Technical Brief

Influence of Porous Inserts and Compact Resonators on Onset of Taconis Oscillations

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ABSTRACT

Taconis oscillations represent excitation of acoustic modes due to large thermal gradients inside narrow tubes penetrating cryogenic vessels from warm ambient environment. These oscillations are usually harmful, as they may drastically increase heat leakage into cryogenic vessels and result in strong vibrations of measuring instruments. Placing a porous material inside a tube with a goal to increase acoustic damping or attaching a small resonator to the main tube are some of possible ways to suppress or mitigate Taconis effects. However, when the porous inserts are positioned in locations with large temperature gradients or the resonator parameters are selected incorrectly, these components may augment thermal-to-acoustic energy conversion and enhance Taconis oscillations. A low-amplitude thermoacoustic model has been extended and applied in this study to determine effects of the insert location and pore radius, as well as the resonator dimensions, on the onset of Taconis phenomena in a hydrogen-filled tube of relevance to lines used in cryogenic hydrogen storage tanks. The presented findings can assist cryogenic specialists interested in suppressing or exciting Taconis oscillations.

INTRODUCTION

Narrow tubes inserted in dewars and used as cryogen transfer lines or pathways for sensing instruments (Fig. 1a) can be prone to high-amplitude acoustic oscillations. This phenomenon, first reported by Taconis et al. [1], represents excitation of the natural acoustic modes of tubes subjected to high thermal gradients that exist due to large temperature differences between warm surrounding environment and cold space inside dewars or cryogenic tanks. Taconis oscillations belong to thermoacoustic instabilities occurring when unsteady heat addition to oscillating fluid takes place in phase with acoustic pressure fluctuations, leading to thermal-to-acoustic energy conversion in accordance with Rayleigh criterion [2]. It can be noted that thermoacoustic instabilities are not limited to cryogenic systems and often appear in rocket motors and other acoustic-resonator-type systems with heat sources [3,4]. Rott [5,6] was the first to develop a theory for heat-sound interactions in tubes to predict the onset conditions for Taconis phenomena. Swift [7] has generalized this low-amplitude theory, including other related phenomena, so it can be applied for modeling thermoacoustic prime movers and refrigerators. Taconis oscillations are usually detrimental for cryogenic systems, as they can drastically increase heat leakage into cryogenic vessels and cause large vibrations undesirable for instruments [8]. Because of these effects, several experimental studies were conducted in the past to characterize Taconis oscillations [9,10].

Currently, significant efforts are being undertaken to develop liquid hydrogen systems, as hydrogen is a renewable and clean fuel that in the liquid form possesses energy density acceptable for fuels. However, very low temperatures (and thus cryogenic vessels) are required to store liquid hydrogen [11]. The transfer and sensor lines penetrating into such systems can be also subjected to Taconis oscillations negatively affecting the tank storage performance (Fig. 1b), which motivates the present study.

Several means for suppressing Taconis oscillations have been reported in the literature [12], but theoretical analyses to determine their effectiveness and results for Taconis phenomena in hydrogen systems are rather limited. Gu and Timmerhaus [13] used a simplified impedance analysis to evaluate reflection coefficients from acoustic components in a tube to judge their effects on Taconis phenomena. Shenton et al. [14] described recent experiments on a benchtop hydrogen system with valved, variable-diameter tube segments and loops aimed at producing and eliminating Taconis oscillations. The objective of the current paper is to present a thermoacoustic model that directly incorporates two acoustic elements that can be used to suppress Taconis phenomena, namely porous inserts placed inside tubes and Helmholtz resonators that can be attached to the main tube (Fig. 2). Parametric results are demonstrated below for the effects of main dimensions of these components (applied separately) on the onset of Taconis oscillations.

MATHEMATICAL MODEL

A geometrical framework for analyzing Taconis oscillations in a tube is schematically shown in Fig. 2, including the original clean tube (Fig. 2a) and two modifications (Fig. 2b,c). The tube end on the warm side is closed, while the other end on the cold side is open. The temperature profile is specified in this study by a piecewise-linear function with constant-temperature regions in the cold and warm portions of the tube and a linear temperature variation in between. In real systems, temperature distributions will deviate from linear functions and depend on thermal properties of surroundings and the system history. For example, in classical open-closed tubes inserted in dewars, temperature profiles are known to be smoother with a steeper temperature gradient in the middle of the transition zone and more gradual at the ends (but the average temperature gradient is the same). In industrial cryogenic tanks, transfer and sensor lines pass through insulation segments of different thermal and geometrical properties, and temperature profiles may be rather complicated. In examples shown below, the simplest (linear) approximation for the temperature profile is employed for simplicity.

