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The Price of Abandoning Dark Matter Is Nonlocality

C. Deffayet[1*] and R. P. Woodard[2†]

[1] *Laboratoire de Physique de l'Ecole Normale Supérieure, ENS, Université PSL, CNRS Sorbonne Université, Université Paris Cité, F-75005 Paris, FRANCE*

[2] *Department of Physics, University of Florida, Gainesville, FL 32611, UNITED STATES*

ABSTRACT

We consider the classic question posed by Pardo and Spergel about the price of abandoning dark matter in the context of an invariant, metric-based theory of gravity. Our answer is that the price is nonlocality. This has been known for some time in the context of the quasi-static regime. We show that it also applies for cosmology and we exhibit a model which reproduces standard CDM successes such as perturbations in the cosmic microwave background, baryon acoustic oscillations and structure formation.

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* e-mail: cedric.deffayet@phys.ens.fr

† e-mail: woodard@phys.ufl.edu

1 Introduction

Despite increasingly aggressive efforts at direct detection [1–7], the evidence for dark matter is still restricted to its impact on gravity. It is therefore worthwhile considering whether or not the same gravitational phenomena might be instead signal some modification of gravity. Milgrom's MOdified Newtonian Dynamics (MOND) [8–10] does an excellent job of explaining, and even predicting observed features of galactic structure which are usually attributed to dark matter [11–23]. MOND has mixed success explaining galactic clusters [24–26], but its real challenge is to reproduce the successes of dark matter in cosmology, including the anisotropies in the cosmic ray microwave background (CMB), baryon acoustic oscillations (BAO) and linearized structure formation.

If dark matter is abandoned in favor of modified gravity, it must be that MOND represents the static limit of some larger, relativistic theory. So it is perhaps not surprising that extrapolations of MOND should encounter problems as one approaches the time-dependent regime of cosmology. And it is significant that relativistic extensions of MOND such as Bekenstein's TeVeS [27] do a much better job of reproducing cold dark matter (CDM) cosmology [28–31]. In particular, the AeST model of Skordis and Zlosnik [32] is internally consistent and agrees with those phenomenological checks which have so far been made [33–36].

The only metric-based modification of gravity which is both stable and generally coordinate invariant consists of changing the Einstein-Hilbert Lagrangian from R to some nonlinear function $f(R)$ [37]. This induces a new scalar degree of freedom which could be regarded as a sort of dark matter [38]. Within the context of locality, the other options for modifying gravity involve either abandoning the metric as the sole gravitational field variable — such as TeVeS [27] and AeST [32] — or else abandoning full general coordinate invariance [39, 40]. We will here explore the option of abandoning locality.

Our result can be regarded as an answer to the question posed in the title of the recent paper by Pardo and Spergel, "What is the price of abandoning dark matter?" [41]. We demonstrate that the price is nonlocality, provided one maintains both general coordinate invariance and the metric's status as the sole gravitational field. This has already been shown for gravitationally bound systems [42, 43], but cosmological extensions of that model involve fitting a free function just to recover the correct expansion history [44], and in any case fail to provide the extra gravitational force need for structure formation without dark matter [45]. In this new effort we exploit the equations which describe cold dark matter (CDM) cosmology in the regime of linearized perturbations, in particular, the fact that there is no strong evidence for self-interactions [46–48]. This means that the CDM stress tensor must be separately conserved, which permits us to express it in terms of linearized scalar gravitational perturbations. We accomplish this in section 2, and also infer a gravitational effective action whose variation gives the CDM stress tensor. In section 3 we devise a generally coordinate invariant expression for this effective action. This approach is guaranteed to reproduce the cosmology of CDM. In our Conclusion (section 4) we discuss the possibility that this action arises from secular interactions between inflationary gravitons which became nonperturbatively strong [49]. We also discuss the prospects for a single effective action which describes both cosmology and gravitationally bound systems.

2 Reconstructing the Effective Action

The purpose of this section is to derive a purely gravitational effective action whose variation is guaranteed to reproduce CDM phenomenology in the linearized regime. We begin by giving the metric perturbations and the two variables which characterize the CDM stress tensor. The fact that the CDM stress tensor is separately conserved permits us to express the density contrast and momentum divergence in terms of the metric. The section closes by constructing a gravitational effective action whose variation gives the CDM stress tensor.

