

Accelerated expansion of an open universe and string theory realizations

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Recently, many works have tried to realize cosmological accelerated expansion in string theory models in the asymptotic regions of field space, with a typical scalar potential $V(\varphi)$ having an exponential falloff $e^{-\gamma\varphi}$. Those attempts have been plagued by the fact that V is too steep, namely $\gamma \geq 2/\sqrt{d-2}$ in a d -dimensional spacetime. We revisit the corresponding dynamical system for arbitrary d and γ and show that for an open universe ($k = -1$) there exists a new stable fixed point P_1 precisely if $\gamma > 2/\sqrt{d-2}$. Building on the recent work [P. Marconnet and D. Tsimpis, Universal accelerating cosmologies from 10d supergravity, *J. High Energy Phys.* **01** (2023) 033], we show in addition that cosmological solutions asymptoting to P_1 exhibit accelerated expansion in various fashions (semi-eternal, eternal, transient with a parametrically controlled number of e -folds, or roller coaster). We finally present realizations in string theory of these cosmological models with asymptotically accelerating solutions, for $d = 4$ or $d = 10$. We also show that these solutions do not admit a cosmological event horizon and discuss the possibility of this being a generic feature of quantum gravity.

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I. INTRODUCTION AND SUMMARY

Recent years have witnessed a renewed interest in string theory for the construction of viable cosmological models. A lot of effort has been put into constructing and testing de Sitter solutions, namely solutions from string theory that exhibit a four-dimensional (4D) spacetime with a positive cosmological constant. Those could reproduce the observed accelerated expansion of our current Universe or serve as a first approximation for an early Universe phase of inflation. The outcome of these attempts is that well-controlled examples of de Sitter solutions are difficult to find, if they exist at all (see e.g. [1], Sec. V, for a recent account). The focus then shifted recently to the study of rolling scalar fields $\{\varphi^i\}$ in a positive scalar potential V , in d -dimensional gravitational models of the form

$$S = \int d^d x \sqrt{|g_d|} \left(\frac{M_p^{d-2}}{2} \mathcal{R}_d - \frac{1}{2} g_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j - V(\varphi^k) \right), \quad (1.1)$$

where M_p is the Planck mass. Phenomenologically, such models in $d = 4$ are known to provide valid cosmological models with accelerated expansion, namely inflation or quintessence models. The question is then whether those can be obtained from string theory.

Since a de Sitter solution would correspond to a critical point of V , a natural guess is that a potential with a small slope would also provide an observationally valid model; while a correct intuition, we will see that this is not the only option. Characterizing the ratio $|\nabla V|/V$, with slope $|\nabla V| = \sqrt{g^{ij} \partial_i V \partial_j V}$, that can be obtained in string theory models, has thus been an important activity leading to the de Sitter conjectures of the swampland program (see e.g. [2] for a recent review). Particular focus has been given to the asymptotics of field space: those typically correspond to regions where string corrections are under control (e.g. small string coupling or large volumes corresponding to negligible higher derivative corrections). In the asymptotics, the following characterization has been proposed: string theory potentials have been conjectured [3,4] to obey

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$$\frac{|\nabla V|}{V} \geq \frac{2}{\sqrt{d-2}}, \quad (1.2)$$

with $M_p = 1$, in a d -dimensional spacetime. This conjecture suffers no known counterexample.

String theory potentials typically have an exponential dependence on the (canonically normalized) fields. Considering that we will do only a single field, an exponential potential can be written as $V = V_0 e^{-\gamma\phi}$, and the above is then nothing but a lower bound on the exponential rate γ . This lower bound on γ seems naively too large for an observationally valid accelerated expansion model. Indeed, it is not compatible with single-field slow-roll inflation and related observational constraints [5], and it is in tension with a valid single-field quintessence model [6]; see also the recent extensive study [7]. More dramatically, a dynamical system analysis [8–13] shows that a stable fixed point in the far future, that we denote P_2 , would only admit acceleration for

$$P_2, \text{acceleration: } \gamma < \frac{2}{\sqrt{d-2}}. \quad (1.3)$$

Note also that the requirement of stability, i.e. P_2 being an attractor, corresponds to the same inequality. Comparing (1.2) and (1.3) appears to ruin hopes of realizing accelerated expansion in string theory models, at least in the asymptotics of field space (and asymptotics in time), for which corrections are under control. This matter has been the topic of many recent works, including [10–22].

However, upon closer examination of the situation, we see two important loopholes. First, having no acceleration at a future fixed point does not mean that acceleration cannot occur before. It can be realized during a transient phase (as is actually expected for inflation), as was first noticed in [23], but also it can be realized for an infinite amount of time, as was pointed out in [24,25], since the fixed point only corresponds to the asymptotic limit $t \rightarrow \infty$. Second, the previously mentioned dynamical system and fixed point analysis was carried out on a spatially flat spacetime. Using a Friedmann-Lemaître-Robertson-Walker (FLRW) metric, this corresponds to $k = 0$. The analysis can be extended to $k = -1$, namely a spatially curved, open universe. This was done in [26] for $d = 4$, and we will provide here a more complete analysis, with arbitrary d and γ . With this extension, one finds a new stable fixed point P_1 leading to different physics. No accelerated expansion is occurring at P_1 itself, but cosmological solutions in its vicinity as well as away from P_1 exhibit accelerated expansion for an infinite amount of time, as described before. Those solutions are in addition able to accumulate an infinite number of e -folds while asymptoting to P_1 . Other solutions also allow for transient acceleration, with a parametric control on the number of e -folds. Last but not least, as we will show in this paper, a necessary condition for P_1 with $k = -1$ to exist is

$$P_1, \text{existence: } \gamma > \frac{2}{\sqrt{d-2}}. \quad (1.4)$$

Note that P_1 is then automatically stable, i.e. an attractor. Given the claim (1.2) and the possibility of having accelerating solutions around P_1 , Eq. (1.4) brings back the hope of realizing asymptotic accelerated expansion in string theory, as was recently discussed in [18]. Note that “asymptotic” does not refer here to the fixed point itself but to the infinite time before it.

Realizing accelerating solutions in string theory is actually not a new idea. Following the time-dependent compactifications of [23], several solutions coming from string/M-theory, and exhibiting accelerated expansion for a finite period of time, were subsequently constructed in [27–32]. It was thus realized that transient acceleration is in fact generic in flux compactifications; see [33] for a review. Until recently, however, all known examples of transient acceleration coming from string/M-theory were only able to produce a number of e -folds of order one and were therefore thought to be unsuitable as models of inflation [24,34]. This common lore was recently found to be false in [18], which showed that cosmologies exhibiting transient accelerated expansion with parametric control of e -folds are in fact generic in flux compactifications of 10D supergravity, provided one allows for an open universe with $k = -1$. In the present paper, we review some models of [18] and also provide a new string theory realization of an accelerated expanding universe. Whether or not such accelerating cosmological solutions are observationally valid remains to be seen; we come back to this matter in the Outlook section.

We start this paper by carrying out in Sec. II a general dynamical system analysis of a single scalar field model, minimally coupled to gravity, in a d -dimensional FLRW spacetime, $d > 2$, with an arbitrary scalar potential $V \geq 0$. The spatial curvature, related to $k = 0, \pm 1$, as well as the ratio $\gamma = -V'/V$ at a given fixed point are left free. We find the fixed points of the system, summarizing them in Table I; study whether they allow for acceleration; and further analyze their stability. This analysis and the corresponding phase space are summarized graphically in Fig. 1.

We then restrict to $k = -1$ and to an exponential potential, with exponential rate γ and study in Sec. III the solutions that asymptote to the stable fixed point P_1 . We first provide, with arbitrary d, γ , analytic expressions for classes of solutions in the vicinity of P_1 , some exhibiting accelerated expansion. We then show numerically the complete solutions in phase space and discuss the various ways this acceleration is realized. We also show that these solutions, asymptoting to P_1 in the future, have no cosmological event horizon; we add a few words on this observation below.

TABLE I. Fixed points $(x, y) = \left(\frac{\phi}{H\sqrt{(d-1)(d-2)}}, \frac{\sqrt{2V}}{H\sqrt{(d-1)(d-2)}} \right)$ in flow of N -folds or time, with $H \neq 0$, $V \geq 0$, and their properties. We denote for $V > 0$ the ratio at a fixed point $\gamma = -\frac{V'}{V}|_0$.

Fixed point (x, y)	Allowed k	Existence constraint	Acceleration
$P_0: (0, 0)$	$k = -1$	$\dot{a}_0^2 = 1$	no ($\ddot{a} = 0$)
$P_{\pm}: (\pm 1, 0)$	$k = 0$	\dots	no ($\ddot{a} < 0$)
$P_1: \left(\frac{2}{\gamma\sqrt{(d-1)(d-2)}}, \pm \frac{2}{\gamma\sqrt{d-1}} \right)$	$k = 0, \pm 1$	$\gamma^2 = \frac{4}{d-2} \left(1 + \frac{k}{d_0} \right)^{-1}$	no ($\ddot{a} = 0$)
$P_2: \left(\frac{\gamma}{2} \sqrt{\frac{d-2}{d-1}}, \pm \sqrt{1 - \frac{\gamma^2}{4} \frac{d-2}{d-1}} \right)$	$k = 0$	$0 \leq \gamma^2 < 4 \frac{d-1}{d-2}$	if and only if $\gamma^2 < \frac{4}{d-2}$

While all the above is a pure cosmological analysis, we finally show in Sec. IV that these solutions can be realized within string theory models. We first review some results of [18]; there the authors obtained appropriate $d = 4$ two-scalar models with $k = -1$, as consistent truncations of 10D type IIA supergravity compactifications, with the 6D compact manifold being Calabi-Yau, Einstein, or Einstein-Kähler. These models always admit consistent subtruncations to a single scalar with an exponential potential. Here, we also show that 10D massive IIA supergravity can itself provide a realization of such a model in $d = 10$, where the scalar field is the dilaton and the exponential potential is generated by the Romans mass. Provided that these settings

can be realized in the classical string regime, as argued for in Sec. IV C, we conclude that cosmological solutions with asymptotic accelerated expansion can be realized in string theory.

The difficulty in string theory, and more generally quantum gravity, is therefore not to get asymptotic or eternal accelerated expansion, contrary to what the attempts with $k = 0$, or with de Sitter solutions, suggest. From the present work, one could rather infer the following claim: in quantum gravity, the difficulty is to get solutions with cosmological event horizons. This is in line with an absence of fully stable (instead of metastable) de Sitter solutions, and with the solutions in asymptotic accelerated expansion obtained here for $d \geq 3$. Interestingly, the notion of cosmological event horizon is intrinsically asymptotic [see e.g. (3.18)]; having a claim related to this concept then fits well with most swampland conjectures, that also tend to be asymptotic. Claiming the absence of cosmological event horizons in quantum gravity is also in line with the absence of a holographic description for a cosmological setting, where a horizon at future infinity would have played the role of holographic boundary. The difficulty in getting such a holographic description is known for a de Sitter spacetime, related there to the nonunitarity of the would-be conformal field theory. A similar conclusion was reached in [35] after investigating power-law accelerating solutions. It would be interesting to investigate in more depth such a *no cosmological horizon conjecture*.

