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Combining Game-Based and Inquiry-Oriented Learning for Teaching Linear Algebra

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Abstract

Linear algebra instruction is an essential competency that is necessary for success in multiple engineering disciplines. Research in realistic mathematics education and the development of an empirically tested curriculum in inquiry oriented practices for teaching linear algebra helps improve the ability of instructors to teach the content via multiple lenses and modes. While there have been good instructional materials and strategies developed to apply inquiry oriented instruction for linear algebra, students struggle to apply and connect the different modes. Game based learning provides a platform to creatively include multiple modes and strategies via a fun and engaging manner. In this paper, we present the addition of game-based learning elements into an existing curriculum that teaches undergraduate linear algebra via an inquiry-oriented pedagogy. The aim of this paper is to discuss the game design strategies used in connecting game based learning to inquiry oriented methods.

1 Introduction

An introduction to linear algebra is widely accepted as an important, albeit being challenging, course for engineering undergraduate students. It is an important foundational course for many engineering students as it provides the ability to apply mathematical constructs in real-word problem based settings that are essential for any engineering discipline [1]. Many strategies have been proposed to help teach linear algebra across various modes, representations and settings. Research suggests that success in linear algebra can be achieved by providing greater flexibility and developing connections across geometric, algebraic and metaphoric ways of relating the content [2]. However, connecting the different modes has been a challenge for instructors as witnessed in the simple example of talking about idea of a basis, where students typically are solving the problems procedurally rather than conceptually [3]. The field of Realistic Mathematics Education (RME) has been active in for the past two decades in bridging this gap and developing new methods and styles of education that can provide students with several learning styles and lenses for teaching the same content [4]. This has led to developing inquiry based practices for teaching linear algebra that have shown success and promise [5]. The emerging area of inquiry oriented linear algebra (IOLA) has undergone many iterations to its pedagogical practice by applying a design based research practice and provides an empirically tested curriculum for linear algebra instructors [6].

1.1 Inquiry Oriented Linear Algebra

The IOLA curriculum draws on RME instructional design heuristics to guide students through various levels of activity and reflection on that activity to leverage their informal, intuitive knowledge into more general and formal mathematics. The first unit of the curriculum, referred to as the Magic Carpet Ride (MCR) sequence, serves as an example of RME instructional design. Specifically, the tasks reflect four levels of activity: situational, referential, general, and formal [7]. Situational activity involves students working toward mathematical goals in an experientially real setting. The first task of MCR sequence shown in Figure 1 serves to engage students in situational activity by asking them to investigate whether it is possible to reach a specific location with two modes of transportation: a magic carpet that, when ridden forward for a single hour, results in a displacement of 1 mile East and 2 miles North of its starting location and a hoverboard, defined similarly along the vector $< 3, 1 >$. As students work through this task and share solutions with classmates, they develop notation for linear combinations of vectors and connections between vector equations and systems of equations, providing support for representing the notion of linear combinations geometrically and algebraically. The second task requires the students to study the situational activity within task 1 and make generalizations about it, for example, where Old Man Guass can hide if they were to use the same two modes of transportation from task 1. Task three now introduces some constraints in the form of scalars and situates the students to use linear combinations of vectors and scalars to reach a destination. The fourth task asks the students to reason in ways there they can create their own vectors and constraints and help generalize the concepts from the previous tasks. These four tasks are collectively summarized as four levels of activity (situational, referential, general and formal) [6].

The Magic Carpet Ride Problem

You are a young traveler, leaving home for the first time. Your parents want to help you on your journey, so just before your departure, they give you two gifts. Specifically, they give you two forms of transportation: a hover board and a magic carpet. Your parents inform you that both the hover board and the magic carpet have restrictions in how they operate:

We denote the restriction on the *hover board's* movement by the vector $\begin{bmatrix}3\\1\end{bmatrix}$.

By this we mean that if the hover board traveled "forward" for one hour, it would move along a "diagonal" path that would result in a displacement of 3 miles East and 1 mile North of its starting location.

We denote the restriction on the *magic carpet's* movement by the vector $\begin{bmatrix}1\\2\end{bmatrix}$.

By this we mean that if the magic carpet traveled "forward" for one hour, it would move along a "diagonal" path that would result in a displacement of 1 mile East and 2 miles North of its starting location.