2.1 Linearized Perturbations

It is simple to reproduce the CDM contribution to the expansion history because this is just a single degree of freedom. The real challenge is getting the infinite number of degrees of freedom characterized by linearized perturbations. Although there are four scalar perturbations about the Friedmann-Lemaître-Robertson-Walker (FLRW) geometry, gauge transformations can be used to set any two to zero. We choose the two nonzero perturbations as,

$$ds^2 = -\Big[1 + 2\Psi(t,\vec{x})\Big] dt^2 + 2\partial_i B(t,\vec{x}) dt dx^i + a^2(t) d\vec{x} \cdot d\vec{x} \; . \qquad (1)$$

The first order connections are,

$$\Gamma^0{}_{00} = \dot{\Psi} \;\;,\;\; \Gamma^0{}_{0i} = \partial_i \Bigl[\Psi + HB\Bigr] \;\;,\;\; \Gamma^0{}_{ij} = Ha^2 \delta_{ij} \Bigl[1 - 2\Psi\Bigr] - \partial_i \partial_j B \;, \qquad (2)$$

$$\Gamma^i{}_{00} = \frac{\partial_i}{a^2} \Bigl[\Psi + \dot{B}\Bigr] \qquad , \qquad \Gamma^i{}_{j0} = H\delta_{ij} \qquad , \qquad \Gamma^i{}_{jk} = -H\partial_i B \delta_{jk} \;. \qquad (3)$$

The CDM stress tensor $\Delta T_{\mu\nu}$ is characterized (at linearized order) by its background energy density $\overline{\rho}(t)$, the density contrast $\delta(t, \vec{x}) \equiv \delta\rho(t, \vec{x}) \div \overline{\rho}(t)$ and the momentum divergence $\theta(t, \vec{x})$,

$$\Delta T_{00} = \overline{\rho}\Bigl[1 + \delta + 2\Psi + \dots\Bigr] \;,\; \Delta T_{0i} = \overline{\rho}\partial_i \Bigl[0 - \theta + \dots\Bigr] \;\;,\;\; \Delta T_{ij} = \overline{\rho}\Bigl[0 + 0 + \dots\Bigr] \;. \qquad (4)$$

2.2 Eliminating CDM

We now exploit the fact that the CDM stress tensor is separately conserved. Relations (2-3) can be used to derive two equations,

$$0 = D^\mu \Delta T_{\mu 0} = -(\partial_t + 3H)\overline{\rho} + \overline{\rho}\Bigl\{-\dot{\delta} + \frac{\nabla^2}{a^2}\Bigl[B - \theta\Bigr]\Bigr\} + \dots \;, \qquad (5)$$

$$0 = D^\mu \Delta T_{\mu i} = 0 + \overline{\rho}\partial_i \Bigl\{\dot{\theta} + \Psi\Bigr\} + \dots \;. \qquad (6)$$

The 0th order part of equation (5) implies that the background CDM energy density has the equation of state of pressureless matter,

$$\overline{\rho}(t) = \frac{\rho_0}{a^3(t)} \;, \qquad (7)$$

where ρ_0 is a constant. Equation (6) allows us to express the momentum divergence as the time integral of the Newtonian potential,

$$\theta(t, \vec{x}) = -\frac{1}{\partial_t} \Psi(t, \vec{x}) \equiv -\int_0^t dt' \, \Psi(t', \vec{x}) \;. \qquad (8)$$

Substituting (8) in equation (5) gives the density contrast,

$$\delta = \frac{1}{\partial_t}\Bigl[\frac{\nabla^2}{a^2}\Bigl(B + \frac{1}{\partial_t}\Psi\Bigr)\Bigr] \;. \qquad (9)$$

2.3 Reconstructing the Effective Action

We seek an effective action ΔS whose variation gives the CDM stress tensor (4), considered as a functional of the metric. The general relation between an induced stress tensor and variations is,

$$\Delta T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \Delta S}{\delta g^{\mu\nu}} \qquad \Longleftrightarrow \qquad \frac{\delta \Delta S}{\delta g^{\mu\nu}} = -\frac{1}{2} \sqrt{-g}\, \Delta T_{\mu\nu} \; . \tag{10}$$

Because our metric (1) depends only on the two scalar perturbations $\Psi(t, \vec{x})$ and $B(t, \vec{x})$ we must specialize relation (10) to variations with respect to these two fields,

$$\frac{\delta \Delta S}{\delta \Psi} = -2 \frac{\partial g^{\mu\nu}}{\partial g_{00}} \times \frac{\partial \Delta S}{\delta g^{\mu\nu}} = -\sqrt{-g}\, g^{0\mu} g^{0\nu} \Delta T_{\mu\nu} \; , \tag{11}$$

$$\frac{\delta \Delta S}{\delta B} = -2 \partial_i \Big\{ \frac{\partial g^{\mu\nu}}{\partial g_{0i}} \times \frac{\partial \Delta S}{\delta g^{\mu\nu}} \Big\} = -\partial_i \Big\{ \sqrt{-g}\, g^{0\mu} g^{i\nu} \Delta T_{\mu\nu} \Big\} \; . \tag{12}$$