Two additional components are considered separately in this study, including a porous insert inside the tube and a Helmholtz-type resonator attached to the tube. These modifications are shown in Fig. 2b and 2c. In the first case, the location of the porous insert center x_p, its length L_p, porosity φ (gas-to-solid volume ratio), and characteristic pore radius R_p are prescribed. In the second case, the length L_n and radius R_n of the resonator neck, as well as the cavity volume V_c, are specified.

In the quasi-one-dimensional analysis of low-frequency sound waves inside narrow tubes, the section-average acoustic pressure $p'(x,t)$ and volumetric velocity $U'(x,t)$ can be conveniently presented in the following form,

$$p'(x,t) = Re\left[p_1(x)e^{i\omega t}\right] \tag{1}$$

$$U'(x,t) = Re\left[U_1(x)e^{i\omega t}\right] \tag{2}$$

where p_1 and U_1 are the complex acoustic amplitudes of the pressure and volumetric velocity, x is the coordinate directed along the tube (Fig. 2), t is the time, i is the imaginary unity, and ω is the angular frequency, which is not known in advance for self-excited systems needs to be found as a part of the solution.

The governing thermoacoustic equations describing spatial evolutions of low-amplitude complex acoustic pressure and volumetric velocity amplitudes in conduits with thermal gradients can be written as follows [7],

$$\frac{dp_1}{dx} = -\frac{i\rho_m\omega/A}{1-f_v}U_1 \tag{3}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A}{\gamma p_m}[1 + (\gamma - 1)f_k]p_1 + \frac{f_k - f_v}{(1 - f_v)(1 - \sigma)}\frac{dT_m}{dx}\frac{U_1}{T_m} \quad (4)$$

where A is the gas-occupied cross-sectional area, ρ_m, p_m and T_m are the mean density, pressure, and temperature, respectively, γ is the ratio of specific heats, and $\sigma = \mu c_p/k$ is the Prandtl number, with μ, c_p and k standing for viscosity, specific heat, and thermal conductivity of the fluid.

The exact forms of thermoacoustic functions f_v and f_k, appearing in Eqs. (3,4), depend on a specific channel geometry. In the present study, circular-type pores are considered as an example, for which the following expressions have been derived [7],

$$f_{k,v} = 2\frac{J_1(y_{k,v})}{y_{k,v}J_0(y_{k,v})} \quad (5)$$

$$y_{k,v} = \frac{(i-1)R}{\delta_{k,v}} \quad (6)$$

where J_1 and J_0 are the first- and zero-order Bessel's functions, and δ_k and δ_v the thermal and viscous penetration depths, which are defined as follows,

$$\delta_k = \sqrt{\frac{2k}{\rho\omega c_p}} \tag{7}$$

$$\delta_v = \sqrt{\frac{2\mu}{\rho\omega}} \tag{8}$$

These depths roughly characterize fluid regions near solid surfaces where time-periodic viscous and thermal effects due to presence of the wall are felt by acoustically oscillating fluid inside the conduit. One can also include effects of thermal waves inside tube walls [15], but their influence is small for common tube materials and thicknesses. In case of clean tube sections (Fig. 2a), R in Eq. (6) represents the tube radius.

The boundary conditions for the acoustic variables include the zero-velocity requirement at the closed end ($x = 0$) and the open-end ($x = L$) relation between the acoustic pressure and velocity assuming large open space inside a vessel [16],

$$U_1(0) = 0 \tag{9}$$

$$p_1(L) = \left(0.61 i\rho\omega R + 0.25\frac{\rho\omega^2 R^2}{c}\right)\frac{U_1(L)}{A} \tag{10}$$

where c is the local speed of sound.

In case of a system with an added porous insert (Fig. 2b), similar boundary conditions and governing equations are applied, but in the expressions for thermoacoustic functions (Eqs. 5,6) inside the insert section, radius R stands for the characteristic pore radius, whereas the tube cross-sectional area in Eqs. (3,4) is multiplied by the insert's material porosity.

For a system with an attached resonator (Fig. 2c), the additional joint conditions for the acoustic pressure and volumetric velocity amplitudes are applied at the junction (Fig. 3), assuming continuity of pressure and conservation of volumetric flow rate,

$$p_{1,1} = p_{1,2} = p_{1,3} \tag{11}$$

$$U_{1,1} = U_{1,2} + U_{1,3} \tag{12}$$

where the second indexes correspond to different system segments that meet at the junction (and the first index 1 is the common designation for the acoustic amplitude). For a compact resonator, one can use Eq. (3) to evaluate pressure drop in a narrow neck, whereas the pressure and volumetric velocity at the entrance of the resonator cavity can be related as follows [7],

$$\frac{p_{1,c}}{U_{1,c}} = i\omega C + \frac{1}{R_k} \tag{13}$$

where the compliance and acoustic resistance are calculated using the cavity geometric parameters and fluid properties,

$$C = \frac{V_c}{\gamma p_m} \tag{14}$$

$$R_k = \frac{2\gamma p_m}{\omega(\gamma - 1)S_c\delta_k} \tag{15}$$

where V_c and S_c are the cavity volume and surface area, respectively, and δ_k is the thermal penetration depth evaluated using Eq. (7).