PROBLEM ONE:
THE MAIDEN VOYAGE

Your Uncle Cramer suggests that your first adventure should be to go visit the wise man, Old Man Gauss. He tells you that Old Man Gauss lives in a cabin that is 107 miles East & 64 miles North of home.

TASK:
Investigate whether or not you can use the hover board and the magic carpet to get to Gauss's cabin. If so, how? If it is not possible to get to the cabin with these modes of transportation, why? Use vector notation for each transportation mode of as part of your explanation. Use a diagram or graphic to help illustrate your point(s).

Figure 1: The Magic Carpet Ride problem from IOLA [8]

1.2 Game Based Learning

The field of game based learning has matured over the past decade with most researchers now having a consensus on what the contributions of the field are in improving learning and literacy [9]. Recent research has demonstrated that there is a strong correlation between student learning outcomes and game based learning approaches when these are designed by integrating into existing pedagogical methods [10, 11, 12]. While some research efforts have been conducted to study the integration of games with the inquiry oriented learning methods [13, 14], additional insights are needed to provide game design strategies that produce successful outcomes. This paper describes the game design strategies used to integrate games with inquiry oriented learning in the context of an undergraduate linear algebra course.

1.3 Burst Game Design

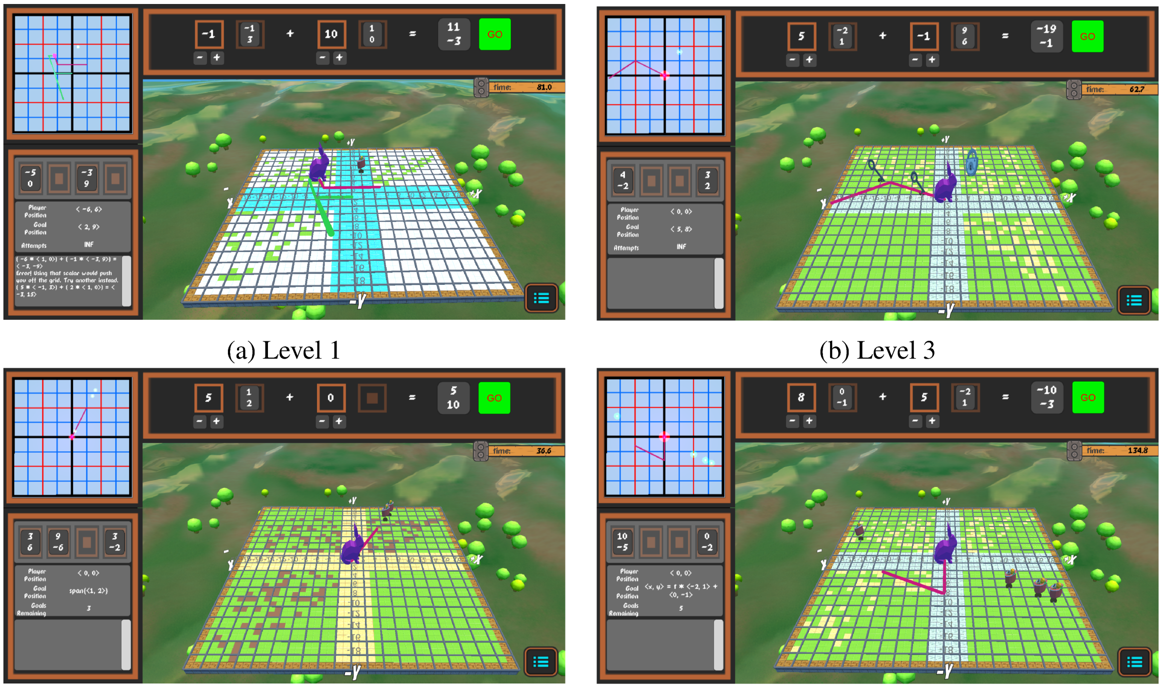
Gee [15] in his groundbreaking work described the following principles that game designers should incorporate when designing educational games: 1) provide information "on-demand" and "just-in-time"; 2) maintain a pleasant challenge and continue to keep that challenge; and 3) be able to form generalizations of the mechanics to then face and solve complex problems. Game mechanics that build upon these above principles also called "learning mechanics" have been classified in the multiple genres of game design depending on the core mechanic employed (narration, quest, skill) which follows traditional game design patterns [16]. We approached the design of our games via the application of a skill based method called "burst game" for incorporating the learning mechanic with inquiry orientation. Burst games are designed to be quick, repetitive and skill-based where the design leads to improvement of skill with repetition [17, 18]. The most famous non-educational example of a burst game is Angry Birds and one could also make the case for the game Tetris to fit this mold.

