode function,

$$(C):\quad 2(a_y a_z)^{D-2}(\overline{\partial}_y\!\cdot\!\overline{\partial}_z)^2 i\Delta_m(x;y)$$
$$\longrightarrow a_z^{D-2}(\overline{\partial}_y\!\cdot\!\overline{\partial}_z)^2\frac{i\delta^D(x-y)}{m^2a_y^2}\longrightarrow -a_z^{D-2}(\overline{\partial}_{z^0})^2 i\delta^D(x-y)\,. \tag{135}$$

5.2.2 Reduction of pure de Sitter terms

Pure de Sitter contributions to $\overline{5a}$ residual diagram consist of twice the contributions in (120), and lines (123) and (124), which added together read,

$$\tfrac{1}{4}(D-2)(a_y a_z)^{D-2}\times\Big\{H(a_y-a_z)\Big[4\overline{\partial}_y\!\cdot\!\overline{\partial}_z\big(\overline{\partial}_{y^0}-\overline{\partial}_{z^0}\big) \tag{136}$$
$$+\overline{\partial}_y\!\cdot\!\overline{\partial}_z\big(\widetilde{\partial}_{y^0}-\widetilde{\partial}_{z^0}\big)+2\overline{\partial}_y\!\cdot\!\widetilde{\partial}_y\,\overline{\partial}_{z^0}-2\overline{\partial}_z\!\cdot\!\widetilde{\partial}_z\,\overline{\partial}_{y^0} \tag{137}$$
$$+(D-2)H(a_y-a_z)\big(2\overline{\partial}_y\!\cdot\!\overline{\partial}_z-\overline{\nabla}_y\!\cdot\!\overline{\nabla}_z\big)\Big]+H^2(a_y^2+a_z^2)\overline{\partial}_y\!\cdot\!\overline{\partial}_z\Big\}\,. \tag{138}$$

Terms in the first line (136) with all derivatives extracted, whose contribution we label by "(D)", are immediately reduced by the Donoghue 3-point identity followed by acting derivatives on external legs. Only the first term in (136) manages to contribute,

$$(D):\qquad (D-2)H(a_y-a_z)(a_ya_z)^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\overline{\partial}_{y^0}i\Delta_m(x;y) \tag{139}$$
$$\longrightarrow (D-2)H(a_y-a_z)a_z^{D-2}\overline{\partial}_y\!\cdot\!\overline{\partial}_z\overline{\partial}_{y^0}\frac{i\delta^D(x-y)}{2m^2a_y^2} \tag{140}$$
$$\longrightarrow \tfrac{1}{2}(D-2)H(a_y-a_z)a_z^{D-2}\overline{\partial}_{z^0}i\delta^D(x-y)\,. \tag{141}$$

The second line (137) contains terms with single internal derivatives that cannot be extracted from the triangular loop. Reduction of such terms requires the derivative 3-point Donoghue identity (36). For the first two terms, their contribution labeled by (E), we can apply this identity immediately after reflecting the internal derivative to act on the y coordinate of the graviton propagator. Acting derivatives on external legs then concludes the reduction,

$$(E): \quad \tfrac{1}{2}(D-2)(a_y a_z)^{D-2} H(a_y - a_z)\Big[\overline{\partial}_y \cdot \overline{\partial}_z \widetilde{\partial}_{y^0} + \overline{\partial}_y \cdot \widetilde{\partial}_y \,\overline{\partial}_{z^0}\Big] i\Delta_m(x;y) \tag{142}$$

$$\longrightarrow \tfrac{1}{2}(D-2) a_z^{D-2} H(a_y - a_z)\Big[\overline{\partial}_y \cdot \overline{\partial}_z \,\overline{\partial}_{y^0} + \overline{\partial}_y \cdot \overline{\partial}_y \,\overline{\partial}_{z^0}\Big] \frac{i\delta^D(x-y)}{2m^2 a_y^2} \tag{143}$$

$$\longrightarrow \tfrac{1}{2}(D-2) a_z^{D-2} H(a_y - a_z) \overline{\partial}_{z^0} i\delta^D(x-y) \,. \tag{144}$$

For the remaining term in line (137), labeled by "(F)", the derivative Donoghue identity should not be applied directly in order not to lose the subleading terms. We first write out the derivatives contracted at the same vertex in terms of d'Alembertian operators keeping track of correction terms up to order H^2,

$$(F): \quad -\tfrac{1}{2}(D-2)(a_y a_z)^{D-2} H(a_y - a_z) \overline{\partial}_z \cdot \widetilde{\partial}_z \,\overline{\partial}_{y^0} i\Delta_m(x;y) \tag{145}$$

$$\longrightarrow -\tfrac{1}{4}(D-2) a_y^{D-2} H(a_y - a_z)\big(\mathcal{D}_z - \overline{\mathcal{D}}_z - \widetilde{\mathcal{D}}_z\big) \overline{\partial}_{y^0} i\Delta_m(x;y) \tag{146}$$

$$-\tfrac{1}{4}(D-2)(a_y a_z)^{D-2} H^2 a_z^2 \widetilde{\partial}_{y^0} \overline{\partial}_{y^0} i\Delta_m(x;y) \,. \tag{147}$$

None of the terms in line (146) can contribute, but the term in line (147) produces a nonvanishing contribution after applying the derivative 3-point Donoghue identity and acting the external leg derivatives,

$$(F): \quad \longrightarrow -\tfrac{1}{4}(D-2) H^2 a_z^D (\overline{\partial}_{y^0})^2 \frac{i\delta^D(x-y)}{2m^2 a_y^2} \longrightarrow \tfrac{1}{4}(D-2) H^2 a_z^D i\delta^D(x-y) \,. \tag{148}$$

Lastly, the remaining terms, given in line (138), do not have enough external derivatives to contribute.

5.2.3 Self-mass contributions

The reduced diagram $\overline{5a}$ collects (A)–(F) contributions from (129), (133), (135), (141), (144), and (148). It is natural to consider it together with the reduced diagram $\overline{5b}$, that is obtained from $\overline{5a}$ by reflecting $x \leftrightarrow y$ in the final results of reductions in sections 5.2.1 and 5.2.2. Together they make up diagrams of the same topology,

$$\big[-iV_{\overline{5a+b}}\big]^{\rm 3pt}_{(I)} = (-i\lambda)^2 \delta^D(x-y) \delta^D(x'-y') (a_x a_{x'})^D \\ \times i\kappa^2 \!\int d^D z \, \Theta_I \Big[a_z^{D-2} i\Delta_C(x;z) i\Delta_A(x;z)\Big] \times i\Delta_A(z;x') \,, \tag{149}$$

where contributions (A)–(F) are accounted for by derivative operators that act to their right,

$$\Theta_{(A)} = \tfrac{1}{2}\mathcal{D}_z a_z^{2-D} + (D-3)H^2 a_z^2\,, \quad \Theta_{(B)} = \partial_x \!\cdot\! \partial_z - (D+2)Ha_x\partial_0^z\,, \qquad (150)$$
$$\Theta_{(C)} = 2(\partial_0^z)^2\,, \quad \Theta_{(D+E)} = 2(D-2)\partial_0^z H(a_x - a_z)\,, \quad \Theta_{(F)} = -\tfrac{1}{2}(D-2)H^2 a_z^2\,.$$

In addition (A) contributions generate diagrams of different topology when scalar d'Alembertians in (129) contract either the graviton or the massless scalar propagator,

$$\bigl[-iV_{\overline{5a+b}}\bigr]^{\rm 4pt}_{(A)} = (-i\lambda)^2 \delta^D(x-y)\delta^D(x'-y')(a_x a_{x'})^D$$
$$\times \tfrac{1}{2}\kappa^2\bigl[i\Delta_C(x;x) - i\Delta_A(x;x)\bigr] \times i\Delta_A(x;x')\,. \qquad (151)$$

Some of the differential operators in (150) can be further rewritten owing to the topology of diagram (149) and the identity,

$$a_x^{D-n_x}\partial_0^z\Bigl[a_z^{D-n_z} i\Delta_C(x;z) i\Delta_A(x;z)\Bigr]$$
$$= a_x^{D-n_x}\bigl(-\partial_0^x + \partial_0^x + \partial_0^z\bigr)\Bigl[a_z^{D-n_z} i\Delta_C(x;z) i\Delta_A(x;z)\Bigr]$$
$$\longrightarrow a_x^{D-n_x}\Bigl[(1-n_x)Ha_x + \partial_0^x + \partial_0^z\Bigr]\Bigl[a_z^{D-n_z} i\Delta_C(x;z) i\Delta_A(x;z)\Bigr]\,, \qquad (152)$$

that is derived by partial integration and accounting for action of time derivatives on external legs,

$$\partial_0\bigl[u_m(x,\vec{k})u_m^*(x,\vec{k}')\bigr] \xrightarrow{m^2\to\infty} -(D-1)Ha_x \times u_m(x,\vec{k})u_m^*(x,\vec{k}')\,. \qquad (153)$$

It is (B) and (C) contributions that benefit from reorganizing derivatives,

$$\Theta_{(B)} \longrightarrow -\mathcal{D}_z a_z^{2-D} - \bigl(\partial_0^z + 2Ha_x\bigr)\bigl(\partial_0^x + \partial_0^z + 2Ha_x\bigr) - (D-2)\partial_0^z H(a_x - a_z)\,,$$
$$\Theta_{(C)} \longrightarrow 2\bigl(\partial_0^z - Ha_x\bigr)\bigl(\partial_0^x + \partial_0^z + 2Ha_x\bigr)\,. \qquad (154)$$