Given that we find interesting cosmological solutions that require $k = -1$, one may wonder how natural this is in string theory; let us make a final comment on this matter. At first glance, it seems that k is an input that one can choose in any string theory construction. However, as formalized with the concept of the string landscape [36,37], it became apparent that string theory may have a huge number of vacua. It was then argued in [38] that in the early Universe, transitions between those vacua would necessarily lead to $k = -1$. The reason for this is the tunneling between vacua as described by Coleman and de Luccia [39], which necessarily requires $k = -1$ after tunneling. This had led to the idea that $k = -1$ is not only possible but actually necessary in string theory. However, this is too strong a

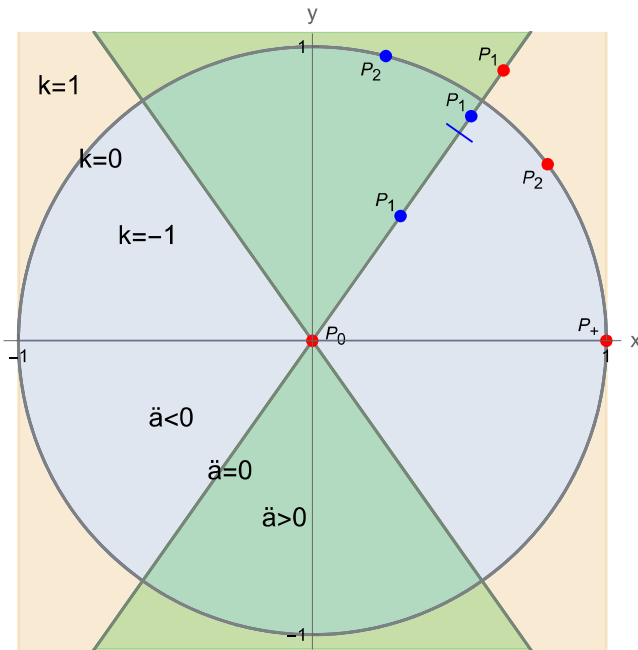


FIG. 1. Illustration of the phase space for the dynamical system considered, here in $d = 4$. The physically relevant regions (value of k , acceleration) are indicated. Restricting to the upper right quarter of the plane, the fixed points P_0, P_+, P_1, P_2 are indicated in blue when stable and red when unstable. The short blue line separates the two different stabilities of P_1 : the node (above) and the spiral (below). We refer to the main text for more details.

claim, as was shown in several more recent papers that find other k values to be possible as well [40–43]. Thus, we conclude that, while not absolutely necessary, an external spacetime with $k = -1$ is still natural in string theory. We come back in the Outlook on how realistic related cosmological solutions can be.

II. SETUP AND FIXED POINT ANALYSIS

In this section, we introduce the d -dimensional theory that will serve as our cosmological model. We then perform a dynamical system analysis of its equations, identifying its fixed points, their stability, and the regions with an accelerating universe. Our results are summarized in Table I and Fig. 1.

A. d -dimensional setup

We consider a d -dimensional theory, $d > 2$, describing a single scalar field minimally coupled to gravity

$$\mathcal{S} = \int d^d x \sqrt{|g_d|} \left(\frac{M_p^{d-2}}{2} \mathcal{R}_d - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right), \quad (2.1)$$

where the field φ is canonically normalized and V is the scalar potential. In the following, we set the Planck mass $M_p = 1$. We focus on solutions with FLRW metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad k = 0, \pm 1, \quad a(t) > 0, \quad (2.2)$$

with scale factor $a(t)$ and k determining the curvature of the $(d-1)$ -dimensional space. We also restrict ourselves to a homogeneous scalar field. Then, the equations of motion (e.o.m.) are given by the two Friedmann equations and the field e.o.m.,¹ namely

$$\begin{aligned} \frac{(d-1)(d-2)}{2} \left(H^2 + \frac{k}{a^2} \right) &= \rho, \\ (d-2) \frac{\ddot{a}}{a} + \frac{d-3}{d-1} \rho + p &= 0 \Leftrightarrow \dot{H} - \frac{k}{a^2} + \frac{\rho + p}{d-2} = 0, \\ \ddot{\varphi} + (d-1)H\dot{\varphi} + V' &= 0, \end{aligned} \quad (2.3)$$

where the Hubble parameter, the energy density, and the pressure are given by

¹A formal linear relation between the three equations can be found in e.g. [44] (A.2). One consequence is that satisfying the first Friedmann equation together with the field e.o.m. implies that the second Friedmann equation is satisfied, as long as $H \neq 0$.

$$H = \frac{\dot{a}}{a}, \quad \rho = \frac{1}{2} \dot{\varphi}^2 + V, \quad p = \frac{1}{2} \dot{\varphi}^2 - V. \quad (2.4)$$

The dot stands for ∂_t , and the prime stands for ∂_φ .

In this context, it is useful to introduce the equation of state parameter $w = \frac{p}{\rho}$, for which we restrict ourselves to $V > 0$. Using the second Friedmann equation, one shows that having an accelerating universe amounts to

$$\ddot{a} \geq 0 \Leftrightarrow w \leq -\frac{d-3}{d-1}, \quad (2.5)$$

where we include for future convenience the case without acceleration, $\ddot{a} = 0$.

B. Dynamical system and fixed points

We perform a dynamical system analysis on the previous equations. Similar analyses were carried out for exponential potentials in $d = 4$ [8,9] and arbitrary d in [10,11]. A more thorough analysis was recently performed in [12,13] in a multifield situation. All these works focused on $k = 0$; the main novelty here is that we allow for $k = 0, \pm 1$, and $k = -1$ will provide interesting new physics. We also allow for a generic positive scalar potential; the only quantity that will matter is the ratio $\gamma = -V'/V|_0$ at a fixed point. This analysis has been performed in [18] for $d = 4$ and specific values for γ coming from certain classes of string theory compactifications, a situation we will come back to; here, we work with arbitrary d , k and γ .

We introduce the following variables:

$$\begin{aligned} N &= \ln a, & x &= \frac{\dot{\varphi}}{H \sqrt{(d-1)(d-2)}}, \\ y &= \frac{\sqrt{2V}}{H \sqrt{(d-1)(d-2)}}, & H &\neq 0, V > 0. \end{aligned} \quad (2.6)$$

Using the field e.o.m. and the second Friedmann equation (without appearance of k), we obtain the following system:

$$\begin{aligned} \frac{dx}{dN} &= -\frac{\sqrt{(d-1)(d-2)} V'}{2V} y^2 - x \left(d-2-x^2(d-2)+y^2 \right), \\ \frac{dy}{dN} &= y \left(\frac{\sqrt{(d-1)(d-2)} V'}{2V} x + 1 + x^2(d-2) - y^2 \right). \end{aligned} \quad (2.7)$$

Because k is not fixed here, we proceed differently than in previous references and do not use the first Friedmann equation when deriving (2.7); we will come back to that equation later.

We now look at the fixed points, given by

$$\frac{dx}{dN} = \frac{dy}{dN} = 0, \quad (2.8)$$

and we introduce the convenient notations at a given fixed point,

$$\gamma \equiv -\frac{V'}{V}\bigg|_0, \quad \lambda = \gamma \frac{\sqrt{(d-1)(d-2)}}{2}. \quad (2.9)$$

To derive the system (2.7), we considered the restriction $V > 0$. It is however possible to extend the fixed point analysis to include the case $V = 0$, i.e. $y = 0$. We get in that case the following fixed points,

$$\begin{aligned} P_0: (x, y) &= (0, 0), \\ P_{\pm}: (x, y) &= (\pm 1, 0), \end{aligned} \quad (2.10)$$

and we refer to [18] for more details about them.

We turn to $y \neq 0$ and rewrite the fixed point equations as

$$\begin{aligned} \lambda y^2 &= x(d-1-\lambda x), \\ y^2 &= x^2(d-2) - \lambda x + 1. \end{aligned} \quad (2.11)$$

If $\lambda = 0$, we get $x = 0$ and $y = \pm 1$. This fixed point will be included below in P_2 . We then consider $\lambda \neq 0$. The equations above get rewritten into

$$\begin{aligned} x^2 \lambda (d-1) + \lambda (-\lambda x + 1) &= x(d-1), \\ (d-1)y^2 &= 1 - \frac{x}{\lambda} \left(\lambda^2 - (d-1)(d-2) \right), \end{aligned} \quad (2.12)$$

which are solved by the following fixed points,

$$\begin{aligned} P_1: (x, y) &= \left(\frac{1}{\lambda}, \pm \frac{\sqrt{d-2}}{\lambda} \right) \\ &= \left(\frac{2}{\gamma \sqrt{(d-1)(d-2)}}, \pm \frac{2}{\gamma \sqrt{d-1}} \right) \end{aligned} \quad (2.13)$$

$$\begin{aligned} P_2: (x, y) &= \left(\frac{\lambda}{d-1}, \pm \sqrt{1 - \frac{\lambda^2}{(d-1)^2}} \right) \\ &= \left(\frac{\gamma}{2} \sqrt{\frac{d-2}{d-1}}, \pm \sqrt{1 - \frac{\gamma^2 d-2}{4(d-1)}} \right), \end{aligned} \quad (2.14)$$

where P_2 exists iff $\lambda^2 < (d-1)^2$, i.e. $\gamma^2 < 4 \frac{d-1}{d-2}$ (P_2 is defined for $y \neq 0$).

We finally turn to the first Friedmann equation; it gets rewritten as

$$x^2 + y^2 = 1 + \frac{k}{\dot{a}^2}. \quad (2.15)$$

Let us consider its impact on each fixed point. To start with, P_{\pm} require $k = 0$, while P_0 requires $k = -1$ together with the value at the fixed point $\dot{a}_0^2 = 1$. P_2 then requires $k = 0$. This fixed point was found in previous references with

$k = 0$, which made use from the start of the resulting first Friedmann equation, contrary to here. Last but not least, P_1 requires the following relation,

$$P_1: \gamma^2 = \frac{4}{d-2} \left(1 + \frac{k}{\dot{a}_0^2} \right)^{-1}, \quad (2.16)$$

allowing for $k = \pm 1, 0$. For $k = 0$, it fixes $\gamma = \pm \frac{2}{\sqrt{d-2}}$, coinciding then with P_2 for this value of γ .

Having determined the fixed points, we finally study the possibility of having acceleration there. Let us start with P_0 and P_{\pm} . As a cosmological solution, P_0 corresponds to a Milne universe, which admits a scale factor linear in t . Therefore, $\ddot{a}(t) = 0$; i.e., there is no acceleration at this fixed point. For P_{\pm} , since $\dot{\phi} \neq 0$, $V = 0$, one obtains there $w = 1$ and $\ddot{a}(t) < 0$, i.e. a decelerating universe. We now turn to P_1, P_2 for which $y \neq 0$ and γ is well defined. By definition, one has $w = \frac{x^2 - y^2}{x^2 + y^2}$, and the condition for acceleration is (2.5). Using Eqs. (2.11) and (2.12), w gets rewritten as

$$w = -1 + \frac{2}{(d-1)} x \lambda = -1 + x \gamma \sqrt{\frac{d-2}{d-1}}. \quad (2.17)$$

The condition (2.5) for acceleration becomes $x \lambda \leq 1$, i.e. $x \gamma \leq 2 / \sqrt{(d-1)(d-2)}$; considered on each fixed point, we obtain

$$P_1: 1 \leq 1, \quad (2.18)$$

$$P_2: \gamma^2 \leq \frac{4}{d-2}. \quad (2.19)$$

The condition for P_2 is well known from previous references. Interestingly, for P_1 , there is no constraint on γ ; the solution at this fixed point is simply never accelerating as the inequality is saturated, i.e. $\ddot{a} = 0$. Note in particular the preservation of this property when P_1 and P_2 coincide with $\gamma = \pm \frac{2}{\sqrt{d-2}}$. We summarize the results in Table I.

We conclude that $k = \pm 1$ allows for new fixed points with respect to $k = 0$. Of particular interest for string theory realizations (see Sec. IV), P_1 with $k = -1$ admits

$$\gamma^2 > \frac{4}{d-2}. \quad (2.20)$$

Even though the fixed point itself does not allow for acceleration, we will see in Sec. III that a (semi)eternally accelerating universe can be found arbitrarily close by. Prior to this, let us study the stability of these fixed points.

