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Research paper

Enhanced Koopman operator-based robust data-driven control for 3 degree of freedom autonomous underwater vehicles: A novel approach

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ABSTRACT

Developing an accurate dynamic model for an Autonomous Underwater Vehicle (AUV) is challenging due to the diverse array of forces exerted on it in an underwater environment. These forces include hydrodynamic effects such as drag, buoyancy, and added mass. Consequently, achieving precision in predicting the AUV's behavior requires a comprehensive understanding of these dynamic forces and their interplay. In our research, we have devised a linear data-driven dynamic model rooted in Koopman's theory. The cornerstone of leveraging Koopman theory lies in accurately estimating the Koopman operator. To achieve this, we employ the dynamic mode decomposition (DMD) method, which enables the generation of the Koopman operator. We have developed a Fractional Sliding Mode Control (FSMC) method to provide robustness and high tracking performance for AUV systems. The efficacy of the proposed controller has been verified through simulation results.

1. Introduction

Autonomous Underwater Vehicles (AUVs) represent a cutting-edge technological innovation in marine exploration and research (Zhang et al., 2024 & Li et al., 2024). These self-propelled, unmanned vehicles navigate the ocean depths with remarkable precision, offering scientists, engineers, and various industries invaluable insights into the underwater world (Er et al., 2023 & Su et al., 2024). Equipped with a myriad of sensors, cameras, and advanced navigation systems, AUVs can conduct a wide range of tasks, from mapping the ocean floor to collecting environmental data and performing underwater inspections. With their ability to operate autonomously for extended periods and reach depths beyond the capabilities of human divers, AUVs are revolutionizing our understanding of the ocean environment and facilitating discoveries in fields such as marine biology, geology, archaeology, and offshore industries. Controlling AUVs is a crucial responsibility in utilizing them for the purposes above.

Obtaining a nonlinear dynamic model for AUVs poses significant challenges due to the complex interaction of hydrodynamics, control systems, and environmental factors. Unlike simpler linear models, nonlinear models must account for varying buoyancy, drag, hydrostatics, and hydrodynamics, making their derivation intricate. Additionally, AUVs operate in an inherently uncertain and dynamic underwater environment, where factors such as currents, waves, and seabed topology can influence their behavior unpredictably. This necessitates sophisticated modeling techniques, often involving numerical simulations and experimental data fusion, to capture the vehicle's dynamics accurately. Moreover, nonlinearities in AUV dynamics can lead to challenges in control design and system identification, requiring advanced methods like adaptive control and nonlinear optimization. Overall, the complexity and nonlinearity of AUV dynamics demand interdisciplinary expertise and computational resources for their accurate modeling and control.

Accuracy in linearizing nonlinear dynamic models using Koopman theory is crucial for ensuring the reliability and effectiveness of the resulting linear approximations (Gong et al., 2022). The accuracy of the linearization process heavily depends on the choice of observable functions used to span the state space and approximate the nonlinear dynamics. Selecting an appropriate set of observables that captures the essential features of the system dynamics is critical for achieving accurate linearizations (Korda and Mezić, 2020). Additionally, the accuracy of the linearized model also depends on the fidelity of the Koopman operator approximation. Techniques such as data-driven approaches and numerical methods play a vital role in estimating the Koopman operator from observed system trajectories, and their accuracy directly influences the quality of the linearized model. Ultimately, ensuring accuracy in linearizing nonlinear dynamic models using Koopman theory enhances the reliability of subsequent control and analysis tasks,

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facilitating more precise predictions and effective system manipulation (Zhang and Wang, 2023; Pan et al., 2023; Abraham and Murphey, 2019)

Dynamic Mode Decomposition (DMD) is a data-driven technique that analyzes dynamical systems to extract coherent structures and patterns from time-series data (Rahmani and Redkar, 2023). It aims to decompose the system's dynamics into spatial modes and their corresponding temporal dynamics. The DMD method involves constructing a low-rank approximation of the data's spatiotemporal dynamics using singular value decomposition (SVD) and then computing the Koopman operator, which is a linear operator that evolves the observables of the system in time (Bakhtiaridoust et al., 2023; Li et al., 2023). The Koopman operator allows for analyzing and predicting system behavior in a transformed space, often revealing underlying dynamical properties that are not evident in the original state space. By leveraging DMD to obtain the Koopman operator, one can gain insights into the dominant modes of behavior, identify essential features, and potentially uncover simplified representations of complex systems for control, forecasting, or modeling purposes. Švec et al. (2023) proposes a predictive algorithm for direct yaw moment control using a finite-dimensional approximation of the Koopman operator called enhanced extended dynamic mode decomposition. This approach reduces model complexity while maintaining accuracy by transforming nonlinear dynamics into a higher-dimensional space. Williams et al. (2016) discuss the growing popularity of data-driven Koopman spectral analysis methods like DMD and EDMD for extracting dynamic features from data. They highlight the limitations of these methods, which assume data from autonomous systems, especially in cases involving system actuation. To address this, they propose a modified version of EDMD that accounts for actuation effects, allowing accurate recovery of Koopman eigenvalues, eigenfunctions, and modes. They demonstrate their approach's effectiveness using periodic forcing examples, such as the Duffing oscillator and a lattice Boltzmann code approximating the FitzHugh-Nagumo equation.