In the present calculations, the warm temperature and the thermal transition zone have been fixed, so that the cold temperature is used as a metric defining the onset of Taconis oscillations. To find this critical value for the cold temperature, two computational loops on cold temperatures and frequencies are implemented. For each temperature profile, a frequency is varied, and for each frequency, the governing Eqs. (3,4) are integrated along the tube, starting with an arbitrary finite pressure amplitude at the warm end, while keeping the warm-end acoustic velocity zero according to Eq. (9). The result of this calculation is an acoustic pressure at the open (cold) end. For each temperature profile, there will be a frequency at which the real part of the open-end pressure amplitude satisfies the real part of the boundary condition given by Eq. (10). However, the imaginary part of the pressure amplitude may or may not satisfy this equation at the selected cold temperature. If it does not, a different cold temperature is tried, and the solution procedure is repeated. Eventually, a temperature profile is found when both real and imaginary components of the acoustic pressure at the open end satisfy Eq. (10). The corresponding cold temperature and frequency are then identified as the neutral (or onset) state of the system.

It can be noted that theoretically, different acoustic modes of the tube with different frequencies can be excited. However, in most practical situations only the lowest frequency of the fundamental acoustic mode is of interest, as higher-frequency oscillations would usually require very cold temperatures for excitation, which for hydrogen would be typically below the saturation values.

RESULTS AND DISCUSSION

For validation purposes, the thermoacoustic model described above has been applied to model Taconis excitations in the experimental system with clean tubes (Fig. 2a) inserted into a helium-filled dewar [10]. The tube length is 1.5 m, whereas some other important parameters, including the tube radius, the ratio of hot-to-cold tube segments, and the warm temperature are shown in Table 1. The cold temperature intervals within which the onset of Taconis oscillations was experimentally observed, as well as the associated frequencies of oscillations, are given in Table 2.

For modeling this helium system and later a hydrogen setup in this study, real fluid properties have been obtained from CoolProp software [17] and arranged in form of look-up tables for numerical modeling. The critical cold temperature and the corresponding onset frequency values determined using the model presented above are also summarized in Table 2. A reasonable agreement between test data and computational predictions indicates the suitability of the employed modeling approach for the onset of Taconis phenomena. Possible causes for discrepancies include a finite volume inside the dewar, real-gas effects, influence of disturbances and system's thermal evolution, and simplifications in the employed model. It should be also noted that experimental frequencies correspond to excited states after the onset and may be slightly different from the actual onset frequencies, whereas the modeling frequencies are computed for the onset conditions. The observed differences between test and model results can roughly indicate an expected accuracy of the present model predictions.

To illustrate effects of a porous insert inside a hydrogen-filled tube with open and closed ends (Fig. 2b), the following system parameters are chosen: mean pressure of 3 bar, tube length of 1.5 m, and tube radius of 3 mm. The warm (300 K) and cold segments occupy 0.6-m-long sections from the closed and open ends of the tube, i.e., x_1 = 0.6 m and x_2 = 0.9 m (Fig. 1a). The linear variation of temperature is assumed in the transition segment between the warm and cold segments. Similar to the validation example, the critical cold temperature, corresponding to the onset of Taconis oscillations, serves as the main excitation threshold determined in this study, while the oscillation frequency is another quantity of interest. A porous insert of length 3 cm, porosity 0.7, and with circular-type pores is analyzed. Its location x_p (Fig. 2) is the main variable in the present investigation, whereas two characteristic pore radii (0.3 and 1.0 mm) are considered.

The computed results for the hydrogen-filled tube with porous inserts are presented in Fig. 4. The critical cold temperature and frequency in a tube without inserts are 29.6 K and 75.7 Hz (shown by solid horizontal lines in Fig. 4). The presence of the porous insert outside the zone with the temperature gradient demonstrates a damping effect (Fig. 4a). For the pore radius of 1 mm, reductions of critical temperature are relatively modest, about 1.5 K. For the insert with tighter pores, the acoustic damping is much more effective, leading to the complete suppression of Taconis oscillations for a large range of locations of the insert in the constant-temperatures zones (as the critical cold temperature that would need to excite sound drops below the saturated hydrogen temperature). Frequencies evaluated for the cases with delayed Taconis onset are found to be slightly lower than for the clean tube (Fig. 3b).