Substituting (1) and (4), and retaining only terms up to first order gives,

$$\frac{\delta \Delta S}{\delta \Psi} = -a^3 \overline{\rho} \Big\{ 1 - \Psi + \delta + \dots \Big\} \;\; , \;\; \frac{\delta \Delta S}{\delta B} = -a^3 \overline{\rho} \frac{\nabla^2}{a^2} \Big\{ -B + \theta + \dots \Big\} \; . \tag{13}$$

There are two ways to reconstruct an effective Lagrangian $\Delta \mathcal{L}$ whose variations reproduce (13). The first is to consider $\delta(t, \vec{x})$ and $\theta(t, \vec{x})$ as auxiliary fields whose equations of motion enforce relations (8-9). In that case the result is,

$$\Delta \mathcal{L}_1 = -\rho_0 \Big\{ 1 + \Psi - \frac{1}{2} \Psi^2 - \Big[B - \theta \Big] \frac{\nabla^2}{2a^2} \Big[B - \theta \Big] + \delta(\dot{\theta} + \Psi) + \dots \Big\} . \tag{14}$$

Note that we have used $\overline{\rho}(t) \times a^3(t) = \rho_0$, and that the variation of (14) with respect to δ enforces (8) while the variation with respect to θ gives (9).

The second way to realize (13) is through a nonlocal Lagrangian which depends only on Ψ and B,

$$\Delta \mathcal{L}_2 = -\rho_0 \Big\{ 1 + \Psi - \frac{1}{2} \Psi^2 - \Big[B + \frac{1}{\partial_t} \Psi \Big] \frac{\nabla^2}{2a^2} \Big[B + \frac{1}{\partial_t} \Psi \Big] + \dots \Big\} . \tag{15}$$

In this case one must interpret the variation of $\frac{1}{\partial_t} \Psi$ in the sense of nonlocal cosmology [50]. That means replacing the "advanced" integral, which is what really results from the variation, with the "retarded" integral which obeys the same differential equation,

$$\int_t^\infty \! dt' F(t') \longrightarrow -\int_0^t \! dt' F(t') \; . \tag{16}$$

The actual genesis of a nonlocal model would be through the Schwinger-Keldysh formalism [51–56], with (16) rigorously justified through the interference between contributions from forward and backwards propagating effective fields [57–59].

3 Invariant Formulation

The purpose of this section is to construct an invariant effective action whose variation produces the full CDM stress tensor. We begin with a convenient representation for the general metric. Then it is shown how the full CDM stress tensor can be expressed in terms of two scalar fields which obey simple, first order differential equations. The section closes by giving an invariant Lagrangian whose variation reproduces the full CDM stress tensor.

3.1 The ADM Metric

Because of the crucial role played by time evolution, it is convenient to carry out this analysis using the representation for the full metric introduced by Arnowitt, Deser and Misner [60],

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\Bigl(dx^i - N^i dt\Bigr)\Bigl(dx^j - N^j dt\Bigr) \; . \qquad (17)$$

The various components are known as the lapse $N(t,\vec{x})$, the shift $N^i(t,\vec{x})$ and the 3-metric $\gamma_{ij}(t,\vec{x})$. In these variables the measure and the inverse metric are,

$$\sqrt{-g} = N\sqrt{\gamma} \qquad , \qquad g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & -\frac{N^n}{N^2} \\ -\frac{N^m}{N^2} & \gamma^{mn} - \frac{N^m N^n}{N^2} \end{pmatrix} \; . \qquad (18)$$

For the scalar perturbation geometry (1) we have,

$$N^2 = 1 + 2\Psi + \frac{\partial_i B}{a}\frac{\partial_i B}{a} \qquad , \qquad N^i = -\frac{\partial_i B}{a^2} \qquad , \qquad \gamma_{ij} = a^2\delta_{ij} \; . \qquad (19)$$

3.2 Representing the Full CDM Stress Tensor

Because cold dark matter has zero pressure and is collisionless, its stress tensor can be written in terms of an energy density $\rho(t,\vec{x})$ and a timelike 4-velocity field $u_\mu(t,\vec{x})$

$$T_{\mu\nu} = \rho\, u_\mu u_\nu \qquad , \qquad g^{\mu\nu} u_\mu u_\nu = -1 \; . \qquad (20)$$

Comparison with (4) and (8) implies an intriguing expansion for the 4-velocity,

$$u_0 = -1 - \Psi + \ldots = -\partial_t\Bigl[t + \frac{1}{\partial_t}\Psi + \ldots\Bigr] \; , \qquad (21)$$

$$u_i = 0 + \partial_i\theta + \ldots = -\partial_i\Bigl[t + \frac{1}{\partial_t}\Psi + \ldots\Bigr] \; . \qquad (22)$$

This suggests that u_ν can be written as minus the gradient of a scalar whose expansion resides within the square brackets.