The utility of this form is that only $\mathcal{D}_z a_z^{2-D}$ term will contain divergences, while all the other terms will be manifestly finite. Collecting all the contributions (A)–(F) to diagrams $\overline{5a} + \overline{5b}$ finally produces a contribution to the β-type diagram from Fig. 2,

$$\bigl[-if_\beta(x;x')\bigr]_{\overline{5}} = \tfrac{\kappa^2}{2}\Bigl[-\mathcal{D}_{x'} a_{x'}^{2-D} + 2\bigl(\partial_0^{x'} - 4Ha_x\bigr)\bigl(\partial_0^x + \partial_0^{x'} + 2Ha_x\bigr)$$
$$+2(D-2)\partial_0^{x'}(a_x - a_{x'}) + (D-4)H^2 a_{x'}^2\Bigr]\Bigl[a_{x'}^{D-2} i\Delta_C(x;x') i\Delta_A(x;x')\Bigr]$$
$$-\tfrac{\kappa^2}{2} i\delta^D(x-x')\Bigl[i\Delta_C(x';x') - i\Delta_A(x';x')\Bigr]\,. \qquad (155)$$

The two remaining diagrams $\overline{5c}+\overline{5d}$ are then inferred by reflecting $x \leftrightarrow x'$ in the β-type contribution above, yielding a γ-type contribution from Fig. 2,

$$\Bigl[-if_\gamma(x;x')\Bigr]_{\overline{5}} = \tfrac{\kappa^2}{2}\Bigl[-\mathcal{D}_x a_x^{2-D} + 2\bigl(\partial_0^x - 4Ha_{x'}\bigr)\bigl(\partial_0^x + \partial_0^{x'} + 2Ha_{x'}\bigr) \\ -2(D-2)\partial_0^x(a_x - a_{x'}) + (D-4)H^2 a_x^2\Bigr]\Bigl[a_x^{D-2} i\Delta_C(x;x') i\Delta_A(x;x')\Bigr] \\ -\tfrac{\kappa^2}{2} i\delta^D(x-x')\Bigl[i\Delta_C(x;x) - i\Delta_A(x;x)\Bigr] . \qquad (156)$$

5.3 $V_{\overline{4}}(x;x';y;y')$ Diagrams

Each of the residual diagrams (100-103) contains a product of truncated vertices from a single graviton coupled to the massive scalar kinetic term. For the case of $-iV_{\overline{4a}}$ these are,

$$\Bigl[a_y^{D-2}\overline{\partial}_y^\mu \partial_y^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Bigr]\Bigl[a_{y'}^{D-2}\overline{\partial}_{y'}^\rho \partial_{y'}^\sigma - \tfrac{1}{4}\eta^{\rho\sigma}\widetilde{\mathcal{D}}_{y'}\Bigr] = (a_y a_{y'})^{D-2}\overline{\partial}_y^\mu \partial_y^\nu \overline{\partial}_{y'}^\rho \partial_{y'}^\sigma \\ -\tfrac{1}{4} a_y^{D-2}\overline{\partial}_y^\mu \partial_y^\nu \widetilde{\mathcal{D}}_{y'}\eta^{\rho\sigma} - \tfrac{1}{4} a_{y'}^{D-2}\widetilde{\mathcal{D}}_y \eta^{\mu\nu}\overline{\partial}_{y'}^\rho \partial_{y'}^\sigma + \tfrac{1}{16}\widetilde{\mathcal{D}}_y \widetilde{\mathcal{D}}_{y'}\eta^{\mu\nu}\eta^{\rho\sigma} . \qquad (157)$$

With the simplification (108) these factors will be contracted into the flat space tensor,

$$\Bigl[a_y^{D-2}\overline{\partial}_y^\mu \partial_y^\nu - \tfrac{1}{4}\eta^{\mu\nu}\widetilde{\mathcal{D}}_y\Bigr]\Bigl[a_{y'}^{D-2}\overline{\partial}_{y'}^\rho \partial_{y'}^\sigma - \tfrac{1}{4}\eta^{\rho\sigma}\widetilde{\mathcal{D}}_{y'}\Bigr]\Bigl[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}\Bigr] \\ = (a_y a_{y'})^{D-2}\Bigl[\overline{\partial}_y \cdot \overline{\partial}_{y'} \partial_y \cdot \partial_{y'} + \overline{\partial}_y \cdot \partial_{y'} \overline{\partial}_{y'} \cdot \partial_y\Bigr] - \frac{2}{D-2}\Bigl[m^2 a_y^D + \tfrac{1}{2}(\mathcal{D}_y - m^2 a_y^D)\Bigr] \\ \times\Bigl[m^2 a_{y'}^D + \tfrac{1}{2}(\mathcal{D}_{y'} - m^2 a_{y'}^D)\Bigr] - \tfrac{1}{4}\widetilde{\mathcal{D}}_y \widetilde{\mathcal{D}}_{y'} . \qquad (158)$$

Note that the last term can be dropped. We label the remaining terms "A", "B" and "C", respectively.

The "C" terms proportional to $-2/(D-2)$ in expression (158) can already be recognized as making 2-point, 3-point and 4-point contributions,

$$2-\text{Point} \Longrightarrow -\tfrac{1}{2(D-2)} \times (\mathcal{D}_y - m^2 a_y^D) \times (\mathcal{D}_{y'} - m^2 a_{y'}^D) , \qquad (159)$$

$$3-\text{Point} \Longrightarrow -\tfrac{1}{D-2} \times (\mathcal{D}_y - m^2 a_y^D) \times m^2 a_{y'}^D - \tfrac{1}{D-2} \times m^2 a_y^D \times (\mathcal{D}_{y'} - m^2 a_{y'}^D) , \qquad (160)$$

$$4-\text{Point} \Longrightarrow -\tfrac{2}{D-2} \times m^2 a_y^D \times m^2 a_{y'}^D . \qquad (161)$$

To reach the four classes of 2-point forms described in Figure 2 we first recall that each term in $-iV_{\overline{4a}}$ shares a common factor of,

$$\kappa^2 \lambda^2 (a_x a_{x'})^D i\Delta_m(x;y) i\Delta_m(x';y') i\Delta_A(x;x') i\Delta_C(y;y') . \qquad (162)$$

One now acts the massive scalar kinetic operators and invokes the appropriate Donoghue Identity. For example, the 2-point contribution (159) gives

$$-\tfrac{\kappa^2\lambda^2}{2(D-2)}(a_x a_{x'})^D \times i\delta^D(x-y)\times i\delta^D(x'-y')\times i\Delta_A(x;x')i\Delta_C(y;y')$$
$$= \delta^D(x-y)\delta^D(x'-y')\times -i\lambda^2(a_x a_{x'})^D\times \tfrac{i\kappa^2}{2(D-2)} i\Delta_A(x;x')i\Delta_C(x;x') \; .(163)$$

The final factor of (163) contributes to $f_\alpha(x;x')$. The 3-point contribution (160) requires the 3-point Donoghue Identity (35) after acting the scalar kinetic operators,

$$-\tfrac{\kappa^2\lambda^2}{D-2}(a_x a_{x'})^D\times i\delta^D(x-y)\times m^2 a_{y'}^D\times i\Delta_m(x';y')i\Delta_A(x;x')i\Delta_C(y;y')$$
$$-\tfrac{\kappa^2\lambda^2}{D-2}(a_x a_{x'})^D\times i\delta^D(x'-y')\times m^2 a_y^D\times i\Delta_m(x;y)i\Delta_A(x;x')i\Delta_C(y;y')$$
$$\longrightarrow -\tfrac{\kappa^2\lambda^2}{D-2}(a_x a_{x'})^D\times i\delta^D(x-y)\times i\delta^D(x'-y')\times i\Delta_A(x;x')i\Delta_C(x;x') \; ,(164)$$
$$= \delta^D(x-y)\delta^D(x'-y')\times -i\lambda^2(a_x a_{x'})^D\times\tfrac{i\kappa^2}{D-2} i\Delta_A(x;x')i\Delta_C(x;x') \; . \quad (165)$$

This is a contribution to $f_\alpha(x;x')$ of twice the size as (163).

The 4-point contribution (161) has no scalar kinetic operators but it requires the 4-point Donoghue Identity (39). The first line contains the factor,

$$\Bigl[-\frac{i}{m^2}+\frac{i}{3m^4}\Bigl(\frac{\overline{\partial}_x\cdot\overline{\partial}_{x'}}{a_x a_{x'}}-m^2\Bigr)\Bigr]\frac{\delta^D(x-y)\delta^D(x'-y')}{(a_x a_{x'})^D} \; . \qquad (166)$$

The external state derivatives can be expanded using (112),

$$\frac{\overline{\partial}_x\cdot\overline{\partial}_{x'}}{a_x a_{x'}} = m^2 - i(D-1)mH - \tfrac12(D-1)^2H^2 + \tfrac12\Bigl\|\frac{\vec{k}_1}{a_x}-\frac{\vec{k}_3}{a_{x'}}\Bigr\|^2 + O\Bigl(\frac{1}{m}\Bigr) \; . \quad (167)$$

It follows that the 4-point contribution (161) reduces to,

$$-\tfrac{2m^4(a_y a_{y'})^D}{D-2}\times\kappa^2\lambda^2(a_x a_{x'})^D i\Delta_m(x;y)i\Delta_m(x';y')i\Delta_A(x;x')i\Delta_C(y;y')$$
$$\longrightarrow \delta^D(x-y)\delta^D(x'-y')\times -i\lambda^2(a_x a_{x'})^D$$
$$\times -\tfrac{\kappa^2}{3(D-2)}\Bigl[3m^2+i(D-1)mH+\tfrac12(D-1)^2H^2-\tfrac12\|\tfrac{\vec{k}_1}{a_x}-\tfrac{\vec{k}_3}{a_{x'}}\|^2\Bigr]$$
$$\times\Biggl\{\int d^Dz\, a_z^D i\Delta_A(x;z)i\Delta_C(x;z)i\Delta_A(z;x') + \Bigl(x^\mu \longleftrightarrow x'^\mu\Bigr)\Biggr\} . (168)$$