C. Stability

We now study the stability of the fixed points listed in Table I. As a nonlinear autonomous system, the stability is determined by looking at the sign of the real parts of the eigenvalues of the Jacobian of the system (2.7), for each fixed point. A stable fixed point is obtained if these real parts are negative. The Jacobian to consider is obtained by acting with ∂_x or ∂_y on the right-hand side of each equation in the system (2.7). It is then given by the following matrix:

$$M = \begin{pmatrix} (3x^2 - 1)(d-2) - y^2 & 2y(\lambda - x) \\ y(2x(d-2) - \lambda) & 1 - \lambda x + x^2(d-2) - 3y^2 \end{pmatrix}. \quad (2.21)$$

In the following, we give its expression at each fixed point, together with its determinant and trace

$$\begin{aligned} \det M &= 2(d-2) \left(1 - \frac{d-1}{\lambda^2}\right) = 2(d-2) \left(1 - \frac{4}{\gamma^2(d-2)}\right), & \text{Tr} M &= -(d-2), \\ P_2: M &= \begin{pmatrix} 1 - d + \lambda^2 \frac{3d-5}{(d-1)^2} & 2y_2 \lambda \frac{d-2}{d-1} \\ y_2 \lambda \frac{d-3}{d-1} & -2 + \lambda^2 \frac{2}{(d-1)^2} \end{pmatrix}, \\ \det M &= 2(d-1-\lambda^2) \left(1 - \frac{\lambda^2}{(d-1)^2}\right) = 2(d-1) \left(1 - \frac{\gamma^2(d-2)}{4}\right) \left(1 - \frac{\gamma^2 d-2}{4(d-1)}\right), \\ \text{Tr} M &= -(d+1) + \frac{3}{d-1} \lambda^2 = -(d+1) + \frac{3(d-2)}{4} \gamma^2. \end{aligned} \quad (2.25)$$

The sign of the real parts of the eigenvalues can be read from $\text{Tr} M$ and $\det M$. Indeed, for a 2×2 matrix M , the eigenvalues are

$$\frac{1}{2} \left(\text{Tr} M \pm \sqrt{(\text{Tr} M)^2 - 4 \det M} \right). \quad (2.26)$$

If $\det M < 0$, then the square root is real, and it is bigger than $|\text{Tr} M|$, so the eigenvalues are real, and there is one positive and one negative eigenvalue. The fixed point is then unstable (a saddle point). If $\det M > 0$, then either the square root is real but smaller than $|\text{Tr} M|$ or it is purely imaginary. In either case, the real parts of the eigenvalues then have the sign of $\text{Tr} M$, with corresponding stability behaviors. Finally, if $\det M = 0$, (at least) one eigenvalue vanishes, and the stability is determined by the sign of the other eigenvalue, that of $\text{Tr} M$.

We apply the above to the fixed points. We start with P_0 for which the situation is clear:

$$P_0: \text{unstable (saddle point)}. \quad (2.27)$$

For P_{\pm} , the two eigenvalues are real and positive, giving

$$P_0: M = \begin{pmatrix} -(d-2) & 0 \\ 0 & 1 \end{pmatrix}, \quad (2.22)$$

$$\begin{aligned} \det M &= -(d-2), & \text{Tr} M &= -(d-3), \\ P_{\pm}: M &= \begin{pmatrix} 2(d-2) & 0 \\ 0 & d-1 \end{pmatrix}, \end{aligned} \quad (2.23)$$

$$\begin{aligned} \det M &= 2(d-1)(d-2), & \text{Tr} M &= 3d-5, \\ P_1: M &= \begin{pmatrix} -(d-2) \left(1 - \frac{2}{\lambda^2}\right) & 2y_1 \lambda \left(1 - \frac{1}{\lambda^2}\right) \\ -\lambda y_1 \left(1 - \frac{2(d-2)}{\lambda^2}\right) & -\frac{2(d-2)}{\lambda^2} \end{pmatrix}, \end{aligned} \quad (2.24)$$

$$P_{\pm}: \text{unstable (unstable node)}. \quad (2.28)$$

We turn to P_2 . There, we recall we must have $\gamma^2 < 4 \frac{d-1}{d-2}$, so the sign of $\det M$ depends solely on the condition for acceleration, $\gamma^2 < \frac{4}{d-2}$. Provided the latter holds, we can show that $\text{Tr} M < 0$. We conclude

$$\begin{aligned} P_2, \gamma^2 &< \frac{4}{d-2} (\ddot{a} > 0): \text{stable (stable node)}, \\ P_2, \gamma^2 &= \frac{4}{d-2} (\ddot{a} = 0): \text{Lyapunov stable (neutral node)}, \\ P_2, \frac{4}{d-2} &< \gamma^2 < 4 \frac{d-1}{d-2} (\ddot{a} < 0): \text{unstable (saddle point)}. \end{aligned} \quad (2.29)$$

For the case $\ddot{a} = 0$, we have one vanishing and one negative eigenvalue, falling in the category of ‘‘Lyapunov stable.’’ In addition, one verifies that

$$P_2: (\text{Tr} M)^2 - 4 \det M = \left(\frac{\gamma^2(d-2)}{4} + d-3 \right)^2 \geq 0, \quad (2.30)$$

so eigenvalues are always real. This is why we get a stable node for the accelerating case and not a stable spiral, and we get a neutral node for $\ddot{a} = 0$ and not a neutral center.

We finally turn to P_1 . The sign of $\det M$ is precisely that of $-k$, which makes obvious the stability since $\text{Tr}M < 0$. In addition, for $\det M \geq 0$, let us as above determine whether the eigenvalues are real or complex conjugate. This is determined by the sign of

$$P_1: (\text{Tr}M)^2 - 4 \det M = (d-2) \left(d - 10 + \frac{32}{\gamma^2(d-2)} \right). \quad (2.31)$$

This quantity is positive or zero for $d \geq 10$, or for

$$\gamma^2 \leq \gamma_s^2 \equiv \frac{32}{(d-2)(10-d)} \quad \text{with } d < 10, \quad (2.32)$$

giving then two real eigenvalues. We conclude

$$\begin{aligned} P_1, k=+1: & \text{unstable (saddle point)} \\ P_1, k=0: & \text{Lyapunov stable (neutral node)} \\ P_1, k=-1, d \geq 10, \text{ or } d < 10, \gamma^2 \leq \gamma_s^2: & \text{stable (stable node)} \\ P_1, k=-1, d < 10, \gamma^2 > \gamma_s^2: & \text{stable (stable spiral)}. \end{aligned} \quad (2.33)$$

We end the discussion by highlighting the new stable fixed point, P_1 with $k = -1$. For $d < 10$, it is a stable node with

$$\frac{4}{d-2} < \gamma^2 \leq \frac{32}{(d-2)(10-d)}, \quad (2.34)$$

and for $d \geq 10$, it is a stable node for any $\gamma^2 > \frac{4}{d-2}$. This is an interesting range for string theory constructions as we will see in Sec. IV.

D. Graphical summary

Inspired by [18] (see e.g. Fig. 8), we provide an illustration of the phase space and fixed points found above. The system depends on the two real variables (x, y) , which can be of either sign, so the phase space can first be represented by the (x, y) -plane. The dynamical system is made of the two differential equations (2.7) and the first Friedmann equation (2.15) which is not differential and thus appears as a constraint. Interestingly, the latter is given in terms of $x^2 + y^2$, i.e. the distance to the origin, and it separates the plane in three zones: the circle $x^2 + y^2 = 1$ is the phase space for $k = 0$, beyond it corresponds to $k = 1$, and inside the circle corresponds to $k = -1$. Note that the fixed points P_2 lie only on the circle.

The dynamical system is symmetric under the following two independent changes of sign,

$$y \rightarrow -y; \quad (2.35)$$

$$x \rightarrow -x, \quad \frac{V'}{V} \rightarrow -\frac{V'}{V}, \quad (2.36)$$

so in the following, when indicating the fixed points, we will restrict to the upper right quarter of the plane. The symmetry in y indicates that for any expanding universe (upper half plane), there is another solution with a contracting one (lower half plane). The symmetry with x and $\frac{V'}{V}$ is related to the freedom in the sign of the scalar field.

Further relevant regions in phase space are given by the acceleration condition (2.5). Using $w = \frac{x^2 - y^2}{x^2 + y^2}$, it gets rewritten as

$$\text{acceleration: } |y| > |x| \sqrt{d-2}. \quad (2.37)$$

The lines saturating this inequality correspond to the no acceleration region (where P_1 lies) and provide further distinct regions of the phase space.

Last but not least, the stability of the fixed points depends on their position. For P_2 , it is directly related to the acceleration or deceleration. For P_1 , in addition to k , the stability behavior depends on the value of γ for $k = -1$. The change happens at a certain distance from the origin given by

$$\sqrt{x_1^2 + y_1^2} \Big|_{\gamma=\gamma_s} = \sqrt{\frac{10-d}{8}}, \quad (2.38)$$

strictly smaller than 1 and greater than 0 as it should.

We illustrate the above in Fig. 1. Such illustration will be useful later to view cosmological solutions around the fixed points, as done already in [18].

III. COSMOLOGICAL SOLUTIONS, ACCELERATION, AND HORIZON

In this section, we determine cosmological solutions in the vicinity of the stable fixed point P_1 for $k = -1$, with a particular interest in those providing acceleration. We then investigate the way acceleration is realized and whether each solution admits a cosmological horizon.

We restrict to a positive exponential potential,

$$V(\varphi) = V_0 e^{-\gamma\varphi}, \quad V_0, \gamma > 0, \quad (3.1)$$

where $\gamma = -V'/V$ is now a given constant everywhere in phase space. An extensive study of such cosmological solutions has been performed in [18] for $d = 4$ and particular values for γ coming from certain classes of string theory compactifications; we will come back to the latter. We work here with a general $d \geq 3$ and an arbitrary

$$\gamma > \frac{2}{\sqrt{d-2}}, \quad (3.2)$$

where this lower bound is due to $k = -1$ [see (2.20)]; we will see that this generically leads to new interesting solutions.

Motivation for studying those solutions can be found in string theory models. In such models, it is a common situation that the scalar potential takes the form (3.1) in field space asymptotics, $\varphi \rightarrow \infty$. Focusing then on asymptotics (in field space but also in time), we are naturally interested in the physics close to attractive fixed points. As summarized in Fig. 1, those are P_1, P_2 for some values of k, γ . P_2 is attractive for $\gamma \leq \frac{2}{\sqrt{d-2}}$ (also corresponding to the accelerating regime). This fixed point was then studied in previous references. However, typical string theory constructions lead to $\gamma \geq \frac{2}{\sqrt{d-2}}$ in field and time asymptotics (see the strong de Sitter conjecture [3,4]), thus forbidding reaching such a P_2 . On the contrary, P_1 allows for γ in agreement with that conjecture as indicated in (3.2), as long as $k = -1$.² In addition, it is then attractive, so this motivates us to focus on that fixed point. Being placed at the boundary of acceleration, solutions that asymptote to it can give accelerating universes, as we will see.

We thus turn to the study of solutions close to P_1 with $k = -1$. As seen in the stability analysis, one needs to distinguish

stable node: $\gamma \leq \gamma_s$

stable spiral: $\gamma > \gamma_s$

$$\text{where } \gamma_s = \frac{4\sqrt{2}}{\sqrt{(10-d)(d-2)}}, \quad (3.3)$$

whenever restricting to $d < 10$. We thus treat those two situations one after the other. For $d \geq 10$, the solutions found for the stable node with $\gamma \leq \gamma_s$ will be valid for any γ .

Before looking at the solutions close to P_1 , let us add a word on the fixed point itself. In general, fixed points correspond by themselves to cosmological solutions. Phase space trajectories between fixed points are also solutions, and those asymptote to each fixed point solution in some late or early time limit. We therefore need to know the solution corresponding to P_1 with $k = -1$, in order to reproduce it in the asymptotics. It can easily be found by solving Eq. (2.3), with the dynamical system variables x, y being constant; we get

$$\begin{aligned} P_1, k = -1: a(t) &= \frac{\gamma(t-t_0)}{\sqrt{\gamma^2 - \frac{4}{d-2}}}, & e^{\gamma\varphi(t)} &= \frac{\gamma^2 V_0 (t-t_0)^2}{2(d-2)} \\ \Leftrightarrow a(t) &= a_0(t-t_0), & \varphi(t) &= \varphi_0 + \varphi_l \log(t-t_0), \\ \text{with } a_0 &= \frac{\gamma}{\sqrt{\gamma^2 - \frac{4}{d-2}}}, & \varphi_0 &= \frac{1}{\gamma} \log\left(\frac{\gamma^2 V_0}{2(d-2)}\right), & \varphi_l &= \frac{2}{\gamma}. \end{aligned} \quad (3.4)$$

This is a Milne universe with an angular defect [18]. The free constant t_0 can be absorbed into a redefinition of t . In the following, we will find solutions around P_1 which asymptote to (3.4) in the limit $t \rightarrow \infty$; for this reason, we also drop t_0 from now on. We now turn to these neighboring solutions.