We can understand the expansions (21-22) using the fact that the CDM stress tensor is separately conserved,

$$D_\mu T^\mu_{\ \nu} = D_\mu \Bigl(u^\mu \rho\Bigr) u_\nu + \rho\, u^\mu D_\mu u_\nu = 0 \; . \tag{23}$$

Conservation implies two distinct equations,

$$\partial_\mu \Bigl(\sqrt{-g}\, g^{\mu\nu} u_\nu \rho\Bigr) \;\; = \;\; 0 \; , \tag{24}$$

$$g^{\mu\rho} u_\mu D_\rho u_\nu \;\; = \;\; 0 \; . \tag{25}$$

Relation (25) can indeed be solved as $u_\nu = -\partial_\nu \phi$,[1] where the scalar field $\phi(t,\vec{x})$ obeys the nonlinear, first order differential equation,

$$g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = -1 \; . \tag{26}$$

The correctness of this solution can be seen by substituting $u_\nu = -\partial_\nu \phi$ in relation (25) and then using (26),

$$g^{\mu\rho} u_\mu D_\rho u_\nu = g^{\mu\rho} \partial_\mu \phi D_\rho \partial_\nu \phi = g^{\mu\rho} \partial_\mu \phi D_\nu \partial_\rho \phi = \frac{1}{2} D_\nu \Bigl[g^{\mu\rho} \partial_\mu \phi \partial_\rho \phi\Bigr] = 0 \; . \tag{27}$$

Expressions (26) and (24) give well-posed evolution equations for $\phi(t,\vec{x})$ and $\rho(t,\vec{x})$, respectively. To see this for $\phi(t,\vec{x})$, substitute (17) into (26) and then solve for $\dot{\phi}$ with an irrelevant sign choice,

$$\dot{\phi} = N \sqrt{1 + \gamma^{ij} \partial_i \phi \partial_j \phi} - N^i \partial_i \phi \; . \tag{28}$$

Equation (28) obviously has a unique solution which will agree with our weak field expansions (21-22) if we choose the initial condition $\phi(0,\vec{x}) = 0$. Substituting (17) into (24) and then eliminating $\dot{\phi}$ using (28) gives a similarly well-posed evolution equation for $\rho(t,\vec{x})$,

$$\partial_t \Bigl[\rho \sqrt{\gamma}\, \sqrt{1 + \gamma^{jk} \partial_j \phi \partial_k \phi}\Bigr] = \partial_i \Bigl[N \rho \sqrt{\gamma}\, \gamma^{ij} \partial_j \phi - N^i \rho \sqrt{\gamma}\, \sqrt{1 + \gamma^{jk} \partial_j \phi \partial_k \phi}\Bigr] \; . \tag{29}$$

This also has a unique solution which will agree with (9) if we begin with,

$$\rho(0,\vec{x}) = \frac{\rho_0}{\sqrt{\gamma(0,\vec{x})}} \; . \tag{30}$$

Note finally that the method of characteristics gives a solution for $\rho(t,\vec{x})$ in terms of $\phi(t,\vec{x})$ and $g_{\mu\nu}(t,\vec{x})$.

[1] A normalized, timelike 4-velocity has three degrees of freedom, so our representation in terms of a single scalar represents the minimal solution.

3.3 Nonlocal Effective Action

We can reproduce the full CDM stress tensor (20), for a general metric, using a local effective Lagrangian with auxiliary fields $\rho(t,\vec{x})$ and $\phi(t,\vec{x})$,

$$\mathcal{L}_1 = -\frac{1}{2}\Big[\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + 1\Big]\rho\sqrt{-g} \; . \tag{31}$$

The variation with respect to ϕ gives the ρ equation (24), while the variation with respect to ρ implies the ϕ equation (26),

$$\frac{\delta S_1}{\delta\phi} = \partial_\mu\Big[\rho\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi\Big] = 0 \; , \tag{32}$$

$$\frac{\delta S_1}{\delta\rho} = -\frac{1}{2}\Big[\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + 1\Big]\sqrt{-g} = 0 \; . \tag{33}$$

The variation with respect to the metric reproduces the full CDM stress tensor (20) when equation (33) is used and we recall that the 4-velocity is $u_\mu = -\partial_\mu\phi$,

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_1}{\delta g^{\mu\nu}} = \rho\,\partial_\mu\phi\partial_\nu\phi = \rho\, u_\mu u_\nu \; . \tag{34}$$

Lagrangian (31), localized in terms of auxiliary scalars ϕ and ρ, is analogous to the weak field result (14) which was localized in terms of auxiliary scalars θ and δ. We stress that the fields $\phi(t,\vec{x})$ and $\rho(t,\vec{x})$ in the CDM stress tensor (34) are not independent dynamical variables but rather nonlocal functionals of the metric which solve the well-posed evolution equations (28) and (29), with initial value data $\phi(0,\vec{x}) = 0$ and (30), respectively. It would be possible to derive a purely gravitational effective action, analogous to the weak field expression (15), as a surface term. However, it is really equation (34) which defines the model.