The 4-point "C" contribution from $-iV_{\overline{\overline{4b}}}$ is almost the same as (168) except the square-bracketed term in the 3rd line is,

$$-iV_{\overline{\overline{4b}}} \Longrightarrow 3m^2 - i(D-1)mH + \tfrac12(D-1)^2H^2 - \tfrac12\|\tfrac{\vec{k}_2}{a_x}-\tfrac{\vec{k}_4}{a_{x'}}\|^2 \; . \qquad (169)$$

The other contributions are simpler,

$$-iV_{\overline{\overline{4c}}} \Longrightarrow -3m^2 + \tfrac{1}{2}\Big\| \tfrac{\vec{k}_1}{a_x} - \tfrac{\vec{k}_4}{a_{x'}} \Big\|^2 \qquad , \qquad -iV_{\overline{\overline{4d}}} \Longrightarrow -3m^2 + \tfrac{1}{2}\Big\| \tfrac{\vec{k}_2}{a_x} - \tfrac{\vec{k}_3}{a_{x'}} \Big\|^2 . \tag{170}$$

Adding (169) and (170) to the 3rd line of (168), and using spatial momentum conservation (116) gives,

$$-iV_{\overline{\overline{4a,b,c,d}}} \Longrightarrow (D-1)^2 H^2 - \frac{\|\vec{q}\,\|^2}{a_x a_{x'}} . \tag{171}$$

This corresponds to Group C contributions to the Class β and γ diagrams of Figure 2,

$$-if_{\overline{\overline{4C}}\beta}(x;x') = \tfrac{\kappa^2 a_{x'}^D}{3(D-2)} \Big[\tfrac{q^2}{a_x a_{x'}} - (D-1)^2 H^2 \Big] i\Delta_A(x;x') i\Delta_C(x;x') , \tag{172}$$

$$-if_{\overline{\overline{4C}}\gamma}(x;x') = \tfrac{\kappa^2 a_{x}^D}{3(D-2)} \Big[\tfrac{q^2}{a_x a_{x'}} - (D-1)^2 H^2 \Big] i\Delta_A(x;x') i\Delta_C(x;x') . \tag{173}$$

In contrast, the C contributions to the Class α diagram of Figure 2 is just four times the sum of the final factors of expressions (163) and (165),

$$-if_{\overline{\overline{4C}}\alpha}(x;x') = \tfrac{6\kappa^2}{D-2}\, i\Delta_A(x;x') i\Delta_C(x;x') . \tag{174}$$

There is no contribution to the Class δ diagram.

The "A" and "B" terms in expression (158) require extensive reduction using momentum conservation (41), the graviton reflection identity (110) and action of the external derivatives (112). This is simplest for the two factors of the "B" term $\overline{\partial}_y \cdot \partial_{y'} \times \overline{\partial}_{y'} \cdot \partial_y$. The first factor gives,

$$\overline{\partial}_y \cdot \partial_{y'} = -\overline{\partial}_y \cdot \Big(\overline{\partial}_{y'} + \widetilde{\partial}_{y'} + \partial_{a_{y'}} \Big) , \tag{175}$$

$$\simeq -\overline{\partial}_y \cdot \overline{\partial}_{y'} + \overline{\partial}_y \cdot \Big(\widetilde{\partial}_y + \tfrac{1}{2}\partial_{a_y} - \tfrac{1}{2}\partial_{a_{y'}} \Big) , \tag{176}$$

$$\simeq -\overline{\partial}_y \cdot \overline{\partial}_{y'} + \tfrac{1}{2}\Big[\partial_y \cdot (\partial_y + \partial_{a_y}) - m^2 a_y^2 \Big] - \tfrac{1}{2}\widetilde{\partial}_y \cdot (\widetilde{\partial}_y + \partial_{a_y}) - (\tfrac{D-2}{2}) a_y a_{y'} H^2 \Delta\eta_{yy'} \overline{\partial}_{y^0} , \tag{177}$$

where $\Delta\eta_{yy'} \equiv y^0 - y'^0$. The second factor produces a similar result,

$$\overline{\partial}_{y'} \cdot \partial_y \simeq -\overline{\partial}_y \cdot \overline{\partial}_{y'} + \tfrac{1}{2}\Big[\partial_{y'} \cdot (\partial_{y'} + \partial_{a_{y'}}) - m^2 a_{y'}^2 \Big] - \tfrac{1}{2}\widetilde{\partial}_{y'} \cdot (\widetilde{\partial}_{y'} + \partial_{a_{y'}}) + (\tfrac{D-2}{2}) a_y a_{y'} H^2 \Delta\eta_{yy'} \overline{\partial}_{y'^0} . \tag{178}$$

If we use $\widetilde{\mathcal{D}}_y i\Delta_C(y;y') = \widetilde{\mathcal{D}}_{y'} i\Delta_C(y;y') \longrightarrow 2(D-3)i\Delta_C(y;y')$ the "B" term in (158) gives,

$$(a_y a_{y'})^{D-2}\overline{\partial}_y\cdot\partial_{y'}\overline{\partial}_{y'}\cdot\partial_y \longrightarrow \Biggl\{a_y^{D-2}\overline{\partial}_y\cdot\overline{\partial}_{y'} + (\tfrac{D-2}{2})a_y^{D-1}a_{y'}H^2\Delta\eta_{yy'}\overline{\partial}_{y^0}$$
$$+(D-3)a_y^D H^2 - \tfrac{1}{2}\Bigl[\mathcal{D}_y - m^2 a_y^D\Bigr]\Biggr\}\Biggl\{a_{y'}^{D-2}\overline{\partial}_y\cdot\overline{\partial}_{y'} - (\tfrac{D-2}{2})a_y a_{y'}^{D-1}H^2\Delta\eta_{yy'}\overline{\partial}_{y'^0}$$
$$+(D-3)a_{y'}^D H^2 - \tfrac{1}{2}\Bigl[\mathcal{D}_{y'} - m^2 a_{y'}^D\Bigr]\Biggr\} . \qquad (179)$$

The 2-point contribution from (179) is easy to read off,

$$2-\text{Point} \Longrightarrow \tfrac{1}{4}\times(\mathcal{D}_y - m^2 a_y^D)\times(\mathcal{D}_{y'} - m^2 a_{y'}^D) \; . \qquad (180)$$

This differs from the "C" contribution (159) only by the replacement of $-\frac{1}{2(D-2)}$ by $\frac{1}{4}$, so we can immediately conclude,

$$-if_{\overline{\overline{4B}}\alpha}(x;x') = -\kappa^2 i\Delta_A(x;x') i\Delta_C(x;x') \; . \qquad (181)$$

Because the 3-point Donoghue Identity (35) goes like $1/m^2$, which can only be compensated by two external derivatives. For $-iV_{\overline{\overline{4a}}}$ the only 3-point contribution which survives for large m is,

$$\overline{\overline{\overline{4aB}}} \Longrightarrow -\tfrac{1}{2}a_{y'}^{D-2}\overline{\partial}_y\cdot\overline{\partial}_{y'}(\mathcal{D}_y - m^2 a_y^D) - \tfrac{1}{2}a_y^{D-2}\overline{\partial}_y\cdot\overline{\partial}_{y'}(\mathcal{D}_{y'} - m^2 a_{y'}^D) \; . \qquad (182)$$

The other three variations give,

$$\overline{\overline{\overline{4bB}}} \Longrightarrow -\tfrac{1}{2}a_{x'}^{D-2}\overline{\partial}_x\cdot\overline{\partial}_{x'}(\mathcal{D}_x - m^2 a_x^D) - \tfrac{1}{2}a_x^{D-2}\overline{\partial}_x\cdot\overline{\partial}_{x'}(\mathcal{D}_{x'} - m^2 a_{x'}^D) \; , \qquad (183)$$
$$\overline{\overline{\overline{4cB}}} \Longrightarrow -\tfrac{1}{2}a_{y'}^{D-2}\overline{\partial}_x\cdot\overline{\partial}_{y'}(\mathcal{D}_x - m^2 a_x^D) - \tfrac{1}{2}a_x^{D-2}\overline{\partial}_x\cdot\overline{\partial}_{y'}(\mathcal{D}_{y'} - m^2 a_{y'}^D) \; , \qquad (184)$$
$$\overline{\overline{\overline{4dB}}} \Longrightarrow -\tfrac{1}{2}a_{x'}^{D-2}\overline{\partial}_x\cdot\overline{\partial}_{x'}(\mathcal{D}_y - m^2 a_y^D) - \tfrac{1}{2}a_y^{D-2}\overline{\partial}_y\cdot\overline{\partial}_{x'}(\mathcal{D}_{x'} - m^2 a_{x'}^D) \; , \qquad (185)$$

Because the external derivatives in (182-183) are either both incoming or outgoing they cancel against the mixed incoming-outgoing derivatives of (184-185) when account is taken of the factors of $\delta^D(x-y)\delta^D(x'-y')$,

$$\overline{\partial}_y\cdot\overline{\partial}_{y'} \longrightarrow +m^2 a_y a_{y'} + O(m) \quad , \quad \overline{\partial}_x\cdot\overline{\partial}_{x'} \longrightarrow +m^2 a_x a_{x'} + O(m) \; , \qquad (186)$$
$$\overline{\partial}_x\cdot\overline{\partial}_{y'} \longrightarrow -m^2 a_x a_{y'} + O(m) \quad , \quad \overline{\partial}_y\cdot\overline{\partial}_{x'} \longrightarrow -m^2 a_y a_{x'} + O(m) \; . \qquad (187)$$