Fractional sliding mode control (FSMC) is a robust control method that extends the classical sliding mode control (SMC) to systems with fractional order dynamics (Rahmani and Rahman, 2021). The system's state trajectory in traditional SMC must slide along a designated manifold to achieve robustness against uncertainties and disturbances. FSMC introduces fractional calculus concepts to handle systems with non-integer order dynamics, familiar with many real-world processes exhibiting complex behaviors. By incorporating fractional derivatives, FSMC offers improved performance in robustness, tracking accuracy, and chattering reduction compared to integer-order SMC. Additionally, FSMC exhibits enhanced adaptability to systems with uncertainties and nonlinearities, making it suitable for a wide range of applications, including robotics, power systems, and biomedical engineering. Its benefits include enhanced robustness, improved tracking accuracy, reduced chattering, and increased adaptability to complex systems with fractional order dynamics. Luo and Liu (2023) propose a novel approach called disturbance observer-based nonsingular fast terminal sliding mode control for guiding an AUV along a desired trajectory. The method incorporates a nonlinear disturbance observer to estimate complex external disturbances, which is then integrated into the proposed control framework. The construction of sliding surfaces involves an expanded parameter selection in the exponential terms of nonsingular fast terminal sliding mode control. The stability of the proposed method is proven using Lyapunov's second method, demonstrating uniformly ultimately bounded stability. Simulation results validate the effectiveness of the proposed controller, showing improved convergence rates compared to existing nonsingular fast terminal sliding mode control methods.

Additionally, the controller exhibits robustness against smooth external disturbances and can effectively track random disturbances, including impulses, showcasing its robust performance. Rong et al. (2022) present an innovative approach to FSMC for an AUV under the influence of random disturbances. The proposed method includes the development of a fractional-order sliding mode disturbance observer that estimates random disturbances and the AUV's unknown model using adaptive and fixed-time techniques. A motion control strategy based on adaptive fractional sliding mode control is then implemented, along with a line-of-sight guidance law featuring a time-varying look-ahead distance for path following. Additionally, a new FSMC is devised by constructing a fractional-order mode surface and introducing a prescribed performance function to ensure control effectiveness. The paper also includes proofs and analyses of closed-loop stability and demonstrates the controller's effectiveness and robustness through numerical simulations. Bingul and Gul (2023) proposes model-free trajectory tracking control for an AUV, addressing challenges such as ocean currents, external disturbances, measurement noise, model uncertainty, initial errors, and thruster malfunctions. It presents a hybrid controller combining intelligent-PID (i-PID) and PD feedforward controllers to improve disturbance rejection, compensate for errors, and maintain precise trajectory tracking. A mathematical AUV model with ocean current dynamics is developed for accuracy. Computer simulations using the LIVA AUV demonstrate the proposed controller's superior performance in trajectory tracking and disturbance rejection compared to other controllers. Wang et al. (2024) proposes a novel control method for an underactuated AUV using adversarial deep reinforcement learning. The method includes a long-short-term-memory neural network for state prediction, a cascaded multilayer perceptron for action mapping, and an adversarial training scheme for robust control strategies. Simulation-based pre-training and experimental validation in a towing tank demonstrate superior robustness compared to traditional PID controllers.

The primary contribution of this research lies in developing an innovative and robust data-driven control methodology tailored for the precise regulation of an AUV system. This methodology integrates advanced theoretical frameworks and computational techniques to enhance the efficacy of control strategies in challenging underwater environments. Specifically, the study employs Koopman theory, a sophisticated mathematical framework renowned for its ability to effectively linearize complex nonlinear dynamical systems to address the inherent nonlinearities within the AUV dynamics. Subsequently, the DMD method is harnessed to accurately estimate the Koopman operator, thereby facilitating a comprehensive understanding of system behavior. The proposed control paradigm leverages Fractional Sliding Mode Control, a robust control technique known for its resilience to uncertainties and disturbances, to govern the linearized dynamic model obtained through Koopman theory. Additionally, the stability of the devised control strategy is rigorously validated utilizing the renowned Lyapunov theory, ensuring its robustness and effectiveness under varying operational conditions. Simulation-based evaluations are conducted to substantiate the superior performance of the proposed control methodology, benchmarked against conventional controllers, thus demonstrating its efficacy in real-world applications. Our proposed control method is designed to handle various sea conditions, including calm waters and turbulence. It applies to multiple disturbances encountered during Autonomous Underwater Vehicle (AUV) movement.

The forthcoming sections of this paper are structured as follows: In Section 2, we delve into the intricate nonlinear dynamics governing the operation of AUV systems. This section aims to provide a comprehensive understanding of the mathematical model underpinning the behavior of AUVs. Section 3 introduces the Koopman theory, offering a robust framework for analyzing the dynamics of nonlinear systems through the lens of linear operators. Section 4 elaborates on the Dynamic Mode Decomposition (DMD) method, a versatile technique for extracting dominant spatiotemporal patterns from high-dimensional datasets. Following this, in Section 5, we present the application of fractional sliding mode control, a robust control strategy capable of handling uncertainties and disturbances inherent in AUV operations. Section 6 is dedicated to presenting simulation results, demonstrating the efficacy of the proposed methodologies in enhancing AUVs' performance and maneuverability. Finally, in Section 7, we conclude from our findings and discuss avenues for future research in AUV dynamics and control.