The effect of a porous insert is drastically different when it is placed in the zone with a strong thermal gradient ($0.6\ m < x_p < 0.9\ m$). Porous materials having an appropriate pore radius are known to enhance the thermal-to-acoustic energy conversion [7], and are utilized intentionally in thermoacoustic engines, where the goal is to produce acoustic power from heat. Consequently, in the considered here setup with porous media subjected to thermal gradients, the critical cold temperature significantly increases, up to about 40 K in the case with wider-pore insert and above 70 K for the narrow-pore insert, albeit in more limited range of the insert positions (Fig. 4a), as the optimal pore radius depends on the thermal penetration depth (which in turn depends on the local temperature and frequency). These temperatures are common for ullage spaces in hydrogen storage tanks, and therefore, Taconis oscillations may be a concern for such applications, as they can increase heat leak into a tank.

In parametric calculations of a tube involving a compact Helmholtz resonator (without a porous insert), the resonator is positioned at the end of the warm region just before the temperature transition zone, as shown in Fig. 2c. While the resonator may be more acoustically effective in the transition or cold zones, it will be difficult to install, service or modify the resonator at those locations in practical cryogenic systems. The resonator neck diameter of 2 mm is fixed in this study, while two neck lengths (L_n = 5 and 8 cm) are investigated. The resonator cavity is considered as a cylindrical volume with a diameter being equal to the cylinder height L_c, which is a variable parameter in this study.

Results for the cold temperature onset and corresponding frequency as functions of the resonator parameters are shown in Fig. 5. One can notice in Fig. 5a that a sufficient cavity volume must be chosen in order to produce a damping effect and delay Taconis onset to lower temperatures and prevent them completely by dropping the onset temperature below the saturated value at large cavity volumes. The longer neck is more effective in damping for the same cavity volume. The oscillation frequencies drop monotonically with increasing cavity volume and are always below the frequency of the clean tube in the studied range (Fig. 5b). The shorter tube produces slightly higher frequency at the same cavity length, as the effective added inertia is smaller. The presented results indicate that both porous inserts and additional resonators can be effectively used for suppression or excitation of Taconis oscillations with the appropriately chosen geometrical parameters.

CONCLUSIONS

The low-amplitude thermoacoustic theory has been applied in this study to model the onset of Taconis oscillations inside a hydrogen-filled tube with porous inserts and added resonators. When an insert is located in constant-temperature portions of the tube or dimensions of a resonator are appropriately selected, these components produce significant damping effect on acoustic oscillations, decreasing the cold temperature required for excitation. However, with an insert positioned in the zone with a large temperature gradient or for improperly selected resonator characteristics, these components can lead to much earlier excitation of Taconis oscillations at significantly higher cold temperatures. Therefore, properly installed inserts and compact resonators can supress Taconic instabilities and associate heat leaks in hydrogen tanks, but incorrectly chosen elements can excite these oscillations at much warmer ullage temperatures in cryogenic vessels, drastically increasing heat leakage into tanks and leading to large boil off losses of hydrogen.

The present model can be extended to piping networks in hydrogen tanks to assess a possibility of excitation of Taconis oscillations (which are not easy to directly detect in practice) that can substantially enhance heat leaks and worsen storage performance. Moreover, in regimes when tank pressurization is needed, modifications of these simple elements can be also be used to intentionally induce Taconis phenomena.

For the future work, exploring effects of different temperature profiles on the Taconis onset can be of interest, whereas development of a coupled heat transfer analysis for the tube surroundings can improve the accuracy of the model predictions, especially for more geometrically complex systems.

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Figure Captions List

Fig. 1 (a) Cryogenic dewar with inserted tube. (b) Liquid hydrogen storage tank.

Fig. 2 Model framework for tube with temperature gradient: (a) clean tube, (b) tube with porous insert, and (c) tube with resonator

Fig. 3 Junction in acoustic network

Fig. 4 (a) Critical cold temperatures and (b) frequencies at onset of Taconis oscillations in the tube with a porous insert. Solid lines, parameters in the tube without insert; dashed line, saturated temperature. Vertical dash-dotted lines indicate tube zone with temperature gradient.

Fig. 5 (a) Critical cold temperatures and (b) frequencies at onset of Taconis oscillations in the tube with added resonator. Solid lines, parameters in the tube without resonator; dashed line, saturated temperature.

Table Caption List

Table 1 Experimental tube parameters.

Tube radius	Warm-to-cold length ratio	Warm temperature
5.84 mm	0.67	296.2 K
2.48 mm	1.4	294.5 K

Table 2 Test and numerical results for onset of Taconis oscillations in helium-filled tube

Experimental data		Modelling results	
Critical cold temperatures	Frequency	Critical cold temperatures	Frequency
15.7-17.4 K	41.8 Hz	14-14.5 K	38.5 Hz
26.5-27.6 K	56.5 Hz	27-27.5 K	58.5 Hz