The 4-point "B" contributions are more complicated because they can involve as many as four external derivatives, and because the 4-point Donoghue Identities (39-40) contain factors of both $1/m^2$ and $1/m^4$, as well as up to two external derivatives. It is therefore best to make a preliminary reduction based on each external derivative potentially contributing a factor of m,

$$4\!-\!\text{Point} \Longrightarrow (a_y a_{y'})^D \Biggl\{ \Bigl(\frac{\overline{\partial}_y \!\cdot\! \overline{\partial}_{y'}}{a_y a_{y'}} \Bigr)^2 \!+ H^2 \Bigl(\frac{\overline{\partial}_y \!\cdot\! \overline{\partial}_{y'}}{a_y a_{y'}} \Bigr) \Bigl[(\tfrac{D-2}{2}) \Delta\eta_{yy'} (\overline{\partial}_{y^0} \!-\! \overline{\partial}_{y'^0}) + (D\!-\!3)(\tfrac{a_y}{a_{y'}} \!+\! \tfrac{a_{y'}}{a_y}) \Bigr] - (\tfrac{D-2}{2})^2 H^4 \Delta\eta_{yy'}^2 \overline{\partial}_{y^0} \overline{\partial}_{y'^0} \Biggr\} . \tag{188}$$

The appropriate factors for the three other permutations are given by making the following coordinate replacements in (188),

$$-iV_{\overline{\overline{4b}}} \quad \Longrightarrow \quad x^{\mu} \longleftrightarrow y^{\mu} \qquad \text{and} \qquad x'^{\mu} \longleftrightarrow y'^{\mu} \; , \tag{189}$$

$$-iV_{\overline{\overline{4c}}} \quad \Longrightarrow \quad x'^{\mu} \longleftrightarrow y'^{\mu} \; , \tag{190}$$

$$-iV_{\overline{\overline{4d}}} \quad \Longrightarrow \quad x^{\mu} \longleftrightarrow y^{\mu} \; . \tag{191}$$

Each of these factors must be multiplied into a factor from the appropriate 4-point Donoghue Identity,

$$-iV_{\overline{\overline{4a}}} \quad \Longrightarrow \quad \Bigl[-\frac{i}{m^2} + \frac{i}{3m^4} \Bigl(\frac{\overline{\partial}_x \!\cdot\! \overline{\partial}_{x'}}{a_x a_{x'}} - m^2 \Bigr) \Bigr] \frac{\delta^D(x\!-\!y) \delta^D(x'\!-\!y')}{(a_x a_{x'})^D} \; , \tag{192}$$

$$-iV_{\overline{\overline{4b}}} \quad \Longrightarrow \quad \Bigl[-\frac{i}{m^2} + \frac{i}{3m^4} \Bigl(\frac{\overline{\partial}_y \!\cdot\! \overline{\partial}_{y'}}{a_y a_{y'}} - m^2 \Bigr) \Bigr] \frac{\delta^D(x\!-\!y) \delta^D(x'\!-\!y')}{(a_y a_{y'})^D} \; , \tag{193}$$

$$-iV_{\overline{\overline{4c}}} \quad \Longrightarrow \quad \Bigl[\frac{i}{m^2} + \frac{i}{3m^4} \Bigl(\frac{\overline{\partial}_y \!\cdot\! \overline{\partial}_{x'}}{a_y a_{x'}} + m^2 \Bigr) \Bigr] \frac{\delta^D(x\!-\!y) \delta^D(x'\!-\!y')}{(a_y a_{x'})^D} \; , \tag{194}$$

$$-iV_{\overline{\overline{4d}}} \quad \Longrightarrow \quad \Bigl[\frac{i}{m^2} + \frac{i}{3m^4} \Bigl(\frac{\overline{\partial}_x \!\cdot\! \overline{\partial}_{y'}}{a_x a_{y'}} + m^2 \Bigr) \Bigr] \frac{\delta^D(x\!-\!y) \delta^D(x'\!-\!y')}{(a_x a_{y'})^D} \; . \tag{195}$$

Then all four permutations are summed and one discards terms which vanish in the large m limit.

It is best to proceed sequentially with each of the four terms in (188), and its permutations, times the two terms from expressions (192-195). The product of the first factors gives,

$$\frac{i}{m^2}\Biggl\{-\Bigl(\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{y'}}{a_y a_{y'}}\Bigr)^2 - \Bigl(\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{x'}}{a_x a_{x'}}\Bigr)^2 + \Bigl(\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{y'}}{a_x a_{y'}}\Bigr)^2 + \Bigl(\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{x'}}{a_y a_{x'}}\Bigr)^2\Biggr\} \longrightarrow i\Bigl[4(D\!-\!2)^2 H^2 - \tfrac{2q^2}{a_x a_{x'}}\Bigr]. \tag{196}$$

The product of the first factor from (188) and its permutations with the final factor of (192-195) is,

$$\frac{i}{3m^4}\Biggl\{\Bigl(\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{y'}}{a_y a_{y'}}\Bigr)^2\Bigl[\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{x'}}{a_x a_{x'}}\!-\!m^2\Bigr] + \Bigl(\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{x'}}{a_x a_{x'}}\Bigr)^2\Bigl[\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{y'}}{a_y a_{y'}}\!-\!m^2\Bigr] + \Bigl(\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{y'}}{a_x a_{y'}}\Bigr)^2\Bigl[\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{x'}}{a_y a_{x'}}\!+\!m^2\Bigr] + \Bigl(\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{x'}}{a_y a_{x'}}\Bigr)^2\Bigl[\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{y'}}{a_x a_{y'}}\!+\!m^2\Bigr]\Biggr\}$$
$$\longrightarrow \frac{i}{3}\Bigl[4(D\!-\!1)(D\!-\!2)H^2 - (D\!-\!1)^2 H^2 + \tfrac{q^2}{a_x a_{x'}}\Bigr] . \tag{197}$$

The product of the second factor of (188) and its permutations with the first factor of (192-195) is,

$$i(\tfrac{D-2}{2})\frac{H^2}{m^2}\Biggl\{-\Bigl(\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{y'}}{a_y a_{y'}}\Bigr)\Delta\eta_{yy'}(\overline{\partial}_{y^0}\!-\!\overline{\partial}_{y'^0}) - \Bigl(\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{x'}}{a_x a_{x'}}\Bigr)\Delta\eta_{xx'}(\overline{\partial}_{x^0}\!-\!\overline{\partial}_{x'^0}) + \Bigl(\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{x'}}{a_y a_{x'}}\Bigr)\Delta\eta_{yx'}(\overline{\partial}_{y^0}\!-\!\overline{\partial}_{x'^0}) + \Bigl(\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{y'}}{a_x a_{y'}}\Bigr)\Delta\eta_{xy'}(\overline{\partial}_{x^0}\!-\!\overline{\partial}_{y'^0})\Biggr\}$$
$$\longrightarrow i\tfrac{3}{2}(D\!-\!1)(D\!-\!2)a_x a_{x'} H^4 \Delta\eta_{xx'}^2 \; . \tag{198}$$

For the second factor of (188) times the second factor of (192-195) it is only the a and b permutations which survive the large m limit,

$$i(\tfrac{D-2}{2})\frac{H^2}{3m^4}\Biggl\{\Bigl(\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{y'}}{a_y a_{y'}}\Bigr)\Delta\eta_{yy'}(\overline{\partial}_{y^0}\!-\!\overline{\partial}_{y'^0})\Bigl[\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{x'}}{a_x a_{x'}}\!-\!m^2\Bigr] + \Bigl(\frac{\overline{\partial}_x\!\cdot\!\overline{\partial}_{x'}}{a_x a_{x'}}\Bigr)\Delta\eta_{xx'}(\overline{\partial}_{x^0}\!-\!\overline{\partial}_{x'^0})\Bigl[\frac{\overline{\partial}_y\!\cdot\!\overline{\partial}_{y'}}{a_y a_{y'}}\!-\!m^2\Bigr]\Biggr\}$$
$$\longrightarrow -\tfrac{2}{3}i(D\!-\!1)(D\!-\!2)a_x a_{x'} H^4 \Delta\eta_{xx'}^2 . \tag{199}$$

The 3rd and 4th factors of (188), and their permutations, contain only two external derivatives so only the first factor of (192-195) survives the large m limit,

$$\frac{iH^2}{m^2}\Biggl\{-\Bigl(\frac{\overline{\partial}_y\cdot\overline{\partial}_{y'}}{a_y a_{y'}}\Bigr)(D-3)\bigl(\tfrac{a_y}{a_{y'}}+\tfrac{a_{y'}}{a_y}\bigr)-\bigl(\tfrac{D-2}{2}\bigr)^2\overline{\partial}_{y^0}\overline{\partial}_{y'^0}H^2\Delta\eta^2_{yy'}+\dots\Biggr\}$$

$$\longrightarrow i4H^2\Bigl[-(D-3)\bigl(\tfrac{a_x}{a_{x'}}+\tfrac{a_{x'}}{a_x}\bigr)+\bigl(\tfrac{D-2}{2}\bigr)^2 a_x a_{x'}H^2\Delta\eta^2_{xx'}\Bigr], \qquad (200)$$

$$= i4H^2\Bigl[-2(D-3)+\tfrac{1}{4}(D^2-8D+28)a_x a_{x'}H^2\Delta\eta^2_{xx'}\Bigr]. \qquad (201)$$