2. AUV nonlinear dynamic model

Certainly, when considering the dynamic behavior of an AUV within the horizontal plane, it's common to simplify the analysis by disregarding motions in heave (vertical movement), roll (rotation around the longitudinal axis), and pitch (rotation around the transverse axis). By doing so, we focus primarily on the movement components of surge (forward and backward motion), sway (sideways motion), and yaw (rotation around the vertical axis). An AUV's kinematic and dynamic model encompasses various factors that influence its motion and behavior in water. This model typically involves a complex interplay of hydrodynamic forces, thruster dynamics, control inputs, and environmental conditions. In the simplified horizontal plane, the model can be delineated more distinctly, highlighting how surge, sway, and yaw interactions govern the AUV's motion. Surge pertains to the AUV's propulsion-driven forward or backward movement along its longitudinal axis, influenced by thrust from propellers or water jets and hydrodynamic resistance. Sway denotes lateral motion perpendicular to the vehicle's longitudinal axis, often arising from propulsion or hydrodynamic forces asymmetries. Yaw refers to the rotational movement around the vertical axis, affecting the AUV's heading and direction of travel, with factors like thruster torque and water currents influencing its behavior. Focusing on these primary motion components in the horizontal plane allows us to develop a simplified yet insightful understanding of the AUV's dynamic behavior, which is essential for control system design, navigation strategies, and overall mission planning in underwater environments (Yan and Yu, 2018).

$$\dot{\eta} = R(\psi)\upsilon \tag{1}$$

$$M\dot{\upsilon} = -C(\upsilon)\upsilon - D(\upsilon)\upsilon + \tau + E(t) \tag{2}$$

Within the dynamics of ship motion, the position vector in the Earth-fixed frame, denoted as $\eta = [x, y, \psi]^T$, encapsulates crucial spatial information. Here, x delineates the surge position, y represents the sway position, while ψ, bounded within [0, 2π], elucidates the ship's heading orientation. Similarly, the velocity vector in the body-fixed frame, $\upsilon = [u, \nu, r]^T$, captures the vessel's dynamic state: u denotes surge velocity, ν signifies sway velocity, and r denotes the vessel's yaw rate. Control inputs, denoted by $\tau = [\tau_1, \tau_2, \tau_3]^T$, encompass τ_1 for surge control forces, τ_2 for sway forces, and τ_3 for yaw moment. Perturbations from external factors are amalgamated in the disturbance vector, $E(t) = [E_1(t), E_2(t), E_3(t)]^T$, where $E_1(t)$ and $E_2(t)$ signify disturbances in surge and sway forces, respectively, and $E_3(t)$ denotes the disturbance moment in the yaw direction. The rotation matrix, pivotal for frame transformations and understanding the vessel's spatial dynamics, remains central to the analysis.

$$R(\psi) = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

It can be considered that the expressions $||R||$ and $||.||$ are indicative of the two-norms associated with a vector or a matrix. Additionally, let's denote *C(υ)* as the function representing the Coriolis and centripetal forces acting on a system, where υ is typically the velocity vector. Similarly, *D(υ)* represents the restoring force vector exerted on the system. Moreover, *M* stands for the inertia matrix, which encapsulates the distribution of mass and rotational inertia of a rigid body or system. In dynamic systems analysis, the inertia matrix plays a crucial role in determining the system's response to applied forces and moments.

$$R(\psi) = \begin{bmatrix} 0 & 0 & -m_\nu \nu \\ 0 & 0 & m_u u \\ m_\nu \nu & -m_u u & 0 \end{bmatrix} \tag{4}$$

$$D(\upsilon) = diag\{d_u, d_\nu, d_r\}, d_u = -X_u - X_{|u|u}|u|, d_\nu = -Y_\nu - Y_{|\nu|\nu}|\nu|, d_r = -N_r - N_{|r|r}|r|, M = diag\{m_u, m_\nu, m_r\}\ m_u = m - X_{\dot{u}}, m_\nu = m - Y_{\dot{\nu}}, m_r = I_z - N_{\dot{r}}$$

In this mathematical model, the symbol m denotes the mass of the AUV. The terms $X_{(.)}, Y_{(.)}$, and $N_{(.)}$ represent the hydrodynamic derivatives of the system, while $d_{(.)}$ characterizes the hydrodynamic damping effect. By deriving the expression from Eq. (1) and subsequently substituting Eq. (2) into Eq. (1), we arrive at the dynamic equation in the following form:

$$\ddot{\eta} = \dot{R}(\psi)\upsilon - M^{-1}R(\psi)C(\upsilon)\upsilon - M^{-1}R(\psi)D(\upsilon)\upsilon + M^{-1}R(\psi)\tau + M^{-1}R(\psi)E(t) \tag{5}$$

According to the fundamental equation represented by Eq. (1), which describes the relationship between variables η, ψ, and *R(ψ)*, where η represents a certain quantity related to the derivative of ψ with respect to *R(ψ)*, the subsequent equation Eq. (5) can be modified by substituting υ with $\upsilon = \dot{\eta}/R(\psi)$.

$$\ddot{\eta} = \left(\dot{R}(\psi)R(\psi)^{-1} - M^{-1}C(\upsilon) - M^{-1}D(\upsilon)\right)\dot{\eta} + \left(M^{-1}R(\psi)\right)\tau + \left(M^{-1}R(\psi)\right)E(t) \tag{6}$$

The derivation of Eq. (6) can be elucidated as follows:

$$\ddot{\eta} = P\dot{\eta} + Q\tau + NE(t) \tag{7}$$

Let $P = \left(\dot{R}(\psi)R(\psi)^{-1} - M^{-1}C(\upsilon) - M^{-1}D(\upsilon)\right)$, $Q = \left(M^{-1}R(\psi)\right)$, and $N = \left(M^{-1}R(\psi)\right)$. The symbols ΔP, ΔQ, and ΔN represent uncertainties arising from parameter variations. Consequently, Equation (7) can be expressed as:

$$\ddot{\eta} = (P + \Delta P)\dot{\eta} + (Q + \Delta Q)\tau + (N + \Delta N)E(t) \tag{8}$$

By definition of *l,u* as lower and upper uncertainty values, the uncertainties can be bounded as:

$\Delta P_l \le |\Delta P| \le \Delta P_u$, and $\Delta Q_l \le |\Delta Q| \le \Delta Q_u$