A. Solutions asymptoting to the stable node

To find solutions that approach the fixed point solution (3.4) for $t \rightarrow \infty$, we make the following ansatz with $p > 0$:

$$\begin{aligned} a(t) &= a_0 t \left(1 + \frac{a_1}{t^p} + \frac{a_2}{t^{2p}} + \frac{a_3}{t^{3p}} + \dots \right), \\ \varphi(t) &= \varphi_0 + \varphi_l \log(t) + \frac{\varphi_1}{t^p} + \frac{\varphi_2}{t^{2p}} + \frac{\varphi_3}{t^{3p}} + \dots \end{aligned} \quad (3.5)$$

² $k = 0$ would allow to saturate the conjecture's bound. It is however difficult to construct a string theory scalar potential precisely matching this bound.

The fixed point corresponds to the leading order terms, given by $a_0, \varphi_0, \varphi_l$ in (3.4). We then solve equations at first subleading order. Since there are only two independent equations, we only fix the two following free parameters,

$$\begin{aligned} p^\pm &= \frac{d-2}{2} \pm \frac{2\sqrt{2}}{\gamma} \sqrt{1 + \gamma^2 \frac{(d-10)(d-2)}{32}}, \\ \varphi_1^\pm &= \frac{d-1}{4} a_1 \gamma p^\pm, \end{aligned} \quad (3.6)$$

meaning we find two solution families labeled with \pm and parametrized by a_1 . These solutions are valid for any $d \geq 3$. Restricting to $d < 10$, we can rewrite p^\pm in terms of $\sqrt{1 - \frac{\gamma^2}{\gamma_s^2}}$, requiring as expected $\gamma \leq \gamma_s$. In the special case $\gamma = \gamma_s$, the two families become only one.

As it should in the ansatz, we verify that $p^\pm > 0 \Leftrightarrow \gamma > 2/\sqrt{d-2}$, which is true here. We can proceed further and solve the equations order by order, with the resulting expressions for the coefficients becoming more

complicated. While solving, we observe however a problem for $p = 1$: in that case, all higher order terms vanish. In other words, the above solution does not exist for $p = 1$. Before focusing on this matter, let us first consider $p^\pm \neq 1$ and study the properties of our solutions.

In the large t limit, properties can be read from the leading and subleading terms above. Of particular interest is the question of acceleration. From (3.5), one has

$$\ddot{a}(t) = a_0 a_1 (p - 1) p \frac{1}{t^{1+p}} + \mathcal{O}(t^{-(1+2p)}). \quad (3.7)$$

Since $a_0 p > 0$, the sign of $\ddot{a}(t)$ for our solutions is fixed by $a_1(p - 1)$. Since we consider $p^\pm \neq 1$, each family (+ or -) of solutions then has an accelerating and a decelerating branch, depending on the sign of a_1 . In between, $\ddot{a}(t) = 0$ when $a_1 = 0$, which corresponds to the fixed point P_1 . We illustrate the two solution families and their branches in Fig. 2. Note that the two branches of one family are actually not continuously connected in terms of time, since they both asymptote to P_1 for $t \rightarrow \infty$.

As a technical remark, we indicate that the tangent vectors to the solution curves in Fig. 2, at the point P_1 , are known analytically. Those correspond to the eigenvectors of the Jacobian (2.24). They agree with the solution above.

We now come back to the case of $p = 1$, where the previous solutions disappear. We verify the following:

$$\begin{aligned} d > 4: & \quad p^+ > 1, \quad p^- = 1 \Leftrightarrow \gamma = \gamma_m \equiv \frac{2\sqrt{2}}{\sqrt{d-1}} < \gamma_s, \\ d = 4: & \quad p^\pm = 1 \Leftrightarrow \gamma = \gamma_s = \gamma_m, \\ d = 3: & \quad p^\pm < 1. \end{aligned} \quad (3.8)$$

It is therefore only for $d \geq 4$ and for a specific $\gamma = \gamma_m$ that one can have $p = 1$. Note the specificity of $d = 4$,

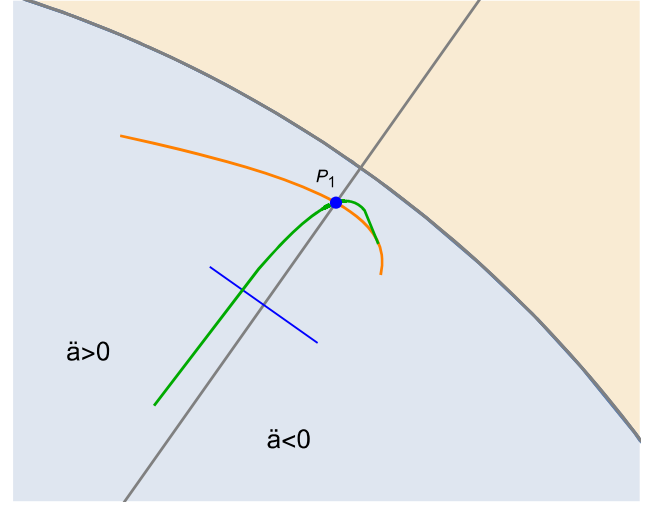


FIG. 2. Evolution of the dynamical variables (x, y) , defined in (2.6), for the solution families + (orange) and - (green). The solutions are given at first order by (3.6), here with $d = 4$, $\gamma = \sqrt{2} + 0.05$. The figure is a zoomed-in part of Fig. 1 around the stable node P_1 (blue point). The solutions are depicted for $a_1 = \pm 1$, allowing us to verify that each branch is accelerating or decelerating.

for which both solution families disappear when $\gamma = \gamma_m$, corresponding in addition to the upper bound γ_s . And it is actually for this very particular value that cosmological solutions around P_1 (stable node) were found from string theory compactifications in [18]. The results of that reference will thus be helpful as we now explain.

We now consider the following ansatz, which goes beyond (3.5) for $p = 1$. This ansatz is inspired from the explicit solutions of [18] in $d = 4$. There, solutions are first expressed in terms of N , the flow variable in the dynamical system. Relating it to t thanks to $a(t)$, one can reach the following ansatz:

$$\begin{aligned} a(t) &= a_0 t + a_l \log(t) + a_c + \frac{1}{t} \left(a_{11} \log(t) + a_{10} \right) + \frac{1}{t^2} \left(a_{22} (\log(t))^2 + a_{21} \log(t) + a_{20} \right) + \dots, \\ \varphi(t) &= \varphi_0 + \varphi_l \log(t) + \frac{1}{t} \left(\varphi_{11} \log(t) + \varphi_{10} \right) + \frac{1}{t^2} \left(\varphi_{22} (\log(t))^2 + \varphi_{21} \log(t) + \varphi_{20} \right) + \dots. \end{aligned} \quad (3.9)$$

Solving the equations at the first subleading order, we find the following solution family (in terms of the parameters a_l, a_c)

$$\begin{aligned} \gamma &= \gamma_m = \frac{2\sqrt{2}}{\sqrt{d-1}}, \\ \varphi_{11} &= \frac{1}{2} a_l \sqrt{\frac{(d-3)(d-1)}{d-2}}, \\ \varphi_{10} &= \frac{1}{2} \left(a_l \frac{5-2d}{d-1} + a_c \right) \sqrt{\frac{(d-3)(d-1)}{d-2}}. \end{aligned} \quad (3.10)$$

The solution family requires us to fix the value of γ to γ_m . It can therefore be considered as replacing the previously lost solutions for $p = 1$, and it does so by including logarithms in the ansatz. Note though that for $d = 4$ and $\gamma = \gamma_m = \gamma_s$, the overall number of solution family is only 1, while all other cases admit 2.

Equations can be solved order by order, and the parameter a_l remains free. As before, it determines the acceleration/deceleration for large t , since we have

$$\ddot{a}(t) = -\frac{a_l}{t^2} + \mathcal{O}\left(\frac{\log(t)}{t^3}\right). \quad (3.11)$$

The solution family has two branches, one accelerating and one decelerating, depending on the sign of a_l . For $a_l = 0$, we recover P_1 with $\ddot{a}(t) = 0$. We illustrate this solution family in Fig. 3.

As a final technical remark, we rewrite the Jacobian (2.24) at P_1 as follows,

$$M_{\pm} = \begin{pmatrix} \frac{\gamma_m^2}{\gamma^2} - (d-2) & \pm \frac{2}{\sqrt{d-2}} \left(d-2 - \frac{\gamma_m^2}{2\gamma^2} \right) \\ \pm \sqrt{d-2} \left(\frac{\gamma_m^2}{\gamma^2} - 1 \right) & -\frac{\gamma_m^2}{\gamma^2} \end{pmatrix}, \quad (3.12)$$

where the sign refers to the sign in y_1 . We recall that the eigenvectors of this matrix give the tangent directions to the solutions, at P_1 . For $\gamma = \gamma_m$, the matrix becomes upper triangular. As a consequence, $(1, 0)$ is an eigenvector. In other words, the tangent to the solution at P_1 is horizontal, which agrees with the orange line in Fig. 3.

B. Solutions asymptoting to the stable spiral

We now look for solutions approaching the fixed point solution (3.4) for $t \rightarrow \infty$, in the case $d < 10$, $\gamma > \gamma_s$, corresponding to a spiral (3.3). Such solutions were found in [18] in $d = 4$ for large γ . We generalize these solutions here to $3 \leq d < 10$, adapting appropriately the ansatz to the following one:

$$\begin{aligned} a(t) &= a_0 t \left(1 + \frac{1}{t^{\frac{d-2}{2}}} \left(a_{c1} \cos [q \log(t)] + a_{s1} \sin [q \log(t)] \right) + \frac{1}{t^{d-2}} \left(a_{c21} \cos [q \log(t)] + a_{c22} \cos [q \log(t)]^2 \right. \right. \\ &\quad \left. \left. + a_{s21} \sin [q \log(t)] + a_{s22} \sin [q \log(t)]^2 + a_{cs} \cos [q \log(t)] \sin [q \log(t)] \right) + \dots \right), \\ \varphi(t) &= \varphi_0 + \varphi_l \log(t) + \frac{1}{t^{\frac{d-2}{2}}} \left(\varphi_{c1} \cos [q \log(t)] + \varphi_{s1} \sin [q \log(t)] \right) + \frac{1}{t^{d-2}} \left(\varphi_{c21} \cos [q \log(t)] + \varphi_{c22} \cos [q \log(t)]^2 \right. \\ &\quad \left. + \varphi_{s21} \sin [q \log(t)] + \varphi_{s22} \sin [q \log(t)]^2 + \varphi_{cs} \cos [q \log(t)] \sin [q \log(t)] \right) + \dots. \end{aligned} \quad (3.13)$$

The leading order is given by the P_1 solution (3.4). At the first subleading order, we find

$$\begin{aligned} q &= \frac{2\sqrt{2}}{\gamma} \sqrt{\frac{\gamma^2}{\gamma_s^2} - 1}, \\ \varphi_{c1} &= \frac{d-1}{8} \gamma (a_{c1}(d-2) - a_{s1}2q), \\ \varphi_{s1} &= \frac{d-1}{8} \gamma (a_{s1}(d-2) + a_{c1}2q). \end{aligned} \quad (3.14)$$

We have a solution family depending on two real parameters: a_{c1}, a_{s1} .³ We also found a solution with $q \rightarrow -q$, but that change of sign can be compensated by $a_{s1} \rightarrow -a_{s1}$, with similar changes at higher orders, so we restrict ourselves to $q > 0$.