The factors of $a_x a_{x'} H^2 \Delta\eta^2_{xx'}$ in expressions (198), (199) and (201) take us beyond the 4-point Donoghue Identities (39-40). These factors must actually be internal to the $d^D z$ integrations on the last lines of expressions (39-40),

$$\int d^D z\, a_z^D \times a_x a_z H^2\Delta\eta^2_{xz}\times i\Delta_A(x;z)i\Delta_C(x;z)i\Delta_A(z;x')$$

$$+\int d^D z\, a_z^D i\Delta_A(x;z)\times a_z a_{x'}H^2\Delta\eta^2_{zx'}\times i\Delta_A(z;x')i\Delta_C(z;x'). \qquad (202)$$

The same comments apply to the factors of $q^2/a_x a_{x'}$ which appear in expressions (196) and (197). These factors must give internal derivatives on the last lines of (39-40),

$$\int d^D z\, \mathcal{D}_z\Bigl[i\Delta_A(x;z)i\Delta_C(x;z)\Bigr]\times i\Delta_A(z;x')$$

$$+\int d^D z\, i\Delta_A(x;z)\times\mathcal{D}_z\Bigl[i\Delta_A(z;x')i\Delta_C(z;x')\Bigr]. \qquad (203)$$

With these understandings we can sum the "B" contributions from expressions (196), (197), (198), (199) and (201) to give,

$$-if_{\overline{\overline{4B}}\beta}(x;x') = \kappa^2\Bigl[-\tfrac{5}{6}\mathcal{D}_{x'}+F_1(D)a_{x'}^D H^2$$

$$+F_2(D)a_x a_{x'}^{D+1}H^4\Delta\eta^2_{xx'}\Bigr]i\Delta_A(x;x')i\Delta_C(x;x'), \qquad (204)$$

$$-if_{\overline{\overline{4B}}\gamma}(x;x') = \kappa^2\Bigl[-\tfrac{5}{6}\mathcal{D}_{x}+F_1(D)a_{x}^D H^2$$

$$+F_2(D)a_x^{D+1} a_{x'}H^4\Delta\eta^2_{xx'}\Bigr]i\Delta_A(x;x')i\Delta_C(x;x'). \qquad (205)$$

The functions $F_1(D)$ and $F_2(D)$ are,

$$F_1(D) \equiv 2(D-2)^2 + \tfrac{2}{3}(D-1)(D-2) - \tfrac{1}{6}(D-1)^2 - 4(D-3) \; , \qquad (206)$$

$$F_2(D) \equiv \tfrac{1}{8}(D-2)^2 + \tfrac{5}{12}(D-1)(D-2) + D-3 \; . \qquad (207)$$

The "A" term of (158) seems simpler than "B" because its first factor of $\overline{\partial}_y \cdot \overline{\partial}_{y'}$ is already reduced. However, the reduction of the second factor is complex,

$$\begin{aligned} \partial_y \cdot \partial_{y'} &= \tfrac{1}{2}(\partial_y + \partial_{y'})^2 - \tfrac{1}{2}\partial_y^2 - \tfrac{1}{2}\partial_{y'}^2 \; , & (208) \\ &\simeq \tfrac{1}{2}\Big[\overline{\partial}_y + \tfrac{1}{2}\partial_{a_y} + \overline{\partial}_{y'} + \tfrac{1}{2}\partial_{a_{y'}}\Big]^2 - \tfrac{1}{2}\partial_y^2 - \tfrac{1}{2}\partial_{y'}^2 \; , & (209) \\ &= \overline{\partial}_y \cdot \overline{\partial}_{y'} - \tfrac{1}{2}\Big[\partial_y \cdot (\partial_y + \partial_{a_y}) - m^2 a_y^2\Big] - \tfrac{1}{2}\Big[\partial_{y'} \cdot (\partial_{y'} + \partial_{a_{y'}}) - m^2 a_{y'}^2\Big] \\ &\qquad + \tfrac{1}{2}(\overline{\partial}_{y'} + \partial_y) \cdot \partial_{a_y} + \tfrac{1}{2}(\overline{\partial}_y + \partial_{y'}) \cdot \partial_{a_{y'}} + \tfrac{1}{8}(\partial_{a_y} + \partial_{a_{y'}})^2 . & (210) \end{aligned}$$

The terms on the last line can be usefully rearranged using momentum conservation,

$$\tfrac{1}{2}(\overline{\partial}_{y'} + \partial_y) \cdot \partial_{a_y} = \tfrac{1}{2}(\overline{\partial}_{y'} - \overline{\partial}_y) \cdot \partial_{a_y} - \tfrac{1}{2}(\widetilde{\partial}_y + \tfrac{1}{2}\partial_{a_y}) \cdot \partial_{a_y} - \tfrac{1}{4}\partial_{a_y}^2 \; , \qquad (211)$$

$$\tfrac{1}{2}(\overline{\partial}_y + \partial_{y'}) \cdot \partial_{a_{y'}} = \tfrac{1}{2}(\overline{\partial}_y - \overline{\partial}_{y'}) \cdot \partial_{a_{y'}} - \tfrac{1}{2}(\widetilde{\partial}_{y'} + \tfrac{1}{2}\partial_{a_{y'}}) \cdot \partial_{a_{y'}} - \tfrac{1}{4}\partial_{a_{y'}}^2 \; . \qquad (212)$$

Substituting (211-212) in (210) and using the reflection identity gives,

$$\begin{aligned} \partial_y \cdot \partial_{y'} \simeq \; & \overline{\partial}_y \cdot \overline{\partial}_{y'} - \tfrac{1}{2}\Big[\partial_y \cdot (\partial_y + \partial_{a_y}) - m^2 a_y^2\Big] - \tfrac{1}{2}\Big[\partial_{y'} \cdot (\partial_{y'} + \partial_{a_{y'}}) - m^2 a_{y'}^2\Big] \\ & - \tfrac{1}{2}(\overline{\partial}_y - \overline{\partial}_{y'}) \cdot (\partial_{a_y} - \partial_{a_{y'}}) - \tfrac{1}{2}(\widetilde{\partial}_y + \tfrac{1}{2}\partial_{a_y}) \cdot (\partial_{a_y} - \partial_{a_{y'}}) - \tfrac{1}{8}(\partial_{a_y} - \partial_{a_{y'}})^2 \; . \qquad (213) \end{aligned}$$

Expression (213) can be further simplified using the relation,

$$\partial_{a_y}^{\mu} - \partial_{a_{y'}}^{\mu} = -(D-2) a_y a_{y'} H^2 \Delta\eta_{yy'}^2 \; . \qquad (214)$$

Multiplying by $(a_y a_{y'})^{D-2} \overline{\partial}_y \cdot \overline{\partial}_{y'}$ gives the total "A" factor,

$$\begin{aligned} (a_y a_{y'})^{D-2} \overline{\partial}_y \cdot \overline{\partial}_{y'} \, \partial_y \cdot \partial_{y'} \simeq \; & (a_y a_{y'})^D \Big(\frac{\overline{\partial}_y \cdot \overline{\partial}_{y'}}{a_y a_{y'}}\Big)^2 - \tfrac{1}{2} a_{y'}^{D-2} \overline{\partial}_y \cdot \overline{\partial}_{y'} \Big[\widetilde{D}_y - m^2 a_y^D\Big] \\ & - \tfrac{1}{2} a_y^{D-2} \overline{\partial}_y \cdot \overline{\partial}_{y'} \Big[\widetilde{D}_{y'} - m^2 a_{y'}^D\Big] + (\tfrac{D-2}{2}) H^2 (a_y a_{y'})^D \Big(\frac{\overline{\partial}_y \cdot \overline{\partial}_{y'}}{a_y a_{y'}}\Big) \Big\{ \Delta\eta_{yy'} (\overline{\partial}_{y^0} - \overline{\partial}_{y'^0}) \\ & + \Delta\eta_{yy'} \Big[\widetilde{\partial}_{y^0} + (\tfrac{D-2}{2}) a_y H\Big] + \tfrac{1}{2} + (\tfrac{D-1}{4}) a_y a_{y'} H^2 \Delta\eta_{yy'}^2 \Big\} . \qquad (215) \end{aligned}$$

There are no 2-point contributions and, like the "B" case, the 3-point contributions cancel. Most of the 4-point contributions are similar to those of (188) and can be read off from expressions (196), (197), (198), (199) and (201). The only exception is penultimate term, which involves a derivative $\widetilde{\partial}_{y^0} + (\frac{D-2}{2}) a_y H \equiv \widetilde{D}_y$ of the C-type propagator. We understand this as acting on the internal propagator in the same sense as the scale factors. The final result is,

$$-if_{\overline{\overline{4A}}\beta}(x;x') = \kappa^2 \Bigl[-\tfrac{5}{6}\mathcal{D}_x + F_3(D) a_{x'}^D H^2 + F_4(D) a_x a_{x'}^{D+1} H^4 \Delta\eta_{xx'}^2\Bigr] \times i\Delta_A(x;x') i\Delta_C(x;x') + (D-2)\kappa^2 H^2 a_{x'}^D i\Delta_A(x;x')\Delta\eta_{xx'} \widetilde{D}_x i\Delta_C(x;x'), \quad (216)$$

$$-if_{\overline{\overline{4A}}\gamma}(x;x') = \kappa^2 \Bigl[-\tfrac{5}{6}\mathcal{D}_x + F_4(D) a_x^D H^2 + F_4(D) a_x^{D+1} a_{x'} H^4 \Delta\eta_{xx'}^2\Bigr] \times i\Delta_A(x;x') i\Delta_C(x;x') + (D-2)\kappa^2 H^2 a_x^D i\Delta_A(x;x')\Delta\eta_{xx'} \widetilde{D}_x i\Delta_C(x;x'). \quad (217)$$