2 Methods

The games were designed via a participatory approach between computer science students in capstone course sequences working with math education and computer science faculty. Over multiple capstone teams the games were refined and readied for use in the classroom. This addressed multiple goals - 1) students are able to apply their skills to address a real world problem and 2) the research team is able to iterate and build a game with very little cost. Two games titled Vector Unknown: 2D or commonly referred to as the "Bunny Game" and Vector Unknown: 3D referred to as the "Pirate Game" were developed over the duration of five years.

2.1 2D Game mechanics to support IOLA

The game Vector Unknown: 2D (Bunny Game) consists of seven levels, each having three levels of difficulty: easy, medium, and hard. The main menu allows you to complete a tutorial level and select the game difficulty before you start playing the game. Each level consists of a mathematical puzzle in which a player must move the bunny to a target location(s) marked by food(s) or key(s). The bunny is located at the origin of the Cartesian coordinate system and the food location is marked as goal position in terms of its $< x, y >$ coordinates. Figure 2a shows the level 1 of the game where the food position is $< 2, -9 >$. To solve the puzzle, a player needs to drag and drop



(a) Level 1

(b) Level 3

(c) Level 4

(d) Level 5

Figure 2: Various levels in Vector Unknown 2D (Bunny Game)

two vectors into appropriate slots and then adjust the vector's factors (scalars) to create a linear combination that can get the bunny to the goal position. In Figure 2a, the available vector choices are $< -5, 0 >$, $< -1, 3 >$, $< -3, 9 >$, $< 1, 0 >$, with $< -1, 3 >$ and $< 1, 0 >$ being selected by the player to reach the target position. A red line in the X-Y plane shows the predicted path that the bunny will follow based on the chosen vectors and their scalar coefficients. In the Figure 2a, based on the chosen vectors and scalars, the bunny will reach a position of $< 11, -3 >$ on hitting the "GO" button. The player is given a visual perspective of the game scene via a bird's eye view of a grid. If the player reaches the target position, then they complete the level.

Figure 2a shows the user interface in the game that features four main sections of heads up display (HUD). Located in the upper left hand corner of the screen is the minimap which provides a top-down view of the game scene. It displays the X-Y axis in the 2D format that students typically associate with linear algebra. Adjacent to the mini-map is the formula tab on the right. It has two blank slots in which players can drag and drop the vector choices. Each blank slot can have a scalar coefficient which can be adjusted with the help of plus and minus buttons located below it. The sum of the vectors is shown to the right of the equals sign and the "GO" button initiates the bunny movement based on the picked vectors and scalars. Game viewport is displayed directly below the formula tab which can be rotated using the arrow keys. The viewport contains the player character with a polygonal bunny model and the target food is depicted using a basket of eggs. It also shows the 2D grid below the player and the trajectory that they will follow based on chosen vectors is displayed by a red path. The path that has already been traversed in previous steps (in case goal was not reached) is shown in green. the viewport also consists of a menu on the lower right corner, which can be used to exit to the level selection screen or restart the level. The timer in the upper right corner shows the real time spent on the current level. The vector choices tab lies to the left of the viewport. It shows four choice vector tiles that can be dragged to the blank slots in the formula tab. It also displays the current position of the player, the goal position, and the number of attempts remaining to solve the puzzle. On easy difficulty there is no limit on the number of attempts, therefore it shows INF (infinity) as shown in Figure 2a. A history of all the vector and scalar choices taken by the player until now are shown in the bottom left space. The four available vector choices consist of two pairs of linearly dependent vectors. For example, in Figure 2a the choices $< -5, 0 >$ and $< 1, 0 >$ are linearly dependent as $< -5, 0 >$ can be obtained by multiplying $< 1, 0 >$ with a scalar -5. Similarly, $< -3, 9 >$ can be obtained by multiplying $< -1, 3 >$ with 3. This is done intentionally to have students make linearly independent vector choices when trying to solve the puzzles.