4 Conclusions

We have derived a nonlocal, metric-based effective action which exactly reproduces cold dark matter. For linearized scalar perturbations in cosmology (1) our result can be expressed either as a localized form (14), using auxiliary scalars θ and δ, or else as a nonlocal, purely gravitational form (15). Either way, the effective CDM stress tensor takes the form (4), with (8-9). For a general metric (17) the localized form is (31), with auxiliary scalars ϕ

and ρ. The nonlocal, purely gravitational form analogous to (15) would be a surface term. Either way, the effective CDM stress tensor takes the form (34), with the auxiliary scalars defined as nonlocal functionals for the metric which obey the well-posed evolution equations (28) and (29), starting from initial values,

$$\phi(0,\vec{x}) = 0 \qquad , \qquad \rho(0,\vec{x}) = \frac{\rho_0}{\sqrt{\gamma(0,\vec{x})}} \; . \tag{35}$$

We stress that $\phi(t,\vec{x})$ and $\rho(t,\vec{x})$ are precisely defined, nonlocal functionals of the metric, without the independent initial value data which local fields would possess.

We stress that there can be no doubt about this model reproducing all the successes of CDM in cosmology. Those successes include the anisotropies of the cosmic microwave background, baryon acoustic oscillations, and linearized structure formation. All of this must come out right because this model was constructed by using the separate conservation of the CDM stress tensor to express it as a nonlocal functional of the metric. The only way this model can be falsified is by showing that CDM interacts with fields other than gravity, the evidence for which is weak [46–48]. This model should put to rest the frequent claims that no modified gravity theory can supplant dark matter. It also demonstrates that the answer to the question of Pardo and Spergel [41] about the price of abandoning dark matter: the price is nonlocality.

A key principle in constructing modified gravity models is that gravitational radiation in vacuum should move at the speed of light so as not to conflict with the simultaneous detection of optical and gravitational radiation from the binary neutron star merger GW170817 approximately 40 Mpc distant [61]. This is usually accomplished by making each term in the effective action involve at least two factors of the Ricci tensor. We did not need to do that here because our model exactly reproduces the result of the CDM stress tensor. So any change our model predicts in the kinematics of gravitational radiation is limited to one already predicted by cold dark matter.

One thing our model does not do is to agree with the nonlocal extension of MOND [42, 43] which was constructed to reproduce the baryonic Tully-Fisher relation [62], with sufficient weak lensing, in gravitationally bound structure. One can see this by simply counting derivatives and weak fields in the CDM stress tensor. At least in cosmological perturbation theory, our model's change in the gravitational field equations is linear in the weak fields

$\Psi(t, \vec{x})$ and $B(t, \vec{x})$, and every inverse differential operator is compensated by a positive power. In contrast, the model which works inside gravitationally bound structures is quadratic in the weak fields and involves three derivatives [42,43]. It would be desirable to have a single formalism which connects both regimes.

It would also be desirable to derive these nonlocal models from fundamental theory. We believe they might arise from resumming the secular logarithms that are induced by loops of inflationary gravitons and which must eventually become nonperturbatively large during a prolonged period of inflation [49]. Such a resummation has been accomplished for the similar logarithms produced by nonlinear sigma models on de Sitter background [63]. The curvature-induced effective potentials which enable the resummation have been generalized from de Sitter to an arbitrary cosmological background which experiences primordial inflation [64]. And when the generalized effective potentials are used to evolve to late times, certain results do retain a strong memory of the inflationary epoch [65]. It remains to generalize the resummation technique from nonlinear sigma models to gravity.

Finally, one should keep in mind the possibility that MOND might be realized as a modification of inertia rather than as a modification of gravity [66]. In this regard it is worth noting that inflationary gravitons not only induce secular changes in the force of gravity [67], they also modify the kinematics (and hence the inertia) of massless fermions [68] and photons [69]. It would be interesting to explore changes to the inertia, and particularly the response to gravity, of massive particles [70].