The functions $F_3(D)$ and $F_4(D)$ are,

$$F_3(D) \equiv 2(D-2)^2 + \tfrac{2}{3}(D-1)(D-2) - \tfrac{1}{6}(D-1)^2 - \tfrac{1}{2}(D-2) , \quad (218)$$

$$F_4(D) \equiv \tfrac{2}{3}(D-1)(D-2) . \quad (219)$$

It is time to sum up all the contributions from $-iV_{\overline{\overline{4}}}$ to the four classes of 2-point forms in Figure 2. The ones of Class α come from expressions (174) and (181),

$$-if_{\overline{\overline{4}}\alpha}(x;x') = -(\tfrac{D-8}{D-2})\kappa^2 i\Delta_A(x;x') i\Delta_C(x;x') . \quad (220)$$

Class β diagrams derive from expressions (172), (204) and (216),

$$-if_{\overline{\overline{4}}\beta}(x;x') = \kappa^2 \Bigl[-\tfrac{(5D-11)}{3(D-2)}\mathcal{D}_{x'} + G_1(D) a_{x'}^D H^2 + G_2(D) a_x a_{x'}^{D+1} H^4 \Delta\eta_{xx'}^2\Bigr] \times i\Delta_A(x;x') i\Delta_C(x;x') + (D-2)\kappa^2 H^2 a_{x'}^D i\Delta_A(x;x')\Delta\eta_{xx'} \widetilde{D}_x i\Delta_C(x;x'), \quad (221)$$

where the functions $G_1(D)$ and $G_2(D)$ are,

$$G_1(D) \equiv -\tfrac{(D-1)^2}{3(D-2)} + 4(D-2)^2 + \tfrac{4}{3}(D-1)(D-2) - \tfrac{1}{3}(D-1)^2 - \tfrac{1}{2}(D-2) - 4(D-3) , \quad (222)$$

$$G_2(D) \equiv \tfrac{1}{8}(D-2)^2 + \tfrac{13}{12}(D-1)(D-2) + D-3 . \quad (223)$$

Expressions (173), (205) and (217) give the Class γ diagrams,

$$-if_{\overline{\overline{4}}\gamma}(x;x') = \kappa^2 \Bigl[-\tfrac{(5D-11)}{3(D-2)}\mathcal{D}_x + G_1(D) a_x^D H^2 + G_2(D) a_x^{D+1} a_{x'} H^4 \Delta\eta_{xx'}^2\Bigr] \times i\Delta_A(x;x') i\Delta_C(x;x') + (D-2)\kappa^2 H^2 a_x^D i\Delta_A(x;x')\Delta\eta_{xx'} \widetilde{D}_x i\Delta_C(x;x'), \quad (224)$$

There are no Class δ diagrams from $-iV_{\overline{\overline{4}}}$.

5.4 Renormalization

Recall the four classes of diagrams described in Figure 2 which contribute to the gauge invariant self-mass,

$$M^2_{\rm inv}(x;x') = \mathcal{D}_x\mathcal{D}_{x'}f_\alpha(x;x') + \mathcal{D}_x f_\beta(x;x') + \mathcal{D}_{x'}f_\gamma(x;x') + f_\delta(x;x') \; . \tag{225}$$

The only contribution to Class δ is from the original, gauge-dependent result $-if_\delta(x;x') = -iM_0(x;x')$. In section 5.3 we derived the $-iV_{\overline{4}}$ contributions to Classes α, β and γ in expressions (220), (221) and (224), respectively. Section 5.2 gives the Class β and γ contributions from $-iV_{\overline{5}}$ in expressions (155) and (156), respectively. (There are no Class α contributions from $-iV_{\overline{5}}$.) Finally, there are Class β and γ contributions from $-iV_{1b}(x;x')$ and $-iV_{1c}(x;x')$, respectively,

$$-if_{1\beta}(x;x') = i\kappa^2 \Big[\tfrac{D}{2}(\tfrac{D-1}{2})i\Delta_A + (\tfrac{D-1}{2})i\Delta_B + \tfrac12 i\Delta_C\Big]\delta^D(x-x') \; , \tag{226}$$

$$-if_{1\gamma}(x;x') = i\kappa^2 \Big[\tfrac{D}{2}(\tfrac{D-1}{2})i\Delta_A + (\tfrac{D-1}{2})i\Delta_B + \tfrac12 i\Delta_C\Big]\delta^D(x-x') \; . \tag{227}$$

To carry out renormalization we must first localize the primitive divergences using expressions (7-9) for the three propagators. The simplest case is the coincidence limits which appear in (226-227),

$$\tfrac{D}{2}(\tfrac{D-1}{2})i\Delta_A + (\tfrac{D-1}{2})i\Delta_B + \tfrac12 i\Delta_C = -\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big[\pi{\rm cot}(\tfrac{D\pi}{2}) - 2\ln(a)\Big] \times\tfrac{D}{2}(\tfrac{D-1}{2}) - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})}\times(\tfrac{D-1}{2}) + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})}\times\tfrac12 \; . \tag{228}$$

There are also divergences which appear in two products of noncoincident propagators,

$$i\Delta_A(x;x')i\Delta_C(x;x') \qquad , \qquad i\Delta_A(x;x')\Delta\eta_{xx'}\widetilde{D}_x i\Delta_C(x;x') \; , \tag{229}$$

where we recall,

$$\widetilde{D}_x \equiv \frac{\partial}{\partial x^0} + (\tfrac{D-2}{2})a_x H \; . \tag{230}$$

There are no divergences in the product,

$$a_x a_{x'} H^2 \Delta\eta^2_{xx'} i\Delta_A(x;x')i\Delta_C(x;x') \; . \tag{231}$$

The two products are reduced by first expanding, retaining only the potentially divergent terms in D dimensions and setting $D=4$ for the others,

$$i\Delta_A i\Delta_C = \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D}\Bigg\{\frac{1}{[aa'\Delta x^2]^{D-2}} - \frac{H^2\ln(\frac14 H^2\Delta x^2)}{2aa'\Delta x^2} + O(D-4)\Bigg\}. \quad (232)$$

The next step is to localize the ultraviolet divergence by extracting a derivative and then adding zero in the form of the flat space propagator equation [48, 49],

$$\frac{1}{\Delta x^{2D-4}} = \frac{\partial^2}{2(D-3)(D-4)}\Bigg[\frac{1}{\Delta x^{2D-6}}\Bigg] , \quad (233)$$

$$= \frac{\mu^{D-4}}{2(D-3)(D-4)}\frac{4\pi^{\frac{D}{2}} i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} + \frac{\partial^2}{2(D-3)(D-4)}\Bigg[\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}}\Bigg], \quad (234)$$

$$= \frac{\mu^{D-4}}{2(D-3)(D-4)}\frac{4\pi^{\frac{D}{2}} i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} - \frac{\partial^2}{4}\Bigg[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigg] + O(D-4) \; . \quad (235)$$

The final result for the first product is,

$$i\Delta_A i\Delta_C \longrightarrow \frac{\mu^{D-4}\Gamma(\frac{D}{2}-1)}{8(D-3)(D-4)\pi^{\frac{D}{2}}}\frac{i\delta^D(x-x')}{(aa')^{D-2}} - \frac{\partial^2}{64\pi^4(aa')^2}\Bigg[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigg] - \frac{H^2\ln(\frac14 H^2\Delta x^2)}{32\pi^4 aa'\Delta x^2}. \quad (236)$$

Using expression (230) in the second product of (229) gives,

$$i\Delta_A \times \Delta\eta \widetilde{D}_x i\Delta_C = \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D} \left\{ \frac{(D-2)\Delta\eta^2}{(aa')^{D-2}\Delta x^{2D-2}} - \frac{H^2\Delta\eta^2 \ln(\frac14 H^2\Delta x^2)}{aa'\Delta x^4} \right\}, \quad (237)$$

$$= \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D} \Bigg\{ -\frac{1}{2(aa')^{D-2}\Delta x^{2D-4}} + \frac{H^2[\ln(\frac14 H^2\Delta x^2)+1]}{2aa'\Delta x^2}$$
$$+ \frac{\partial_0^2}{4(aa')^2}\Big[\frac{1}{\Delta x^2}\Big] + \frac{H^2\partial_0^2}{8aa'}\Big[\ln^2(\tfrac14 H^2\Delta x^2) + 2\ln(\tfrac14 H^2\Delta x^2)\Big] \Bigg\}, (238)$$

$$\longrightarrow -\frac{\mu^{D-4}\Gamma(\frac{D}{2}-1)}{16(D-3)(D-4)\pi^{\frac{D}{2}}} \frac{i\delta^D(x-x')}{(aa')^{D-2}}$$
$$+ \frac{\partial^2}{128\pi^4(aa')^2}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + \frac{H^2[\ln(\frac14 H^2\Delta x^2)+1]}{32\pi^4 aa'\Delta x^2}$$
$$+ \frac{\partial_0^2}{64\pi^4(aa')^2}\Big[\frac{1}{\Delta x^2}\Big] + \frac{H^2\partial_0^2}{128\pi^4 aa'}\Big[\ln^2(\tfrac14 H^2\Delta x^2) + 2\ln(\tfrac14 H^2\Delta x^2)\Big]. \quad (239)$$

The 1-loop divergences of the invariant self-mass are canceled by three counterterms,

$$\Delta\mathcal{L} = -\tfrac12 \alpha_1 \Box\phi\Box\phi\sqrt{-g} - \tfrac12\alpha_2 R\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \tfrac12\alpha_3 R\partial_0\phi\partial_0\phi g^{00}\sqrt{-g}\,, \quad (240)$$

where the Ricci scalar in de Sitter is $R = D(D-1)H^2$. The associated contribution to the renormalized self-mass is,

$$-iM^2_{\rm ctm}(x;x') = -i\alpha_1 \mathcal{D}_x\mathcal{D}_{x'}\Big[\frac{\delta^D(x-x')}{a_x^D}\Big]$$
$$+ i\alpha_2 R\mathcal{D}_x\delta^D(x-x') - i\alpha_3 R\partial_0\Big[a^{D-2}\partial_0\delta^D(x-x')\Big]. (241)$$