2.2 Game levels and difficulty

There are seven levels with three difficulty settings leading to 21 total possible combinations. In the easy difficulty settings, one of the vectors usually has a zero to simplify the addition and multiplication. As the difficulty increases, zero disappears and is replaced by harder numbers which will take more effort do mental math. In level 1, the future bunny trajectory is displayed in red while the past trajectory is shown in green. The game goal is visible at all the times. Level 2 is similar to the level 1, except that the red trajectory is not shown to the player, only the green path is shown for route followed in the previous steps. This would push the player to use math instead of just guessing and checking the correct combination of scalars and vectors. In level 3, players are expected to pick up key(s) to unlock a lock. They need to pick up 1, 2 or 3 keys depending on the game difficulty prior to reaching the lock position. Lock position is indicated as the final goal position, while key positions are not explicitly indicated in the HUD. Figure 2b shows the level 3 on medium difficulty in which player needs to pick up two keys before unlocking the lock at position $< 5, 8 >$. A player must figure out the key coordinates using the grid system in the game and then pick up the keys. However, the player need not necessarily stop at the key's position in order to pick them up, so that they can pick up multiple or even all of the keys in just one "GO". Once you have picked the keys is very likely that the player will not be located at the origin anymore. Hence, this level introduces the challenge of navigating to the target when you are displaced from the origin. Further, this level displays both green and red trajectories of the bunny. Level 4 consists of multiple egg baskets that lie along a given vector span. The goal position is indicated using a vector span. Figure 2c shows the level 4 on the hard difficulty setting which says the number of egg baskets (3) that the bunny needs to pick up and the vector span $(< 1, 2 >)$ along which they lie. Level 5 is slightly different from the level 4 in terms of goal positioning. In level 5, the goal positions are indicated by the egg baskets scattered along a line which are shown using the parametric equation of a line. The example in Figure 2d shows that the goal baskets lie along the line $< x, y >= t* < -2, 1 > + < 0, -1 >$. Level 6 is similar to level 5, the only difference being that the egg basket locations are not indicated in the viewport and the player simply needs traverse by reconstructing the equation of the line indicated in goal.

2.3 Analytics to support learning proficiency

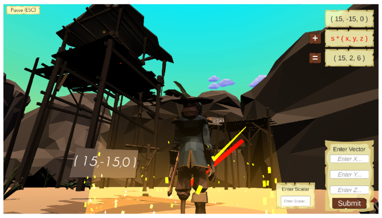
The game develops an algorithm to generate the available vector choices and to determine if the vectors and scalars chosen by the player are ideal or not. The fraction of ideal to non-ideal steps would indicate which strategy (guessing and checking or mental math) is being used by the student. A low ratio would imply guessing at work while high ratio would indicate otherwise. See Section 3 and Figure 4 for representative sample plots for the strategies. The algorithm picks two vectors a and b randomly, such that, $a =< n, m >$ and $b =< p, q >$. Then the other two vectors c and d are chosen to be linearly dependent on them, $c = ka$ and $d = jb$. Based on these the goal position is fixed as $xc + yd$ which is the ideal solution to the given puzzle. However, $xka + yd$, $xc + ylb$, and $xka + ylb$ are other possible solutions. Apart from the vectors chosen by the player, the scalars also affect the output, and therefore, sign(x), sign(y), sign(xk), sign(yj) play and important role. Given the above vectors, the step taken by the player to solve the puzzle can be non-ideal due to following reasons:

- If the step involved two linearly dependent vectors, i.e. choosing both a,c or b,d.
  - a and c are chosen as vectors
  - b and d are chosen as vectors
- If the scalar is chosen such that it takes them away from the goal position. For example, if they choose a as an option and its corresponding scalar is adjusted in the direction opposite to where it should go. If the sign of the scalar is opposite to sign of xk
  - sign of vector a's scalar is different from sign of xk
  - sign of vector b's scalar is different from sign of x
  - sign of vector c's scalar is different from sign of yj
  - sign of vector d's scalar is different from sign of y

For helping the instructor understand proficiency, the game further provides data analytics based on the above algorithm per student as well as per class level. This helps for further interventions by the instructor in creating inquiry based questioning indicative of the produced results.