We now turn to the question of acceleration. We compute at first subleading order

$$\begin{aligned} \ddot{a}(t) &= \frac{\gamma}{4\sqrt{\gamma^2 - \frac{4}{d-2}}} \frac{1}{t^{\frac{d}{2}}} \left(\left(a_{c1}4(d-3)q + a_{s1}((d-6)d + 8 - 4q^2) \right) \sin [q \log(t)] \right. \\ &\quad \left. - \left(a_{s1}4(d-3)q - a_{c1}((d-6)d + 8 - 4q^2) \right) \cos [q \log(t)] \right). \end{aligned} \quad (3.15)$$

For any given time t , if one solution is accelerating, flipping simultaneously the signs of a_{c1}, a_{s1} gives a decelerating solution. We then have four branches of solutions depending on the signs of a_{c1}, a_{s1} , as illustrated in Fig. 4.

Whether a solution is accelerating or decelerating is time dependent. More precisely, for any given fixed parameters a_{s1} and a_{c1} , the above solutions undergo an infinite number of accelerating and decelerating phases. This is easiest to see if a_{s1} and a_{c1} are chosen such that for example the prefactor of the cosine function vanishes. Then, $\ddot{a}(t) \propto \sin [q \log(t)]$ changes

³One could consider introducing in the ansatz (3.13) a phase in the cosine and sine functions. Such a phase can however be absorbed in a redefinition of the coefficients a_{c1}, a_{s1} , etc. Such a phase would then be a redundant parameter.

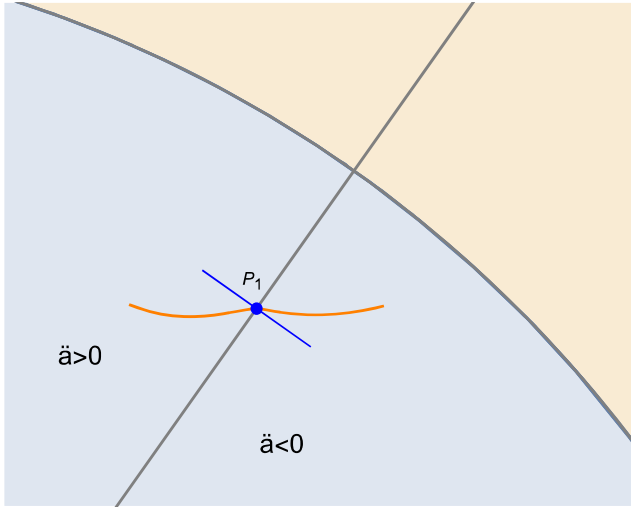


FIG. 3. Evolution of the dynamical variables (x, y) , defined in (2.6), for the solution family (in orange) given at first order by (3.10), here with $d = 4$, $\gamma = \gamma_m$. The figure is a zoomed in part of Fig. 1 around the stable node P_1 (blue point). The solutions are depicted for $a_l = \pm 1$, $a_c = 0$, allowing us to verify that each branch is accelerating or decelerating. We also observe the particularity of $d = 4$ for which $\gamma_m = \gamma_s$, thus placing P_1 on the blue line; beyond this line, the stable node turns into a stable spiral.

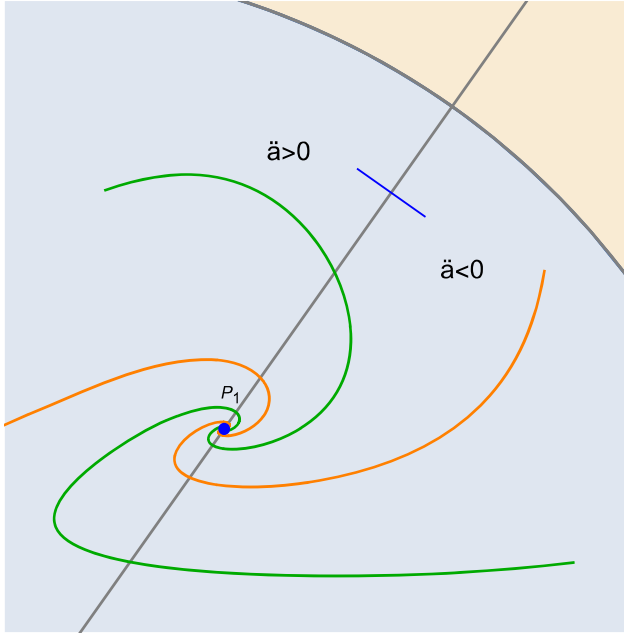


FIG. 4. Evolution of the dynamical variables (x, y) , defined in (2.6), for the solution family given at first order by (3.14), here with $d = 4$, $\gamma = \gamma_s + 1.2$. The figure is a zoomed-in part of Fig. 1 around the stable node P_1 (blue point). We first verify that P_1 is beyond the blue line and is approached by spirals. The solutions are depicted for $(a_{c1}, a_{s1}) = \pm(1, 1)$ (orange) and $\pm(1, -1)$ (green). This allows us to verify each branch of the same color is accelerating or decelerating.

its sign infinitely many times for $t \rightarrow \infty$. The same holds with both prefactors being nonzero.

C. Horizon and acceleration

As a cosmological solution, the stable fixed point P_1 with $k = -1$ is given in (3.4). In Sec. III A and III B, we have determined cosmological solutions approaching the latter for $t \rightarrow \infty$. While P_1 is not a universe undergoing accelerated expansion, several of the solutions asymptoting to it are actually accelerating for an infinitely long time. Those can be completed numerically into full solutions, asymptoting in the past to another fixed point. Their acceleration phases are then realized in various manners that we describe below. Prior to this, let us first discuss an important common property to all these solutions, which is the absence of an event horizon.

1. No event horizon

Given a cosmological solution with an $a(t)$, it is interesting to determine whether it admits an event horizon at a time t_i . To that end, one should compute the distance d_e traveled by light until the end of time, denoted by t_f

$$d_e = a_i \int_{t_i}^{t_f} \frac{dt}{a(t)}, \quad (3.16)$$

where $a_i = a(t_i)$ and t_f is the largest possible time; we take here $t_f = \infty$. If d_e is finite, then the Universe admits an event horizon of size d_e . This is the case for the de Sitter solution with $a(t) = a_i e^{H(t-t_i)}$ and a constant H , or for an accelerated expansion with $a(t) = a_i (t/t_i)^p$ and $p > 1$. The boundary case $p = 1$ without acceleration has no horizon, or equivalently has an infinite horizon size.

Let us now evaluate the horizon size d_e for the cosmological solutions considered previously, that asymptote to P_1 . The horizon is evaluated at a finite time t_i . We do not take t_i to correspond to another asymptotic time, where the solution would match another fixed point or hit a singularity. Then, we can cut the integral in two pieces, the first one being between two finite times t_i and $t_a > t_i$; the integral between those two is finite. We choose t_a sufficiently large, in such a way that the solution between t_a and ∞ can be approximated by an expansion around $t \rightarrow \infty$. More precisely, one verifies for the solution ansatz (3.5), (3.9), and (3.13) that at a sufficiently large t one has

$$a(t) = a_0 t (1 + \text{corr}), \quad |\text{corr}| < 1. \quad (3.17)$$

Picking t_a in this way, we get

$$\begin{aligned}
d_e &= a_i \int_{t_i}^{\infty} \frac{dt}{a(t)} = a_i \int_{t_i}^{t_a} \frac{dt}{a(t)} + a_i \int_{t_a}^{\infty} \frac{dt}{a(t)} \\
&= (\text{finite}) + \frac{a_i}{a_0} \int_{t_a}^{\infty} \frac{dt}{t(1 + \text{corr})} \\
&> (\text{finite}) + \frac{a_i}{a_0} \int_{t_a}^{\infty} \frac{1}{2} \frac{dt}{t} = \infty.
\end{aligned} \tag{3.18}$$

We conclude there is no event horizon for the solutions asymptoting to P_1 , i.e. the horizon size is infinite. This is true despite the fact that some solutions are accelerating at any finite time. Thus, we conclude that eternal accelerated expansion does not necessarily lead to horizons. In Sec. IV below, we will discuss that eternally accelerating cosmologies do arise in quantum gravity. The difficulty in finding de Sitter solutions could then be attributed to the existence of a finite event horizon instead of eternal accelerated expansion.

2. Stable node: Semi-eternal or eternal acceleration and transient acceleration with parametric control of e -folds

In Sec. III A, we have obtained various solution families asymptoting to P_1 when the latter is a stable node. Each solution family was found to have locally an accelerating and a decelerating branch, depending on which side of the acceleration cone they approached P_1 . This was illustrated in Figs. 2 and 3. Numerically, one can solve the system of equations and find complete solutions evolving in phase space. Those necessarily asymptote in the past to one of the other fixed points appearing in Fig. 1. We display in Fig. 5 these complete solutions. As we will see, they can have transient or (semi)eternal acceleration phases. Most of our discussion is drawn from the results of [18], even though the solutions considered here are more general.

Let us start with a specific solution which asymptotes to P_1 (at $t \rightarrow \infty$) and asymptotes in the past to P_0 (at $t \rightarrow 0$). This phase space trajectory never leaves the acceleration cone, thus exhibiting eternal acceleration. While the solution at P_0 is a regular Milne universe ($a(t) \sim t$), the solution in its vicinity [$a(t)$ corrected at $\mathcal{O}(t^3)$] is nothing but a de Sitter universe. Indeed, it was shown in [18] that, up to and including terms of order $\mathcal{O}(t^4)$, the spacetime metric becomes that of de Sitter space in the vicinity of P_0 ,

$$ds_{dS}^2 = -dt^2 + l^2 \sinh^2\left(\frac{t}{l}\right) (d\chi^2 + \sinh^2(\chi) d\Omega^2), \tag{3.19}$$

where the radius l is related to the scalar curvature \mathcal{R}_4 of de Sitter via $l^2 = 12/\mathcal{R}_4$. Note that this is the de Sitter metric in hyperbolic slicing (with $k = -1$), different from the more familiar one with an exponential scale factor and $k = 0$. Recall this is not an *asymptotic* de Sitter, since $t = 0$, where the spacetime becomes a regular Milne universe, is reached at finite proper time in the past.

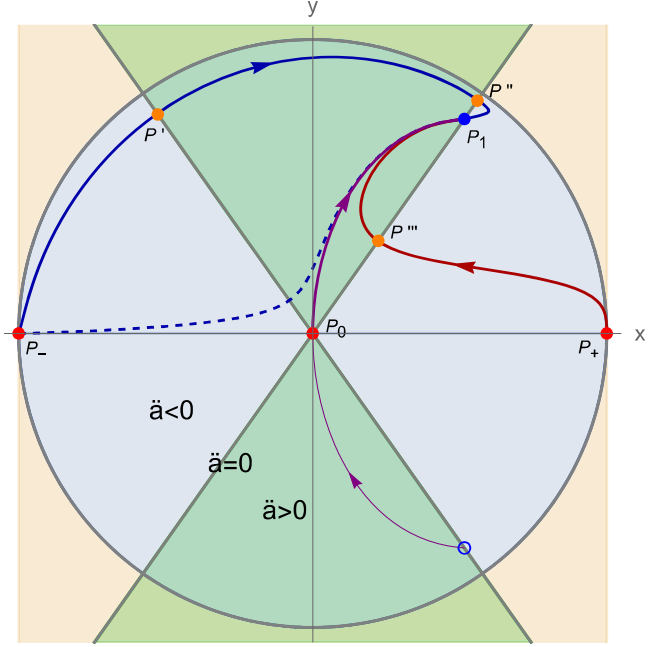


FIG. 5. Complete cosmological solutions asymptoting to P_1 as a stable node, displayed as trajectories in the phase space of Fig. 1, here with $d = 4$ and $\gamma = \sqrt{\frac{8}{3}} - 0.05$. The purple solution exhibits eternal acceleration, the red ones exhibit semi-eternal acceleration, and the blue ones exhibit transient acceleration. The plain versus dashed blue curves correspond to solutions with different numbers of e -folds, on which we have parametric control. The thin purple curve corresponds to the geodesic completion of the thicker one. We refer to the main text for more details.