From relations (228), (236) and (239) we see that all divergences can be expressed in terms of two constants,

$$K \equiv \frac{\mu^{D-4}\Gamma(\frac{D}{2}-1)}{8(D-3)(D-4)\pi^{\frac{D}{2}}} = \frac{\mu^{D-4}}{8\pi^2}\times\frac{1}{D-4} + O\Big((D-4)^0\Big)\,, \quad (242)$$

$$\mathcal{K} \equiv \frac{\mu^{D-4}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\times\tfrac{\pi}{2}\cot(\tfrac{D\pi}{2}) = \frac{\mu^{D-4}}{8\pi^2}\times\frac{1}{D-4} + O\Big((D-4)^0\Big)\,. \quad (243)$$

The counterterms required to renormalize the original, gauge-dependent self-mass are $(\alpha_1)_0 = 0$ and [13],

$$R(\alpha_2)_0 = -\kappa^2 H^2 \times (D-2) K \quad , \quad R(\alpha_3)_0 = -\kappa^2 H^2 \Bigl[2D\mathcal{K} - (D-2)(D+4) K\Bigr] . \tag{244}$$

From expressions (226-227) we see that the $-iV_{1b,c}$ diagrams have $(\alpha_1)_1 = 0 = (\alpha_3)_1$ and,

$$R(\alpha_2)_1 = \kappa^2 H^2 \times D(D-1)\mathcal{K} . \tag{245}$$

The $-iV_{\overline{4}}$ diagrams (220), (221) and (224) imply $(\alpha_3)_4 = 0$ and,

$$(\alpha_1)_4 = -\tfrac{\kappa^2}{3}(\tfrac{13D-46}{D-2}) K \qquad , \qquad R(\alpha_2)_4 = \kappa^2 H^2 \Bigl[-2G_1(D) + (D-2)\Bigr] K , \tag{246}$$

where $G_1(D)$ was defined in expression (222). Finally, the $-iV_{\overline{5}}$ diagrams (155-156) imply $(\alpha_3)_5 = 0$ and,

$$(\alpha_1)_5 = -\kappa^2 K \qquad , \qquad R(\alpha_2)_5 = 2\kappa^2 H^2 \mathcal{K} . \tag{247}$$

Summing the contributions (244-247) gives,

$$\alpha_1 = -\frac{2\kappa^2}{8\pi^2} \times \frac{\mu^{D-4}}{D-4} + O(1) , \tag{248}$$

$$R\alpha_2 = -\frac{15\kappa^2 H^2}{8\pi^2} \times \frac{\mu^{D-4}}{D-4} + O(1) , \tag{249}$$

$$R\alpha_3 = +\frac{8\kappa^2 H^2}{8\pi^2} \times \frac{\mu^{D-4}}{D-4} + O(1) . \tag{250}$$

6 Conclusions

No one disputes that an epoch of primordial inflation would produce a vast ensemble of cosmological scale gravitons [2]. At some level these quanta must interact with themselves and with other particles to induce changes in kinematics and in long range forces. A fascinating aspect of these changes is that the continual production of infrared gravitons causes them to grow without bound in time and sometimes also in space. During a very long period of inflation, this growth can overwhelm the small coupling constants to produce significant effects which might persist to the present day [50]. However, explicit computations [9–14] have been criticized as potentially unphysical owing to gauge dependence [23–26].

Opinions differ on how to deal with gauge dependence. Some people favor replacing gauge-dependent Green's functions with expectation values of generally coordinate invariant operators [35, 51–60]. We have instead pursued the approach of combining bits and pieces of gauge-dependent Green's functions so as to achieve a gauge-independent result based on the flat space S-matrix.

In our view the usual effective field equations are gauge-dependent because they ignore quantum gravitational correlations with the source, which excites the effective field, and with the observer which detects it. We restore these correlations by first forming the position-space amplitudes that represent quantum gravitational corrections to the t-channel scattering between two massive fields by the exchange of the light field under study (in this case, a massless, minimally coupled scalar). These amplitudes involve 2-point, 3-point and 4-point correlators. Our second step is to simplify the 3-point and 4-point correlators to 2-point form using a series of relations for extracting the t-channel contributions, first derived in flat space by Donoghue and collaborators [3, 43–45]. Our final step is to insert exchange propagators of the light field so that we can regard each of the 2-point contributions as a correction to the 1PI 2-point function of the light field. This has been explicitly carried out on flat space background, and verified to be independent of the gauge, for 1-graviton loop corrections to a massless, minimally coupled scalar [41] and for 1-graviton loop corrections to electromagnetism [42]. The current work generalizes this procedure to de Sitter background for 1-graviton loop corrections to a massless, minimally coupled scalar.

Our de Sitter analysis has been enormously simplified by correspondence with its flat space analog [41]. In particular, we used the very same Lagrangian (3), and the very same five diagrams (Figures 4-8) which can potentially enhance t-channel scattering between massive sources. The de Sitter generalizations (35-36) and (39-40) of the Donoghue Identities were motivated by minimally preserving the transformation properties of the flat space results. One new feature of our analysis is the recognition in section 4 of two classes of the five diagrams which can be consolidated to give two residual amplitudes, $-iV_{\overline{4}}(x;x';y;y')$ — given in expressions (100-103) — and $-iV_{\overline{5}}(x;x';y;y')$ — given in expressions (104-107). This consolidation is exact, independent of the Donoghue Identities, and valid for any gauge on any cosmological background. Of course the final reduction of these diagrams to 2-point form in sections 5.2 and 5.3 does require a specific gauge on de Sitter and the Donoghue Identities. Our final primitive results (155-156), (220), (221), (224) and (226-227) are expressed in terms of the four classes of 2-point diagrams defined in Figure 2. Renormalization is carried out in section 5.4 using the same three counterterms (240) of the original, gauge-dependent computation [13]. Our final results for the gauge-independent coefficients of these counterterms are given in equations (248-250).

Of course the reason for purging the scalar effective field equation of gauge dependence is to confirm or refute the reality of the large logarithmic correction that was found (using the simplest gauge) to the scalar exchange potential [13]. Recall the relation that was found in the gauge-dependent calculation between large logarithms in the exchange potential and the curvature-dependent field strength renormalization $-\frac{1}{2}\alpha_2 R\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g}$ [13]. Because the renormalized, gauge invariant self-mass has the same functional form as its gauge-dependent ancestor, this relation should continue to pertain. That means we can read off the gauge-independent logarithm from our result (249) for the coefficient $\alpha_2 R$. The fact that it is nonzero contradicts the hypothesis that graviton-induced logarithms are a gauge artifact.

We stress the importance of making a direct check of gauge dependence by re-doing the computation in the de Sitter generalization [61] of the same 2-parameter family of gauges which was employed to check the flat space result [41]. One reason is that there may be diagrams, such as external line corrections, which do not enhance t-channel exchange in flat space but may do so in de Sitter. Another reason is that we know of no way to derive the de Sitter Donoghue Identities (35-36) and (39-40). Their flat space antecedents could be (and were) derived [3,43–45] by computing the scattering amplitudes in momentum space and then extracting the leading contributions for small t. What we did is to first express the same relations in flat position space (which is exact) and then generalize them to de Sitter using minimal coupling (which is a guess). We will not feel completely confident until the gauge independence of the final result has been confirmed.

Acknowledgements

DG was supported by the European Union and the Czech Ministry of Education, Youth and Sports (Project: MSCA Fellowship CZ FZU I — CZ.02.01.01/00/22_010/0002906). SPM was supported by Taiwan NSTC grants 111-2112-M-006-038 and 112-2112-M-006-017. TP was supported by the D-ITP consortium, a program of the Netherlands Organization for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW). RPW was supported by NSF grant PHY-2207514 and by the Institute for Fundamental Theory at the U. of Florida.

References

[1] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532-535 (1981)

[2] A. A. Starobinsky, JETP Lett. **30**, 682-685 (1979)

[3] J. F. Donoghue, Phys. Rev. D **50**, 3874-3888 (1994) doi:10.1103/PhysRevD.50.3874 [arXiv:gr-qc/9405057 [gr-qc]].

[4] J. F. Donoghue, [arXiv:gr-qc/9512024 [gr-qc]].

[5] C. P. Burgess, Living Rev. Rel. **7**, 5-56 (2004) doi:10.12942/lrr-2004-5 [arXiv:gr-qc/0311082 [gr-qc]].

[6] J. F. Donoghue, AIP Conf. Proc. **1483**, no.1, 73-94 (2012) doi:10.1063/1.4756964 [arXiv:1209.3511 [gr-qc]].

[7] J. F. Donoghue, Scholarpedia **12**, no. 4, 32997 (2017) doi:10.4249/scholarpedia.32997

[8] J. F. Donoghue, [arXiv:2211.09902 [hep-th]].

[9] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 024021 (2006) doi:10.1103/PhysRevD.74.024021 [arXiv:gr-qc/0603135 [gr-qc]].

[10] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. **31**, 175002 (2014) doi:10.1088/0264-9381/31/17/175002 [arXiv:1308.3453 [gr-qc]].

[11] C. L. Wang and R. P. Woodard, Phys. Rev. D **91**, no.12, 124054 (2015) doi:10.1103/PhysRevD.91.124054 [arXiv:1408.1448 [gr-qc]].

[12] L. Tan, N. C. Tsamis and R. P. Woodard, Phil. Trans. Roy. Soc. Lond. A **380**, 0187 (2021) doi:10.1098/rsta.2021.0187 [arXiv:2107.13905 [gr-qc]].