2.4 Scaffolding from 2D to 3D

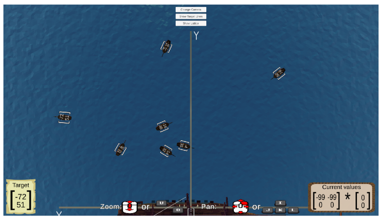
The 2D game evolved in a 3D format and followed similar design strategy, albeit adding an extra dimension into the mix. A narrative was also added to the game where the player is a pirate who is marooned on an island. By completing four unique levels the pirate can get the missing parts for his ship and sail away from the island. In the first level, the player needs to pick up the ship's mast lying at the top of a tower and therefore needs to build bridges using mathematical skills. The path to the top of tower is broken but can be accessed by multiple platforms. Figure 3b shows a top-down view of the first level. The player must reach these platforms from their current position by using a displacement vector and a scalar coefficient. In the Figure 3a, the player's starting position is $< -15, -15, 0 >$ and they need to reach the target platform located at the position $< 15, 2, 6 >$. On reaching the $< 15, 2, 6 >$, it becomes the new current position and they need to reach the next platform by using another vector and scalar. In the second level, the player



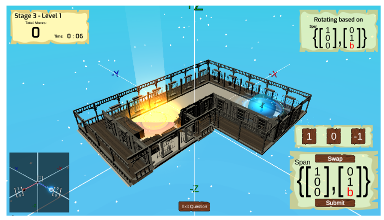
(a) Level 1

(b) Level 1: top-down view

(c) Level 2



(d) Level 2: top-down view



(e) Level 3

(f) Level 4

Figure 3: Various levels in Vector Unknown 3D (Pirate Game)

finds themselves in another dysfunctional ship that has working cannons. This ship is surrounded by other pirate ships which the player needs to destroy using the cannons. These pirate ships are located at several positions that are indicated by means of vector notation, as indicated in the Figure 3d. There are multiple cannons and each of them represents a 1x2 vector. The ship also has an inventory of cannonballs represented by a 2x2 matrix. The player then has to pick one of the cannonballs and place it in a cannon. The choices made by the player are shown in the bottom right corner as shown in Figure 3c. When fired, the result of cannonball matrix multiplication with the cannon vector will determine the landing position of the cannonball. If this landing position vector matches the vector that governs the pirate ship's location, then that pirate ship is destroyed. After sinking all the pirate ships, the player can take the cannons from the broken ship and use them to rebuild his ship. The third level is about vector spans in which the player has to retrieve the gold stolen by other pirates. It consists of a rotating labyrinth in which player has to choose a vector span that can tilt such that the rolling blue ball falls into the goal location to solve the puzzle. For the span vector, the player is given a variable that they can chose to modify. This variable is indicated by the letter b in red color, as shown in the Figure 3e . In the fourth level the player needs to cross a series of chasms by catapulting across them. They must reach a target vector position with the help of given vector choices. They need to drag two vectors from the available four choices on the left side and drag them to the available slots on bottom right of screen. Then they must adjust their scalar coefficients such that the pirate is able to reach the target position. Once they cross all the chasms, they can obtain the sail needed for their ship. Figure 3f shows the first puzzle chasm where the player chose the vectors $< -3, 1, 1 >$ and $< 0, -3, -2 >$ with their scalar coefficients as 3 and -2 respectively, to reach the target position $< -9, 9, 7 >$. Finally, after obtaining all the missing parts for the ship and retrieving the stolen gold, the player is able to leave the island.