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Note Added

As this paper was going to press we learned of "Mimetic Matter" models [71, 72] which have the same field equations (24) and (26) as the invariant model of section 3. The chief difference is that our auxiliary scalars $\rho(t, \vec{x})$ and $\phi(t, \vec{x})$ are functionals of the metric, with fixed initial value data (35), whereas the mimetic fields have independent (and sometimes ghost) initial value data.

References

[1] X. Cui *et al.* [PandaX-II], Phys. Rev. Lett. **119**, no.18, 181302 (2017) doi:10.1103/PhysRevLett.119.181302 [arXiv:1708.06917 [astro-ph.CO]].

[2] E. Aprile *et al.* [XENON], Phys. Rev. Lett. **121**, no.11, 111302 (2018) doi:10.1103/PhysRevLett.121.111302 [arXiv:1805.12562 [astro-ph.CO]].

[3] Y. Meng *et al.* [PandaX-4T], Phys. Rev. Lett. **127**, no.26, 261802 (2021) doi:10.1103/PhysRevLett.127.261802 [arXiv:2107.13438 [hep-ex]].

[4] C. Bartram *et al.* [ADMX], Phys. Rev. Lett. **127**, no.26, 261803 (2021) doi:10.1103/PhysRevLett.127.261803 [arXiv:2110.06096 [hep-ex]].

[5] J. Aalbers *et al.* [LZ], Phys. Rev. Lett. **131**, no.4, 041002 (2023) doi:10.1103/PhysRevLett.131.041002 [arXiv:2207.03764 [hep-ex]].

[6] D. Perez Adan [ATLAS and CMS], [arXiv:2301.10141 [hep-ex]].

[7] E. Aprile *et al.* [XENON], Phys. Rev. Lett. **131**, no.4, 041003 (2023) doi:10.1103/PhysRevLett.131.041003 [arXiv:2303.14729 [hep-ex]].

[8] M. Milgrom, Astrophys. J. **270**, 365-370 (1983) doi:10.1086/161130

[9] M. Milgrom, Astrophys. J. **270**, 371-383 (1983) doi:10.1086/161131

[10] M. Milgrom, Scholarpedia, 9(6):31410 (2014) doi:10.4249/scholarpedia.31410

[11] M. Milgrom, Astrophys. J. **270**, 384-389 (1983) doi:10.1086/161132

[12] M. Milgrom, Astrophys. J. **338**, 121-127 (1989) doi:10.1086/167184

[13] R. Brada and M. Milgrom, Astrophys. J. **519**, 590 (1999) doi:10.1086/307402 [arXiv:astro-ph/9811013 [astro-ph]].

[14] R. Brada and M. Milgrom, Astrophys. J. Lett. **512**, L17 (1999) doi:10.1086/311871 [arXiv:astro-ph/9812117 [astro-ph]].

[15] M. Milgrom, Mon. Not. Roy. Astron. Soc. **398**, 1023 (2009) doi:10.1111/j.1365-2966.2009.15255.x [arXiv:0909.5184 [astro-ph.CO]].

[16] S. McGaugh and M. Milgrom, Astrophys. J. **775**, 139 (2013) doi:10.1088/0004-637X/775/2/139 [arXiv:1308.5894 [astro-ph.CO]].

[17] M. Milgrom, Mon. Not. Roy. Astron. Soc. **437**, no.3, 2531-2541 (2014) doi:10.1093/mnras/stt2066 [arXiv:1212.2568 [astro-ph.CO]].

[18] F. Lelli, S. S. McGaugh, J. M. Schombert and M. S. Pawlowski, Astrophys. J. Lett. **827**, no.1, L19 (2016) doi:10.3847/2041-8205/827/1/L19 [arXiv:1607.02145 [astro-ph.GA]].

[19] M. Milgrom, Phys. Rev. Lett. **117**, no.14, 141101 (2016) doi:10.1103/PhysRevLett.117.141101 [arXiv:1607.05103 [astro-ph.GA]].

[20] S. McGaugh, F. Lelli and J. Schombert, Phys. Rev. Lett. **117**, no.20, 201101 (2016) doi:10.1103/PhysRevLett.117.201101 [arXiv:1609.05917 [astro-ph.GA]].

[21] F. Lelli, S. S. McGaugh, J. M. Schombert and M. S. Pawlowski, Astrophys. J. **836**, no.2, 152 (2017) doi:10.3847/1538-4357/836/2/152 [arXiv:1610.08981 [astro-ph.GA]].

[22] S. McGaugh, Galaxies **8**, no.2, 35 (2020) doi:10.3390/galaxies8020035 [arXiv:2004.14402 [astro-ph.GA]].

[23] I. Banik and H. Zhao, Symmetry **14**, no.7, 1331 (2022) doi:10.3390/sym14071331 [arXiv:2110.06936 [astro-ph.CO]].