[13] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, JHEP **03**, 088 (2022) doi:10.1007/JHEP03(2022)088 [arXiv:2112.00959 [gr-qc]].

[14] L. Tan, N. C. Tsamis and R. P. Woodard, Universe **8**, no.7, 376 (2022) doi:10.3390/universe8070376 [arXiv:2206.11467 [gr-qc]].

[15] R. Jackiw, Phys. Rev. D **9**, 1686 (1974) doi:10.1103/PhysRevD.9.1686

[16] N. C. Tsamis and R. P. Woodard, Commun. Math. Phys. **162**, 217-248 (1994) doi:10.1007/BF02102015

[17] R. P. Woodard, [arXiv:gr-qc/0408002 [gr-qc]].

[18] B. Allen and M. Turyn, Nucl. Phys. B **292**, 813 (1987) doi:10.1016/0550-3213(87)90672-9

[19] S. W. Hawking, T. Hertog and N. Turok, Phys. Rev. D **62**, 063502 (2000) doi:10.1103/PhysRevD.62.063502 [arXiv:hep-th/0003016 [hep-th]].

[20] A. Higuchi and S. S. Kouris, Class. Quant. Grav. **18**, 4317-4328 (2001) doi:10.1088/0264-9381/18/20/311 [arXiv:gr-qc/0107036 [gr-qc]].

[21] A. Higuchi and R. H. Weeks, Class. Quant. Grav. **20**, 3005-3022 (2003) doi:10.1088/0264-9381/20/14/303 [arXiv:gr-qc/0212031 [gr-qc]].

[22] S. P. Miao, N. C. Tsamis and R. P. Woodard, J. Math. Phys. **52**, 122301 (2011) doi:10.1063/1.3664760 [arXiv:1106.0925 [gr-qc]].

[23] A. Higuchi, D. Marolf and I. A. Morrison, Class. Quant. Grav. **28**, 245012 (2011) doi:10.1088/0264-9381/28/24/245012 [arXiv:1107.2712 [hep-th]].

[24] S. P. Miao, N. C. Tsamis and R. P. Woodard, Class. Quant. Grav. **28**, 245013 (2011) doi:10.1088/0264-9381/28/24/245013 [arXiv:1107.4733 [gr-qc]].

[25] I. A. Morrison, [arXiv:1302.1860 [gr-qc]].

[26] S. P. Miao, P. J. Mora, N. C. Tsamis and R. P. Woodard, Phys. Rev. D **89**, no.10, 104004 (2014) doi:10.1103/PhysRevD.89.104004 [arXiv:1306.5410 [gr-qc]].

[27] N. C. Tsamis and R. P. Woodard, Annals Phys. **253**, 1-54 (1997) doi:10.1006/aphy.1997.5613 [arXiv:hep-ph/9602316 [hep-ph]].

[28] N. C. Tsamis and R. P. Woodard, Phys. Rev. D **54**, 2621-2639 (1996) doi:10.1103/PhysRevD.54.2621 [arXiv:hep-ph/9602317 [hep-ph]].

[29] N. C. Tsamis and R. P. Woodard, Annals Phys. **321**, 875-893 (2006) doi:10.1016/j.aop.2005.08.004 [arXiv:gr-qc/0506056 [gr-qc]].

[30] S. P. Miao and R. P. Woodard, Class. Quant. Grav. **23**, 1721-1762 (2006) doi:10.1088/0264-9381/23/5/016 [arXiv:gr-qc/0511140 [gr-qc]].

[31] E. O. Kahya and R. P. Woodard, Phys. Rev. D **76**, 124005 (2007) doi:10.1103/PhysRevD.76.124005 [arXiv:0709.0536 [gr-qc]].

[32] S. P. Miao, Phys. Rev. D **86**, 104051 (2012) doi:10.1103/PhysRevD.86.104051 [arXiv:1207.5241 [gr-qc]].

[33] K. E. Leonard and R. P. Woodard, Class. Quant. Grav. **31**, 015010 (2014) doi:10.1088/0264-9381/31/1/015010 [arXiv:1304.7265 [gr-qc]].

[34] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. **32**, no.19, 195014 (2015) doi:10.1088/0264-9381/32/19/195014 [arXiv:1504.00894 [gr-qc]].

[35] S. P. Miao, N. C. Tsamis and R. P. Woodard, Phys. Rev. D **95**, no.12, 125008 (2017) doi:10.1103/PhysRevD.95.125008 [arXiv:1702.05694 [gr-qc]].

[36] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Phys. Rev. D **101**, no.10, 106016 (2020) doi:10.1103/PhysRevD.101.106016 [arXiv:2003.02549 [gr-qc]].

[37] S. Boran, E. O. Kahya and S. Park, Phys. Rev. D **90**, no.12, 124054 (2014) doi:10.1103/PhysRevD.90.124054 [arXiv:1409.7753 [gr-qc]].

[38] S. Boran, E. O. Kahya and S. Park, Phys. Rev. D **96**, no.2, 025001 (2017) doi:10.1103/PhysRevD.96.025001 [arXiv:1704.05880 [gr-qc]].

[39] P. J. Mora, N. C. Tsamis and R. P. Woodard, J. Math. Phys. **53**, 122502 (2012) doi:10.1063/1.4764882 [arXiv:1205.4468 [gr-qc]].

[40] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. **34**, no.8, 085002 (2017) doi:10.1088/1361-6382/aa61da [arXiv:1609.00386 [gr-qc]].

[41] S. P. Miao, T. Prokopec and R. P. Woodard, Phys. Rev. D **96**, no.10, 104029 (2017) doi:10.1103/PhysRevD.96.104029 [arXiv:1708.06239 [gr-qc]].

[42] S. Katuwal and R. P. Woodard, JHEP **10**, 029 (2021) doi:10.1007/JHEP10(2021)029 [arXiv:2107.13341 [gr-qc]].

[43] J. F. Donoghue, Phys. Rev. Lett. **72**, 2996-2999 (1994) doi:10.1103/PhysRevLett.72.2996 [arXiv:gr-qc/9310024 [gr-qc]].

[44] N. E. J. Bjerrum-Bohr, Phys. Rev. D **66**, 084023 (2002) doi:10.1103/PhysRevD.66.084023 [arXiv:hep-th/0206236 [hep-th]].

[45] J. F. Donoghue and T. Torma, Phys. Rev. D **54**, 4963-4972 (1996) doi:10.1103/PhysRevD.54.4963 [arXiv:hep-th/9602121 [hep-th]].

[46] W. Beenakker and A. Denner, Nucl. Phys. B **338**, 349-370 (1990) doi:10.1016/0550-3213(90)90636-R

[47] R. K. Ellis and G. Zanderighi, JHEP **02**, 002 (2008) doi:10.1088/1126-6708/2008/02/002 [arXiv:0712.1851 [hep-ph]].

[48] V. K. Onemli and R. P. Woodard, Class. Quant. Grav. **19**, 4607 (2002) doi:10.1088/0264-9381/19/17/311 [arXiv:gr-qc/0204065 [gr-qc]].

[49] V. K. Onemli and R. P. Woodard, Phys. Rev. D **70**, 107301 (2004) doi:10.1103/PhysRevD.70.107301 [arXiv:gr-qc/0406098 [gr-qc]].

[50] R. P. Woodard and B. Yesilyurt, JHEP **08**, 124 (2023) doi:10.1007/JHEP08(2023)124 [arXiv:2305.17641 [gr-qc]].

[51] N. C. Tsamis and R. P. Woodard, Annals Phys. **215**, 96-155 (1992) doi:10.1016/0003-4916(92)90301-2

[52] G. Modanese, Riv. Nuovo Cim. **17N8**, 1-62 (1994) doi:10.1007/BF02724514 [arXiv:hep-th/9410086 [hep-th]].

[53] C. Rovelli, Phys. Rev. D **65**, 044017 (2002) doi:10.1103/PhysRevD.65.044017 [arXiv:gr-qc/0110003 [gr-qc]].

[54] S. B. Giddings, D. Marolf and J. B. Hartle, Phys. Rev. D **74**, 064018 (2006) doi:10.1103/PhysRevD.74.064018 [arXiv:hep-th/0512200 [hep-th]].

[55] D. R. Green, Phys. Rev. D **78**, 064066 (2008) doi:10.1103/PhysRevD.78.064066 [arXiv:0804.4450 [hep-th]].

[56] W. Donnelly and S. B. Giddings, Phys. Rev. D **93**, no.2, 024030 (2016) [erratum: Phys. Rev. D **94**, no.2, 029903 (2016)] doi:10.1103/PhysRevD.93.024030 [arXiv:1507.07921 [hep-th]].

[57] D. Marolf, Class. Quant. Grav. **32**, no.24, 245003 (2015) doi:10.1088/0264-9381/32/24/245003 [arXiv:1508.00939 [gr-qc]].

[58] M. B. Fröb, Class. Quant. Grav. **35**, no.3, 035005 (2018) doi:10.1088/1361-6382/aa9ad1 [arXiv:1706.01891 [hep-th]].

[59] M. B. Fröb and W. C. C. Lima, Class. Quant. Grav. **35**, no.9, 095010 (2018) doi:10.1088/1361-6382/aab427 [arXiv:1711.08470 [gr-qc]].

[60] J. Wilson-Gerow and P. C. E. Stamp, Annals Phys. **442**, 168898 (2022) doi:10.1016/j.aop.2022.168898 [arXiv:2011.05305 [hep-th]].

[61] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, JHEP **10**, 096 (2019) doi:10.1007/JHEP10(2019)096 [arXiv:1908.06064 [gr-qc]].