As well as, if we consider *τ(t)* to be equal to *u(t)*, then dynamic Equation (8) can be expressed as follows:

$$\ddot{\eta} = (P + \Delta P)\dot{\eta} + (Q + \Delta Q)u(t) + E(t) \tag{9}$$

3. Koopman Theory

In the realm of Koopman operator theory, the pivotal strategy lies in transforming a nonlinear dynamical system into a linear counterpart by embedding it within an infinite-dimensional state space. This intricate maneuver entails mapping the system's original form into a framework where linearity prevails, facilitating a more tractable analysis and solution methodology (Ping et al., 2021). The discrete-time definition of the dynamic evolution of a system is characterized by how the state variables transition from one time step to the next, encapsulated in the recursive equation:

$x_{k+1} = F(x_k)$, where x_k represents the state at time step n, and f denotes the transition function governing the system's behavior.

$$F(x(t_0)) = x(t_0) + \int_{t_0}^{t_0+t} f(x(\tau))d\tau \tag{10}$$

The dynamics of the original system undergo a transformative shift towards linearity when the intricate behavior of a finite-dimensional nonlinear system is transposed into the expansive realm of an infinite-dimensional function space. Within this vast domain, often represented by an infinite-dimensional Hilbert space, the function g emerges as a pivotal observable, serving as a conduit for real-valued scalar measurements. Through the lens of this observable, the Koopman operator unfolds its transformative prowess. The Koopman operator, a powerful mathematical construct, operates on the space of observables, providing a systematic framework for analyzing the evolution of dynamical systems. It maps the evolution of functions representing system observables, effectively capturing the essence of the system's dynamics within an infinite-dimensional context.

$$Kg = g \circ F \tag{11}$$

Continuous systems, characterized by their continuous-time processes and smooth evolution of state variables according to differential equations or integral equations, are widely utilized to accurately model and implement smooth dynamics across various fields such as control theory, physics, engineering, and computer graphics as:

$$\frac{d}{dt}g(x) = Kg(x) = \nabla g(x).f(x) \tag{12}$$

The Koopman operator, denoted by K, operates on functions of the state space, transforming them according to the dynamics of the system. This operator has an infinite-dimensional representation, which can make its practical application and representation both important and troublesome. Applied Koopman analysis employs roughly approximates the evolution of a subspace covered by a limited number of measurement functions rather than detailing the development of all measurement functions in a Hilbert space. By constraining the operator to an invariant subspace, the Koopman operator may be represented as a matrix with limited dimensions. Any combination of the Koopman operator's eigenfunctions can cover a Koopman invariant subspace (Kaiser et al., 2021). when the Koopman model's eigenfunction $\varphi(x)$ satisfies eigenvalue λ.

4. DMD method

The DMD for Koopman Theory presents a robust framework for analyzing and understanding complex dynamical systems. Koopman's theory, rooted in functional analysis and dynamical systems theory, offers a unique perspective by treating the evolution of observables as linear operators acting on a function space. However, traditional methods for extracting dynamic modes from data often need help handling high-dimensional and nonlinear systems. In response, the DMD introduces an innovative approach that leverages the Koopman operator's spectral properties to decompose the dynamics into orthogonal modes, revealing underlying patterns and simplifying the analysis of complex systems. This method holds significant promise across various disciplines, from engineering and physics to biology and finance, offering insights into the fundamental dynamics governing diverse phenomena.

$$X' \approx AX \tag{13}$$

Shifting the elements of matrix X along the time dimension as:

$$X = [x_1 \quad x_2 \quad ....]$$

The equation labeled as (13) can be utilized to calculate the value of A in the following manner:

$$A = X'X^{+} \tag{14}$$

The paragraph suggests using Singular Value Decomposition (SVD) to compute the Moore-Penrose pseudoinverse X^{+} of a matrix X. This approach is chosen to avoid the computational complexity that arises when directly computing X^{+}, especially for matrices with large dimensions. By employing SVD on the data snapshots, the dominant characteristics of X^{+} can be identified more efficiently, making computations more manageable (Snyder and Song, 2021).

$$X \approx U\Sigma V^{*} \tag{15}$$

In Singular Value Decomposition (SVD), the matrices are defined as follows:

$U \in R^{n \times r}$ represents the left singular vectors,

$\Sigma \in R^{r \times r}$ is a diagonal matrix containing the singular values,

$V \in R^{n \times r}$ signifies the right singular vectors.

The symbol * denotes the conjugate transpose operation. In SVD, the reduced rank for approximating a matrix Z is r. The eigenvectors corresponding to Z can be defined as:

$$\Phi = X'V\Sigma^{-1}W \tag{16}$$

The vector W represents the eigenvectors of dynamic systems that are characterized by having full rank.

If λ is an eigenfunction, then it implies:

$$KW = \lambda W \tag{17}$$

where K is the Koopman operator.

The demonstration of the linearized dynamic model is as follows:

$$\frac{d}{dt}z = Kz + u \tag{18}$$

5. Robust Koopman control

FSMC is an advanced control technique that integrates the principles of SMC with fractional calculus. SMC is a robust control method designed to ensure system stability and performance in the presence of uncertainties and disturbances. It achieves this by driving the system states onto a designated sliding surface, where the dynamics are constrained to remain, effectively ignoring the uncertainties. However, conventional SMC may exhibit chattering phenomena, leading to undesirable high-frequency oscillations in control signals. Fractional calculus offers a powerful mathematical tool for describing systems with non-integer order dynamics, allowing for more flexible modeling of complex physical processes. In FSMC, fractional-order derivatives and integrals are employed within the sliding mode control framework to enhance system robustness and reduce chattering effects. By incorporating fractional calculus, FSMC offers improved performance, robustness, and stability compared to traditional SMC, making it particularly suitable

for applications requiring precise control in the presence of uncertainties and disturbances (see Fig. 1). Additionally, FSMC has shown promising results in various fields, including robotics, aerospace, and automotive systems, highlighting its potential as a versatile control strategy for complex dynamical systems. The diagram in Fig. 2 depicts the proposed control method. The fractional sliding mode surface, which is derived from the linearized dynamic model using Koopman theory, can be formulated as follows:

$$s_k(t) = e_k(t) + \alpha D^{\mu} e_k(t) \tag{19}$$

where the e_k is the tracking error as:

$$e_k(t) = z_d - z \tag{20}$$

where z_d is desired trajectory tracking. To compute the derivative of a fractional sliding mode surface, it's essential to elaborate on the concept of a fractional sliding mode surface itself.