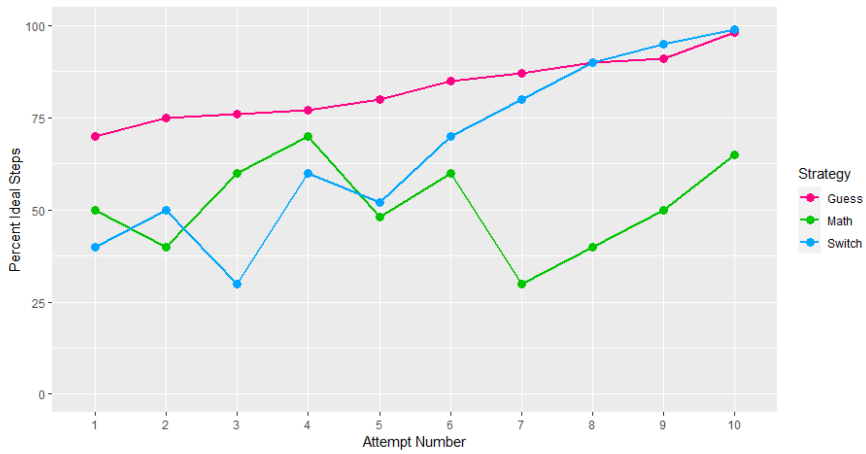


Figure 4: Representative plots for the strategies used

3 Results

In Section 2, the burst game design aimed to connect with an existing inquiry oriented curriculum to teach linear algebra. We were able to successfully address several IOLA based goals (from Figure 1) via the design of these games: 1) students are actual producers of mathematics via these games by making decisions and trying to solve the puzzles; 2) the puzzles gradually increase in complexity (2D - 3D) and also provide multiple challenge modes; 3) the completion of initial levels (bunny game) helps build good generalizations (linear dependency, span, scalar / vector multiplication) that help solve complex mathematical situations (matrix multiplication in the pirate game); 4) The game provides simple and accurate on-demand and just-in-time feedback (visual elements, star-rating, winning conditions) to create inquiry based discussions in the classroom. The games developed have been a source for multiple research studies over the course of their development. A doctoral thesis that studied the bunny game in an IOLA based classroom setting found that the students went from “no idea what I just did” to “mindlessly clicking” to a growing understanding of vector algebra, over repeated attempts [19], supporting the burst game design paradigm. They understood the meaning of linear dependence of vectors when they picked up two dependent vectors and realized that the predicted path follows the same line irrespective of the scalar coefficients chosen by them. The predicted path helped them connect the numeric and graphical idea of linear independence. Player strategies evolved during the course of gameplay and helped them in mathematical realization. The strategies followed were classified in several different categories: numeric, geometric, button-pushing, quadrantal, focus on one coordinate, and focus on one vector [20]. Numeric strategy involved focusing on the vectors while geometric strategy focused more on the predicted path. Button-pushing strategy involved rapidly pressing the + and - buttons to adjust the scalar coefficients and switching the vectors. It evolved from being less anticipatory where students mindlessly clicked the buttons to more anticipatory where they clicked it only if led them towards the goal. Students following the quadrantal theme chose vectors based on the sign of the goal coordinates, such that the starting vector is in the same quadrant as the goal. The strategy of focus on one coordinate involved matching the coordinate of the result with that of the goal while the strategy of focus on one vector focused on bringing one vector close to the goal before choosing another. This study revealed that even though the students begin with less-anticipatory strategies, they shift to more-anticipatory game play overtime. It was further used to inform the current game development by introducing easy, medium, and hard difficulty levels. Figure 4 shows representative sample plots of the three strategies observed (mental math, guess and check, starting with guess and check and switching to mental math). The percentage of ideal steps calculated by our analytics algorithm (see Section 2.3) gradually increases over time, however it's more consistent with mathematical strategies compared to guess and check ones. This is an important observation that helps the learning mechanics for the games as it reduces false positives that educational games can produce purely based on chance. Another study [21] with the bunny game incorporated as homework assignments revealed that the game can promote multiple ways of thinking about linear independence and vector span, thus helping them understand the meaning of it. IOLA's MCR task [22] involved only two vectors which were linearly independent, and the bunny game introduced two additional vector choices that were linearly dependent on the other two. A study [22] comparing the two helped inform the game design further, because of which all the four vector choices in the pirate game were linearly independent of each other. This supported the design of the games where they move from generalizable concepts to more complex ones. The study found that practicing the puzzles in the bunny game had a positive effect in solving puzzles in the pirate game.

4 Discussion

We have presented the design of two games that have found success in the classroom to help understand university level linear algebra curriculum. The games were designed to work with an existing inquiry oriented curriculum and demonstrated that games can be successfully integrated with inquiry oriented pedagogy. In the next steps we plan to run a mixed-methods based proficiency measuring study that looks at students of linear algebra from multiple demographics (novice/experienced; undergraduates/pre-service teachers) and collect data on decision choices made in both the bunny and pirate games. We are particularly interested in learning how the decisions evolve with game repetition and if there is a relationship between time spent in the game and mastery of the content.

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