One can further show [18] that the flow parameter N , corresponding to the number of e -folds, is related to time via

$$\frac{t}{l} \sim e^N [1 + \mathcal{O}(e^{2N})], \tag{3.20}$$

so that $t = 0$ corresponds to $N \rightarrow -\infty$. This allows one to conclude that the number of e -folds accumulated between $t = 0$ and any finite $t > 0$ is infinite. Likewise, we can conclude that asymptoting to P_1 leads to an infinite number of e -folds since $t \sim e^N$ and thus for $t \rightarrow \infty$ we accumulate an infinite number of e -folds.

Finally, let us point out that the solution between P_1 and P_0 is actually not geodesically complete. It can be completed beyond the point P_0 in the past to $t < 0$, by gluing together its mirror trajectory in the lower half of the phase space disk. The scale factor vanishes as $t \rightarrow 0$, but it can be shown that the total 4D spacetime remains smooth in this limit. The phase trajectory actually goes through P_0 , instead of asymptoting to it. We refer to [18] for more details.

Having understood that the solution between P_0 and P_1 allows for eternal acceleration and an infinite number of e -folds, it is easier to understand the features of the other solutions. As displayed in Fig. 5, there are two possible types of solutions: those asymptoting to P_1 accelerating (and asymptoting in the past to P_+) or those asymptoting to P_1 decelerating (and asymptoting in the past to P_-). One such solution of the former type starts accelerating at the point P''' , when entering the acceleration cone, and from there is eternally accelerating since P_1 is only an asymptotic point; we then qualify such solutions as having semi-eternal acceleration and leading to an infinite number of e -folds. The other solutions accelerate between two points only, P' and P'' , and thus have a transient acceleration. Interestingly though, the point P' can be brought as close as desired to P_0 , with the result of increasing the number of e -folds. This was shown numerically in [18] and can be understood by the fact that the phase space trajectory becomes close to the one with eternal acceleration, which has an infinite number of e -folds coming from the vicinity of P_0 . For all these solutions, we then have a parametric control of the number of e -folds.

3. Stable spiral: Roller coaster cosmology and transient acceleration with parametric control of e -folds

In Sec. III B, we have obtained solution families asymptoting to P_1 , the latter being a stable spiral fixed point. As above, we now solve numerically the system of equations to find the complete solutions, which can asymptote to the various other fixed points indicated in Fig. 1. We display in Fig. 6 these complete solutions.

Due to the spiraling, each solution asymptoting P_1 successively enters and exits the acceleration cone. This alternation of acceleration and deceleration phases is reminiscent of the so-called roller coaster cosmology [45]. Each cycle contributes a finite number of e -folds, computed in first approximation from (3.13) to be

$$\Delta N = \frac{2\pi}{q} = \frac{\pi}{\sqrt{2}} \frac{\gamma\gamma_s}{\sqrt{\gamma^2 - \gamma_s^2}}. \quad (3.21)$$

The “initial” phases of the solutions, closer to the other asymptotics, however differ. The solution passing through P_0 is special, and we refer to the above discussion about it. This solution accelerates for a finite amount of time between a finite t and $t=0$. As before, it admits an infinite number of e -folds between any finite time and $t=0$. The other solutions rather have a first transient acceleration phase between points P' and P'' . As before, bringing P' closer to P_0 increases at will the number of e -folds accumulated during this first phase. This provides again to these solutions a parametric control on the number of e -folds.

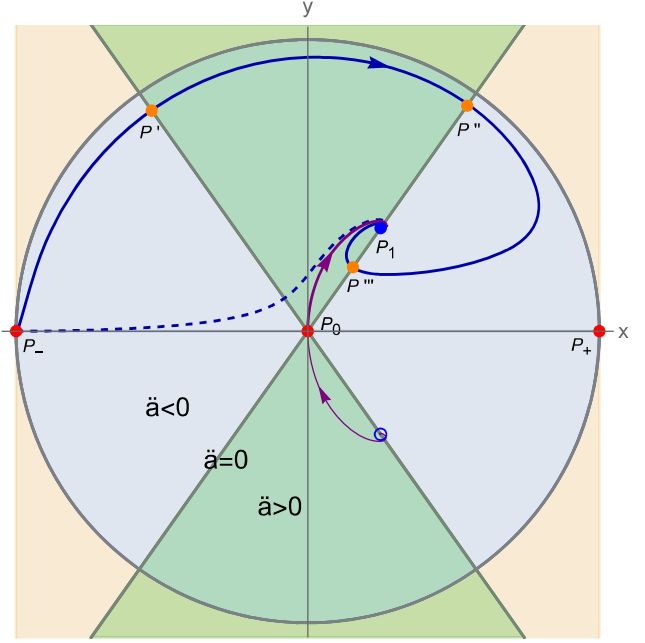


FIG. 6. Complete cosmological solutions asymptoting to P_1 as a stable spiral, displayed as trajectories in the phase space of Fig. 1, here with $d = 4$ and $\gamma = \sqrt{\frac{32}{3}}$. The spiraling leads to an alternation of accelerating and decelerating phases as in a rollercoaster cosmology. In addition, the blue solutions have a “first” transient acceleration phase, whose number of e -folds can be increased arbitrarily by moving P' toward P_0 as for the dashed curve. We refer to the main text for more details.

IV. STRING THEORY REALIZATIONS

In this section, we discuss how the cosmological solutions studied in the previous section can arise in string theory. In Sec. IV A, we first discuss string compactifications to four dimensions that were studied in [18]. We review how they give rise to consistent truncations in four dimensions that realize the $d = 4$ theory (2.1) with $k = -1$ and with an exponential potential (3.1). One example provides the fixed point P_1 as a stable node, and another one provides it as a stable spiral. Both the node and the spiral cases allow for transient acceleration with parametric control of e -folds, as discussed above. The node also allows for eternal or semi-eternal acceleration. We then show in Sec. IV B that massive type IIA supergravity itself can match (2.1) and give rise to $d = 10$ cosmologies that are undergoing accelerated expansion for an infinite amount of time.

A. 4D realizations as string compactifications

We review here some results of [18] that show that a string theory origin can be provided for certain 4D models of the form (2.1) with $k = -1$ and an exponential potential (3.1). Remarkably, as argued in Sec. III, this allows for 4D cosmological solutions, realized in string

theory, that are accelerating for an infinite amount of time and are accumulating an infinite number of e -folds or for transient acceleration with a parametric control of e -folds. This is true even though one has $\gamma > \sqrt{2}$ for the exponential rate of the potential in $d = 4$, as discussed in the beginning of Sec. III.

The framework of [18] is a compactification of 10D type II supergravities with a 10D spacetime of the form (warped) $M_4 \times M_6$. The 6D compact manifold M_6 can be either Calabi-Yau (CY), Einstein, or Einstein-Kähler (EK). Reference [18] first constructed several classes of 10D solutions, where M_4 is a FLRW 4D spacetime with $k = -1$ [see (2.2)], meaning with a negatively curved space giving an open universe. Then, a subset of these 10D solutions can equivalently be realized as 4D solutions of the following 4D theory

$$\mathcal{S}_{4d} = \int d^4x \sqrt{g} \left(R - 24g^{\mu\nu} \partial_\mu A \partial_\nu A - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(A, \phi) \right), \quad (4.1)$$

which contains gravity and two scalar fields A, ϕ . In other words, \mathcal{S}_{4d} is a two-scalar consistent truncation of 10D

type II supergravities for cosmological solutions; all cosmological solutions of \mathcal{S}_{4d} , with a specific V given below in (4.4), lift to 10D solutions of type II supergravities.

To see this, it is enough to show that given a certain compactification ansatz (or a certain class of 10D solutions), the 10D equations of motion and Bianchi identities of type II supergravities boil down to exactly the equations of motion of (4.1), with an FLRW metric; those are given by

$$\begin{aligned} \frac{\ddot{a}}{a} &= \frac{1}{6} V - 8\dot{A}^2 - \frac{1}{6} \dot{\phi}^2 \\ 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{2k}{a^2} &= \frac{1}{3} V + 8\dot{A}^2 + \frac{1}{6} \dot{\phi}^2 \\ \ddot{A} + 3\frac{\dot{a}}{a}\dot{A} &= -\frac{1}{48} \partial_A V \\ \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} &= -\partial_\phi V, \end{aligned} \quad (4.2)$$

where a, A, ϕ are assumed to depend only on time. Note that we use for now the notations of [18] and will eventually give a translation to conventions of the previous sections.⁴

Some cases where such a 4D consistent truncation is possible were found in [18]⁵ to be given by

$$V = \begin{cases} 72b_0^2 e^{-\phi-12A} + \frac{3}{2} c_0^2 e^{\frac{\phi}{2}-14A} \\ \frac{1}{2} c_\varphi^2 e^{-\frac{\phi}{2}-18A} + \frac{1}{2} m^2 e^{\frac{5\phi}{2}-6A} - 6\lambda e^{-8A} \\ \frac{3}{2} c_0^2 e^{\frac{\phi}{2}-14A} + \frac{1}{2} m^2 e^{\frac{5\phi}{2}-6A} - 6\lambda e^{-8A} \\ \frac{1}{2} c_\varphi^2 e^{-\frac{\phi}{2}-18A} + \frac{3}{2} c_f^2 e^{\frac{3\phi}{2}-10A} - 6\lambda e^{-8A} \end{cases}, \quad (4.4)$$

CY with internal 3- and 4-form fluxes

Einstein with external 4-form flux

EK with internal 4-form flux

EK with internal 2-form, external 4-form

where the right column describes the compactification ansatz (M_6 , fluxes). Those examples are all type IIA compactifications, to which we also restrict in the following. The different parameters appearing in the potential are constants coming from the fluxes of the solution, whose 10D origin is summarized in Table II.

TABLE II. List of the constant coefficients entering the potential (4.4) of the 4D consistent truncation and their 10D origin as type IIA supergravity fluxes or curvature. A form is called external (internal), if all its legs are along the external 4D (internal 6D) directions.

m	0-form (Romans mass)
c_f	Internal 2-form
b_0	Internal 3-form
c_φ	External 4-form
c_0	Internal 4-form
λ	Scalar curvature of M_6

⁴For the reader interested in a further comparison of conventions, let us mention that the dynamical system variables x, z of [18] match the x, y of Sec. II B. Also, the dynamical flow parameter ω of [18] should be identified with the number of e -folds N of (2.6). Moreover, the scale factor $a(t)$ was denoted in [18] by S .

⁵The full set of 10D solutions presented in [18] is much richer and allows for more fluxes to be turned on. All the type II 10D supergravity solutions presented therein can be derived using an appropriate 1D consistent truncation with potential $U(A, B, \phi)$ [see [18], Eq. (15)]. This is indeed possible due to the fact that all field profiles only depend on one variable, the time. Such 1D Lagrangians can be used as a starting point for a minisuperspace treatment. The potential $V(A, \phi)$ of the 4D consistent truncations given in (4.4) is related to the potential $U(A, B, \phi)$ of the 1D consistent truncations of [18] via

$$V(A, \phi) = e^{-24A-6B} U(A, B, \phi). \quad (4.3)$$

However, as already mentioned, not all the solutions of [18] admit 4D consistent truncations. For that to be the case, the right-hand side of the equation above must be B independent. This is indeed satisfied for the compactifications listed in (4.4).

Given a 4D consistent truncation from 10D type IIA supergravity with $k = -1$, giving rise to the 4D theory (4.1) with scalar potential (4.4), one still has to get rid of one scalar field to match (2.1). Note that the potential (4.4) is already exponential, so one eventually reaches the scalar potential (3.1). Reaching a single-scalar theory is achieved by a further consistent truncation, i.e. a subtruncation of (4.1), that we will describe in two examples below.

Prior to this, let us present the ansatz for the 10D metric, common to the two 10D solutions and their 4D realizations to be considered. The 10D metric in the 10D Einstein frame reads

$$ds_{10}^2 = e^{2A} (e^{2B} g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n), \quad (4.5)$$

where the scalars A, B are assumed to only depend on time, while y^m are coordinates of the internal 6D space. The unwarped 4D metric is assumed to be of the form⁶

$$g_{\mu\nu} dx^\mu dx^\nu = -d\eta^2 + d\Omega_k^2, \quad (4.6)$$

where η is the conformal time, and the spatial 3D part of the metric is locally isometric to a maximally symmetric 3D space of scalar curvature $6k$, namely

$$d\Omega_k^2 = \gamma_{ij}(\vec{x}) dx^i dx^j, \quad R_{ij}^{(3)} = 2k\gamma_{ij}, \quad (4.7)$$

with $i, j = 1, 2, 3$ and where $R_{ij}^{(3)}$ is the Ricci tensor of the metric γ_{ij} . In the following, we will take the 3D metric to be locally that of hyperbolic space ($k < 0$). We further set $k = -1$ without loss of generality, by redefining $B \rightarrow B + \text{constant}$. Note also that, eventually, the degree of freedom B will be traded for the scale factor a .