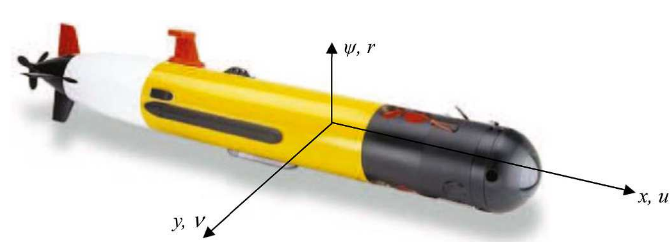

Fig. 1. An AUV body-referenced coordinate system (Rahmani and Rahman, 2021).

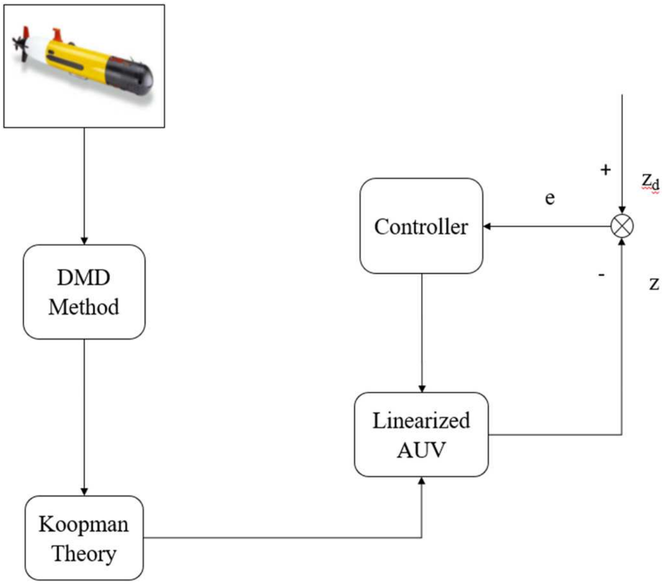

Fig. 2. The proposed controller block diagram.

$$\dot{s}_k(t) = \dot{e}_k(t) + \alpha\mu D^{\mu+1}e_k(t) \tag{21}$$

Taking the derivative of Eq. (20) and subsequently substituting it into Eq. (21) leads to the derivation of a new expression as:

$$\dot{s}_k(t) = \dot{z}_d - \dot{z} + \alpha\mu D^{\mu+1}e_k(t) \tag{22}$$

Integrating Eq. (18) into Eq. (22) introduces a novel representation, amalgamating the interplay of variables delineated within each equation as:

$$\dot{s}_k(t) = \dot{z}_d - Kz - u + \alpha\mu D^{\mu+1}e_k(t) \tag{23}$$

Equating Eq. (23) to zero and then proceeding to simplify it results in a streamlined and more comprehensible form as:

$$u_{eq}(t) = \dot{z}_d - Kz + \alpha\mu D^{\mu+1}e_k(t) \tag{24}$$

The equivalent control system demonstrates an inability to adequately counteract external disturbances. Consequently, we introduce the concept of reaching control, which can be characterized as:

$$u_{rk}(t) = K_{rk}s_k(t) \tag{25}$$

where K_{rk} represents the reaching control gain, which is a positive constant value. The new control las based on Koopman theory (KOFSMC) is characterized by the following definition:

$$u_{KOFSMC}(t) = u_{rk}(t) + u_{eq}(t) \tag{26}$$

Lyapunov's theory serves as a fundamental framework for analyzing the stability of dynamical systems. At its core, it utilizes Lyapunov functions, which are scalar functions that quantify the system's energy or a related metric. The theory posits that if a Lyapunov function exists for a dynamical system, and if it satisfies certain conditions, then the system is deemed stable. Specifically, the function must be positive definite, its derivative must be negative semi-definite, and it must converge to zero as time progresses. By assessing how the Lyapunov function evolves over time, one can ascertain whether the system's trajectories tend towards equilibrium, signifying stability, or diverge, indicating instability. This approach provides a rigorous mathematical framework for stability analysis across various domains, from control theory to differential equations and beyond. Therefore, Lyapunov theory in sliding mode control assesses stability by evaluating the convergence properties of a Lyapunov function along the sliding surface. It establishes stability by demonstrating that the derivative of the Lyapunov function along trajectories remains negative definite or nonpositive, confirming the system's ability to maintain robust stability despite uncertainties or disturbances. The stability of the proposed control approach can be validated utilizing Lyapunov theory, wherein it is demonstrated that a Lyapunov function exists and satisfies certain conditions, thereby ensuring the system's robust stability as:

$$V(t) = \frac{1}{2}s_k^T(t)s_k(t) \tag{27}$$

Deriving the derivative of Eq. (27) yields a new mathematical expression or equation.

$$\dot{V}(t) = s_k^T(t)\dot{s}_k(t) \tag{28}$$

Incorporating Eq. (23) into Eq. (28) generates a revised equation or expression.