[24] A. Aguirre, J. Schaye and E. Quataert, Astrophys. J. **561**, 550 (2001) doi:10.1086/323376 [arXiv:astro-ph/0105184 [astro-ph]].

[25] M. Milgrom, Phys. Rev. D **99**, no.4, 044041 (2019) doi:10.1103/PhysRevD.99.044041 [arXiv:1811.12233 [astro-ph.GA]].

[26] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones and D. Zaritsky, Astrophys. J. Lett. **648**, L109-L113 (2006) doi:10.1086/508162 [arXiv:astro-ph/0608407 [astro-ph]].

[27] J. D. Bekenstein, Phys. Rev. D **70**, 083509 (2004) [erratum: Phys. Rev. D **71**, 069901 (2005)] doi:10.1103/PhysRevD.70.083509 [arXiv:astro-ph/0403694 [astro-ph]].

[28] C. Skordis, D. F. Mota, P. G. Ferreira and C. Boehm, Phys. Rev. Lett. **96**, 011301 (2006) doi:10.1103/PhysRevLett.96.011301 [arXiv:astro-ph/0505519 [astro-ph]].

[29] G. W. Angus, B. Famaey and H. Zhao, Mon. Not. Roy. Astron. Soc. **371**, 138 (2006) doi:10.1111/j.1365-2966.2006.10668.x [arXiv:astro-ph/0606216 [astro-ph]].

[30] S. Dodelson and M. Liguori, Phys. Rev. Lett. **97**, 231301 (2006) doi:10.1103/PhysRevLett.97.231301 [arXiv:astro-ph/0608602 [astro-ph]].

[31] T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. **513**, 1-189 (2012) doi:10.1016/j.physrep.2012.01.001 [arXiv:1106.2476 [astro-ph.CO]].

[32] C. Skordis and T. Zlosnik, Phys. Rev. Lett. **127**, no.16, 161302 (2021) doi:10.1103/PhysRevLett.127.161302 [arXiv:2007.00082 [astro-ph.CO]].

[33] C. Skordis and T. Zlosnik, Phys. Rev. D **106**, no.10, 104041 (2022) doi:10.1103/PhysRevD.106.104041 [arXiv:2109.13287 [gr-qc]].

[34] P. Verwayen, C. Skordis and C. Bœhm, [arXiv:2304.05134 [astro-ph.CO]].

[35] M. Bataki, C. Skordis and T. Zlosnik, [arXiv:2307.15126 [gr-qc]].

[36] A. Durakovic and C. Skordis, [arXiv:2312.00889 [astro-ph.CO]].

[37] R. P. Woodard, Lect. Notes Phys. **720**, 403-433 (2007) doi:10.1007/978-3-540-71013-4_14 [arXiv:astro-ph/0601672 [astro-ph]].

[38] J. A. R. Cembranos, Phys. Rev. Lett. **102**, 141301 (2009) doi:10.1103/PhysRevLett.102.141301 [arXiv:0809.1653 [hep-ph]].

[39] P. Horava, Phys. Rev. D **79**, 084008 (2009) doi:10.1103/PhysRevD.79.084008 [arXiv:0901.3775 [hep-th]].

[40] A. O. Barvinsky, D. Blas, M. Herrero-Valea, S. M. Sibiryakov and C. F. Steinwachs, Phys. Rev. D **93**, no.6, 064022 (2016) doi:10.1103/PhysRevD.93.064022 [arXiv:1512.02250 [hep-th]].

[41] K. Pardo and D. N. Spergel, Phys. Rev. Lett. **125**, no.21, 211101 (2020) doi:10.1103/PhysRevLett.125.211101 [arXiv:2007.00555 [astro-ph.CO]].

[42] C. Deffayet, G. Esposito-Farese and R. P. Woodard, Phys. Rev. D **84**, 124054 (2011) doi:10.1103/PhysRevD.84.124054 [arXiv:1106.4984 [gr-qc]].

[43] C. Deffayet, G. Esposito-Farese and R. P. Woodard, Phys. Rev. D **90**, no.6, 064038 (2014) doi:10.1103/PhysRevD.90.089901 [arXiv:1405.0393 [astro-ph.CO]].

[44] M. Kim, M. H. Rahat, M. Sayeb, L. Tan, R. P. Woodard and B. Xu, Phys. Rev. D **94**, no.10, 104009 (2016) doi:10.1103/PhysRevD.94.104009 [arXiv:1608.07858 [gr-qc]].

[45] L. Tan and R. P. Woodard, JCAP **05**, 037 (2018) doi:10.1088/1475-7516/2018/05/037 [arXiv:1804.01669 [gr-qc]].