Regarding the 6D manifold M_6 , we take it for now to be Einstein, namely

$$R_{mn} = \lambda g_{mn}, \quad (4.8)$$

where R_{mn} is the Ricci tensor associated to g_{mn} and 6λ is the scalar curvature of M_6 .

The rest of the ansatz for the 10D type IIA supergravity solutions has to do with the dilaton and the fluxes. We will specify those for each of the two examples below and show how they admit an interpretation as solutions of the 4D consistent truncation (4.1) with (4.4) and further as a subtruncation to a single-scalar model (2.1) with (3.1).

1. Stable node example

This first example has been studied in [25] and more recently in [18], Sec. 6.1. Considering the above 10D metric

⁶On the other hand, the 4D Einstein-frame metric is given by $e^{8A+2B}(-d\eta^2 + d\Omega_k^2)$.

ansatz, we further specify to a 6D Einstein manifold with negative curvature, meaning $\lambda < 0$. We may set $\lambda = -1$ without loss of generality by redefining $A \rightarrow A + \text{constant}$. Furthermore, we take all the 10D fluxes to vanish. Finally, we also set the dilaton to a constant; we will see that this corresponds to a consistent subtruncation of the 4D theory (4.1).

This ansatz trivially satisfies the 10D equations of motion and Bianchi identities for all the fluxes, as well as the dilaton equation of motion. The remaining equations of motion are as follows. The internal components of the Einstein equations reduce to

$$d_\tau^2 A = -\lambda e^{16A+6B}, \quad (4.9)$$

where we have introduced a new time variable τ defined by $d\eta = e^{8A+2B} d\tau$. The external Einstein equations reduce to the following two equations,

$$\begin{aligned} \lambda e^{16A+6B} - 2k e^{16A+4B} &= d_\tau^2 B \\ -12\lambda e^{16A+6B} - 12k e^{16A+4B} &= 144(d_\tau A)^2 + 12(d_\tau B)^2 \\ &\quad + 96d_\tau A d_\tau B, \end{aligned} \quad (4.10)$$

while the mixed Einstein equations (along time and internal directions) are automatically satisfied.

The three equations (4.9) and (4.10) can be “integrated” into the 4D action (4.1); in other words, they correspond to its equations of motion (4.2) with a subtruncation. To see this, let us first introduce the cosmological (standard) time coordinate t and the standard scale factor a via

$$\frac{dt}{d\tau} = a^3, \quad a = e^{4A+B}. \quad (4.11)$$

Taking suitable linear combinations thereof, the three equations (4.9) and (4.10) can then be written equivalently as the 4D ones (4.2), with $\phi = \text{constant}$, and $V = 6|\lambda|e^{-8A}$ as in (4.4). Setting the dilaton to a constant is a consistent truncation of the 4D theory⁷; indeed, since the potential is ϕ independent, the equation $\ddot{\phi} + \frac{3}{a}\dot{a}\dot{\phi} = -\partial_\phi V$ is trivially satisfied. This leaves only three 4D equations to match the three 10D equations. It follows that a solution of equations (4.9) and (4.10) is also automatically a solution of the subtruncated 4D action (4.1).

It is then straightforward to see that this realizes the single-scalar 4D theory (2.1) with potential (3.1). The 4D actions (4.1) and (2.1) then match with an overall rescaling as follows:

⁷Note that this subtruncation is simply the restriction to the invariant plane $y = 0$ of the phase space of the corresponding dynamical system [see [18], Eq. (91)]. This is a special instance of a more general result: the existence of the invariant plane [[18] (68)], which therefore guarantees that all models considered in that reference admit a consistent one-scalar subtruncation.

$$2\sqrt{6}A \leftrightarrow \varphi, \quad 3e^{-8A} \leftrightarrow V(\varphi) = 3e^{-\frac{4}{\sqrt{6}}\varphi}, \quad \gamma = -\frac{V'(\varphi)}{V} = \frac{4}{\sqrt{6}}. \quad (4.12)$$

As mentioned in Sec. III, this value of γ is precisely the boundary value γ_s for $d = 4$. The fixed point P_1 is then a stable node [see e.g. (3.3)], and one gets in its vicinity the accelerating solutions previously discussed.

2. Stable spiral example

In this second example, M_6 is a CY space such that the internal metric is Ricci flat: $R_{mn} = 0$. In (4.8), we then take $\lambda = 0$. We keep for now a nontrivial dilaton ϕ . All fluxes are taken to vanish, except for the internal Ramond-Ramond (RR) 2-form which is given by

$$F = c_f J, \quad (4.13)$$

where c_f is a constant and J is the Kähler form of M_6 .

This ansatz can be seen to automatically satisfy all 10D flux equations of motion and Bianchi identities. Moreover, the mixed Einstein equations are automatically satisfied. The four remaining 10D equations, namely the dilaton and the internal and external Einstein equations [see [18], Eqs. (425) and (426)], can be put in the form of the four 4D equations (4.2). This is achieved with the potential $V = \frac{3}{2}c_f^2 e^{3\phi/2-10A}$, a particular case of (4.4) with EK and $\lambda = 0$ (CY), and no external 4-form. This way, the 10D solutions can also be thought of as solutions of the 4D action (4.1).

We are left with a subtruncation to reach a single-scalar model. With the previous potential, the right-hand sides of the last two 4D equations (4.2) become

$$-\frac{1}{48}\partial_A V = \frac{10}{48}V, \quad -\partial_\phi V = -\frac{3}{2}V = -\frac{36}{5} \times \frac{10}{48}V. \quad (4.14)$$

It is then straightforward to see that those two equations become identical when considering⁸

$$\phi = -\frac{36}{5}A. \quad (4.15)$$

As can be checked directly from the equations of motion (4.2), this choice gives a consistent subtruncation of the action (4.1) to a single scalar A ,

⁸Note that this subtruncation is simply the restriction to the invariant plane [see [18], Eq. (427)] of the phase space of the corresponding dynamical system.

$$S_{4d} = \int d^4x \sqrt{g} \left(R - \frac{1248}{25} g^{\mu\nu} \partial_\mu A \partial_\nu A - \frac{3}{2} c_f^2 e^{-\frac{104}{5}A} \right). \quad (4.16)$$

We then reach the single field 4D theory (2.1) with (3.1). This is done with an overall rescaling and the following matching from (4.16):

$$\frac{4}{5}\sqrt{78}A \leftrightarrow \varphi, \quad \frac{3}{4}c_f^2 e^{-\frac{104}{5}A} \leftrightarrow V(\varphi) = \frac{3}{4}c_f^2 e^{-\sqrt{\frac{26}{3}}\varphi}, \quad \gamma = -\frac{V'(\varphi)}{V} = \sqrt{\frac{26}{3}}. \quad (4.17)$$

We conclude that we get $\gamma > \gamma_s = \sqrt{\frac{8}{3}}$. As mentioned around (3.3), the fixed point P_1 is then a stable spiral. This leads to 4D accelerating solutions of the kind discussed in Sec. III.

B. 10D massive type IIA supergravity

We now show that one of the simplest examples of accelerated cosmology of the type previously discussed is realized in massive type IIA supergravity (mIIA) in ten dimensions [46]. Type IIA supergravity is a low energy limit of IIA string theory, and the mass parameter of mIIA appears naturally when considering T-duality between type IIA and type IIB string theory [47]. As is well known, mIIA does not admit any straightforward covariant 11-dimensional uplift; see e.g. [48] for a recent discussion. It is one of the two possible massive deformations of 10D IIA supergravity [49]. The action of massive type IIA supergravity is given in 10D Einstein frame by

$$\begin{aligned} S_{\text{mIIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|g_{10}|} & \left(\mathcal{R}_{10} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right. \\ & \left. - \frac{1}{2} (e^{-\phi} |H_3|^2 + e^{\frac{5}{2}\phi} F_0^2 + e^{\frac{3}{2}\phi} |F_2|^2 + e^{\frac{1}{2}\phi} |F_4|^2) \right) \\ & - \frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4. \end{aligned} \quad (4.18)$$

We are interested in 10D solutions whose 10D metric is the FLRW metric with $k = -1$, as given in (2.2). In order to preserve the symmetry group of the maximally symmetric 9D spatial part, we set all fields except the metric, the dilaton ϕ , and the mass parameter F_0 to zero. It can readily be verified that this is indeed a consistent truncation. The action then reduces to

$$S_{\text{mIIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|g_{10}|} \left(\mathcal{R}_{10} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} e^{\frac{5}{2}\phi} F_0^2 \right), \quad (4.19)$$

which is exactly of the type (2.1) and (3.1) studied above. We can read off

$$\varphi = -\frac{1}{\sqrt{2}}\phi, \quad V(\phi) = \frac{1}{4}F_0^2 e^{-\frac{5}{\sqrt{2}}\phi}, \quad (4.20)$$

where we set as before $M_p = \kappa_{10}^{-\frac{1}{4}} = 1$. Note that the Bianchi identity $dF_0 = 0$ gives us a constant F_0 . So, we get an exponential potential with $\gamma = \frac{5}{\sqrt{2}}$, which is larger than the lower bound $2/\sqrt{d-2}$ for $d = 10$.

This setup then provides a stringy realization of the above analysis with $k = -1$. Interestingly, we recall from Sec. III that $d = 10$ is special, because the fixed point P_1 is then a stable node for any $\gamma > 2/\sqrt{d-2}$. We deduce that this setup gives rise to cosmologies that undergo (semi) eternal accelerated expansion. This is to be contrasted with the absence of de Sitter solutions in $d = 10$ [see e.g. [11], Eq. (31)].

C. Accelerated expansion in the classical regime of string theory

We have just considered examples based on 10D type II supergravities and compactifications thereof, that admit cosmological solutions with accelerated expansion. Knowing whether those are realized in string theory requires one to verify that they are in a classical string regime, i.e. that the supergravity approximations are justified.

Before doing so, let us recall the existence of other cosmological solutions with accelerated expansion in type II supergravity compactifications; those are de Sitter solutions, a sample of which has been obtained recently in [50]. Having analyzed few examples of such solutions as well as considered asymptotic behaviors of related potentials [51–55], it seems however that these de Sitter solutions do not fall in a classical string regime. One reason is the numerous constraints they have to obey [55], one being the requirement of having orientifolds (often together with D -branes) leading to the tadpole constraint. Having these extended objects is problematic for another reason: in the solution, one would like to take into account their backreaction. But this is a complicated problem, especially in the compactification settings considered, so the de Sitter solutions mentioned only include such objects as smeared. This refers to the fact that their contribution to the equations is taken as integrated. Although recent progress has been made on localizing these solutions [1,56,57], this problem is not solved.

As we will see, the 10D supergravity solutions considered in this paper and in [18] face none of these issues; this is a major improvement toward ensuring control and a string theory origin. First, *these solutions do not have orientifolds nor D -branes*, so the above difficulties (and related criticisms) are avoided. Second, *the classical string regime appears to be easily reached* as we now detail.

The 4D solutions discussed in Sec. IV A have an internal volume that grows as e^{6A} when $A \rightarrow \infty$. In

addition, volumes of internal p -cycles grow as e^{pA} . So, 4D α' -corrections can legitimately be neglected. Turning to the string coupling constant g_s governing loop corrections, we verify that it can be made small. Indeed, in the stable node case, the dilaton is fixed to an arbitrary constant that can be picked to give a small g_s . In the stable spiral case, the relation (4.15) ensures that in the limit where the internal volume becomes large, g_s becomes small. In other words, the time evolution goes into the direction of the classical regime.