$$\dot{V}(t) = s_k^T(t)\left(\dot{z}_d - Kz - u + \alpha\mu D^{\mu+1}e_k(t)\right) \tag{29}$$

Substituting Eq. (26) into Eq. (29) produces an altered form or representation.

$$\dot{V}(t) = s_k^T(t)\left(\dot{z}_d - Kz - u_{rk}(t) - u_{eq}(t) + \alpha\mu D^{\mu+1}e_k(t)\right) \tag{30}$$

Integrating Eq. (24) into Eq. (30) introduces a novel element or factor into the equation.

$$\dot{V}(t) = s_k^T(t)\left(\dot{z}_d - Kz - u_{rk}(t) - \dot{z}_d + Kz - \alpha\mu D^{\mu+1}e_k(t) + \alpha\mu D^{\mu+1}e_k(t)\right) \tag{31}$$

Simplifying Eq. (31) leads to a more refined or condensed representation of the equation.

$$\dot{V}(t) = s_k^T(t)(-u_{rk}(t)) \tag{32}$$

Integrating Eq. (25) into Eq. (32) yields an updated expression or equation.

$$\dot{V}(t) = s_k^T(t)(-K_{rk}s_k(t)) \tag{33}$$

The expression presented in Eq. (33) provides evidence of the stability inherent in the proposed control methodology, affirming its efficacy in maintaining robust performance across varied operational scenarios.

6. Simulation results

The newly proposed control methods are applied to the lateral dynamics model of an AUV system. The initial states are selected as follows:

$$x(0) = 0, u(0) = 0$$
$$y(0) = 0, v(0) = 0$$
$$\psi(0) = 0, r(0) = 0$$

To assess the effectiveness of the proposed control method, reference trajectories are designated as follows:

$$x_d(t) = 2t, y_d(t) = 3\,sin(0.5t)$$

The design parameters for the proposed control method are carefully selected based on a thorough analysis of the system dynamics and performance requirements as:

$$\alpha = 1, \mu = 0.75, h = 0.01, K_{rk} = diag\{2, 2\}$$

Table 1
Parameters of AUV dynamics model (Rahmani and Rahman, 2021).

$m = 185kg$	$I_z = 50kgm^2$	
$X_u = -70kg/s$	$Y_v = -100kg/s$	$N_r = -50kgm^2/s$
$X_{\dot{u}} = -30kg$	$Y_{\dot{v}} = -80kg$	$N_{\dot{r}} = -30kgm^2$
$X_{u\lvert u\rvert} = -100kg/m$	$Y_{v\lvert v\rvert} = -200kg/m$	$N_{r\lvert r\rvert} = -100kgm^2$

The detailed model parameters of the AUV system are meticulously documented and presented in Table 1 for comprehensive insight and reference purposes. Fig. 3 presents a comparative analysis of position tracking performance along the x and y axes, showcasing the outcomes achieved using both the FSMC and the newly devised control approach. Notably, the FSMC implementation exhibits a discernible chattering phenomenon, characterized by irregular fluctuations in the system's behavior. This chattering effect can potentially compromise the system's overall efficiency and stability. Consequently, the pursuit of an innovative compound control strategy is imperative, aiming to mitigate or eliminate the observed chattering phenomenon while enhancing the system's tracking accuracy and robustness. Fig. 4 shows the position

tracking error of x and y under FSMC and KOFSMC. When comparing the FSMC with the KOFSMC controller, several significant differences emerge regarding their effectiveness in addressing chattering reduction, achieving high tracking performance, and minimizing tracking errors. Firstly, KOFSMC exhibits superior capabilities in chattering reduction compared to FSMC. Chattering, characterized by rapid and erratic oscillations in control signals, can significantly degrade system performance and induce wear and tear on mechanical components. KOFSMC leverages its Koopman approach to smooth out control signals more effectively, thereby mitigating chattering phenomena and ensuring smoother operation of the controlled system compared to FSMC.

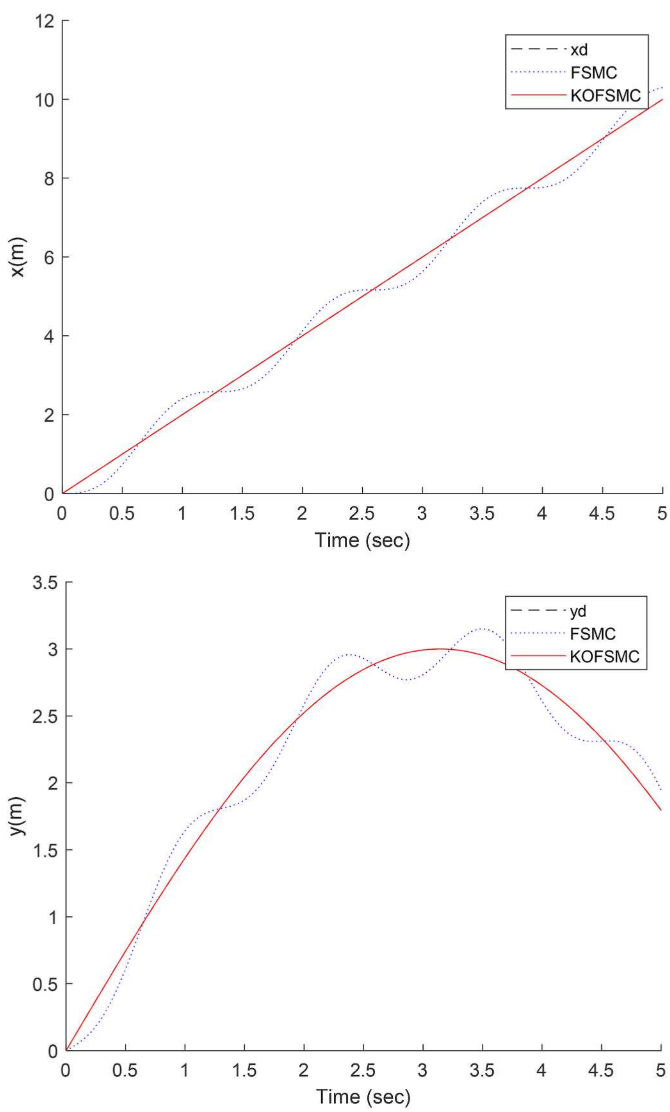

Fig. 3. Position tracking of x and y under FSMC and KOFSMC.