[46] S. Tulin and H. B. Yu, Phys. Rept. **730**, 1-57 (2018) doi:10.1016/j.physrep.2017.11.004 [arXiv:1705.02358 [hep-ph]].

[47] S. Sarkar, Nature Astron. **2**, no.11, 856-857 (2018) doi:10.1038/s41550-018-0598-6

[48] S. Adhikari, A. Banerjee, K. K. Boddy, F. Y. Cyr-Racine, H. Desmond, C. Dvorkin, B. Jain, F. Kahlhoefer, M. Kaplinghat and A. Nierenberg, *et al.* [arXiv:2207.10638 [astro-ph.CO]].

[49] R. P. Woodard, Universe **4**, no.8, 88 (2018) doi:10.3390/universe4080088 [arXiv:1807.01791 [gr-qc]].

[50] S. Deser and R. P. Woodard, Phys. Rev. Lett. **99**, 111301 (2007) doi:10.1103/PhysRevLett.99.111301 [arXiv:0706.2151 [astro-ph]].

[51] J. S. Schwinger, J. Math. Phys. **2**, 407-432 (1961) doi:10.1063/1.1703727

[52] K. T. Mahanthappa, Phys. Rev. **126**, 329-340 (1962) doi:10.1103/PhysRev.126.329

[53] P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. **4**, 1-11 (1963) doi:10.1063/1.1703883

[54] P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. **4**, 12-16 (1963) doi:10.1063/1.1703879

[55] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515-1527 (1964)

[56] K. c. Chou, Z. b. Su, B. l. Hao and L. Yu, Phys. Rept. **118**, 1-131 (1985) doi:10.1016/0370-1573(85)90136-X

[57] R. D. Jordan, Phys. Rev. D **33**, 444-454 (1986) doi:10.1103/PhysRevD.33.444

[58] E. Calzetta and B. L. Hu, Phys. Rev. D **35**, 495 (1987) doi:10.1103/PhysRevD.35.495

[59] L. H. Ford and R. P. Woodard, Class. Quant. Grav. **22**, 1637-1647 (2005) doi:10.1088/0264-9381/22/9/011 [arXiv:gr-qc/0411003 [gr-qc]].

[60] R. L. Arnowitt, S. Deser and C. W. Misner, Phys. Rev. **116**, 1322-1330 (1959) doi:10.1103/PhysRev.116.1322

[61] S. Boran, S. Desai, E. O. Kahya and R. P. Woodard, Phys. Rev. D **97**, no.4, 041501 (2018) doi:10.1103/PhysRevD.97.041501 [arXiv:1710.06168 [astro-ph.HE]].

[62] S. S. McGaugh, J. M. Schombert, G. D. Bothun and W. J. G. de Blok, Astrophys. J. Lett. **533**, L99-L102 (2000) doi:10.1086/312628 [arXiv:astro-ph/0003001 [astro-ph]].

[63] S. P. Miao, N. C. Tsamis and R. P. Woodard, JHEP **03**, 069 (2022) doi:10.1007/JHEP03(2022)069 [arXiv:2110.08715 [gr-qc]].

[64] E. Kasdagli, M. Ulloa and R. P. Woodard, Phys. Rev. D **107**, no.10, 105023 (2023) doi:10.1103/PhysRevD.107.105023 [arXiv:2302.04808 [gr-qc]].

[65] R. P. Woodard and B. Yesilyurt, JHEP **08**, 124 (2023) doi:10.1007/JHEP08(2023)124 [arXiv:2305.17641 [gr-qc]].

[66] M. Milgrom, [arXiv:2310.14334 [astro-ph.GA]].

[67] L. Tan, N. C. Tsamis and R. P. Woodard, Universe **8**, no.7, 376 (2022) doi:10.3390/universe8070376 [arXiv:2206.11467 [gr-qc]].

[68] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 024021 (2006) doi:10.1103/PhysRevD.74.024021 [arXiv:gr-qc/0603135 [gr-qc]].

[69] C. L. Wang and R. P. Woodard, Phys. Rev. D **91**, no.12, 124054 (2015) doi:10.1103/PhysRevD.91.124054 [arXiv:1408.1448 [gr-qc]].

[70] S. P. Miao, Phys. Rev. D **86**, 104051 (2012) doi:10.1103/PhysRevD.86.104051 [arXiv:1207.5241 [gr-qc]].

[71] A. H. Chamseddine and V. Mukhanov, JHEP **11**, 135 (2013) doi:10.1007/JHEP11(2013)135 [arXiv:1308.5410 [astro-ph.CO]].

[72] A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP **06**, 017 (2014) doi:10.1088/1475-7516/2014/06/017 [arXiv:1403.3961 [astro-ph.CO]].