The same holds true in the 10D realization in Sec. IV B. The asymptotics considered are $\phi \rightarrow -\infty$, corresponding to the classical regime. Regarding 10D α' -corrections, one may argue that in a 10D expanding open universe, such higher derivative corrections can be neglected at a sufficiently late time. Note also that in mIIA as well as in the 4D stable spiral example, flux needs to be quantized in string theory, a point for which we do not see any difficulty.

For completeness, one could complain about the lack of control over the truncated modes in the 4D consistent truncations, which needs further investigation and is beyond the scope of this paper. One may also worry about the (non-) low energy effectiveness of the truncation (see e.g. [58], Sec. V). In the case of the CY compactification at least, this criticism may however be lifted: as the modes left after the truncation are part of a universal consistent truncation, which is known to be a subsector of the CY effective action [59,60], we expect them to be light. However, there could in general be additional light fields that are not taken into account by the truncation. Another legitimate criticism could be that we do not address moduli stabilization. At least in the massive type IIA example in $d = 10$, there are no moduli to stabilize. In addition, specifying further the above $d = 4$ examples, many flat field directions can be avoided. Indeed, for the stable node example, one can pick the Einstein manifold to be the 6D Poincaré plane divided by a discrete subgroup of $SO(1,6)$. Due to Moscow's rigidity theorem (in other words thanks to the maximal symmetry), the resulting compact 6D manifold has only one modulus, its overall volume, which is the only one rolling. Similarly, for the stable spiral example, one can choose a rigid CY manifold. The RR 2-form flux proportional to the Kähler form generates a potential for all Kähler moduli except for the overall volume, the latter contributing again to the running field. In these examples, there is thus little concern about moduli stabilization, or at least flat directions.

Finally, let us mention that, as can easily be seen in both these examples, the Hubble scale goes to zero at future infinity faster than the overall inverse size of the internal manifold, as measured in the 10D (and 4D) Einstein frame. So, these solutions can be viewed as (parametrically scale separated) 4D cosmologies.

Overall, the “string theory realizations” presented previously appear to offer a good control over possible

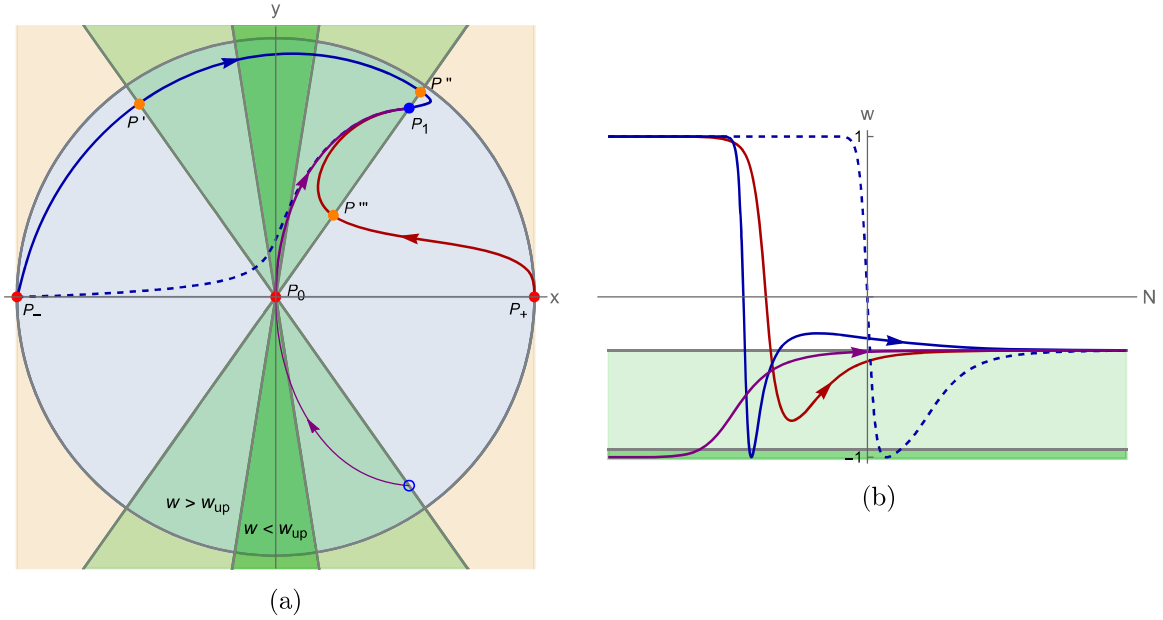


FIG. 7. Equation of state parameter w for the cosmological solutions of Fig. 5. Figure 7(a) reproduces Fig. 5 together with the observationally favored region in which $-1 \leq w \leq w_{up}$, with an upper bound $w_{up} = -0.95$; this region is the narrower, darker green double cone. Figure 7(b) gives the evolution of w along the cosmological solutions depicted in Fig. 7(a): the acceleration light green region corresponds to $w < -\frac{1}{3}$, while the dark green region corresponds to $-1 \leq w \leq -0.95$. Many asymptotic features of these solutions, discussed in the main text, can be identified in Fig. 7(b). That figure is plotted for $-10 \leq N \leq 10$.

corrections, avoiding typical problems of other constructions, thus ensuring their string theory origin.

V. OUTLOOK

In this paper, we have presented cosmological solutions describing a universe undergoing accelerated expansion, with various realizations in string theory models; our results are summarized in the Introduction. Such solutions are obtained provided the spatial curvature is negative, namely $k = -1$. One may wonder how realistic these solutions are, starting with the choice $k = -1$; we indeed recall that the minimal Λ CDM cosmological model does not consider the curvature density parameter $\Omega_k = -k/(aH)^2$. This is in agreement with the fact that this quantity is observationally constrained to be very close to 0 (see e.g. [61,62]). Given its definition, Ω_k can however get very much diluted in a universe undergoing accelerated expansion; this is the standard argument in favor of inflation to solve the flatness problem. For this reason, it is very difficult to exclude the possibility of $k \neq 0$ and of having $k = -1$ as here. Nevertheless, it would be surprising to access the new physics described in this work thanks to $k = -1$, while having a negligible Ω_k compatible with $k = 0$. Actually, we get an expression in terms of the phase space variables: $\Omega_k = 1 - (x^2 + y^2)$ (i.e. it is given by the distance to the circle with radius 1 in Fig. 1). In most solutions considered here, Ω_k is then not negligible. Regarding today's dark

energy, this discussion remains meaningless without including nonrelativistic matter captured by Ω_m . We therefore hope to extend our analysis to include matter in future investigations.

A further question is how well the new cosmological solutions reproduce the acceleration observed, focusing first on the late universe. This is measured by the equation of state parameter w . If we consider that Ω_k is indeed negligible, we can use recent constraints on w . Let us take for instance from [61,63] the observational upper bound $w \leq w_{up} = -0.95$ (see also [64] for a recent account). We recall that $w = -1$ corresponds to having a cosmological constant, and theorists typically consider $w \geq -1$ to avoid scenarios with phantom energy for which no good theoretical models exist. Having $-1 < w \leq w_{up}$ would then correspond to having a valid rolling field scenario, i.e. a viable quintessence model. As discussed in the Introduction, the ratio $|\nabla V|/V$ or γ can sometimes be related to w , but this is not enough in the complete cosmological solutions. We therefore simply obtain the value of w numerically, along these solutions, and show the result in Fig. 7.

We first see in Fig. 7(a) that the region for which $-1 \leq w \leq -0.95$ is only crossed for a finite amount of time by the various solutions,⁹ and sometimes not at all,

⁹One verifies that this region corresponds to $|y| \geq |x| \sqrt{(1 - w_{up})/(1 + w_{up})}$, a double cone, provided $-1 < w_{up} < 1$.

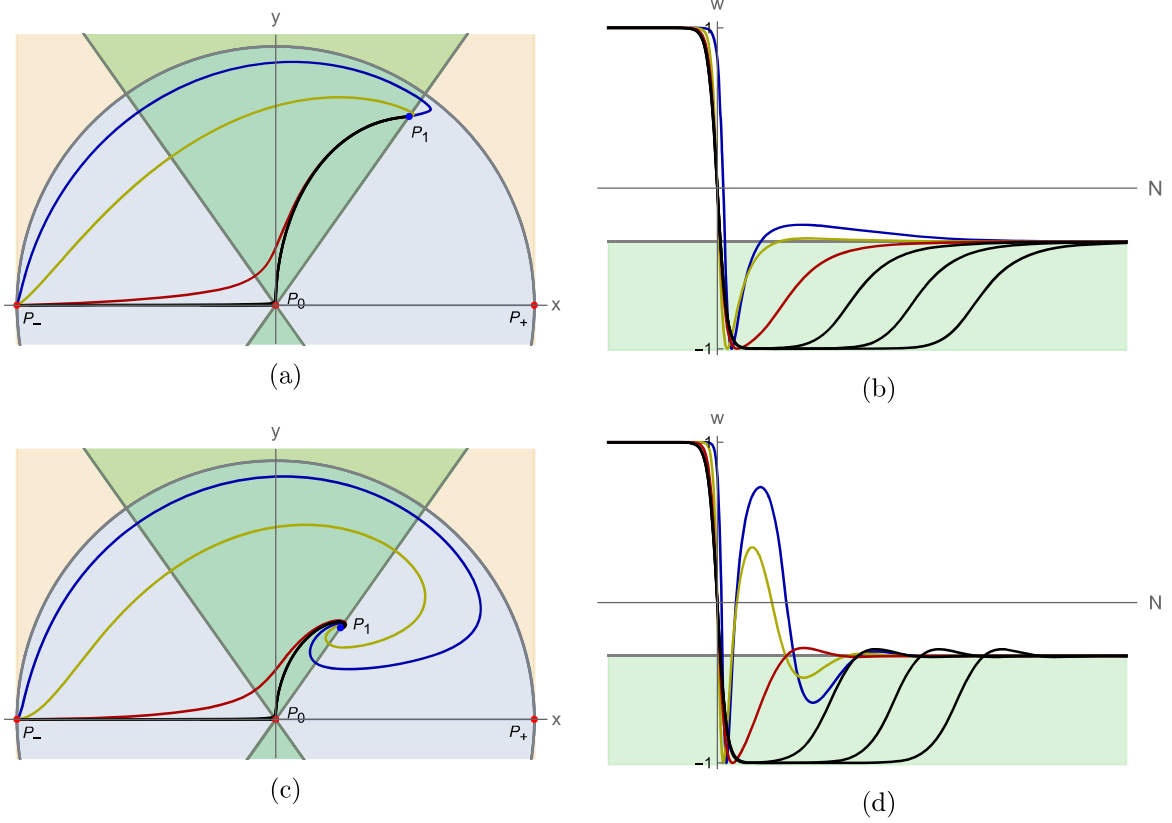


FIG. 8. A realistic single-field slow-roll inflation model should admit an equation of state parameter $w \approx -1$ during a certain number of e -folds. For various solutions asymptoting to P_1 , we show how this number of e -folds gets tuned when approaching the eternally accelerating solution, that includes a quasi-de Sitter phase. Solutions in phase space are depicted for P_1 being a stable node in Fig. 8(a), respectively a stable spiral in Fig. 8(c), and the corresponding w are given in Fig. 8(b), respectively Fig. 8(d). The black solutions cannot be visually distinguished in the phase space figures. Figures 8(b) and 8(d) are plotted for $-4 \leq N \leq 30$.

even though the solutions certainly go through the wider acceleration region ($w < -\frac{1}{3}$). This is made clearer in Fig. 7(b). It would be interesting to investigate in more detail how long this duration is, whether this can be enhanced for the various solutions, and how this compares with observations.

Last but not least, the accelerating solutions could also be considered as models of inflation in the early Universe. It would be interesting to run similar comparisons with observational constraints. Let us recall that solutions with transient acceleration allow for a parametric control on the number of e -folds, an appealing feature when attempting to construct realistic inflation models; we illustrate this point in Fig. 8. Of course, not only should the number of e -folds be in agreement with constraints on inflation, but also the power spectrum predicted by the model should be

consistent with observational data. Work on these questions is in progress [65].

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