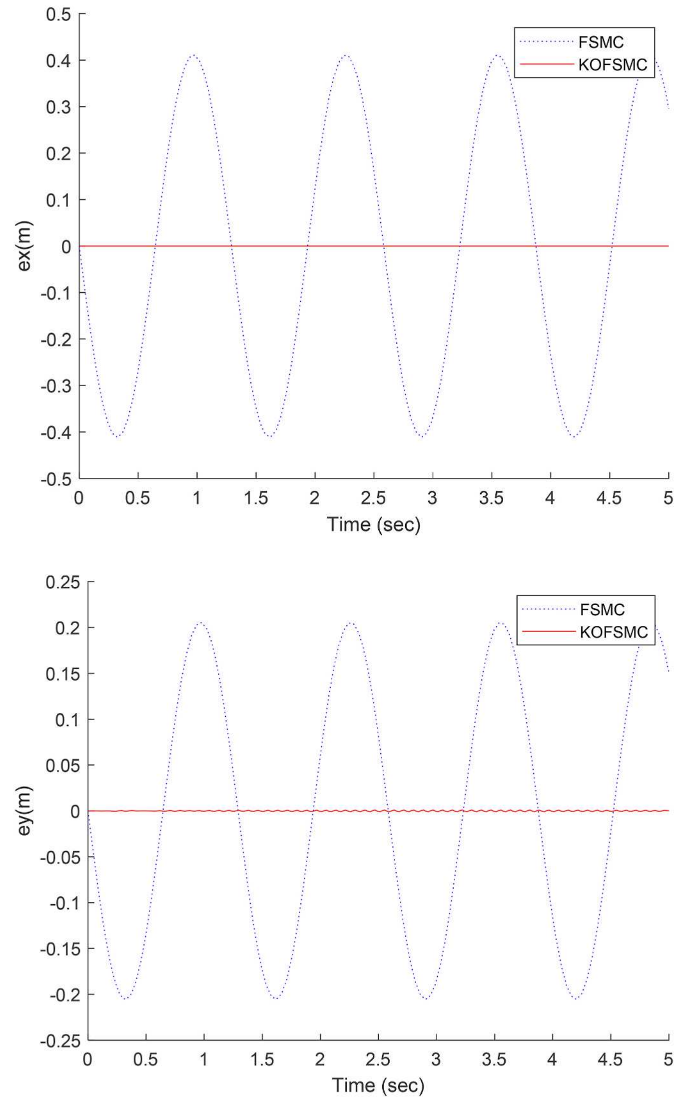

Fig. 4. Position tracking error of x and y under FSMC and KOFSMC.

Moreover, KOFSMC demonstrates a notable advantage in achieving high tracking performance and minimizing tracking errors over FSMC. The linearization in KOFSMC enables more accurate modeling of complex system dynamics and nonlinearities, thereby facilitating enhanced tracking precision. By effectively capturing the intricate relationships between input and output variables, KOFSMC can adapt more dynamically to varying system conditions, resulting in superior tracking performance and reduced tracking errors compared to FSMC. Overall, the comparative analysis highlights the KOFSMC controller's superiority over FSMC in addressing chattering reduction, achieving high tracking performance, and minimizing tracking errors, making it a more favorable choice for advanced control applications. Fig. 5 shows Velocity tracking of AUV versus time using FSMC and KOFSMC.

6.1. Robustness verification

In the intricate domain of underwater operations, an AUV system continuously grapples with a plethora of external disturbances as it maneuvers through the dynamic and often unpredictable water environment. This reality underscores the critical necessity of devising a robust control methodology that can withstand and counteract these persistent disturbances effectively. In order to thoroughly assess the resilience and noise mitigation capabilities inherent in the proposed

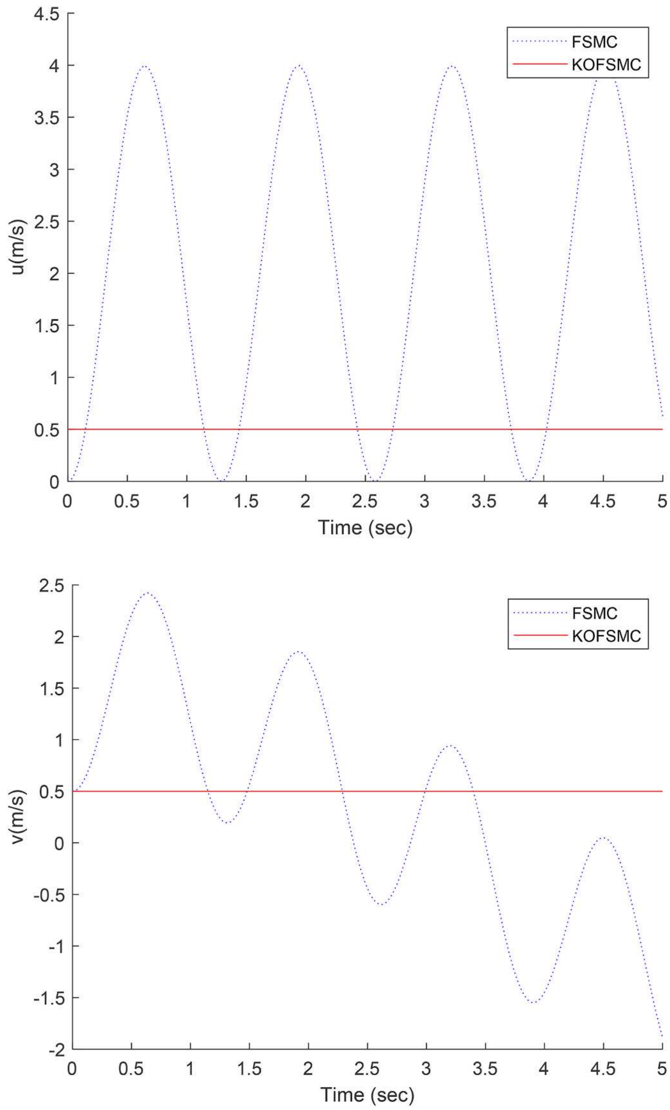

Fig. 5. Velocity tracking of AUV versus time using FSMC and KOFSMC.

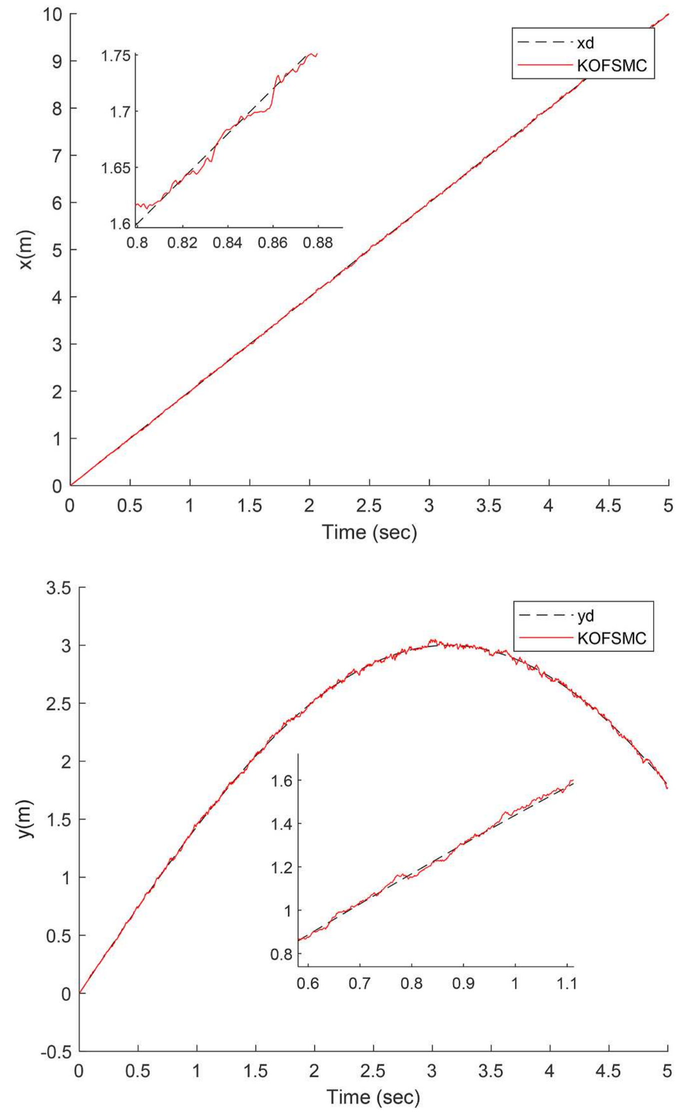

Fig. 6. Robustness verification of the proposed controller using random noise.

control method, a deliberate strategy is employed wherein random noise with a standard deviation of 2 is meticulously introduced into the system. This controlled noise injection is mathematically characterized by equation $E(t) = 2 * randn(1, 1)$, where $E(t)$ represents the applied noise at time t. The ensuing simulation endeavors yield a comprehensive and insightful depiction of the control method's performance. Fig. 6 serves as a visual testament, elucidating the control method's prowess in seamlessly suppressing the injected noise. This discernible suppression not only underscores the control method's robustness but also underscores its reliability and effectiveness in real-world operational scenarios, thereby affirming its potential for practical implementation in challenging underwater environments.

7. Conclusion

The research presents a novel data-driven control methodology tailored specifically for managing the intricate dynamics of an AUV system. This innovative approach stems from the inherent challenges in accurately modeling the nonlinear dynamics of the AUV, primarily due to the diverse and complex forces acting upon the system. To address this, the study leverages the Koopman theory to derive a linearized data-driven model, which serves as a foundational framework for control design. Utilizing the Koopman operator generated through the DMD method, the research team implements a FSMC to control the linearized model. This FSMC not only enhances trajectory performance but also bolsters the system's robustness against external disturbances, leading to improved trajectory tracking and minimized tracking errors. To validate the robustness and efficacy of the proposed control methodology, the study employs random noise simulations, showcasing its capability to mitigate disturbances and maintain stability. Looking ahead, the authors aim to transition this innovative control strategy into real-time AUV systems, anticipating enhanced performance and adaptability in practical applications. The research substantiates the effectiveness of the proposed control approach through comprehensive simulation results, underscoring its potential for real-world implementation and impact in autonomous underwater operations.

CRediT authorship contribution statement

Mehran Rahmani: Writing – original draft, Software, Methodology, Formal analysis, Conceptualization. **Sangram Redkar:** Writing – review & editing, Validation, Supervision, Methodology, Conceptualization.

Declaration of competing interest

There is no conflict of interest or competing interest in this research.

Data availability

No data was used for the research described in the article.

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